applied sciences

MDPI

Communication

One-Dimensional Nonlinear Consolidation for Soft Clays with Continuous Drainage Boundary Considering Non-Darcy Flow

Jin Wu [1,2,3], Ruichen Xi [1], Rongzhu Liang [1,*], Mengfan Zong [1,4] and Wenbing Wu [1,5,*]

1 Faculty of Engineering, Zhejiang Institute, China University of Geosciences, Wuhan 430074, China
2 Wuhan Shuchuang Technology Co., Ltd., Wuhan 430063, China
3 Wuhan Metro Group Co., Ltd., Wuhan 430000, China
4 School of Civil and Ocean Engineering, Jiangsu Ocean University, Lianyungang 222005, China
5 Research Center of Coastal Urban Geotechnical Engineering, Zhejiang University, Hangzhou 310058, China
* Correspondence: liangcug@163.com (R.L.); wuwb@cug.edu.cn (W.W.)

Abstract: Adopting the non-Darcy flow presented by Hansbo and considering the nonlinear compression and permeability characteristics of soils, the one-dimensional nonlinear consolidation problem of soft clays is investigated by means of a continuous drainage boundary. The numerical solutions of average consolidation degrees defined by settlement and excess pore water pressure are derived by using the finite difference method, and the correctness of these solutions is verified by comparing them with existing analytical and numerical solutions. Based on the proposed solutions, a parametric study is conducted to study the influence of interface parameter, non-Darcy flow parameter and soil nonlinearity on the consolidation behavior of soft clays. The results show that the solutions based on the continuous drainage boundary can be degenerated into the solutions based on the Terzaghi drainage boundary if the interface parameter is taken as a reasonable value. The soil consolidation behavior considering both non-Darcy seepage and nonlinear characteristics of soil is very complex.

Keywords: one-dimensional consolidation; continuous drainage boundary; non-Darcy flow; nonlinear consolidation; average consolidation degree

check for updates

Citation: Wu, J.; Xi, R.; Liang, R.; Zong, M.; Wu, W. One-Dimensional Nonlinear Consolidation for Soft Clays with Continuous Drainage Boundary Considering Non-Darcy Flow. *Appl. Sci.* **2023**, *13*, 3724. https://doi.org/10.3390/app13063724

Academic Editor: Tiago Miranda

Received: 25 February 2023
Revised: 12 March 2023
Accepted: 13 March 2023
Published: 15 March 2023

1. Introduction

With the increase of engineering construction in coastal areas, the long-term settlement caused by nonlinear consolidation of soil has attracted more and more attention all over the world [1–5]. Scholars have found that the seepage form in soil and the compressibility of soil have a great effect on the nonlinear consolidation characteristics of soil. Therefore, since the 1960s, many scholars have investigated the influence of non-Darcy seepage and nonlinear compression and permeability characteristics of soil on the consolidation process of soil [6,7]. Among the existing non-Darcy flow models, the non-Darcy flow of Hansbo is widely utilized [8]. Schmidt and Westmann applied Hansbo's flow to obtain an analytical solution to the 1D consolidation problem of soil with a small time factor [9]. Hansbo further investigated the 1D consolidation problem of saturated soil with Hansbo's flow [10]. Subsequently, Liu et al. solved the 1D consolidation and rheological consolidation problems of saturated clay under instantaneous load by using Hansbo's flow [11,12]. Jiu et al. investigated the influence of self-weight stress on the 1D consolidation of soil with Hansbo's flow [13]. In addition, Li et al. [14,15] and Cui et al. [16] studied the 1D consolidation of elastic and fractional viscoelastic saturated soils by using the exponential seepage form in Hansbo's flow model.

The aforementioned works have deepened the understanding of the influence of non-Darcy characteristics on the consolidation process of soil. However, the compressibility and permeability of soil will change during the consolidation process, so the nonlinear consolidation of soil should also be paid more attention [3,5]. In view of this, assuming that the permeability coefficient and the compression coefficient change synchronously during the consolidation process and the self-weight stress remains unchanged along the depth, Davis and Raymond first derived an analytical for the 1D nonlinear consolidation of saturated soil under instantaneous loading [17]. Based on the currently recognized e-lgσ' and e-lgk_v relations, Mesri and Rokhsar obtained a solution for the 1D nonlinear consolidation of saturated soil by using the finite difference method [18]. Soon after, many scholars investigated the nonlinear consolidation problems of soil by considering variable load [19,20] and layered characteristics of soils [21,22]. In order to consider the influence of both soil nonlinear characteristics and non-Darcy seepage on the soil consolidation process, Liu et al. solved the 1D nonlinear consolidation problem of saturated clay under non-Darcy seepage [23]. Li et al. investigated the influence of variable load on the 1D nonlinear consolidation of soil with non-Darcy flow [24]. Soon after, Li et al. further investigated the 1D nonlinear consolidation of soil with non-Darcy flow by considering the influence of variable load, large deformation and so on [25,26]. Due to the complexity of the soil consolidation problem considering both soil nonlinearity and non-Darcy seepage, the drainage boundary especially may change with time, and further in-depth study on this topic is still needed.

The classic Terzaghi consolidation theory assumes that the drainage boundary conditions are fully pervious and impervious, but in engineering practice, the drainage capacity of a soil boundary always changes between fully permeable and fully impermeable. To simulate the soil drainage boundary more realistically, Gray [27] originally proposed the impeded drainage boundary, which was introduced by Schiffman and Stein [28] to investigate the 1D consolidation problem of layered saturated soil with variable coefficients of permeability and compressibility. Subsequently, many scholars have introduced Gray's impeded drainage boundary to study the 1D problems of single-layered [29–32], bilayered [33–35] and multilayered soils [15,36]. Nevertheless, owing to the complexity of the impeded drainage boundary, it is not easy to obtain explicit solutions for most consolidation problems. Further, the above two models do not consider the relationship between drainage boundary and time, so they cannot reflect the fact that the drainage capacity of a boundary changes with the consolidation process [37]. In order to consider the time effect of a drainage boundary, Mei et al. [38] originally developed a continuous drainage boundary (CBD) expressed as an exponential function over time, which can reflect the dissipation of excess pore water pressure (EPWP) at the boundary over time during the consolidation process. The greatest advantage of the CDB is that it can reflect the boundary drainage capacity between fully permeable and impervious, and its expression is very simple. After the work of Mei et al. [38], investigations on consolidation theory with the CDB have been carried out for saturated soils with a single layer [39] and multiple layers [40] undergoing instantaneous load, and unsaturated soil with a single layer [41]. Recently, Zong et al. introduced the CDB to separately study the 1D consolidation characteristics of soil with nonlinear compressibility [42] and non-Darcy flow [43]. Although the research on consolidation theory based on the CDB is relatively rich, the existing work is still insufficient because of the complexity of consolidation problems.

This paper aims to study the 1D consolidation of soft clays with Hansbo's flow based on the CDB. The finite difference method is utilized to derive the solutions of the 1D nonlinear consolidation problem of soft clays with Hansbo's flow. Comparing with the existing analytical and numerical solutions, the rationality of the present solutions is verified. Furthermore, a parametric study is carried out to study the influence of soil characteristics on the consolidation characteristics of soft clays.

2. Problem Description and Governing Equations

Figure 1 illustrates the schematic diagram of a soft clay foundation. H is the thickness of the clay layer; q_0 is the instantaneous uniformly distributed load applied on the top surface of the clay layer; v depicts the seepage velocity; z and z_0 represent the downward vertical coordinate originated from the top boundary and the depth of soil unit, respectively. In Hansbo's flow, the relationship between seepage velocity and the hydraulic gradient of soil is composed of the exponential segment for the conditions of the small hydraulic gradient, and the linear segment for the cases of the large hydraulic gradient. Therefore, the seepage law in soil can be expressed as [9]:

$$v = \begin{cases} k_{\rm v} i^m / (m i_1^{m-1}), \ i < i_1 \\ k_{\rm v}(i - i_0), \ i \geq i_1 \end{cases}, \tag{1}$$

where $k_{\rm v}$ denotes the permeability coefficient in the linear section, i, i_0 and i_1 represent the hydraulic gradient, initial hydraulic gradient of seepage calculation in linear section and initial hydraulic gradient of linear seepage, respectively, and m is a constant determined by test. The expression of i_0 can be written as:

$$i_0 = \frac{i_1(m-1)}{m}. \tag{2}$$

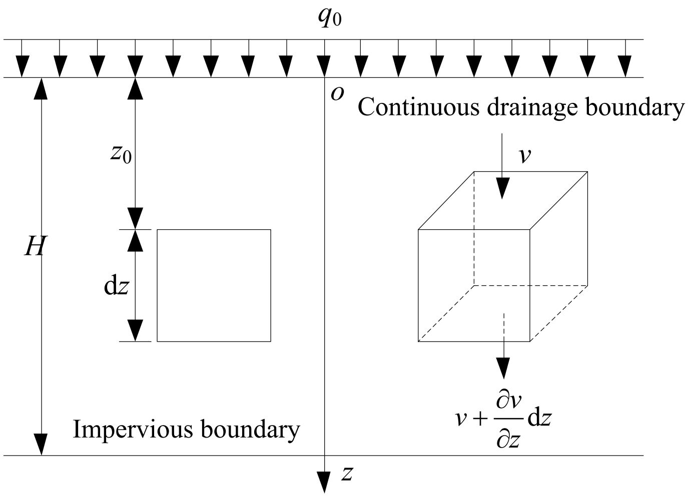

Figure 1. Schematic diagram of soft clay layer.

Introducing the relationship between soil compression and seepage proposed by Mesri and Rokhsar [18], the nonlinear characteristics of soil can be obtained as:

$$e - e_0 = c_{\rm c} \lg\left(\frac{\sigma_0'}{\sigma'}\right), \tag{3}$$

$$e - e_0 = c_{\rm k} \lg\left(\frac{k_{\rm v}}{k_{\rm v0}}\right), \tag{4}$$

where e and e_0 represent the void ratio and initial void ratio, respectively. σ' and σ_0' denote the effective stress and initial effective stress, respectively. $k_{\rm v}$ and $k_{\rm v0}$ are the permeability coefficient and initial permeability coefficient, respectively. $c_{\rm c}$ and $c_{\rm k}$ represent the compression index and permeability index, respectively.

Combining Equation (3) with Equation (4) yields:

$$k_{\rm v} = k_{\rm v0} \left(\frac{\sigma_0'}{\sigma'}\right)^{\frac{c_{\rm c}}{c_{\rm k}}}, \tag{5}$$

$$m_{\rm v} = -\frac{1}{1+e_0} \frac{\partial e}{\partial \sigma'} = \frac{m_{\rm v0} \sigma_0'}{\sigma'}, \tag{6}$$

where m_{v} and m_{v0} denote the compressibility and initial compressibility coefficients, respectively. The expression of m_{v0} can be written as:

$$m_{\mathrm{v0}} = -\frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'}\bigg|_{\sigma'=\sigma_0'} = \frac{c_{\mathrm{c}}}{(1+e_0)\sigma_0' \ln 10}. \tag{7}$$

Then, the consolidation coefficient of soil can be obtained as:

$$c_{\mathrm{v}} = \frac{k_{\mathrm{v}}}{\gamma_{\mathrm{w}} m_{\mathrm{v}}} = c_{\mathrm{v0}}\left(\frac{\sigma_0'}{\sigma'}\right)^{\frac{c_{\mathrm{c}}}{c_{\mathrm{k}}}-1}, \tag{8}$$

where γ_{w} denotes the unit weight of water. c_{v0} represents the initial consolidation coefficient of soil and can be written as:

$$c_{\mathrm{v0}} = \frac{k_{\mathrm{v0}}(1+e_0)\sigma_0' \ln 10}{\gamma_{\mathrm{w}} c_{\mathrm{c}}}. \tag{9}$$

According to the continuous condition of seepage in soil, the following equation can be obtained:

$$\frac{\partial v}{\partial z} = -\frac{1}{1+e_0}\frac{\partial e}{\partial t}, \tag{10}$$

where t is time.

Assuming that the instantaneous uniformly distributed load and the initial effective stress are uniformly distributed along the depth, and introducing the effective stress principle, the following equation can be gained:

$$\sigma' = q_0 + \sigma_0' - u, \tag{11}$$

where u represents the excess pore water pressure.

Combining Equations (1), (3), (5), (6) and (10) with Equation (11), the governing equations for the 1D nonlinear consolidation problem of soil with Hansbo's flow can be obtained as [24]:

$$\begin{cases} c_{\mathrm{v0}}\frac{\sigma_0'+q_0-u}{\sigma_0'}\frac{\partial}{\partial z}\left[\frac{1}{m i_1^{m-1}}\left(\frac{\sigma_0'}{\sigma_0'+q_0-u}\right)^{\frac{c_{\mathrm{c}}}{c_{\mathrm{k}}}}\left(\frac{1}{\gamma_{\mathrm{w}}}\frac{\partial u}{\partial z}\right)^{m-1}\frac{\partial u}{\partial z}\right] = \frac{\partial u}{\partial t}, & \frac{1}{\gamma_{\mathrm{w}}}\frac{\partial u}{\partial z} < i_1 \\ c_{\mathrm{v0}}\frac{\sigma_0'+q_0-u}{\sigma_0'}\frac{\partial}{\partial z}\left[\left(\frac{\sigma_0'}{\sigma_0'+q_0-u}\right)^{\frac{c_{\mathrm{c}}}{c_{\mathrm{k}}}}\left(1-i_0/(\frac{1}{\gamma_{\mathrm{w}}}\frac{\partial u}{\partial z})\right)\frac{\partial u}{\partial z}\right] = \frac{\partial u}{\partial t}, & \frac{1}{\gamma_{\mathrm{w}}}\frac{\partial u}{\partial z} \geq i_1 \end{cases} \tag{12}$$

According to the CDB, the continuous conditions of soil can be obtained as follows. The initial condition:

$$u(z,0) = q_0. \tag{13}$$

The boundary condition:

$$\left.\begin{array}{l} u(0,t) = q_0 \mathrm{e}^{-\alpha\frac{c_{\mathrm{v}}t}{H^2}} \\ \frac{\partial u}{\partial z}\Big|_{z=H} = 0 \end{array}\right\}, \tag{14}$$

where α denotes the interface parameter of the soil top surface, which can be obtained by model test or field test data inversion.

3. Solutions to Governing Equations

In order to simplify the solution process, the following dimensionless parameters are given: $U = \frac{u}{\sigma_0'}$, $Z = \frac{z}{H}$, $T_{\mathrm{v}} = \frac{c_{\mathrm{v0}}t}{H^2}$, $c = c_{\mathrm{c}}/c_{\mathrm{k}}$, $b = (q_0 + \sigma_0')/\sigma_0'$, $I = (i\gamma_{\mathrm{w}}H)/\sigma_0'$

and $I_1 = (i_1\gamma_w H)/\sigma_0'$. Substituting these dimensionless parameters into the governing equations and continuous conditions, the corresponding equations can be given as:

$$\begin{cases} (b-U)\frac{\partial}{\partial Z}\left[\frac{(b-U)^{-c}}{mI_1^{m-1}}\left(\frac{\partial U}{\partial Z}\right)^{m-1}\frac{\partial U}{\partial Z}\right] = \frac{\partial U}{\partial T_v}, I \le I_1 \\ (b-U)\frac{\partial}{\partial Z}\left[(b-U)^{-c}\frac{\partial U}{\partial Z}\right] = \frac{\partial U}{\partial T_v}, I \ge I_1 \end{cases}, \tag{15}$$

$$U(0,T_v) = (b-1)e^{-\alpha T_v}, \tag{16}$$

$$\left.\frac{\partial U}{\partial Z}\right|_{Z=1} = 0, \tag{17}$$

$$U(Z,0) = b-1. \tag{18}$$

The soil layer is discretized in space and time, and $i = 0,1,2,\ldots,n$ and $j = 0,1,2,\ldots,N$ represent the space and time notes, respectively. The 0th space note means the top surface of the soil layer, and the 0th time note mean the initial moment. The time and space steps are denoted as τ and h, respectively. Setting $\lambda = \frac{\tau}{h^2}$ and using the implicit scheme of a quasilinear diffusion equation, the following difference equation can be obtained:

$$U_i^{j+1} = U_i^j + \lambda(b-U_i^j)\left[\alpha_{i+1/2}^{j+1}\left(U_{i+1}^{j+1} - U_i^{j+1}\right), -\alpha_{i-1/2}^{j+1}\left(U_i^{j+1} - U_{i-1}^{j+1}\right)\right], \tag{19}$$

where

$$\alpha_{i+1/2}^{j+1} = \begin{cases} \frac{1}{mI_1^{m-1}}\left[b - \frac{U_{i+1}^{j+1}+U_i^{j+1}}{2}\right]^{-c}\left(\frac{U_{i+1}^{j+1}-U_i^{j+1}}{h}\right)^{m-1}, I_i^{j+1} \le I_1 \\ \left[b - \frac{U_{i+1}^{j+1}+U_i^{j+1}}{2}\right]^{-c}, I_i^{j+1} > I_1 \end{cases}, \tag{20}$$

$$\alpha_{i-1/2}^{j+1} = \begin{cases} \left[b - \frac{U_i^{j+1}+U_{i-1}^{j+1}}{2}\right]^{-c}\left(\frac{U_i^{j+1}-U_{i-1}^{j+1}}{h}\right)^{m-1}, I_i^{j+1} \le I_1 \\ \left[b - \frac{U_i^{j+1}+U_{i-1}^{j+1}}{2}\right]^{-c}, I_i^{j+1} > I_1 \end{cases}, \tag{21}$$

$$I_i^{j+1} = \frac{\left|U_{i+1}^{j+1} - U_{i-1}^{j+1}\right|}{2h}. \tag{22}$$

Substituting Equation (19) into Equation (15) yields:

$$-\lambda(b-U_i^j)\alpha_{i+1/2}^{j+1}U_{i+1}^{j+1} + [1+\lambda(b-U_i^j)\alpha_{i+1/2}^{j+1} + \lambda(b-U_i^j)\alpha_{i-1/2}^{j+1}]U_i^{j+1} - \lambda(b-U_i^j)\alpha_{i-1/2}^{j+1}U_{i-1}^{j+1} = U_i^j \tag{23}$$

The continuous conditions are rewritten as:

$$U_0^j = (b-1)e^{-\alpha\Delta T_v j}, \tag{24}$$

$$U_{n+1}^j = U_{n-1}^j, \tag{25}$$

$$U_i^0 = b-1. \tag{26}$$

Then, Equation (23) can be obtained by using matrix as:

$$\boldsymbol{A}^{j-1}\boldsymbol{V}^j = \boldsymbol{B}^{j-1}, \tag{27}$$

where

$$\boldsymbol{A} = \begin{bmatrix} A_{11} & A_{12} & & & & & \\ A_{21} & A_{22} & A_{23} & & & & \\ & A_{32} & A_{33} & A_{34} & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ & & & & A_{(n-1)(n-2)} & A_{(n-1)(n-1)} & A_{(n-1)n} \\ & & & & & A_{n(n-1)} & A_{nn} \end{bmatrix}, \tag{28}$$

$$\boldsymbol{V} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \vdots \\ U_{n-1} \\ U_n \end{bmatrix}, \tag{29}$$

$$\boldsymbol{B} = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_{n-1} \\ B_n \end{bmatrix}, \tag{30}$$

$$A_{ii} = 1 + \lambda(b - U_i^j)\alpha_{i+1/2}^{j+1} + \lambda(b - U_i^j)\alpha_{i-1/2}^{j+1}, \tag{31}$$

$$A_{i(i-1)} = -\lambda(b - U_i^j)\alpha_{i-1/2}^{j+1}, \tag{32}$$

$$A_{i(i+1)} = -\lambda(b - U_i^j)\alpha_{i+1/2}^{j+1}, \tag{33}$$

$$A_{nn} = 1 + \lambda(b - U_n^j)\alpha_{n+1/2}^{j+1} + \lambda(b - U_n^j)\alpha_{n-1/2}^{j+1}, \tag{34}$$

$$A_{n(n-1)} = -\lambda(b - U_n^j)\alpha_{n-1/2}^{j+1} - \lambda(b - U_n^j)\alpha_{n+1/2}^{j+1}, \tag{35}$$

$$B_i = U_i, \tag{36}$$

$$B_1 = U_1^{j-1} + \lambda(b - U_1^j)\alpha_{1-1/2}^{j+1}U_0^j, \tag{37}$$

$$\alpha_{n+1/2}^{j-1} = \left(b - \frac{U_{n-1}^{j-1} + U_n^{j-1}}{2}\right)^{-c}, \tag{38}$$

$$\alpha_{n-1/2}^{j-1} = \left(b - \frac{U_n^{j-1} + U_{n-1}^{j-1}}{2}\right)^{-c}. \tag{39}$$

It can be seen that the unknowns of Equation (23) are linear, and the tridiagonal matrix can be solved by the catch-up method with Matlab. In order to speed up the solving process of the difference equation, h is set as 0.01. When $T_\mathrm{v} < 0.2$, τ is set as 0.000001; when $T_\mathrm{v} > 0.2$, τ is set as 0.0001. Therefore, when $T_\mathrm{v} < 0.2$, λ is set as 0.01; when $T_\mathrm{v} > 0.2$, λ is set as 1. With these values, the solution of EPWP can be obtained.

Furthermore, the average consolidation degree defined by settlement (ACDS) can be derived as:

$$U_\mathrm{s} = \frac{\int_0^H (e_0 - e)\mathrm{d}z}{\int_0^H (e_0 - e_\mathrm{f})\mathrm{d}z} = \frac{\sum\limits_{i=1}^{n} \lg\left(b - \frac{\overline{u}_{i-1} + \overline{u}_i}{2}\right)}{n\lg b}. \tag{40}$$

The average consolidation degree defined by excess pore water pressure (ACDEPWP) can be derived as:

$$U_\mathrm{p} = \frac{\int_0^1 (b - 1 - U)\mathrm{d}Z}{\int_0^1 (b-1)\mathrm{d}Z} = 1 - \frac{\sum\limits_{i=1}^{n} (U_{i-1}^j + U_i^j)}{2n(b-1)}. \tag{41}$$

It can be seen from Equations (40) and (41) that after considering the nonlinear characteristics of soil, the expressions of the ACDS and ACDEPWP are no longer the same.

4. Verification of the Present Solutions

First, the derived solutions are compared with the existing solutions and models. The comparison of analytical and numerical solutions is depicted in Figure 2, in which $m = 1$ and $c = 1$, thus Davis's solution [17] and Zong's solution [44] are degenerated into 1D nonlinear analytical solutions under the Terzaghi drainage boundary (TDB) and the CDB, respectively. As shown in Figure 2, when $\alpha = 8$, the derived solution of ACD obtained by the finite difference method is consistent with Zong's analytical solution [44] and when $\alpha = 10^6$, the present solution of ACD derived by the finite difference method is consistent with Davis's analytical solution [17]; thus, the correctness of the present solution obtained by the finite difference method is preliminarily verified. In addition, in Davis's solution [17], the value of b has no effect on the ACDS, while in the present solution, U_s increases with the increase of the value of b. When the interface parameter α is large enough (i.e., $\alpha = 10^6$ in this case), the present solution with the CDB is consistent with Davis's solution [17], which indicates that the CDB can be degenerated into the TDB. In other words, the consolidation solution based on the TDB is a special case of the consolidation solution based on the CDB.

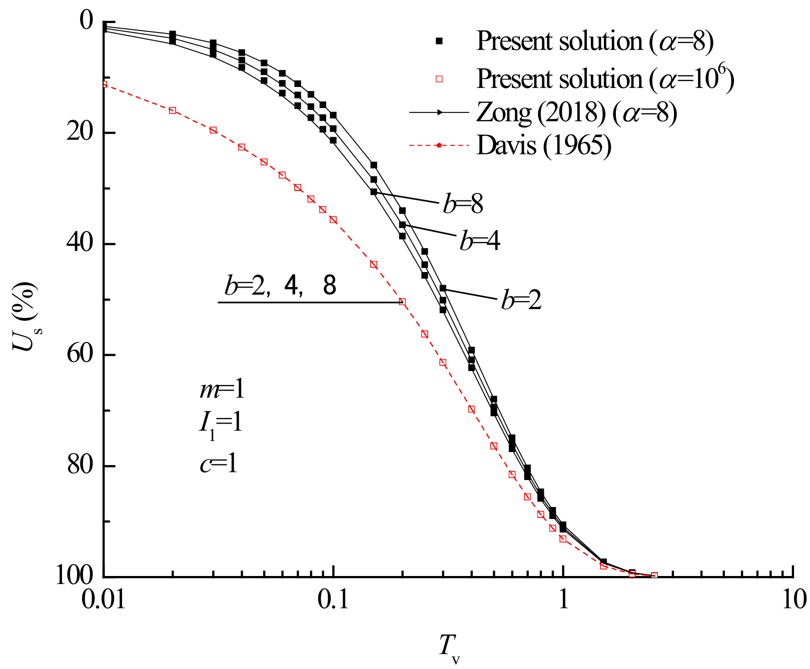

Figure 2. Comparison of analytical solutions and numerical solutions [17,44].

The comparison of ACDs obtained from different drainage boundaries is illustrated in Figure 3. When $\alpha = 10^6$, the consolidation solutions based on the CDB are consistent with Liu's solution [23] based on the TDB, which again verifies the rationality of the derived solutions. Furthermore, Figure 3 also illustrates the influence of I_1 on U_s and U_p, and the value of I_1 depends on the ratio of soil thickness to external load. It is found that both U_s and U_p decrease as the value of I_1 increases, which indicates that the smaller the ratio of soil thickness to external load, the faster the soil consolidation process. According to Equation (11), with the increase of I_1, the coefficient on the left side of Equation (11) will decrease, so the soil consolidation rate will decrease. In addition, it can also be found that when the values of model parameters are the same, U_s is greater that U_p, indicating that the soil settlement rate is greater than the dissipation rate of EPWP during the consolidation process.

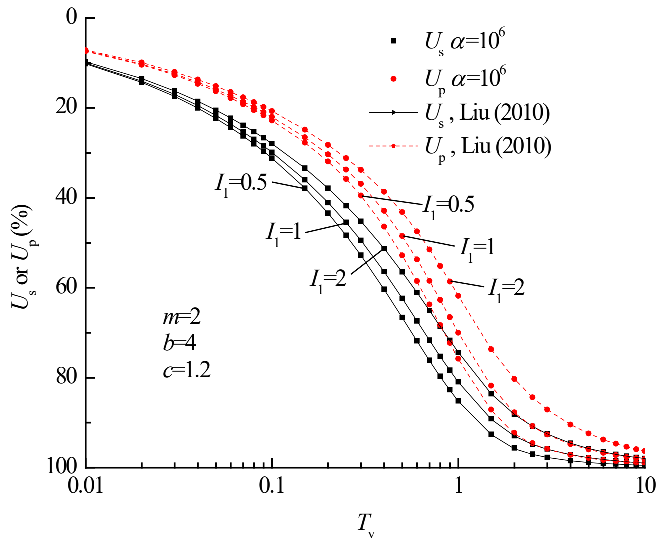

Figure 3. Comparison of ACDs obtained from different drainage boundaries [23].

5. Analysis of Consolidation Degree

In this section, a parametric is conducted to study the 1D nonlinear consolidation characteristics of soft clays with Hansbo's flow. Figure 4 shows the influence of interface parameter α on the ACDS. It is found that with the increase of the interface parameter, the soil consolidation rate gradually increases, and the gap between the present solution and Liu's solution gradually decreases. When the value of α is relatively small, the increase of α will greatly improve the ACDS. However, when the value of α is relatively large, the increase of α has little effect on the ACDS. In the early stage of consolidation, the soil consolidation rate of the present solution is slower than that of Liu's solution [23]. In the middle and late stages of consolidation, the soil consolidation rate of the present solution is significantly faster than that of Liu's solution [23]. The time required for complete consolidation of soil calculated by the present solution and Liu's solution [23] is roughly the same, which shows that when the improvement of soil is designed according to the CDB in engineering practice, although the soil consolidation rate is slow in the early stage of consolidation, the soil is more fully consolidated in the middle and late stages of consolidation, and the soil consolidation rate can be controlled by choosing the interface parameter.

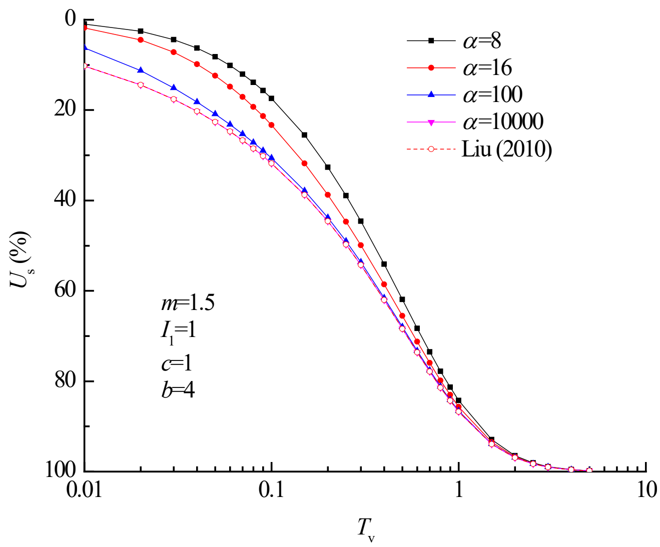

Figure 4. Influence of the interface parameter on ACDS [23].

Figure 5 depicts the influence of the m value on the ACDS and ACDEPWP. According to Equation (1), when $i \leq i_1$, $v = \frac{Ki}{m}\left(\frac{i}{i_1}\right)^{m-1}$, the seepage velocity decreases with the increase of the m value; when $i > i_1$, $v = K\left(i - i_1\frac{m-1}{m}\right)$, the seepage velocity also decreases with the increase of the m value. During the consolidation process, the larger the m value, the slower the seepage velocity in the soil. As shown in Figure 5, the ACDs decrease with the increase of the m value, which shows that the calculated soil consolidation rate is faster when the non-Darcy flow is not considered. In addition, it can be seen that U_s is always greater than U_p, indicating that the soil settlement rate is greater than the dissipation rate of EPWP during the consolidation process.

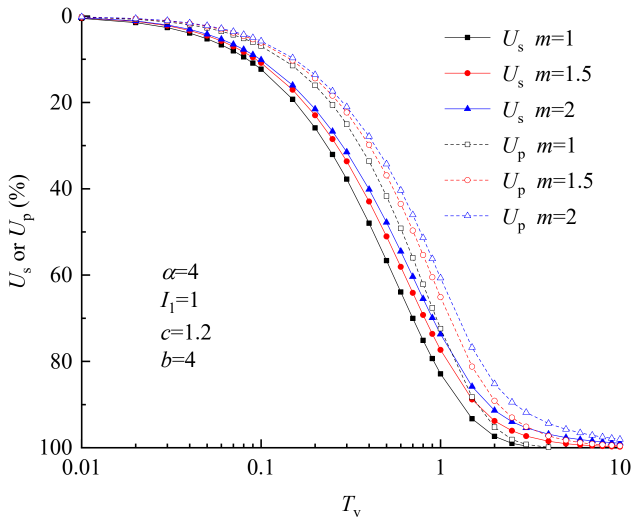

Figure 5. Influence of the m value on ACDs.

Figure 6 illustrates the influence of the c value on the ACDS and ACDEPWP. It is found that both U_s and U_p decrease with the increase of the c value. According to Equation (11), with the increase of the c value, the value of $(b - U)^{-c}$ gradually decreases, thus the soil consolidation rate will decrease. In addition, for any c value, the ACDS is greater than that defined by settlement, indicating that the soil settlement rate is greater than the dissipation rate of EPWP during the consolidation process.

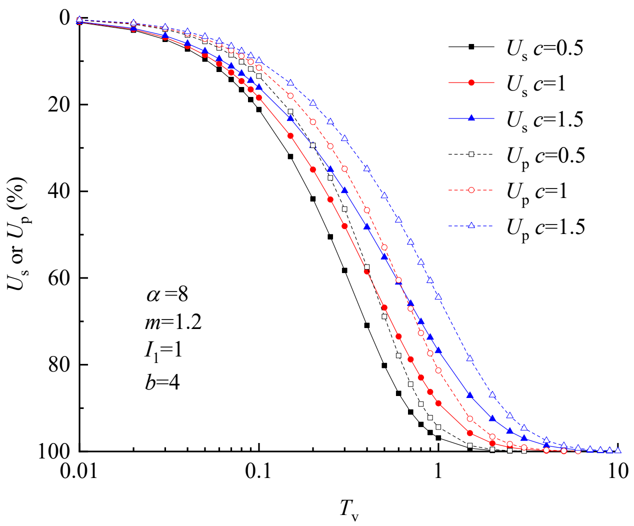

Figure 6. Influence of the c value on ACDs.

Setting $I_1 = 1$ and $c = 0.5$, Figures 7 and 8 show the influence of the b value on the ACDS when $m = 1$ and $m = 1.5$, in which $m = 1$ means Darcy seepage and $m = 1.5$ means non-Darcy flow, respectively. The value of b is the ratio of the final effective stress to the initial effective stress, and the greater the external load, the greater the value of b. It is found that in both the ACDs obtained by the present solution and Liu's solution [23], U_s increases as the b value increases, which indicates that when the c value is relatively small (i.e., $c = 0.5$ in this case), the soil settlement rate increases with the increase of external load. In addition, the value of U_s calculated by the present solution is always smaller than that of Liu's solution [23]. At the early stage of consolidation, the two solutions have a large gap. With the increase of time, the two solutions gradually approach, and the time required for complete consolidation of soil is roughly the same.

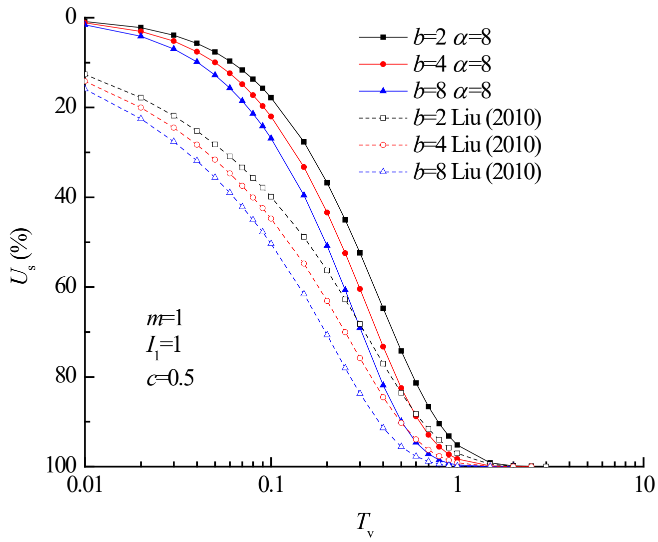

Figure 7. Influence of the b value on ACDS ($m = 1,\ c = 0.5$) [23].

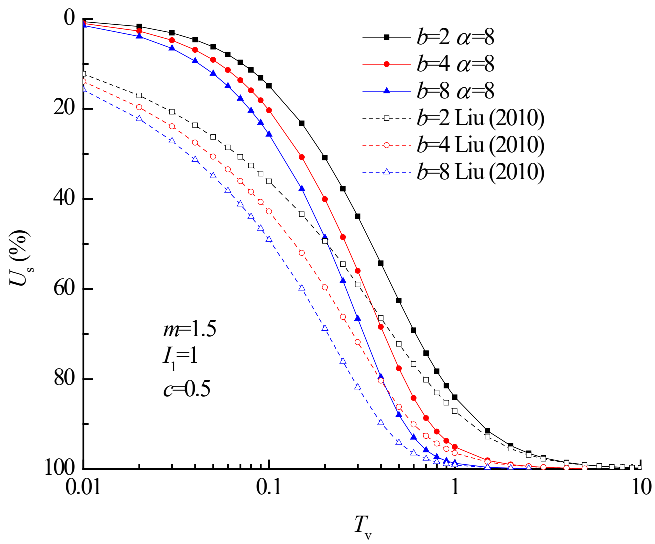

Figure 8. Influence of the b value on ACDS ($m = 1.5,\ c = 0.5$) [23].

Letting $I_1 = 1$ and $c = 1$,Figures 9 and 10 illustrate the influence of the b value on the ACDS when $m = 1$ and $m = 1.5$, respectively. As shown in Figure 9, the ACD calculated by Liu's solution [23] does not change with the variation of the b value, namely, the external load has no effect on soil consolidation. However, the ACD calculated by the present solution increases with the increase of the b value. The different consolidation curves calculated by the present solution almost coincide in the late stage of consolidation, indicating that the larger the external load, the faster the soil settlement in the early stage of consolidation, but the impact of the external load on the late stage of consolidation is very small. These phenomena show that the boundary condition has a great influence on the consolidation characteristics of soil. As shown in Figure 10, in both the ACDs obtained by the present solution and Liu's solution [23], U_s increases as the b value increases, but the b value of Liu's solution [23] has little effect on the early stage of soil consolidation. The m value in Figures 9 and 10 are different, and the influence of the b value on the ACD by Liu's solution [23] is also different, which reflects that the different seepage forms of soil will lead to different consolidation laws. Meanwhile, the c value in Figure 9 is different from that in Figure 7, and the influence of the b value on the ACD by Liu's solution [23] is different, which reflects that the different nonlinear characteristics of soil would lead to different consolidation laws.

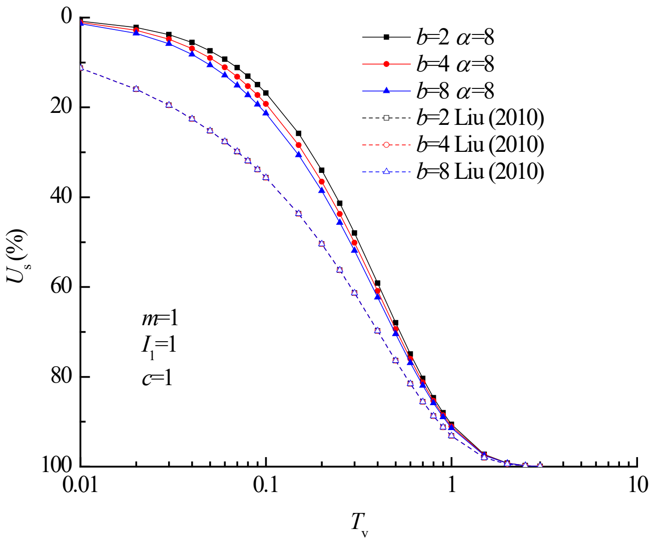

Figure 9. Influence of the b value on ACDS ($m = 1,\ c = 1$) [23].

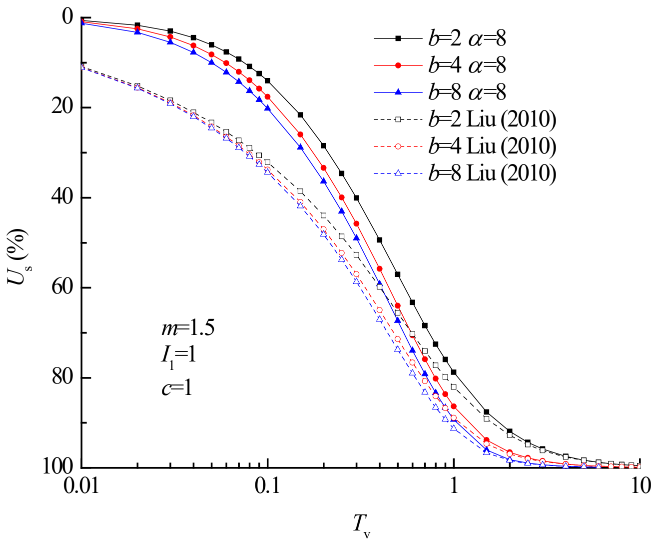

Figure 10. Influence of the b value on ACDS ($m = 1.5,\ c = 1$) [23].

Letting $I_1 = 1$ and $c = 1.5$, Figures 11 and 12 depict the influence of the b value on the ACDS when $m = 1$ and $m = 1.5$, respectively. As shown in Figure 11, the ACD calculated by Liu's solution [23] decreases with the increase of the b value, which is quite different from the change of U_s with the b value when $c = 0.5$. The ACD obtained by the present solution increases with the increase of the b value in the early stage of consolidation, and decreases with the increase of the b value in the middle and late stages of consolidation. As shown in Figure 12, considering both the non-Darcy seepage and nonlinear characteristics of soil, the ACD calculated by Liu's solution [23] decreases with the increase of the b value in the early stage of consolidation, but the variation of U_s with the b value is not obvious in the late stage of consolidation. However, the ACD calculated by the present solution increases with the increase of the b value in the early stage of consolidation, decreases with the increase of the b value in the middle stage of consolidation and increases with the increase of the b value in the late stage of consolidation. Figure 12 shows that the soil consolidation law considering both non-Darcy seepage and nonlinear characteristics of soil is very complex.

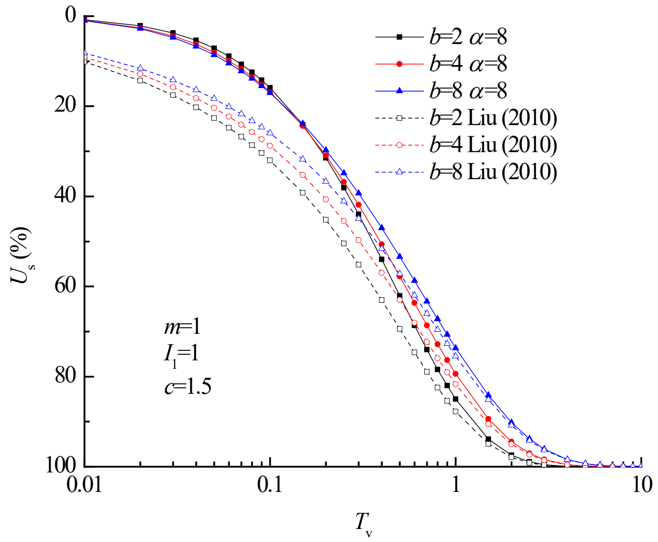

Figure 11. Influence of the b value on ACDS ($m = 1,\ c = 1.5$) [23].

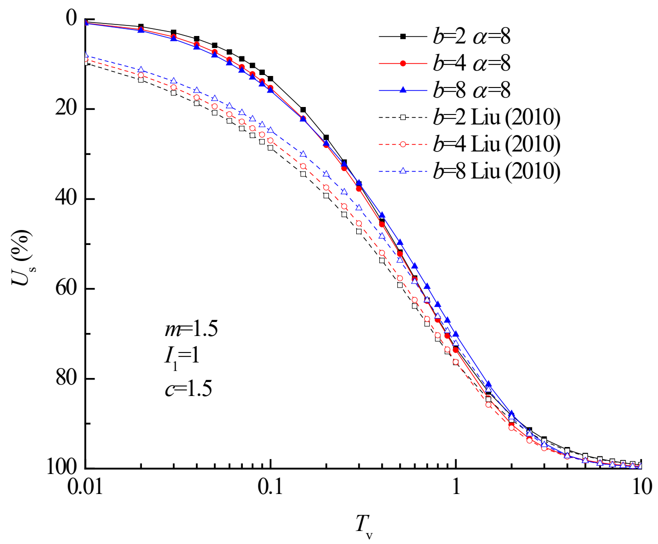

Figure 12. Influence of the b value on ACDS ($m = 1.5,\ c = 1.5$) [23].

6. Conclusions

In this paper, the solutions of the 1D nonlinear consolidation problem of soft clays with Hansbo's flow are derived based on the CDB. The present solutions are verified by comparing them with existing solutions and models. Then, a parametric study is carried out to investigate the consolidation behavior of soil, and the following conclusions can be obtained:

(1) When the interface parameter is large enough, the solutions based on the continuous drainage boundary can be degenerated into the solutions based on the Terzaghi drainage boundary.

(2) With the increase of the interface parameter, the soil consolidation rate increases, but the gap between the present solution and Liu's solution [23] gradually decreases. When the interface parameter is small, increasing the interface parameter will greatly improve the average consolidation degree defined by settlement, while when the interface parameter is large, increasing the interface parameter will not affect the average consolidation degree defined by settlement.
(3) When non-Darcy seepage is not considered or the ratio of soil thickness to external load is smaller, the soil consolidation rate is slower.
(4) The soil consolidation behavior considering both non-Darcy seepage and nonlinear characteristics of soil is very complex.

Author Contributions: Conceptualization, J.W., R.X. and M.Z.; methodology, R.L. and W.W.; software, J.W.; validation, J.W., R.X. and M.Z.; formal analysis, J.W., M.Z. and W.W.; investigation, J.W. and R.L.; resources, W.W.; data curation, J.W. and W.W.; writing—original draft preparation, J.W. and R.X.; writing—review and editing, R.L. and W.W.; visualization, J.W. and R.X.; supervision, W.W.; project administration, W.W.; funding acquisition, R.L. and W.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant Nos. 52178371, 52168046, 52178321, 52108347), the Exploring Youth Project of Zhejiang Natural Science Foundation (LQ22E080010), the Outstanding Youth Project of Natural Science Foundation of Zhejiang Province (Grant No. LR21E080005), the China Postdoctoral Science Foundation Funded Project (Grant No. 2022M712964), the Research Project of Engineering Research Centre of Rock-Soil Drilling & Excavation and Protection, Ministry of Education (Grant No. 202203), the Zhejiang Province's 2022 Key R&D Plan Project-'Lingyan' Project (2022C03151), the Science and Technology Project of Zhejiang Provincial Communication Department (202305-2), the Construction Research Founds of Department of Housing and Urban-Rural Development of Zhejiang Province (Grant No. 2021K256).

Institutional Review Board Statement: Not appliable.

Informed Consent Statement: Not appliable.

Data Availability Statement: All the data used in this research are easily accessible by downloading the various documents appropriately cited in the paper.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Hu, A.F.; Xia, C.Q.; Cui, J.; Li, C.X.; Xie, K.H. Nonlinear consolidation analysis of natural structured clays under time-dependent loading. *Int. J. Geomech.* **2018**, *18*, 4017140. [CrossRef]
2. Zhang, Y.P.; Wang, Z.Q.; El Naggar, M.H.; Wu, W.B.; Wang, L.X.; Jiang, G.S. Three-dimensional wave propagation in a solid pile during torsional low strain integrity test. *Int. J. Numer. Anal. Methods Geomech.* **2022**, *46*, 2398–2411. [CrossRef]
3. Xu, C.; Pan, S.L. Finite element study on calculation of nonlinear soil consolidation using compression and recompression indexes. *Appl. Sci.* **2022**, *10*, 4737. [CrossRef]
4. Wen, M.J.; Tian, Y.; Li, L.C.; Wang, K.H.; Wu, W.B. An imperfect thermal contact problem for consolidation of bilayered saturated soil subjected to a ramp-type heating. *Int. J. Heat Mass Transf.* **2022**, *190*, 122755. [CrossRef]
5. Li, C.X.; Xue, Q.; Jin, D.D. Analytical solutions for one-dimensional large-strain nonlinear consolidation of soft soils by considering a time-dependent drainage boundary. *Int. J. Geomech.* **2023**, *23*, 04023006. [CrossRef]
6. Chen, L.B.; Wu, W.B.; Liu, H.; Li, J.X.; Newson, T.; El Naggar, M.H. Analytical solution for kinematic response of offshore piles under vertically propagating S-waves. *Ocean Eng.* **2022**, *262*, 112018. [CrossRef]
7. Li, J.S.; Jiang, W.H.; Ge, S.Q.; Chen, X.H.; Chen, X.; Liu, L. General analytical solutions for one-dimensional nonlinear consolidation of saturated clay under non-isothermal distribution condition. *Int. J. Numer. Anal. Methods Geomech.* **2022**, *46*, 1811–1830. [CrossRef]
8. Hansbo, S. Consolidation of clay with special reference to influence of vertical drains. *Swed. Geotech. Inst. Proceeding Stockh. Swed. Geotech. Inst.* **1960**, *18*, 45–50.
9. Schmidt, J.D.; Westmann, R.A. Consolidation of porous media with non-Darcy flow. *J. Eng. Mech.* **1973**, *99*, 1201–1216. [CrossRef]
10. Hansbo, S. Deviation from Darcy's law observed in one-dimensional consolidation. *Geotechnique* **2003**, *53*, 601–605. [CrossRef]
11. Liu, Z.Y.; Sun, L.Y.; Yue, J.C.; Ma, C.W. One-dimensional consolidation theory of saturated clay based on non-Darcy flow. *Chin. J. Rock Mech. Eng.* **2009**, *28*, 973–979. (In Chinese)

12. Liu, Z.Y.; Yan, F.Y.; Wang, X.J. One-dimensional rheological consolidation analysis of saturated clay considering non-Darcy flow. *Chin. J. Rock Mech. Eng.* **2013**, *32*, 1937–1944. (In Chinese)
13. Jiu, Y.Z.; Liu, Z.Y.; Yue, J.C.; Sun, L.Y. One-dimensional consolidation with a consideration of non-Darcy flow and self-gravity stress. *J. Tongji Univ. Nat. Sci.* **2012**, *40*, 541–548. (In Chinese)
14. Li, C.X.; Xie, K.H.; Wang, K. Analysis of 1D consolidation with non-Darcian flow described by exponent and threshold gradient. *J. Zhejiang Univ. Sci. A* **2010**, *11*, 656–667. [CrossRef]
15. Li, C.X.; Xie, K.H.; Hu, A.F.; Hu, B.X. Analysis of one-consolidation of double-layered soil with non-Darcian flow described by exponent and threshold gradient. *J. Cent. South Univ.* **2012**, *19*, 562–571. [CrossRef]
16. Cui, P.L.; Cao, W.G.; Xu, Z.; Wei, Y.B.; Zhang, J.C. Analysis of one-dimensional consolidation of fractional viscoelastic saturated soils under continuous drainage boundary conditions. *J. Cent. South Univ.* **2022**, *29*, 3745–3756. [CrossRef]
17. Davis, E.H.; Raymond, G.P. A non-linear theory of consolidation. *Geotechnique* **1965**, *15*, 161–173. [CrossRef]
18. Mesri, G.; Rokhsar, A. Theory of consolidation for clays. *J. Geotech. Eng.* **1974**, *100*, 889–903.
19. Liu, X.; Wu, W.B.; El Naggar, M.H.; Wang, K.H.; Mei, G.X.; Liu, H.; Wang, L.X.; Sun, J. A simplified non-axisymmetric pile-soil interaction model for pile integrity testing analysis. *Appl. Math. Model.* **2023**, *119*, 137–155. [CrossRef]
20. Li, C.X.; Huang, J.S.; Wu, L.Z.; Lu, J.F.; Xia, C.Q. Approximate analytical solutions for one-dimensional consolidation of a clay with variable compressibility and permeability under a ramp loading. *Int. J. Geomech.* **2018**, *18*, 6018032. [CrossRef]
21. Xie, K.H.; Xie, X.Y.; Jiang, W. A study on one-dimensional nonlinear consolidation of double-layered soil. *Comput. Geotech.* **2002**, *29*, 151–168. [CrossRef]
22. Zhang, Y.P.; El Naggar, M.H.; Wu, W.B.; Wang, Z.Q.; Yang, X.Y.; Jiang, G.S. Dynamic torsional impedance of large-diameter pipe pile for offshore engineering: 3D analytical solution. *Appl. Math. Model.* **2022**, *111*, 664–680. [CrossRef]
23. Liu, Z.Y.; Jiu, Y.Z.; Yue, J.C.; Sun, L.Y. One-dimensional nonlinear consolidation analysis of saturated clay based on non-Darcy flow. *Chin. J. Rock Mech. Eng.* **2010**, *29*, 2348–2355. (In Chinese)
24. Li, C.X.; Xie, K.H. One-dimensional nonlinear consolidation of soft clay with the non-Darcian flow. *J. Zhejiang Univ. Sci. A* **2013**, *14*, 435–446. [CrossRef]
25. Li, C.X.; Xie, K.H. Analysis of nonlinear large-strain consolidation of soft clay with exponential seepage flow law. *Rock Soil Mech.* **2013**, *34*, 2991–2996. (In Chinese)
26. Li, C.X.; Xie, K.H. Large-strain consolidation of soft clay with non-Darcian flow by considering time-dependent load. *Chin. J. Geotech. Eng.* **2015**, *37*, 1002–1009. (In Chinese)
27. Gray, H. Simultaneous consolidation of contiguous layers of unlike compressible soils. *Trans. Am. Soc. Civ. Eng.* **1945**, *110*, 1327–1356.
28. Schiffman, R.L.; Stein, J.R. One-dimensional consolidation of layered systems. *J. Soil Mech. Found.* **1970**, *96*, 1499–1504. [CrossRef]
29. Mesri, G. One-dimensional consolidation of a clay layer with impeded drainage boundaries. *Water Resour. Res.* **1973**, *9*, 1090–1103. [CrossRef]
30. Wang, L.; Sun, D.A.; Qin, A.F.; Yu, Y.F. Semi-analytical solution to one-dimensional consolidation for unsaturated soils with semi-permeable drainage boundary under time-dependent loading. *Int. J. Numer. Anal. Met.* **2017**, *41*, 1636–1655. [CrossRef]
31. Wang, L.; Sun, D.A.; Li, L.Z.; Li, P.C.; Xu, Y.F. Semi-analytical solutions to one-dimensional consolidation for unsaturated soils with symmetric semi-permeable drainage boundary. *Comput. Geotech.* **2017**, *89*, 71–80. [CrossRef]
32. Zhou, W.H.; Zhao, L.S. One-dimensional consolidation of unsaturated soil subjected to time-dependent loading with various initial and boundary conditions. *Int. J. Geomech.* **2014**, *14*, 291–301. [CrossRef]
33. Xie, K.H.; Xie, X.Y.; Gao, X. Theory of one dimensional consolidation of two-layered soil with partially drained boundaries. *Comput. Geotech.* **1999**, *24*, 265–278. [CrossRef]
34. Zhang, Y.P.; Di, T.Y.; El Naggar, M.H.; Wu, W.B.; Liu, H.; Jiang, G.S. Modified Rayleigh-Love rod model for 3D dynamic analysis of large-diameter thin-walled pipe pile embedded in multilayered soils. *Comput. Geotech.* **2022**, *149*, 104853. [CrossRef]
35. Wen, M.J.; Tian, Y.; Li, L.C.; Qiu, X.C.; Wang, K.H.; Wu, W.B.; Mei, G.X.; Xu, M.J. A general interfacial thermal contact model for consolidation of bilayered saturated soils considering thermos-osmosis effect. *Int. J. Numer. Anal. Met.* **2022**, *46*, 2375–2397. [CrossRef]
36. Wu, W.B.; Wang, Z.Q.; Zhang, Y.P.; El Naggar, M.H.; Wu, T.; Wen, M.J. Semi-analytical solution for negative skin friction development on deep foundations in coastal reclamation areas. *Int. J. Mech. Sci.* **2023**, *241*, 107981. [CrossRef]
37. Yang, X.Y.; Zong, M.F.; Tian, Y.; Jiang, G.S.; El Naggar, M.H.; Wu, W.B.; Xu, M.J. One-dimensional consolidation of layered soils under ramp load based on continuous drainage boundary. *Int. J. Numer. Anal. Met.* **2021**, *45*, 738–752. [CrossRef]
38. Mei, G.X.; Lok, T.M.H.; Xia, J.; Wu, S.S. One-dimensional consolidation with asymmetrical exponential drainage boundary. *Geomech. Eng.* **2014**, *6*, 47–63. [CrossRef]
39. Mei, G.X.; Chen, Q.M. Solution of Terzaghi one-dimensional consolidation equation with general boundary conditions. *J. Cent. South Univ.* **2013**, *20*, 2239–2244. [CrossRef]
40. Liu, J.C.; Lei, G.H. One-dimensional consolidation of layered soils with exponentially time-growing drainage boundaries. *Comput. Geotech.* **2013**, *54*, 202–209. [CrossRef]
41. Wang, L.; Sun, D.A.; Qin, A.F. Semi-analytical solution to one-dimensional consolidation for unsaturated soils with exponentially time-growing drainage boundary conditions. *Int. J. Geomech.* **2018**, *18*, 4017144. [CrossRef]

42. Zong, M.F.; Tian, Y.; Liang, R.Z.; Wu, W.B.; Xu, M.J.; Mei, G.X. One-dimensional nonlinear consolidation analysis of soil with continuous drainage boundary. *J. Cent. South Univ.* **2022**, *29*, 270–281. [CrossRef]
43. Zong, M.F.; Wu, W.B.; El Naggar, M.H.; Mei, G.X.; Zhang, Y. Analysis of one-dimensional consolidation for double-layered soil with non-Darcian flow based on continuous drainage boundary. *Int. J. Geomech.* **2023**, *23*, 04022306. [CrossRef]
44. Zong, M.F.; Wu, W.B.; Mei, G.X.; Liang, R.Z.; Tian, Y. An analytical solution for one-dimensional nonlinear consolidation of soils with continuous drainage boundary. *Chin. J. Rock Mech. Eng.* **2018**, *37*, 2829–2838. (In Chinese)