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Analyzing Parking Demand Characteristics Using a Bayesian Model Averaging

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Abstract: Parking duration analysis is an important aspect of evaluating parking demand. Identifying accurate distribution characteristics of parking duration can not only enhance parking efficiency and parking facility planning, but also provide essential support for parking delicacy management. Previous studies have proposed various statistical distributions to depict parking duration data. However, it is difficult to find a certain type of distribution to describe the characteristics of parking duration in diverse parking facilities, since model uncertainty is caused by stochastic parking behaviors and diverse parking environments. To address the model uncertainty, a Bayesian model averaging (BMA) was applied to integrate the advantages of different statistical distributions to depict parking duration characteristics. The parking dataset was collected from a commercial parking lot in Chengdu, China, and the dataset was categorized into two groups (i.e., temporary users and long-term users) to analyze. A set of statistical distributions was chosen as candidate models, and their corresponding unknown parameters were estimated. The posterior model probability for each candidate model was calculated according to the goodness-of-fit (GOF) metric. The findings of the study illustrate that there is no universally applicable distribution form (e.g., log-normal distribution) to depict the parking duration distribution for both user types, whereas the BMA approach assigns weights to candidate models and always provides an accurate description of the parking duration characteristics. The parking duration analysis is useful for improving parking management strategies and optimizing parking pricing policies.

Keywords: applied sciences; traffic management; parking delicacy management; parking demand; parking duration; Bayesian model averaging

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1. Introduction

Amidst rapid socioeconomic development, there has been a notable upsurge in private car ownership. This surge, however, has not been met with a corresponding expansion in parking infrastructure. Consequently, the parking supply–demand imbalance is becoming increasingly acute, resulting in a pervasive issue of parking challenges [1–3]. Park-and-ride facilities provide a solution to parking problems in cities [4]. They are typically located at the outskirts of cities, which allows users to park and transfer to public transport to enter the city center. However, the choice of park-and-ride facilities is a complicated issue, influenced by numerous factors. Macioszek et al. [5] attempted to identify and quantify the factors determining the choice of park-and-ride facilities using a multinomial logit model.

Another solution to address the parking supply–demand disparity is to improve the requirement for comprehensive, secure, and delicate management of parking facilities [6,7]. However, the delicacy of parking management necessitates more precise and comprehensive parking demand information. Therefore, accurately depicting parking demand characteristics, such as parking arrival and departure times, parking duration, parking peak ratio, etc., can provide crucial support for delicacy parking management, and has garnered much attention from scholars [8–10].

Parking duration, a crucial parking demand parameter, is defined as the amount of time a vehicle parks in a parking lot. Parking duration modeling serves as an essential procedure in parking demand analysis, as it allows for the discovery of intrinsic parking demand patterns [11–13]. Nevertheless, due to the sensitivity of parking duration, it is susceptible to various factors [14], such as the socioeconomic status of travelers, travel cost, parking fee, driving characteristics, etc. Parmar et al. [15] proposed an artificial neural network (ANN) to capture the interrelationship between driver characteristics and parking duration considering two land uses. These reveal that parking duration exhibits considerable uncertainty, and accurately capturing its statistical characteristics presents a significant challenge [16–18]. In addition, an accurate description of parking duration distribution can improve parking efficiency, parking facility planning and management, etc.

In the context of analyzing parking characteristics, parking duration frequently serves as a metric to elucidate parking traits by delineating statistical distribution characteristics [19,20]. To depict the parking characteristics exhibited by urban streets in commercial areas, Parmar et al. [21] conducted statistical modeling to analyze parking indexes, parking occupancy, parking duration, etc. Chen et al. [22] used parking indexes, peak parking ratio, parking turnover rate, and parking duration to explore the parking characteristics of different types of land use (i.e., market, business-and-office-oriented, and food-and-drink-oriented areas) in central Shanghai city. Wang et al. [23] conducted an investigation of parking duration and parking turnover in street parking, delving into the impact of parking pricing policies on street parking characteristics. Nie et al. [24] focused on curb parking and designed a parking demand estimation framework to accurately predict the parking demand distribution over different parking durations for a road section. Sun et al. [25] investigated the influence of parking time and duration on the choice of parking location and revealed increased turnover rates in parking lots that have higher parking fees. However, while these studies investigated the impacts of parking duration and described the parking demand characteristics, they do not provide a precise description of the parking duration distribution.

In the area of statistical modeling of parking duration, several studies have endeavored to capture the distribution characteristics through both parametric and non-parametric approaches [26,27]. Mesfin et al. [28] considered the impacts of COVID-19 using a non-parametric approach to characterize the parking arrival, departure, and duration distribution of an off-street parking area. Ran et al. [29] sought to employ the gamma distribution and a gamma mixture model for delineating the parking duration distribution across distinct periods, with the additional objective of delving into the dynamic parking demand characteristics. Li et al. [30] used the parking duration distribution to estimate the probability of parking durations using survival analysis, thereby facilitating nighttime parking demand forecasting. Similarly, Zheng et al. [31] explored the distribution patterns of parking arrival and departure times, and proposed using the Markov birth-and-death process to model the distribution of short-term parking demand. Kalahasthi et al. [32] investigated the parking patterns of trucks in urban freight loading zones by concurrently modeling vehicle arrival rates and parking duration, where the parking duration was characterized using survival analysis based on Weibull distribution. However, statistical distributions (e.g., log-normal distribution, log-logistic distribution, etc.) were not employed to depict the parking duration characteristics, and there was no specific distribution that could adequately fit the diverse parking duration data.

In summary, previous studies have endeavored to characterize parking duration distribution patterns using statistical models like the gamma distribution and log-normal distribution. However, it is important to note that the appropriateness of these distributions may vary depending on the parking duration data collected from the various parking facilities. More specifically, because of the stochastic parking behavior and the diversity of parking facilities, distinct distributions may be more appropriate for representing parking duration, which is referred to as model uncertainty. Bayesian model averaging (BMA) is renowned for its capacity to mitigate model uncertainty by generating a set of candidate models and assigning weights to each component, where the weights reflect the contributions of candidate modes to the ensemble over the given data [33–35]. Previous studies have demonstrated that the BMA approach can offer a more dependable depiction of the overall predictive uncertainty and produce a more accurately derived probability density function and cumulative density function in probabilistic forecasts [36]. Wang et al. [37] introduced a hybrid forecasting method for wind power, applying the BMA approach to amalgamate three distinct machine learning techniques, thereby enhancing predictive accuracy. Wu et al. [38] employed the BMA approach to delineate the characteristics of time headway distribution in diverse traffic facilities, taking into account the model uncertainty.

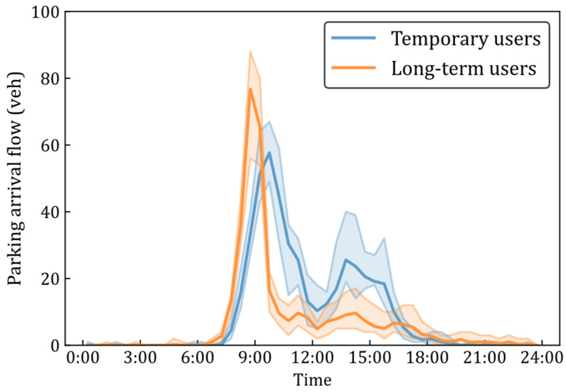
This study applies a BMA approach to depict the characteristics of parking duration distribution. The parking dataset was gathered within a commercial district, and it was categorized into two user groups—temporary users and long-term users—for comprehensive analysis. The rest of this paper is organized as follows. Section 2 provides a detailed description of the data preparation. Section 3 describes the methodology, encompassing the theory of the BMA approach and its implementation. Section 4 presents the experiments and results, and the conclusion is summarized in Section 5.

2. Data Preparation

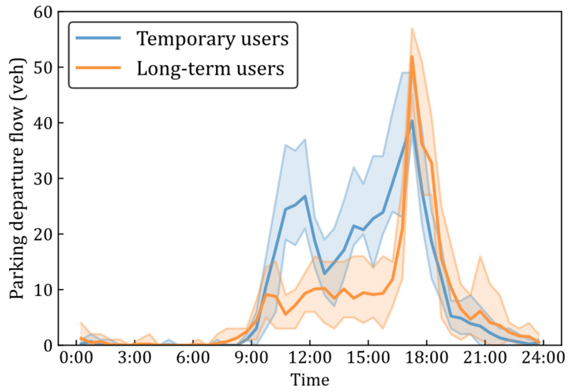
The parking demand dataset was collected in the parking lot of a High-tech International Plaza in Chengdu from 1 to 31 January 2015. The plaza is a business office space with a total area of 2.3×10^5 m^2 and 632 parking spots. The dataset provides details including the username and specific time of entry and exit from the parking lot. The parking lot users can be divided into two types: temporary users and long-term users. The temporary users primarily comprise individuals needing short-term parking, such as shoppers and occasional visitors, whereas the majority of long-term users are employees working in this plaza. For a business office space, the parking demand is affected by whether it is a working day. Therefore, the data collected under holiday and moderate/heavy rain conditions were removed.

For exploring the characteristics of parking demand, the time-varying curves depicting parking arrival and departure patterns are illustrated in Figure 1. Note that the statistical time interval is 30 min, and 48 data points are obtained per day. The curves indicate the fluctuation trends of the mean value of parking arrival and departure vehicles over statistical days during the time interval, and the shaded regions denote the upper and lower range. It can be seen that the daily patterns of parking arrival and departure present similar trends. For the parking arrival pattern, the majority of long-term users arrive between 8:00 and 10:00 for work. The curve of temporary users demonstrates a bimodal trend, with peak hours occurring between 9:00 to 11:00 and 13:00 to 16:00. Regarding the parking departure pattern, long-term users' peak hours are from 17:00 to 19:00, whereas temporary users' peak periods are from 11:00 to 12:00 and 14:00 to 18:00. These patterns reveal that the parking duration of long-term users tends to exceed that of temporary users, and the predominant work hours of long-term users typically commence at 9:00 and conclude at 18:00. This observation aligns with real-world practices.

For a better understanding of the characteristics of the parking arrivals and departures, several metrics (e.g., minimum, maximum, mean, etc.) are summarized in Table 1 to depict the statistical characteristics. Note that the metrics are derived from data points between 8:00 a.m. and 8:00 p.m. using mean values, as data points from other time periods frequently include a considerable number of zeros. From Table 1, it can be observed that the statistical characteristics of the parking arrival and departures for temporary users exhibit similar trends, with comparable minimum, maximum, mean, and median values. Similarly, these characteristics are also observed for long-term users. However, the standard deviation for the parking departure is smaller than that for the parking arrival (the standard deviations of temporary users are 10.307 for departure and 15.723 for arrival, and those of long-term users are 11.411 and 15.723 for departure and arrival, respectively), which can be attributed to the more dispersed nature of the parking arrival.



(a)



(b)

Figure 1. Temporal patterns of parking arrival and departure. (**a**) Arrival. (**b**) Departure.

Table 1. Summary of parking arrivals and departures (unit: vehicle).

	User Type	Minimum	Maximum	Mean	Median	S.D. [1]
Arrival	Temporary users	0.000	57.667	18.403	16.278	15.723
	Long-term users	1.111	76.778	13.106	7.222	18.803
Departure	Temporary users	0.000	40.333	18.093	18.667	10.307
	Long-term users	1.333	51.889	12.903	9.333	11.411

[1] S.D. denotes standard deviation.

Based on the recorded parking arrival and departure times, the parking duration can be calculated. To enhance data quality, parking durations shorter than 0.25 h were considered outliers and removed. Several statistical indicators (i.e., mean, median, standard deviation) were employed to briefly analyze the statistical characteristics. The mean, median, and standard deviation for both the temporary users and long-term users are as follows: 2.683, 1.390, and 2.845 h for temporary users, and 7.352, 6.549, and 7.077 h for long-term users. It is evident that the parking duration of long-term users exceeds that of temporary users. Moreover, Figure 2 illustrates the kernel density function of the parking duration. It can be observed that the parking duration of temporary users follows a unimodal distribution, with the majority of parking duration samples falling within the range of 0 to 5 h. Conversely, the parking duration of long-term users exhibits a bimodal distribution, with parking duration samples spanning from 0 to 12 h.

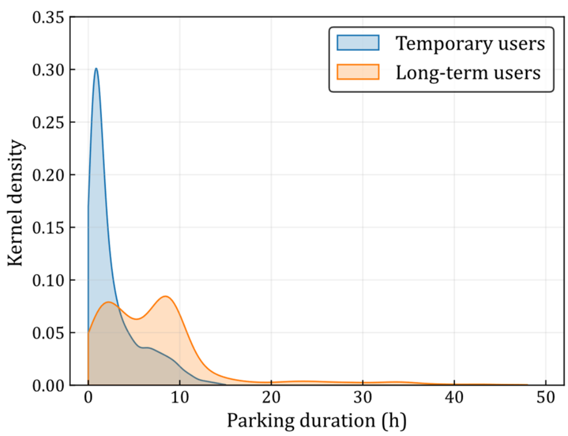


Figure 2. Kernel density function of parking duration.

3. Methodology

3.1. Bayesian Model Averaging

To more effectively address the uncertainty of parking demand arising from stochastic parking behavior, a BMA approach was applied to depict the characteristics of the parking demand (i.e., parking duration). For a predefined model space denoted as M consisting of K components (i.e., candidate models), represented as $M_k\{k = 1, 2, \cdots, K\}$, the BMA approach weights individual candidate models and integrates the results into a deterministic model. Let x denote the quantity of interest (i.e., an observation of parking duration) and D denote an observed dataset; the posterior distribution (i.e., derived probability density function of BMA) can be formulated as

$$p(x|D) = \sum_{k=1}^{K} p(x|M_k, D)p(M_k|D) \tag{1}$$

where $p(x|M_k, D)$ represents the probability density function of x within the candidate model M_k, and $p(M_k|D)$ is the posterior model probability, which represents the likelihood of the candidate model M_k being a correct model for the given observational dataset D. For a specific model space M, $\sum_{k=1}^{K} p(M_k|D) = 1$, and $p(M_k|D)$ can be calculated using Bayes rules:

$$p(M_k|D) = \frac{p(M_k)p(D|M_k)}{\sum_{i=1}^{K} p(M_i)p(D|M_i)} \tag{2}$$

where $p(M_k)$ denotes the prior probability of the candidate model M_k when it is considered as a "true" model, and $p(D|M_k)$ represents the corresponding marginal likelihood, which is formulated by

$$p(D|M_k) = \int p(D|\theta_k, M_k)p(\theta_k|M_k)d\theta_k \tag{3}$$

where θ_k is a parameter vector of the candidate model M_k, $p(\theta_k|M_k)$ denotes the prior probability distribution of θ_k under candidate model M_k, and $p(D|\theta_k, M_k)$ is the likelihood under the candidate model M_k and parameter θ_k. Then, the posterior mean $E[x|D]$ and variance $Var[x|D]$ of the BMA approach are formulated as

$$E[x|D] = \sum_{k=1}^{K} E(x|D, M_k)p(M_k|D) \tag{4}$$

$$Var[x|D] = \sum_{k=1}^{K} \left(Var[x|D, M_k] + E\left[x \middle| D, M_k\right]^2 \right) p(M_k|D) - E\left[x \middle| D\right]^2 \tag{5}$$

where $E(x|D, M_k)$ and $Var[x|D, M_k]$ denote the mean and variance of candidate model M_k under the given dataset D, respectively.

3.2. Difficulties in Implementing BMA

While BMA holds considerable theoretical appeal, its implementation encounters two pivotal challenges [39,40]. One pertains to judiciously determining the model space, which entails the meticulous selection of a set of candidate models. The straightforward approach entails encompassing all possible models within the model space, yet potentially entailing substantial time consumption due to the large model space. Another alternative approach, such as Occam's window method and Markov Chain Monte Carlo Model Composition (MC3), determines a set of appropriate candidate models using predefined criteria (e.g., Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), etc.). This approach helps conserve resources by eliminating models that do not perform effectively.

The second practical challenge posed by the BMA approach revolves around the arduous computation of marginal likelihood. The calculation of marginal likelihood may prove to be analytically intractable, particularly in cases where closed-form integrals are not attainable (see Equation (3)). Consequently, a variety of approaches have been developed to calculate or approximate the marginal likelihood, such as Laplace approximation, harmonic mean estimator, and so on. The Laplace approximation calculates the marginal likelihood at either the posterior mode or parameter estimates aligned with the maximum likelihood estimation. The harmonic mean estimator employs a Monte Carlo (MC) numerical approach to appraise the marginal likelihood or the ratio thereof. Monte Carlo integration draws samples from the specified distribution and aggregates these samples to approximate expectations.

3.3. Model Space Determination

The initial phase in executing the BMA approach involves the meticulous selection of a set of candidate models as the model space. According to the empirical histograms of the observed data, the parking duration of temporary users shows a unimodal trend, while that of long-term users displays a bimodal trend. Therefore, several commonly used single distributions were chosen as the candidate models to analyze the characteristics of the parking duration of temporary users, including normal, log-normal, gamma, Weibull, log-logistic, Burr, and generalized extreme value (GEV) distributions, as shown in Table 2.

Table 2. Summary of candidate models.

Distribution	Probability Density Function	Parameter
Normal	$f(x\|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	$\mu \in (-\infty,\infty), \sigma > 0$
Log-normal	$f(x\|\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln\ x-\mu)^2}{2\sigma^2}}$	$\mu \in (-\infty,\infty), \sigma > 0$
Gamma [1]	$f(x\|\alpha,\beta) = \frac{1}{\beta^\alpha \cdot \Gamma(\alpha)}x^{\alpha-1}e^{-\frac{x}{\beta}}$	$\alpha > 0, \beta > 0$
Weibull	$f(x\|\alpha,\beta) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-(\frac{x}{\beta})^\alpha}$	$\alpha > 0, \beta > 0$
Log-logistic	$f(x\|\mu,\sigma) = \frac{1}{\sigma x} \cdot \frac{e^{\frac{\ln\ x-\mu}{\sigma}}}{\left(1-e^{\frac{\ln\ x-\mu}{\sigma}}\right)^2}$	$\mu > 0, \sigma > 0$
Burr	$f(x\|\alpha,\beta,\gamma) = \frac{\beta\gamma(\frac{x}{\alpha})^\beta}{x\left[1+(\frac{x}{\alpha})^\beta\right]^{\gamma+1}}$	$\alpha > 0, \beta > 0, \gamma > 0$
GEV	$f(x\|k,\mu,\sigma) = \frac{1}{\sigma}\left(1+k\frac{x-\mu}{\sigma}\right)^{-1-\frac{1}{k}} \cdot e^{-(1+k\frac{x-\mu}{\sigma})^{-\frac{1}{k}}}$	$k \neq 0, 1+k\frac{x-\mu}{\sigma} > 0$
GIG [2]	$f(x\|\mu,\sigma,\nu) = \left(\frac{c}{\mu}\right)^\nu \cdot \left[\frac{x^{\nu-1}}{2\kappa_v\left(\frac{1}{\sigma_2}\right)}\right]e^{-\frac{1}{2\sigma^2}(\frac{cx}{\mu}+\frac{\mu}{cx})}$	$\mu > 0, \sigma > 0$, $\nu \in (-\infty,\infty)$

[1] $\Gamma(\alpha)$ represents the gamma function. [2] κ_v is the modified Bessel function of the third kind with order ν, and $c = \left[\kappa_{v+1}\left(\frac{1}{\sigma^2}\right)\right]\left[\kappa_v\left(\frac{1}{\sigma^w}\right)\right]^{-1}$.

In addition, several mixture models were chosen as candidate models to depict the parking duration characteristics of long-term users. The general formulation of the mixture model encompassing two components is formulated as

$$f(x|\theta) = \sum_{i=1}^{2} \omega_1 \cdot f_1(x|\theta_1) + \omega_2 \cdot f_2(x|\theta_2) \tag{6}$$

where $f(x|\theta)$, $f_1(x|\theta_1)$, and $f_2(x|\theta_2)$ denote the probability density function of the mixture model and the two components under the corresponding parameter vectors (i.e., θ, θ_1, and θ_2), respectively. ω_1 and ω_2 are the weights for the two components, and $\omega_1 + \omega_2 = 1$. Please note the parameter vector θ, which is represented as $\theta = [\omega_1, \omega_2, \theta_1, \theta_2]$. In this study, Gaussian (normal), log-normal, gamma, Weibull, log-logistic, and generalized inverse Gaussian (GIG) distributions were chosen as components (the probability density functions are provided in Table 1), and the corresponding mixture models are denoted as the Gaussian mixture model (GauMM), log-normal mixture model (LognMM), gamma mixture model (GamMM), Weibull mixture model (WeiMM), log-logistic mixture model (LoglMM), and GIG mixture model (GIGMM).

3.4. Posterior Model Probability Calculation

The final step in implementing the BMA approach is to calculate the posterior model probability (see Equation (2)), but this is hard to obtain due to the difficulty in calculating the marginal likelihood. In addition to the approaches outlined in Section 3.2 for the approximate estimation of marginal likelihoods, an alternative approach is to use information criteria (e.g., Watanabe–Akaike's information criterion (WAIC), Pareto-smoothed importance sampling leave-one-out cross-validation (LOO)) to calculate the posterior model probability [41]. Nevertheless, the parameters of the candidate models were estimated using maximum likelihood estimation, which rendered the computation of WAIC and LOO challenging. To sum up, AIC was employed for calculating the posterior model probability [42], as shown in Equation (7):

$$p(M_k|D) = \frac{e^{-\frac{1}{2}(AIC_k - AIC_{min})}}{\sum_{i=1}^{K} e^{-\frac{1}{2}(AIC_i - AIC_{min})}} \tag{7}$$

where $AIC_{min} = \min\{AIC_i\}, i = 1, 2, \cdots, K$.

3.5. BMA Implementation Procedure

For a better understanding of the principles of the BMA approach, we provide the implementation procedures of BMA:

Step 1: Determination of model space. This step is to determine a set of candidate models as the model space. The details are described in Section 3.3.

Step 2: Parameter estimation and evaluation of candidate models. According to the parking duration dataset, the parameters of candidate models are estimated using different methods. More specifically, the maximum likelihood estimation and the expectation-maximum methods are applied to estimate the parameters of single models and mixture models, respectively. Then, several goodness-of-fit metrics are calculated, including the log-likelihood, Akaike information criterion (AIC), and Bayesian information criterion.

Step 3: Calculate the posterior model probability. According to the AIC values from step 2, the posterior model probability of each candidate model can be calculated using Equation (7).

Step 4: Generate the probability density function of BMA. As per the posterior model probability and the probability density functions of candidate models, the derived probability density function of BMA can be obtained using Equation (1).

To sum up, the derived probability density function of BMA integrates the probability density functions of candidate models by assigning weights (i.e., posterior model probability) that can overcome the model uncertainty in depicting parking duration distribution characteristics.

4. Experiments and Results

4.1. Parameter Estimation of Candidate Models

In the process of fitting candidate models, the maximum likelihood estimation was applied to estimate the parameters of the candidate models for temporary users, and the expectation-maximum algorithm was adopted to determine the parameters of the mixture models. The results of the parameter estimation are summarized in Tables 3 and 4. For single distributions (see Table 3), the parameters of most distributions display a noticeable distinction. For instance, the parameters of the normal distribution are $\mu = 2.638$ and $\sigma = 2.884$, while the parameters of the log-normal distribution are $\mu = 0.466$ and $\sigma = 1.031$. However, the distributions with equivalent parameters obtained similar estimation results. For example, the parameters of the gamma distribution and Weibull distribution are $\alpha = 1.097, \beta = 2.446$ and $\alpha = 1.008, \beta = 2.693$, respectively. Here, α is the shape parameter, and β denotes the scale parameter.

Table 3. Parameter estimation results of candidate models for temporary users.

Distribution	Parameter	Estimate	Distribution	Parameter	Estimate
Normal	μ σ	2.683 2.844	Log-logistic	μ σ	0.430 0.618
Log-normal	μ σ	0.466 1.031	Burr	α β γ	0.839 2.233 0.496
Gamma	α β	1.097 2.446	GEV	k μ σ	0.902 0.864 0.963
Weibull	α β	1.008 2.693			

Table 4. Parameter estimation results of candidate models for long-term users.

Distribution	Parameter	Estimate	Distribution	Parameter	Estimate
GauMM	(ω_1, ω_2) (μ_1, μ_2) (σ_1, σ_2)	(0.273, 0.727) (1.890, 9.406) (1.128, 7.278)	WeiMM	(ω_1, ω_2) (α_1, α_2) (β_1, β_2)	(0.256, 0.744) (10.843, 0.943) (9.044, 6.710)
LognMM	(ω_1, ω_2) (μ_1, μ_2) (σ_1, σ_2)	(0.283, 0.717) (2.171, 1.298) (0.107, 1.163)	LoglMM	(ω_1, ω_2) (μ_1, μ_2) (σ_1, σ_2)	(0.298, 0.702) (2.172, 1.299) (0.068, 0.971)
GamMM	(ω_1, ω_2) (α_1, α_2) (β_1, β_2)	(0.248, 0.752) (87.373, 0.982) (0.100, 7.007)	GIGMM	(ω_1, ω_2) (μ_1, μ_2) (σ_1, σ_2) (ν_1, ν_2)	(0.271, 0.729) (8.729, 6.816) (0.111, 1.548) (46.972, 0.263)

For mixture models (see Table 4), the weight parameters ω_i for each component are non-negligible and exhibit relatively similar values. Take log-normal distribution as an illustrative example; the parameters exhibit the following values: $\omega_1 = 0.283$, $\omega_2 = 0.717$, $\mu_1 = 2.171$, $\mu_2 = 1.298$, $\sigma_1 = 0.107$, and $\sigma_2 = 1.163$. Here, ω_1 and ω_2 correspond to the weights of the two components. Similar results are obtained from other mixture models. This implies that these mixture models can effectively capture the bimodal trend in the data.

4.2. BMA Results

After fitting the candidate models, the log-likelihood can be obtained according to the given data. Then, AIC and BIC can be calculated, and the posterior model probability is calculated using Equation (7). The results are summarized in Table 5. Note that a higher posterior model probability indicates superior fitting performance for parking duration, while the values equal to 0 indicate that they are extremely small, specifically less than 0.001.

For the parking duration of temporary users, the log-normal distribution shows the best performance in describing the characteristics of parking duration, since it has the highest posterior model probability with a value of 0.910. However, normal distribution displays the worst fitting performance in the parking duration modeling, attributable to an exceptionally low posterior model probability (i.e., below 0.001), the minimum log-likelihood (with a value of -3710.85), and the maximum AIC and BIC values (measuring 7425.69 and 7436.20, respectively). Furthermore, GEV distribution is a viable option for characterizing parking duration features, exhibiting a posterior model probability of 0.090. According to the parking duration of long-term users, GIGMM obtains the maximum posterior model probability with a value of 0.897, indicating that GIGMM is the best model for depicting the characteristics of parking duration. In addition, it is noteworthy that multiple models exhibit substantial posterior model probabilities. The LognMM model, with a posterior model probability of 0.103, is also a suitable choice for modeling parking duration among long-term users. This indicates that both GIGMM and LognMM are viable options for modeling parking duration data (the values of posterior model probability are both larger than 0.001). These models illustrate that there is model uncertainty in describing parking duration distribution among different user groups.

Table 5. Summary of BMA results for temporary users and long-term users.

User Type	Distribution	Log-Likelihood	AIC [1]	BIC [1]	PMP [2]
Temporary users	Normal	−3710.85	7425.69	7436.20	0.000
	Log-normal	−2884.97	5773.94	5784.45	0.910
	Gamma	−2988.53	5981.06	5991.57	0.000
	Weibull	−2992.44	5988.88	5999.39	0.000
	Log-logistic	−2941.78	5887.56	5898.07	0.000
	Burr	−2933.35	5872.69	5881.20	0.000
	GEV	−2887.29	5780.58	5789.09	0.090
Long-term users	GauMM	−4761.90	9533.81	9560.62	0.000
	LognMM	−4447.30	8904.60	8931.41	0.103
	GamMM	−4497.93	9005.86	9032.68	0.000
	WeiMM	−4494.62	8999.23	9026.05	0.000
	LoglMM	−4472.90	8955.80	8982.62	0.000
	GIGMM	−4445.13	8900.27	8927.08	0.897

[1] $AIC = 2m - 2ln(L)$, $BIC = mln(n) - 2ln(L)$, where m is the number of parameters, n denotes the number of data samples, and L represents the likelihood. [2] PMP is posterior model probability.

In summary, it is difficult to find a certain distribution form to describe the characteristics of parking duration for both temporary and long-term users. Due to the uncertainty in parking duration, it exhibits various distributional characteristics, making it challenging to characterize its properties with a specific distribution. It is important to highlight that the dissimilarity between log-normal distribution and GEV distribution for temporary users (LognMM and GIGMM for long-term users) is minimal, and one must exercise caution when considering a distribution as the best model, as a distinct goodness-of-fit function (e.g., WAIC, LOO) may yield different best models in depicting parking duration characteristics. However, the BMA approach always integrates the strengths of acceptable candidate models based on posterior model probability and provides universally reliable and robust modeling of parking duration distribution with high accuracy.

According to the posterior model probability, the derived probability density function and cumulative density function of the BMA approach can be determined using Equation (1). The fitting performance of the parking duration of temporary and long-term users is illustrated in Figures 3 and 4. The histogram and dotted line (black) are the probability density function and cumulative density function of the observed data, and the dotted lines (green) and the curves (red) represent those of the best model and derived BMA approach. It can be observed that the parking duration distribution of temporary users is skewed and unimodal; the distribution of long-term users is long-tailed and exhibits a bimodal skewness. The log-normal distribution provides a precise fitting performance to parking duration data for temporary users compared with other candidate models. In contrast, GIGMM exhibits superior fitting performance compared with other models for long-term users. The derived BMA approach seamlessly integrates acceptable candidate models and provides accurate depictions of parking duration data for both temporary and long-term users. These findings illustrate the universal reliability and robustness of the BMA approach in modeling parking duration distribution.

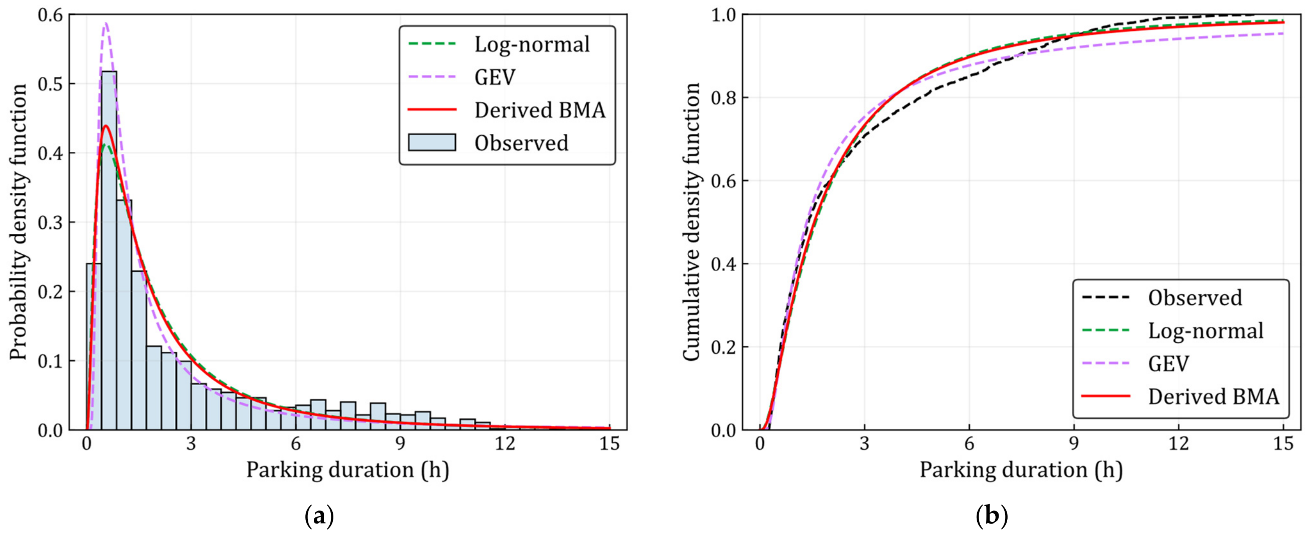


Figure 3. Fitting performance of parking duration for temporary users. (**a**) Probability density function. (**b**) Cumulative density function.

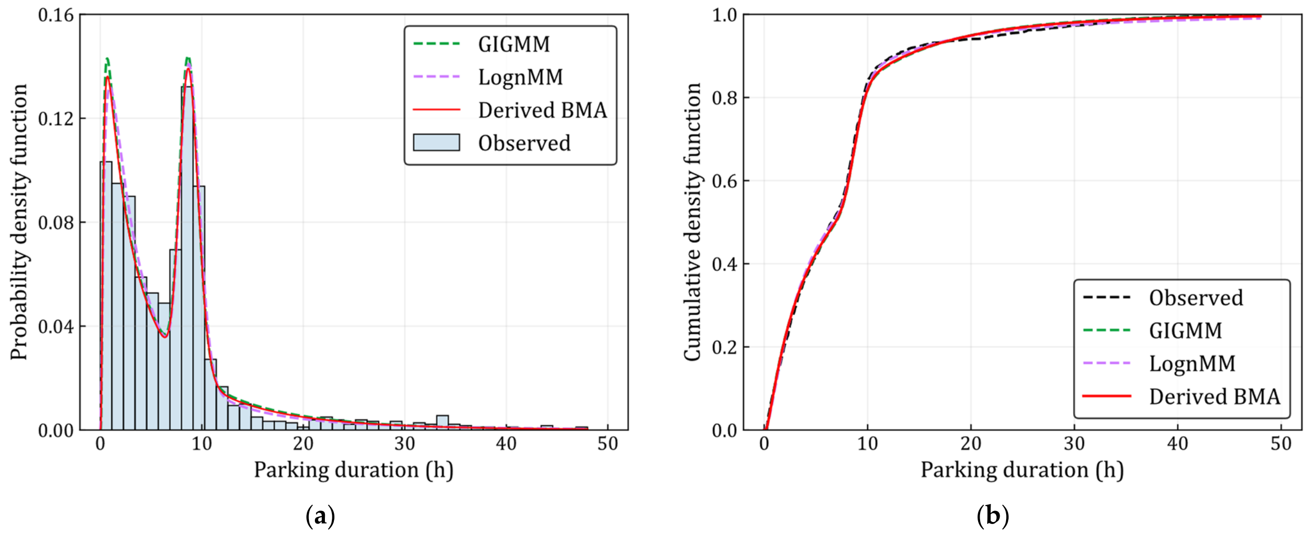


Figure 4. Fitting performance of parking duration for long-term users. (**a**) Probability density function. (**b**) Cumulative density function.

5. Conclusions

An accurate description of parking duration distribution serves as a specific reflection of parking demand characteristics, which can enhance parking efficiency and parking facility planning and support parking delicacy management. This study aimed to describe the characteristics of parking duration distribution using the BMA approach for two groups of users (i.e., temporary users and long-term users). The main conclusions are summarized as follows: (1) The log-normal distribution is the best model for depicting the parking duration of temporary users, with a posterior model probability of 0.910, while GIGMM is more suitable for characterizing the parking duration distribution of long-term users due to obtaining the largest posterior model probability of 0.897. (2) The BMA approach consistently exhibits universal reliability in accurately characterizing parking duration distribution by assigning weights to candidate models based on their posterior model probability, which can integrate the advantages of acceptable models. The accurate description of parking duration distribution is essential not only for supporting the planning and maneuverability of parking resources, but also for guiding policies related to parking fees. More specifically, the proportion of short-term and long-term parking spots can be reasonably allocated according to the parking duration distribution, and differential charging strategies can be implemented to encourage short-term parking or restrict long-term parking.

Although the BMA approach yields favorable results in modeling parking duration and offers an intriguing alternative for exploring parking demand characteristics, there is a minor limitation that the candidate models considered in modeling the parking duration may not be exhaustive. In future work, additional parking demand datasets from different parking facilities and locations (e.g., surface parking lots in business zones) and a broader selection of suitable distributions will be considered as candidate models. This will serve to further validate the effectiveness and robustness of the BMA approach in the accurate depiction of parking duration distribution characteristics.

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