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Calibration of a Rotary Encoder and a Polygon Using a Two-Autocollimator Method

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Abstract: In this work, we propose a two-autocollimator method in which all pitch angle deviations of a polygon and angle errors of a rotary encoder can be calibrated simultaneously. A polygon with any number of faces can be calibrated. Any face of the polygon is a measurement cycle of one. Compared to a traditional method, cross-calibration calibrates a rotary encoder and a polygon. This method can simultaneously calibrate all pitch angle deviations of the polygon and angle errors of the rotary encoder. The measurement cycle depends on how many faces the polygon has. There are 24 measurement cycles for a 24-faced polygon. In the experiment, we use two autocollimators to calibrate a 24-faced polygon and the SelfA rotary encoder to conduct the proposed two-autocollimator method. According to the uncertainty evaluation, the expanded uncertainty is 0.46″. For a 95% confidence level, the coverage factor is 2.00. To verify all pitch angle deviations, the shift-angle method, based on cross-calibration, uses one autocollimator to measure the same polygon. The difference in pitch angle deviations is smaller than ±0.28″. The maximum *E*n-value is 0.58. The SelfA rotary encoder comprises 12 read heads and calibrates using self-calibration. The difference in angle errors is smaller than ±0.27″. The maximum *E*n-value is 0.59. The two *E*n-values mean that the proposed two-autocollimator method is practical.

Keywords: angle standards; angle calibration; two-autocollimator; 24-faced polygon; rotary encoder

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1. Introduction

Polygons and rotary encoders are considered to be full-circle angle standards. A polygon can be treated as a circumference divided by an equal angle, such as 15°, 30°, and 90°. These are named 24-faced, 12-faced, or 4-faced polygons, respectively. A rotary encoder is made up of a glass disc with a suitable reader. The reader can determine the angular positions of the glass disc. Calibration of polygons and rotary encoders is essential to determine their accuracy at most national standards laboratories worldwide. The calibration values are standard angle values to transfer other instruments. Polygons and rotary encoders are implemented for various tasks such as accuracy testing and calibration of angle-measuring instruments, rotary tables, and so on. The calibration is based on the circle closure principle: the sum of all angle errors of a full circle must be zero. Therefore, the circle closure principle is widely used to calibrate polygons and rotary encoders.

For the calibration of rotary encoders, many previous papers proposed using self-calibration. For the self-calibration method, it is unnecessary to use an external standard. Sensor heads and a rotary encoder are installed inside a rotary table. The rotary encoder is rotated, and the sensor heads measure angle values at different angle positions corresponding to the rotary encoder. According to the circle closure principle, the rotary encoder can self-calibrate. Many self-calibration methods have been proposed [1–3]. Probst et al. [4,5] developed an angle comparator that was constructed using a rotary encoder and 16 read heads to calibrate all angle errors of the rotary encoder. Watanabe et al. [6–11] developed the SelfA rotary encoder for self-calibrating the rotary encoder. The SelfA rotary encoder consists of a rotary encoder and several groups of sensor heads. The SelfA rotary encoder can calibrate all angle errors of the rotary encoder. In contrast to self-calibration, using an external standard is another method to calibrate a rotary encoder. Many studies have demonstrated the use of a polygon as an external standard [12,13]. Kim et al. [14] and Huang et al. [15] used a 36-faced polygon, Jia et al. [16] used a 24-faced polygon, and Pisani et al. [17] used a 12-sided polygon and a 6-sided polygon. This method has to use an autocollimator to measure the polygon. The problem with this method is that the polygon must be calibrated before calibrating a rotary encoder.

For the calibration of polygons, many papers proposed using a rotary encoder as an external standard [18,19]. The rotary encoder has to calibrate by using self-calibration to transfer standard angle values. These standard angle values can calibrate a polygon. This method also has the disadvantage of using two experiments to calibrate the rotary encoder and the polygon. Using two autocollimators is another method to only calibrate a polygon [20–22]. The polygon is mounted on a rotary table. The two autocollimators measure the polygon. The angle errors of the rotary table do not influence the calibration of the polygon. Cross-calibration is used simultaneously, calibrating a rotary encoder and a polygon [23]. With this method, a rotary encoder is the external angle standard, and a polygon is the calibrated standard. All angle errors of the rotary encoder and all pitch angle deviations of the polygon are unknown. Based on the circle closure principle, the rotary encoder and the polygon can be calibrated simultaneously. However, the measurement cycle depends on how many faces the polygon has. Hsieh et al. [24] proposed a 24-sided polygon as the calibrated angle standard. The SelfA rotary encoder is the external angle standard. The polygon and SelfA rotary encoder can be calibrated simultaneously. This method is time-consuming depending on how many faces the polygon has. The 24-sided polygon is used so the measurement cycle is 24. Therefore, we propose the two-autocollimator method. The experiment setup is the same as in the previous studies [20–22]. These previous studies only proposed the calibration of the polygon. However, the proposed method can simultaneously calibrate the polygon and the rotary encoder. The measurement cycle is one.

This paper presents the new two-autocollimator method for calibrating the rotary encoder and the polygon. First, the introduction section presents the related research papers and techniques. Next, the configuration of the two-autocollimator method shows two autocollimators, a 24-faced polygon, and the SelfA rotary encoder. The theory of the two-autocollimator method is described in detail using the circle closure principle. In the experiment, the two-autocollimator method, using the two autocollimators and the 24-faced polygon, is conducted to calibrate the SelfA rotary encoder. A comparison of calibration results, the uncertainty evaluation, and the *E*n-value calculation are presented. Finally, we present our discussion and conclusions.

2. Configuration and Theory of the Two-Autocollimator Method

Figure 1a shows a schematic of the calibration setup. A 24-faced polygon is mounted at the center of a rotary encoder. The 24-faced polygon and the rotary encoder are rotated at the same time. In this paper, the 24-faced polygon is used to illustrate the related theory and conduct experiments. A polygon with any number of faces can be used. This polygon has 24 measuring faces. Two autocollimators are placed and fixed in front of the rotary encoder. The no. 1 autocollimator is collimated to the first face of the polygon. The no. 2 autocollimator is collimated to the second face of the polygon.

As shown in Figure 1b, the polygon has the measuring faces (R) and the nominal faces (N). In the ideal polygon, the measuring faces are the same as the nominal faces. Between the measuring face and the nominal face is a slight deviation, called the pitch angle deviation, β_{i}. Each measuring face has a different pitch angle deviation. For the 24-faced polygon, one full circle is assumed to be divided into 24 nominal faces and 24 measuring faces. The angle interval between two consecutive nominal faces is 15°, called the nominal pitch angle. According to the circle closure principle, the summation of all pitch angle deviations must be zero:

$$\sum_{i=1}^{24} \beta_i = 0 \tag{1}$$

The rotary encoder is calibrated to determine the angle error α (θ), which is an angle deviation between the measured and true angles. The angle error α (θ) is the angle error of the rotary encoder at θ. The 24-faced polygon can be measured at rotary encoder angles θ of ((i − 1)*15°), where i is from 1 to 24.

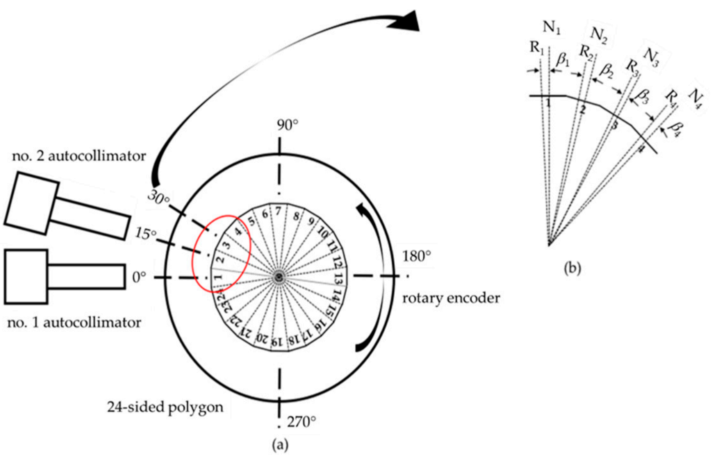

Figure 1. (**a**) Diagram of the 24-faced polygon. (**b**) Enlarged view of the red circle highlighted in (**a**).

Figure 2 shows the calibration value determination. The measured values contain the angle error of the rotary encoder, the pitch angle deviation of the 24-faced polygon, and the setup error. The setup error is the misalignment between the rotary encoder and the polygon, which remains constant at different angles. Different autocollimators have different setup errors. The setup error U_1 is for the no. 1 autocollimator, and the setup error U_2 is for the no. 2 autocollimator. As shown in Figure 2a, the rotary encoder is rotated at the starting angle of 0°. The first face of the 24-faced polygon is adjusted to a starting angle of 0°. The no. 1 autocollimator measures the angle error α (0°), the pitch angle deviation β_1, and the setup error U_1. The rotary encoder is rotated in one step to an angle of 15°. The no. 1 autocollimator collimates the second face of the 24-faced polygon. The no. 1 autocollimator then measures the angle error α (15°), the summation of the pitch angle deviations (β_1 + β_2), and the setup error U_1. The rotary encoder is rotated in two steps to an angle of 30°. The no. 1 autocollimator collimates the third face of the 24-faced polygon. The no. 1 autocollimator then measures the angle error α (30°), the summation of the pitch angle deviations (β_1 + β_2 + β_3), and the setup error U_1. Overall, the rotary encoder is rotated in 23 steps to an angle of 345°. The no. 1 autocollimator collimates the 24th face of the 24-faced polygon. The no. 1 autocollimator measures the angle error α (345°), the summation of the pitch angle deviations of the first to the 24th faces, and the setup error U_1. As shown in Figure 2b, the rotary encoder is at the starting angle of 0°. The no. 2 autocollimator collimates the second face of the 24-faced polygon. The no. 2 autocollimator measures the angle error α (0°), the pitch angle deviation β_2, and the setup error U_2. The rotary encoder is rotated in one step at 15°. The no. 2 autocollimator collimates the third face of the 24-faced polygon. The no. 2 autocollimator then measures the angle error α (15°), the summation of the pitch angle deviations (β_2 + β_3), and the setup error U_2. The rotary encoder is rotated in two steps at 30°. The no. 2 autocollimator collimates the fourth face of the 24-faced polygon. The no. 2 autocollimator then measures the angle error α (30°), the summation of the pitch angle deviations (β_2 + β_3 + β_4), and the setup error U_2. At the final angle, the rotary encoder is rotated in 23 steps to an angle of 345°. The no. 2 autocollimator collimates the first face of the 24-faced polygon. The no. 2 autocollimator measures the angle error α (345°), the summation of the pitch angle deviations of the first to the 24th faces, and the setup error U_2. The measured value contains the angle error of the rotary encoder, the summation of the pitch angle deviations, and the setup error, as follows:

$$\varepsilon_{i,j}=\alpha\{(i-1)\times 15°\}+\sum_{k=j}^{i}\beta_k+U_j \quad (2)$$

where $\varepsilon_{i,j}$ is the value measured by the autocollimator at an angle of i × 15°, and i is the number of steps of rotation of the rotary encoder. The value of i equals 1 at a starting angle of 0°; j denotes the no. j autocollimator; β_k is the pitch angle deviation at *k*th face of the 24-faced polygon; and U_j denotes the setup error for the no. j autocollimator.

$$\sum_{i=1}^{24}\alpha\{(i-1)\times 15°\}=0 \quad (3)$$

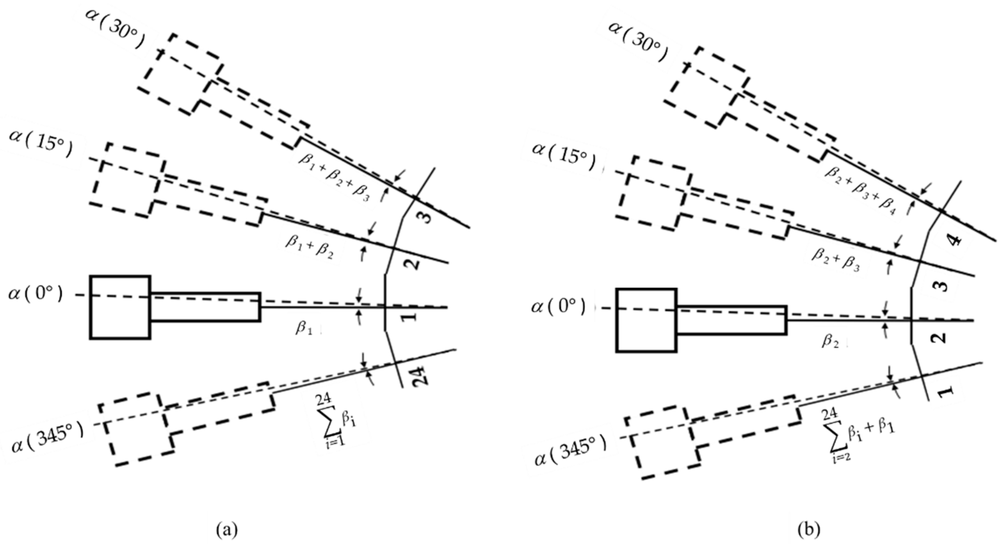

Figure 2. Schematic of the calibration value determination. (**a**) The no. 1 autocollimator. (**b**) The no. 2 autocollimator.

The measured values of the no. 1 autocollimator at different rotated angles are as follows:

$$\begin{gathered}\varepsilon_{1,1}=\alpha(0°)+\beta_1+U_1\\ \varepsilon_{2,1}=\alpha(15°)+\sum_{k=1}^{2}\beta_k+U_1\\ \vdots\\ \varepsilon_{23,1}=\alpha(330°)+\sum_{k=1}^{23}\beta_k+U_1\\ \varepsilon_{24,1}=\alpha(345°)+\sum_{k=1}^{24}\beta_k+U_1\end{gathered} \quad (4)$$

The measured values of the no. 2 autocollimator at different rotated angles are as follows :

$$\begin{gathered}\varepsilon_{1,2} = \alpha(0^\circ) + \beta_2 + U_2 \\ \varepsilon_{2,2} = \alpha\ (15^\circ\ ) + \sum_{k=2}^{3} \beta_k + U_2 \\ \vdots \\ \varepsilon_{23,2} = \alpha\ (330^\circ) + \sum_{k=2}^{24} \beta_k + U_2 \\ \varepsilon_{24,2} = \alpha\ (345^\circ\ ) + \sum_{k=1}^{24} \beta_k + U_2\end{gathered} \tag{5}$$

The difference measured values between the measured values of the no. 1 autocollimator and the measured values of the no. 2 autocollimator at the same rotated angles are as follows :

$$\begin{gathered}\varepsilon_{1,2} - \varepsilon_{1,1} = \beta_2 - \beta_1 + U_2 - U_1 \\ \varepsilon_{2,2} - \varepsilon_{2,1} = \beta_3 - \beta_1 + U_2 - U_1 \\ \vdots \\ \varepsilon_{23,2} - \varepsilon_{23,1} = \beta_{24} - \beta_1 + U_2 - U_1 \\ \varepsilon_{24,2} - \varepsilon_{24,1} = U_2 - U_1\end{gathered} \tag{6}$$

For Equation (6), all difference measured values reduce the difference measured value, $\varepsilon_{24,2} - \varepsilon_{24,1}$, at the final rotated angle. Equation (6) can be rewritten as follows:

$$\begin{gathered}\varepsilon_{1,2} - \varepsilon_{1,1} - (\varepsilon_{24,2} - \varepsilon_{24,1}) = \beta_2 - \beta_1 \\ \varepsilon_{2,2} - \varepsilon_{2,1} - (\varepsilon_{24,2} - \varepsilon_{24,1}) = \beta_3 - \beta_1 \\ \vdots \\ \varepsilon_{22,2} - \varepsilon_{22,1} - (\varepsilon_{24,2} - \varepsilon_{24,1}) = \beta_{23} - \beta_1 \\ \varepsilon_{23,2} - \varepsilon_{23,1} - (\varepsilon_{24,2} - \varepsilon_{24,1}) = \beta_{24} - \beta_1\end{gathered} \tag{7}$$

The summation of Equation (7) is as follows:

$$\sum_{i=1}^{24} \varepsilon_{i,2} - \varepsilon_{i,1} - (\varepsilon_{24,2} - \varepsilon_{24,1}) = \sum_{k=1}^{24} \beta_k - 24 \times \beta_1 \tag{8}$$

According to the circle closure principle, the summation of all the pitch angle deviations must be zero, as in Equation (1). The first term on the right-hand side of Equation (8) must be zero. The first pitch angle deviation β_1 is calculated. According to Equation (7), the pitch angle deviations from β_2 to β_{24} are calculated. Until this calculation, all pitch angle deviations from β_1 to β_{24} are calculated. Then, the summation of Equation (4) is as follows:

$$\sum_{i=1}^{24} \varepsilon_{i,1} = \sum_{k=1}^{24} \alpha\ \{(i\ -\ 1)\ \times\ 15^\circ\} + \sum_{k=1}^{24} (25 - k) \times \beta_k + 24 \times U_1 \tag{9}$$

According to Equation (2), the summation of all angle errors is zero. As in the previous discussion, all pitch angle deviations are known in the second term on the right-hand side of Equation (9). In the third term on the right-hand side of Equation (9), the setup error of the no. 1 autocollimator U_1 is calculated.

The summation of Equation (5) is as follows:

$$\sum_{i=1}^{24} \varepsilon_{i,1} = \sum_{k=1}^{24} \alpha\ \{(i\ -\ 1)\ \times\ 15^\circ\} + \sum_{k=2}^{24} (26 - k) \times \beta_k + \beta_1 + 24\ \times\ U_2 \tag{10}$$

According to Equation (2), the summation of all angle errors is zero. As in the previous discussion, all pitch angle deviations are known in the second and third terms on the right-hand side of Equation (10). In the fourth term on the right-hand side of Equation (10), the setup error of the no. 2 autocollimator U_2 is calculated. Until this calculation, all pitch angle deviations from β_1 to β_{24}, the setup error of the no. 1 autocollimator U_1, and the setup error of the no. 2 autocollimator U_2 are calculated. According to Equations (4) and (5), all angle errors of the rotary encoder from α (0°) to α (345°) are calculated. As in the previous calculation procedure, all angle errors of the rotary encoder and all pitch angle deviations are calculated simultaneously by using measured values of the two autocollimators.

3. Experiments

3.1. The/Two-Autocollimator Method

Figure 3a shows a photograph of the experimental setup. The 24-faced polygon is used in the experiment, as expressed in Figure 3b. The polygon is mounted at the center of the SelfA rotary encoder (e-motionsystem, Inc., SCMS-127). A servo motor with an air bearing controls the SelfA rotary encoder. The SelfA rotary encoder and the 24-faced polygon are rotated at different angles from 0° to 345° in steps of 15°. The no. 1 autocollimator (MOLLER-WEDEL OPTICAL GmbH, ELCOMAT 2000) and the no. 2 autocollimator (MOLLER-WEDEL OPTICAL GmbH, ELCOMAT 3000) are fixed outside the SelfA rotary encoder. Both the two autocollimators have the same specifications. The measuring range is from +1000″ to −1000″, and the resolution is 0.001″. The SelfA rotary encoder is rotated at the starting angle of 0°. The no. 1 autocollimator is collimated to the first face of the 24-faced polygon. The no. 2 autocollimator is collimated to the second face of the 24-faced polygon.

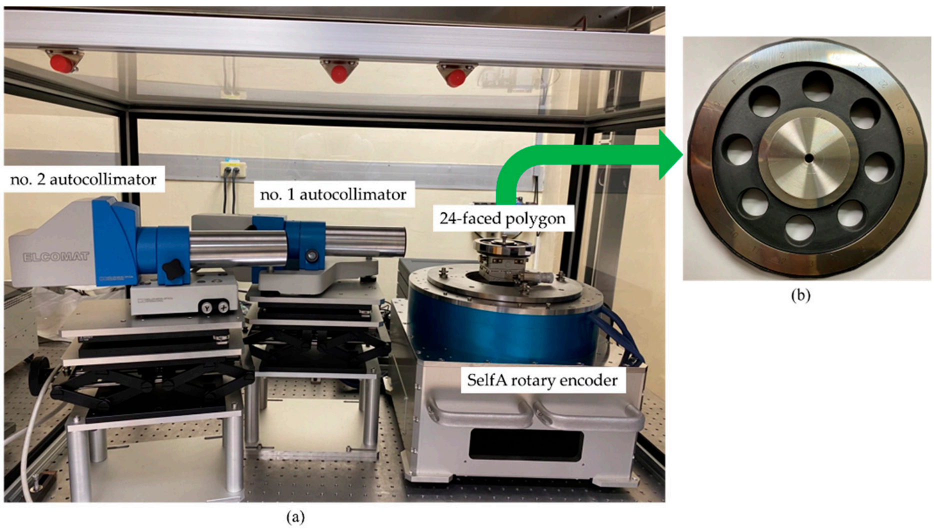

Figure 3. (**a**) Photograph of the experimental setup. (**b**) Photograph of the 24-sided polygon.

Calibration procedures are as follows. The SelfA rotary encoder is located at 0°. The no. 1 autocollimator is aligned to the first face of the 24-faced polygon. The angle of the no. 1 autocollimator is adjusted to approximately 0. The no. 2 autocollimator is aligned to the second face. The angle of the no. 2 autocollimator is adjusted to approximately 0. Then, the SelfA rotary encoder is rotated by 15° in one step. The no. 1 autocollimator collimates the second face of the polygon. The no. 2 autocollimator collimates the third face of the polygon. The two autocollimators collimate successive faces of the polygon. Finally, the SelfA rotary encoder is rotated by 345° in 23 steps. The no. 1 autocollimator collimates the 24th face of the polygon. The no. 2 autocollimator collimates the first face of the polygon. Both the no. 1 autocollimator and the no. 2 autocollimator measure 24 values corresponding to 0°, 15°, . . . , 345° in steps of 15°.

The measured values of the no. 1 autocollimator are as per Equation (4). The measured values of the no. 2 autocollimator are as per Equation (5). These 48 measured values are used to calculate the difference measured values in Equations (6) and (7). All pitch angle deviations of the polygon can be calculated using Equations (7) and (8), as depicted in

Figure 4. According to Equations (9) and (10), the setup errors, U_1 and U_2, are calculated as $-0.02''$ and $0.71''$, respectively. A shift-angle method, based on cross-calibration, for calibrating the 24-sided polygon is used to verify the calibration results. The detailed calculation is presented in the reference [24]. All pitch angle deviations of the polygon can be calculated using the shift-angle method, as depicted in Figure 4. Figure 5 depicts the difference in calibration results using the two-autocollimator method and shift-angle methods. The difference in calibration results is less than $\pm 0.28''$.

Figure 6 shows the angle errors, from α (0°) to α (345°) in steps of 15°, using Equations (4) and (5). The SelfA rotary encoder is used to verify the proposed two-autocollimator method. The theory of the SelfA rotary encoder is described in the literature [6–11]. A previous study proposed the SelfA rotary encoder configuration [24] so we do not describe the details in this paper. The SelfA rotary encoder consists of twelve optical sensors and a rotary encoder. All optical sensors are divided into three groups which consist of three optical sensors, four optical sensors, and seven optical sensors, respectively. Optical sensors of each set are installed for the same rotary encoder at an equal interval, at angles of 120°, 90°, and 51.42°, respectively. The SelfA rotary encoder can calibrate the angle errors using self-calibration, as depicted in Figure 6. The calibration uses the two-autocollimator method and self-calibration. Figure 7 shows a comparison of the calibration results obtained using the two methods. The angle errors are calibrated from 0° to 345° in steps of 15°. The differences in calibration results are less than $\pm 0.27''$.

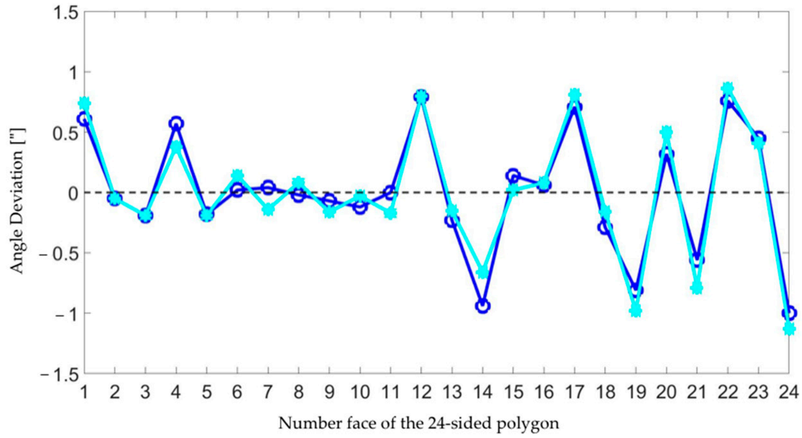

Figure 4. Calibration results for pitch angle deviations of the 24-faced polygon using the two-autocollimator method (circle label) and the shift-angle method (star label).

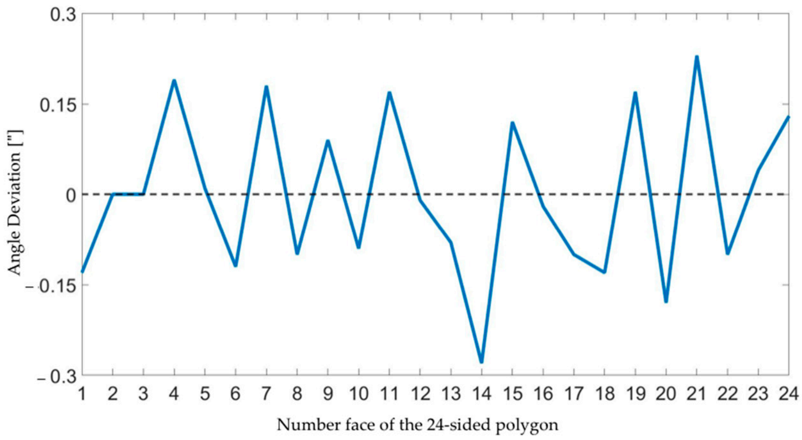

Figure 5. Difference in calibration results for the 24-faced polygon between the two-autocollimator method and the shift-angle method.

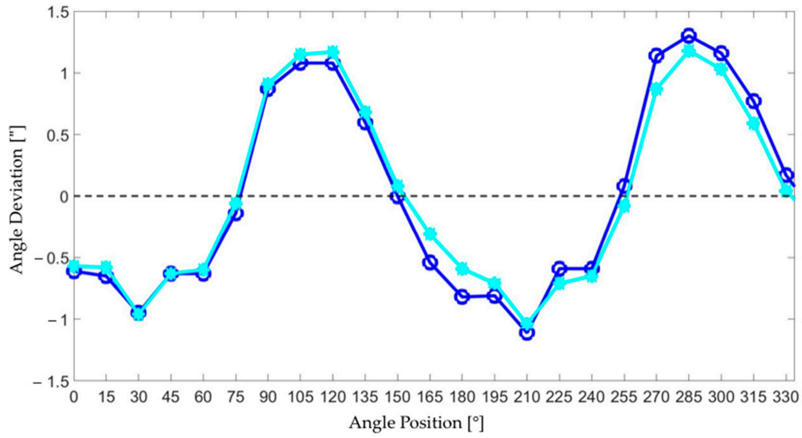

Figure 6. Calibration results for the SelfA rotary encoder are obtained using the two-autocollimator method (circle label) and self-calibration (star label).

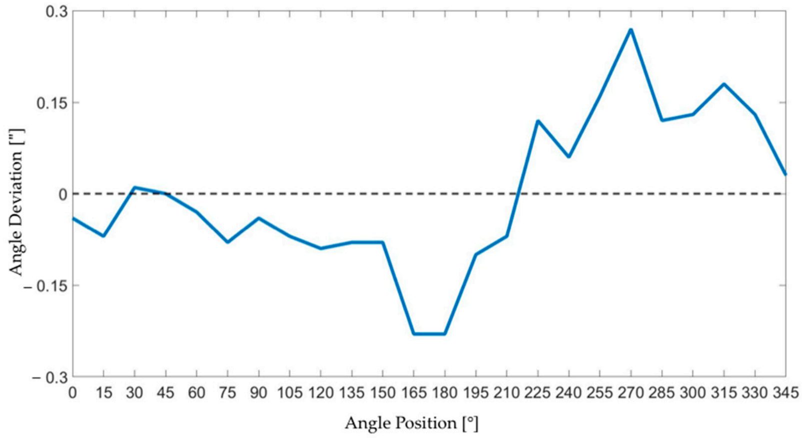

Figure 7. Difference in calibration results for the SelfA rotary encoder between the two-autocollimator method and self-calibration.

3.2. Uncertainty Evaluation

The uncertainty evaluation is based on the ISO Guide to the Expression of Uncertainty in Measurement [25]. The measured value ε is related to n independent and uncorrelated influence sources, X_i, $k = 1, 2, \ldots, n$, via the functional relation. According to Equation (3), the measurement uncertainty of the two-autocollimator method originates from several influence sources to contribute to the measured value ε, including the angle error α, the pitch angle deviation β, and the setup error U.

$$\varepsilon = f\,(\alpha,\ \beta,\ U) \tag{11}$$

Table 1 shows the uncertainty budget for the proposed two-autocollimator method. The influence sources contribute to calibrating the polygon and the rotary encoder. Some influence quantities are directly measured and estimated, called Type A. Other influence quantities use specification knowledge to estimate using a set of probability distributions, called Type B. The combined standard uncertainty is expressed as follows:

$$U_c^2\,(\varepsilon) = \left(\frac{\partial\varepsilon}{\partial\alpha}\right)^2 u\,(\alpha)^2 + \left(\frac{\partial\varepsilon}{\partial\beta}\right)^2 u(\beta)^2 + \left(\frac{\partial\varepsilon}{\partial U}\right)^2 u(U)^2 \tag{12}$$

where $u(\alpha)$, $u(\beta)$, and $u(U)$ are called standard uncertainties. The parameters $\partial\varepsilon\,/\,\partial\alpha$, $\partial\varepsilon\,/\,\partial\beta$ and $\partial\varepsilon\,/\,\partial U$ are known as both sensitivity coefficients and weighting factors for the standard uncertainties. All sensitivity coefficients equal 1.

Table 1. Uncertainty budget for the two-autocollimator method.

Standard Uncertainty	Type	$U\ (X_i)$	$\frac{\partial\alpha}{\partial X_i}$	$\left\lvert\frac{\partial\alpha}{\partial X_i}\right\rvert u\ (X_i)$	Degree of Freedom
Measured value, ε		0.0910″	1	0.0910″	43
Repeatability of angle error measurement	A	0.0450″	1	0.0450″	5
Resolution of the autocollimator	B	0.0003″	1	0.0003″	50
Traceability of the autocollimator	B	0.0050″	1	0.0050″	60
Angle error of autocollimator	B	0.0780″	1	0.0780″	50
Resolution of the rotary encoder	B	0.0116″	1	0.0116″	50
pitch angle deviation, β	A	0.0450″	1	0.0450″	5
setup error, U	B	0.0205″	1	0.0205″	50

Combined standard uncertainty (u): 0.23; Effective degrees of freedom (ν_{eff}): 74; Expanded uncertainty (95% confidence level): 0.46″ (k = 2.00).

The angle error standard uncertainty $u(\alpha)$ is evaluated using five uncertainty sources:

(1) Repeatability of angle error measurement.

Repeatability is an uncertainty source from six multiple measurements. The measured results of the two-autocollimator method, ranging from 0° to 345° in steps of 15°, are computed as the average measurement results and the standard deviations. The standard uncertainty is a maximum standard deviation of 0.11″ by a square root of six, called type A.

$$u = 0.11''/\sqrt{6} = 0.0450''$$

The degree of freedom is 5.

(2) Resolution of the autocollimator.

According to the specification, the resolution of the no. 1 autocollimator and the no. 2 autocollimator is 0.001″. The standard uncertainty distribution assumes a rectangular probability by a square root of twelve, called type B.

$$u = 0.001''/\sqrt{12} = 0.0003''$$

(3) Traceability of the autocollimator.

The calibration certificates of the no. 1 autocollimator and the no. 2 autocollimator declare the same expanded uncertainty of 0.01″ with a coverage factor k of 2 (95% confidence level). The standard uncertainty is obtained by dividing 0.01″ by 2, called type B.

$$u = 0.01''/2 = 0.0050''$$

According to the t distribution [25], the coverage factor k of 2 equals the degrees of freedom 60.

(4) Angle error of the autocollimator.

The no. 1 autocollimator and no. 2 autocollimator are calibrated in the same measurement range of ±1000″. According to two calibration reports, the calibration error of the no. 1 autocollimator is 0.11 (peak-to-valley), and the calibration error of the no. 2 autocollimator is 0.27 (peak-to-valley). The maximum error is 0.27″. The uncertainty distribution assumes a rectangular probability by a square root of twelve, called type B.

$$u = 0.27''/\sqrt{12} = 0.0780''$$

(5) Resolution of the SelfA rotary encoder.

The resolution of the SelfA rotary encoder is 0.04″. The uncertainty distribution assumes a rectangular probability by a square root of twelve, called type B.

$$u = 0.04''/\sqrt{12} = 0.0116''$$

The four uncertainty sources, (2), (4), and (5), are type B. All relative standard uncertainty is estimated as 10% with 50 degrees of freedom. The angle error standard uncertainty $u\ (\alpha)$ is 0.0910″, the root sum square of standard uncertainty sources, from (1) to (5). Using the Welch–Satterthwaite equation [25], the degree of freedom ν_{eff} of the measured value uncertainty is 43.

The pitch angle deviation uncertainty $u\ (\beta)$ is evaluated by the six repeated measurements and standard deviations. The maximum standard deviation for all pitch angle deviations is 0.11″. The standard uncertainty is $0.11''/\sqrt{6}= 0.0450''$. The degree of freedom is 5, called type A. The misalignment between the rotary encoder and the polygon causes the setup error uncertainty $u\ (U)$. According to Equations (9) and (10), the setup errors, U_1 and U_2, are calculated as −0.01″ and 0.09″, respectively. The maximum setup error is 0.09″. The uncertainty distribution assumes a rectangular probability by a square root of twelve, called type B. The standard uncertainty is $0.71''/\sqrt{12}= 0.2050''$ with 50 degrees of freedom, using the 10% relative standard uncertainty.

In Table 1, the combined standard uncertainty of the two-autocollimator method is 0.23″, the root sum square of all uncertainty contributions, including the measured value uncertainty, the pitch angle deviation standard uncertainty, and the setup error standard uncertainty. The degree of freedom ν_{eff} of the combined standard uncertainty is 74. According to the t distribution [25], the coverage factor is 2.00. The expanded uncertainty for a confidence level of 95% in $U_{\text{two-autocollimator}}$ is given as follows:

$$U_{\text{two-autocollimator}} = 0.46'' \ (\mathrm{k} = 2.00) \tag{13}$$

In order to investigate the consistency of calibration results, the En-value is computed. The En-value is less than 1, which indicates an acceptance criterion of comparison. The En-value for pitch angle deviations of the 24-sided polygon is shown as follows:

$$E_{\mathrm{n}} = \frac{\left|\beta_{\text{two-autocollimator}} - \beta_{\text{shift-angle}}\right|}{\sqrt{U^2_{\text{two-autocollimator}} - U^2_{\text{shift-angle}}}} \tag{14}$$

where $\beta_{\text{two-autocollimator}}$ is a pitch angle deviation using the two-autocollimator method with the expanded uncertainty $U_{\text{two-autocollimator}}$. $\beta_{\text{shift-angle}}$ is a calibration result using the shift-angle-method, and $U_{\text{shift-angle}}$ is the expanded uncertainty of the shift-angle-method. According to the reference paper [24], the expanded uncertainty of the shift-angle-method $U_{\text{shift-angle}}$ is 0.15″. Figure 5 shows the different calibration results between the two-autocollimator method and the shift-angle-method. The maximum En-value is 0.58, which is less than 1. The En-value for angle errors of SelfA rotary encoder is shown as follows:

$$E_{\mathrm{n}} = \frac{|\alpha(\theta) - \mu|}{\sqrt{U^2_{\text{two-autocollimator}} - U^2_{\text{self-calibraion}}}} \tag{15}$$

where $\alpha\ (\theta)$ is an angle error using the two-autocollimator method with the expanded uncertainty $U_{\text{two-autocollimator}}$, μ is a self-calibration result, and $U_{\text{self-calibration}}$ is the expanded uncertainty of self-calibration. According to the reference paper [24], the expanded uncertainty of self-calibration $U_{\text{self-calibration}}$ is 0.02″. Figure 7 shows the different calibration results between self-calibration and the two-autocollimator method. The maximum En-value is 0.59, which is less than 1. The En-values for the angle error and the pitch angle deviation mean that the proposed two-autocollimator method is reliable.

4. Discussion

Table 2 makes comparisons between the proposed two-autocollimator method and related previous studies, as mentioned in the Introduction. This study proposes the two-autocollimator method for simultaneously calibrating a rotary encoder and a polygon. The previous studies show that the two-autocollimator only can calibrate a polygon [20–22].

This paper uses two autocollimators to calibrate a polygon and a rotary encoder. The proposed two-autocollimator method can calibrate any polygon with any number of faces. Any face of the polygon is where the measurement cycle is one. The previous studies show the shift-angle method, based on cross-calibration, in which the measurement cycles depend on how many faces the polygon has [23,24]. Self-calibration is a method to only calibrate a rotary encoder [1–3].

Table 2. Comparisons between the proposed two-autocollimator method and related previous studies.

Method	Calibration	Measurement Cycle
The proposed two-autocollimator method	A rotary encoder and a polygon	One
The previous two-autocollimator studies	A polygon	One
Self-calibration	A rotary encoder	One
The shift-angle method (cross-calibration)	A rotary encoder and a polygon	It depends on how many faces of a polygon (ex.: a 24-faced polygon is 24 measurement cycles)

5. Conclusions

The proposed two-autocollimator method uses two autocollimators to calibrate the 24-sided polygon and the SelfA rotary encoder. The two autocollimators are set up outside the SelfA rotary encoder. The 24-faced polygon is on the SelfA rotary table. As discussed in the Theory of The Two-Autocollimator Method section, all pitch angle deviations of the 24-faced polygon and angle errors of SelfA rotary encoder can be calibrated simultaneously. To verify the proposed two-autocollimator method, the shift-angle method, based on cross-calibration, is used to measure the same 24-faced polygon, and self-calibration is used to measure the same SelfA rotary encoder. According to the shift-angle method, based on cross-calibration, an autocollimator calibrates all pitch angle deviations of the 24-faced polygon. The difference in pitch angle deviations is smaller than $\pm 0.28''$. The SelfA rotary encoder comprises 12 read heads and calibrates using self-calibration. Angle errors of SelfA rotary encoder can be calibrated using the proposed two-autocollimator method and self-calibration. The difference in angle errors is smaller than $\pm 0.27''$.

Following the ISO Guide to the Expression of Uncertainty in Measurement [25], the evaluated expanded uncertainty of the proposed two-autocollimator method is 0.46″. For a 95% confidence level, the coverage factor is 2.00. The difference in pitch angle deviations and expanded uncertainty are used in the En-value for the proposed two-autocollimator and shift-angle methods. The maximum En-value is 0.58. The difference in angle errors and expanded uncertainty are used in the En-value to compare the proposed two-autocollimator method and self-calibration. The maximum En-value is 0.59. Both the En-values are less than 1, meaning that the proposed two-autocollimator method is practical.

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