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A New Automated Algorithm for Optimization of Measurements for Achieving the Required Accuracy of a Geodetic Network

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Abstract: The optimization of measurements in a geodetic network (second-order design) has been investigated in the past; however, the practical usability of the outcomes of most of such studies is doubtful. Hence, we have proposed a new automated optimization algorithm, taking into account the practical aspects of total station measurements. The algorithm consists of four parallel partial algorithms, of which one is subsequently automatically selected—the one meeting the geodetic network accuracy requirements with the lowest number of necessary measurements. We tested the algorithm (and individual partial algorithms) on four geodetic networks designed to resemble real-world networks with 50–500 modifications to each of those networks in individual tests. The results indicate that (i) the results achieved by the combined algorithm are close to the optimal results and (ii) none of the four partial algorithms universally performs the best, implying that the combination of the four partial algorithms is necessary for achieving the best possible results of geodetic network optimization.

Keywords: optimization; geodetic measurements; measurement accuracy planning; second-order design; geodetic network

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1. Introduction

The optimization of geodetic networks is an important component of geodetic practice. Although methods such as laser scanning or photogrammetry gradually take over in the collection of spatial data over larger extents, measurement using total station remains crucial where local measurements for construction purposes are concerned. This method also remains indispensable in the measurement of deformations of, e.g., human-built objects [1,2] or natural scenes [3].

The sufficient and known accuracy of survey results [4,5] is a fundamental prerequisite for the successful construction process, and the aim of geodetic network optimization is to minimize the costs of constructing and maintaining the network while maintaining the required accuracy and reliability. Many methods have been proposed for this purpose; however, not all of them are suitable for the relatively complex mathematical problem of geodetic networks.

The geodetic optimization method should be applied wherever the required accuracy of the measurement result is not easily met by the simple use of geodetic instruments, and where the quality of the subsequent construction process or the safety of the building operation depends on the accuracy achieved. These are, e.g., surveying activities in bridge construction [6], the determination of coordinates of staking out geodetic network [7], and the monitoring or measuring of structures above ground [8,9] or below ground [10–12].

The basics of optimization in the field of geodetic networks have been laid down in the 1950s to 1970s. The general objective function for geodetic network optimization was introduced by Schaffrin [13]. Recently, Bagerbandi followed up on his work [14], comparing the suitability of single-criterion or multi-criterion optimization approaches.

Scalar objective functions describing the accuracy and the properties of these functions were investigated in detail by Grafarend [15]. Together with Schaffrin [16], they defined a matrix objective function using the Taylor–Karman structure of the covariance matrix. Baarda then laid down the principles of objective functions for reliability [17].

Grafarend classified the optimization of geodetic networks into four types, depending on the optimized parameters [15]. Namely, these are termed zero-order design (ZOD) to third-order design (TOD). The ZOD, the basics of which are described by Teunnissen [18], focuses on the coordinate system definition; it has, however, not gained much popularity. The first-order design (FOD) deals with the optimization of the network configuration, i.e., the locations of the network points. It was investigated in detail, in particular by Koch [19], who optimized the position of the points through very small coordinate changes using the Taylor–Karman structure as the objective function. Berne and Baselga [20] presented a different approach to the FOD, using a heuristic method of simulated annealing for the optimization of the covariance matrix determinant [21]. They demonstrated this approach using two examples; however, in the more complicated of these examples, the optimization led to the convergence of the optimized points towards the center of gravity, leading to their final position on the borders predefined to limit their further convergence to the center of gravity. This indicates that even though the optimization of the network shape to maximize the accuracy might have seemed an interesting concept, it is not practically applicable as the mathematical solution necessarily always ends up on predefined borders designating the smallest allowed network (as seen, e.g., in [20] or [22]). The cause is simple: the errors generally grow with distance, and the simplest mathematical solution for improving accuracy, therefore, reduces the distances. Without setting the borders and continuing in optimization according to the FOD, the entire network would eventually converge into a single point.

The second-order design (SOD) optimizes the weight matrix (determining measurements to be taken and the number of repetitions of these measurements) and the third-order design attempts to optimize the network through its densification (i.e., the addition of new point(s) or observation(s)) [23]. From the perspective of practical use, SOD (determining what measurements need to be taken and how many times) is the most important of these designs. However, the available literature [20,22,24–28] mostly deals with determining weights of individual metrics (typically slope distance, zenith angle, and horizontal direction) separately. It should be strongly emphasized that in terms of the operation of geodetic instruments, when performing terrain measurements, the total station provides all these values at the same time and, in effect, recording only one or two of these values would not make much sense. Also, many of these studies work with measurement weights in the optimization procedure, which is also suboptimal as the weights must be eventually converted to the actual number of measurements. This usually does not yield integers and the numbers must be rounded up, which negatively affects the optimality of the solution (the number of measurements actually taken may then be higher than what could be sufficient). The same issues can be found in many other papers as well [29–33].

In this paper, we aim to propose a method of second-order design optimization that would be applicable in everyday geodetic practice. The requirement of practicability disqualifies all previously discussed methods as it implies the following: (i) the numbers of measurements are integers; (ii) total station always simultaneously measures three values (slope distance, zenith angle, and horizontal direction), which must be considered as a group (it would make no sense to discard one or two measurements from the same position if they are taken anyway); (iii) all points measured from the same position of the total station are measured with the same number of repetitions (which is the way the software in total stations work); and (iv) an optimized network is defined as a network, whose points are determined with an accuracy parameter equal or better than a set (required) value and which is obtained using the minimum effort (i.e., number of measurements). For ease of use, it would also be optimal if the accuracy characteristic could be chosen from a pool of metrics (such as standard deviation of the coordinate, of the position, or the longest semiaxis of the error ellipsoid (hereinafter abbreviated as LSEE)).

2. Materials and Methods

The requirements on practical use detailed above have been previously applied by our group on the SOD of a geodetic network determined by geometric leveling [34]. However, that method is not usable for measurements using the total station. For this reason, we can use the method proposed for geometrical leveling as a basis of the new algorithm and amend it to make it compatible with total station measurement specifics.

2.1. The Model of Geodetic Network Accuracy

The method proposed in this paper is based on the "greedy algorithm" [35]. Considering the principle of this algorithm (i.e., maximizing immediate gain; in our case, the accuracy increment), its use could lead to the identification of a local rather than global optimum. Several strategies to overcome this problem (or minimize its effects) have been proposed and will be tested in this paper on four generated 3D geodetic networks.

The inputs for the proposed method of optimization include (a) the approximate coordinates of the points that will form the geodetic network, (b) the information on what measurements can be made, and (c) the accuracy characteristics of the device (total station) that will be used for measurements. The algorithm itself then consists, in principle, of four steps:

1. The measurement (or, rather, a group of measurements) is added into the network if it causes the greatest improvement in the used accuracy characteristic of the (at the time) least accurate point in the network.
2. This is repeated until the required accuracy is achieved for all points.
3. Subsequently, the backward analysis follows, in which a group of measurements is removed from the network if its deletion has the least negative effect on the respective accuracy characteristic of all possible measurement deletions.
4. This is repeated until the removal of the next group of measurements would worsen the accuracy of any point in the network below the required limit.

This approach uses the geodetic network adjustment by the least squares method. The accuracy of the network is characterized by a covariance matrix $\boldsymbol{M_x}$. In the case of a geodetic network with two or more fixed points, the covariance matrix can be calculated as

$$\boldsymbol{M_x} = c\left(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\right)^{-1}, \tag{1}$$

where c is an arbitrary constant, which has to be identical for all elements in the matrix, $\boldsymbol{A}$ is the Jacobi's matrix, i.e., a matrix of partial derivations of the observation equations by network point coordinates (see, e.g., [36] for more details, dimensions are $m \times u$, where m is number of measurement in the network and u is the number of unknowns), and $\boldsymbol{P}$ is the weight matrix (dimensions $m \times m$). Covariance matrix dimension is $u \times u$.

To be able to use integers as the numbers of measurement repetitions instead of measurement weights, we have modified the definition of the weight matrix that takes them into account as follows:

$$\boldsymbol{P} = \boldsymbol{N} \cdot \boldsymbol{P}_0 = \begin{pmatrix} n_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & n_k \end{pmatrix} \cdot \begin{pmatrix} p_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & p_k \end{pmatrix}, \tag{2}$$

where $\boldsymbol{N}$ is a diagonal matrix containing the numbers of measurement repetitions, and $\boldsymbol{P_0}$ is a weight matrix with weights for a single repetition of each measurement, both with dimension ($m \times m$). Each p_i (i = 1 to k) is then defined as

$$p_i = \frac{c}{\sigma_i^2}, \tag{3}$$

where p_i is the weight of the ith measurement and σ_i is the standard deviation of the ith measurement.

However, if the network is solved as a free network (without a sufficient number of fixed points) additional conditions must be added into the calculation. In this work, conditions for Helmert transformation to all points of the network ($\boldsymbol{B}$, with dimensions $4 \times 3l$, where l is number of points in the network) were added to transform the network so that the sum of squares of displacements (for all points) is minimized as follows:

$$\boldsymbol{B} = \begin{pmatrix} Y_{1,0} & X_{1,0} & 0 & Y_{1,0} \cdots & Y_{l,0} & X_{l,0} \\ 1 & 0 & 0 & 1 \cdots & 1 & 0 \\ 0 & 1 & 0 & 0 \cdots & 0 & 1 \\ 0 & 0 & 1 & 0 \cdots & 0 & 0 \end{pmatrix}, \tag{4}$$

where $Y_{j,0}$ and $X_{j,0}$ are the approximate coordinates of the j-th point of the network with l points in total. The covariance matrix describing the accuracy of the geodetic network is then defined as

$$\boldsymbol{M_x} = submatrix_{1..u,1..u}\left(c \begin{pmatrix} \boldsymbol{A}^T \boldsymbol{N}\,\boldsymbol{P}_0 \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B^T} & 0 \end{pmatrix}^{-1} \right) \tag{5}$$

This covariance matrix serves as a source for all accuracy characteristics of the network and any such characteristic that can be calculated from it can be used as the criterion for subsequent optimization (e.g., the standard deviation of the coordinate, of the position, or the longest semiaxis of the error ellipsoid). As various characteristics can be used, the symbol K will be used for simplicity to describe any accuracy characteristic and K_T the target accuracy.

2.2. The Optimization Method

The optimization algorithm looks for the result with the lowest effort (i.e., with the minimum sum of the diagonal elements in the $\boldsymbol{N}$ matrix, indicating the minimum number of measurements) that yields the accuracy characteristic K equal or better than K_T for all points of the network.

The optimization is performed in several steps. The first step, which may or may not be used (see below) lies in creating the initial configuration of the measurements. Subsequently, the aforementioned maximum accuracy increment method followed by the minimum accuracy decrease method are applied. In this paper, we tested four algorithms combining these steps as shown in the flowchart in Figure 1. All four algorithms are employed in parallel to each geodetic network undergoing optimization and, finally, the result requiring the least effort is automatically selected as the optimal one. Components of the algorithm are described in the following paragraphs, the full algorithm in pseudocode is given in the Appendix B.

2.2.1. Initial Network Configuration

In some cases, starting the optimization from a (near-)zero number of all repetitions reduces the likelihood of finding the optimal solution as it could lead rather to the identification of a local optimum instead of a global one. This is caused by the fact that the algorithm would prefer adding further measurements from the same standpoint from which the first measurement was performed to adding measurements from new standpoints—just because the second (or any subsequent) measurement from the same standpoint always contains one piece of additional information (namely, the horizontal angle between the previous measurement and the new one). In effect, the algorithm could keep adding measurements from the same standpoint only. One of the possible solutions to this issue lies in determining the initial measurement configuration of the network.

This initial measurement configuration is determined by declaring the measured horizontal directions to be bearings in the first step. This will allow the algorithm to move to another point in the network without the penalization stemming from the necessity to measure the first horizontal direction on the new standpoint. Subsequently, the above-described maximum accuracy increment method is applied to the network formed by the points, zenith angles, distances, and bearings. This will yield a preliminary (virtual) optimization of the network, which will provide us with measurements that need to be taken (i.e., measurements for which this preliminary optimization yielded the number of repetitions ≥ 1). This information is then adopted to determine measurements for the actual network containing horizontal directions instead of bearings, i.e., the initial measurement configuration (with the initial number of repetitions for each of these adopted measurements set to 1).

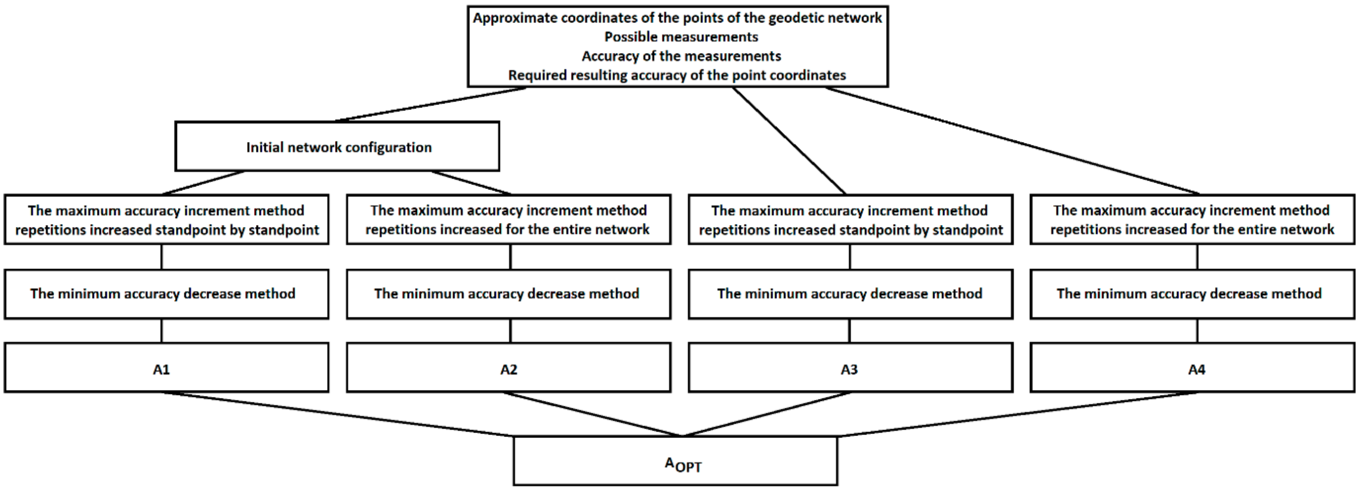

Figure 1. The four tested optimization algorithms/strategies: A1—numbers of measurement repetitions increased standpoint by standpoint with initial network configuration; A2—numbers of measurement repetitions increased for the entire network simultaneously with initial network configuration; A3—numbers of measurement repetitions increased standpoint by standpoint without initial network configuration; A4—numbers of measurement repetitions increased for the entire network simultaneously without initial network configuration; A_{OPT}—the final optimal result selected as the best-performing result from all four algorithms.

2.2.2. The Maximum Accuracy Increment Method

In this step, the algorithm adds the most suitable group of measurements to the arbitrary starting configuration of the network that does not meet the target accuracy K_T in all points. The most suitable group of measurements is defined as the group of measurements that most improves the K of the point with the lowest K. This is repeated until the K_T is achieved for all points of the network.

The number of repetitions for all possible measurements in the matrix can be, in the beginning, set to virtual 0 or, to be more exact, to $\varepsilon = 10^{-12}$, which is sufficiently low to serve as a "practical zero" and, at the same time, meets the requirements of matrix regularity in Equations (1) and (5). The initial matrix $\boldsymbol{N}$ is then defined as

$$\boldsymbol{diag}(\boldsymbol{N}) = \begin{pmatrix} n_1 & \cdots & n_m \end{pmatrix} = \begin{pmatrix} \varepsilon & \cdots & \varepsilon \end{pmatrix}, \quad (6)$$

2.2.3. The Minimum Accuracy Decrease Method

This method starts from a network configuration that meets the K_T criterion, i.e., follows after the previous part of the algorithm had produced a network with sufficient accuracy. In this step, the decrease in the accuracy characteristic K after the removal of each group of measurement is evaluated and the group the removal of which leads to the lowest decrease in accuracy is removed.

2.3. Testing of Optimization Results

The quality of the results of our optimization method was tested on four spatial geodetic networks, with random variations in the coordinates of the points within the network, total station accuracies, and requirements for the resulting point accuracies. The longest semiaxis of the error ellipsoid (LSEE) was employed as the accuracy characteristic K in our testing.

Hereinafter, the five employed algorithms will be designated (in line with Figure 1) as follows:

- A1—The numbers of measurement repetitions are increased standpoint by standpoint; initial network configuration is used.
- A2—The number of measurement repetitions is increased for the entire network simultaneously; initial network configuration is used.
- A3—The numbers of measurement repetitions are increased standpoint by standpoint without initial network configuration.
- A4—The number of measurement repetitions is increased for the entire network simultaneously without initial network configuration.
- A_{OPT}—The final optimal result selected as the best-performing result from all four algorithms.

2.3.1. Geodetic Networks Used for Testing

Four testing geodetic networks were designed, simulating practical types of geodetic networks that can be employed in practice:

- The first network was designed as a simple regular square-like network, with observations possible from all four points of the network, using the forced centering for both instrument and target.
- The second network is triangular and also allows observations from all points of the network. It simulates a network for staking out a line construction, also using forced centering.
- The third network (“bridge”) simulates an accurate network for staking out, e.g., a highway bridge. In the vicinity of the construction, there are four points with forced centering, while at greater distances, there are additional reference points from where no observations are possible.
- The last network (“building”) represents a network for staking out a building. Reference points are stabilized on the perimeter of the construction site and determined from free standpoints.

The coordinates of all networks are detailed in Table A1. Figure 2 provides visualization of all four networks and Table 1 shows the basic parameters of the networks.

Table 1. Basic characteristics of individual geodetic networks used for testing.

Network	Points to Be Determined	Points Eligible to Serve as Standpoints	Max Distance between Points [m]	Number of Possible Measurements
Square-like	4	4	158.7	36
Bridge	8	4	175.3	60
Triangular	8	8	595.5	78
Building	13	3	220.2	78

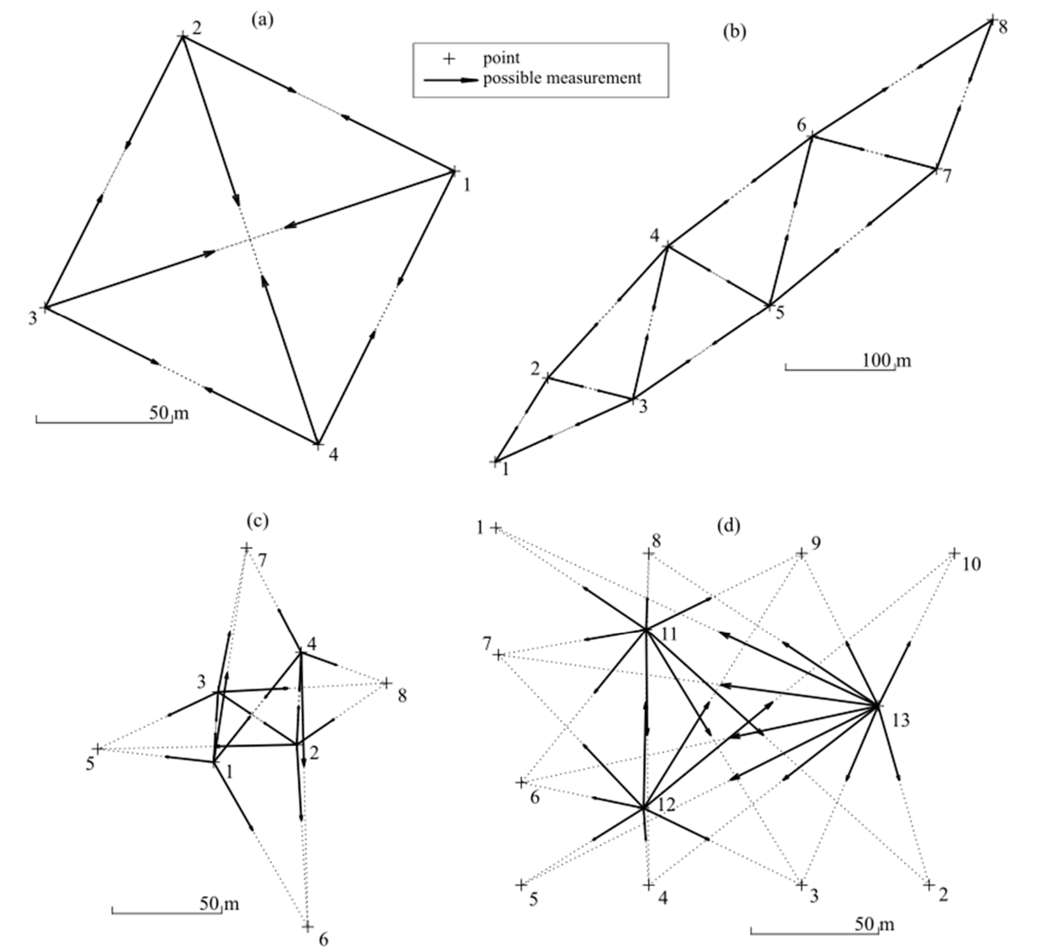

Figure 2. Testing networks: (**a**) square-like network; (**b**) triangular network; (**c**) bridge network; (**d**) building network; the coordinates of the points can be found in the Table A1 according to the point numbers.

2.3.2. Optimization Testing

Most available studies on geodetic network optimization use only one or two sites/ networks. This can, however, yield falsely favorable and optimistic results. Such a situation could arise even when using four networks as originally proposed in this paper. For this reason, we performed 500 modifications of each of the basic networks (hereinafter referred to as network variants), which differ in the position of individual points. As shown in the example in Figure 3 (square-like network), these positions were generated pseudo-randomly within a certain range around the original points (detailed in Table A1).

For each of the network variants, a calculation using all four algorithms (described in detail in Section 2.2) was performed. In the first test, the optimal number of measurements was determined by brute force, i.e., by scanning all possible combinations and selecting the one that contains the fewest number of measurements (groups of measurements) and meets the desired precision. In subsequent tests, the optimal number of measurements given the computational complexity of the brute force solution was determined as the smallest one found by our algorithm. To be able to compare the efficiency of individual algorithms, the results need to be normalized. In our study, this normalization was calculated as the ratio of the number of measurements acquired using the respective algorithm divided by the optimal number of measurements (if known) or by the best result of all four algorithms, expressed as the percentage. In other words, the optimal solution is 100%, and suboptimal solutions are characterized by numbers >100%.

Statistical evaluation was then performed for all algorithms using the results from all 500 network variants for each network and the results are presented as the mean normalized performance (MNP) of the algorithm for the particular network and the percentual success in optimal result detection (ORD). We chose 500 variants as an essentially extreme number in order to make the statistical uncertainty of the results practically negligible. In addition, the best result achieved by the combination of all four algorithms was also evaluated. Multiple tests were performed for each of the networks as will be described below.

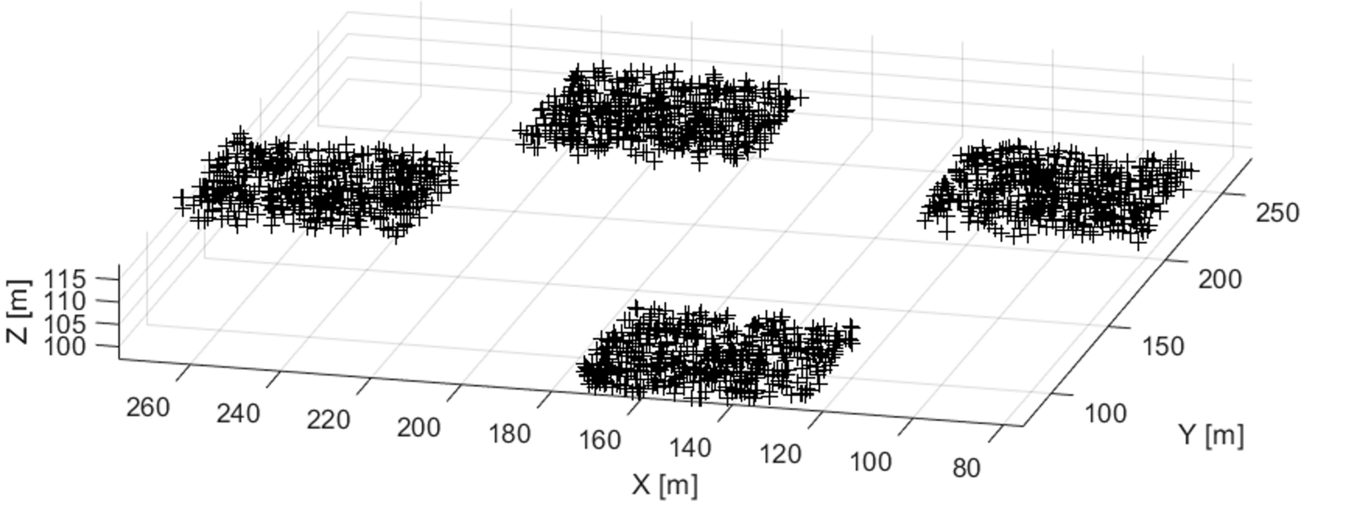

Figure 3. Example of the generation of network variants—in this case, 500 variants of the square-like network.

(1) Brute force evaluation

To be able to objectively evaluate the success of the optimization method, we need to know the absolute optimal solution, which can only be achieved using brute force calculation, i.e., by calculating all possible solutions and selecting the one that provides the required accuracy with a minimum effort (absolute optimum). This approach is, however, computationally demanding and, as a consequence, it is applicable only for the square-like network (the computational demands grow exponentially with the number of possible measurements in the network), and even this was performed only for 50 network variants. In all, four tests with different required accuracies and measurement accuracies were performed (see Table 2).

Table 2. Parameters and accuracy requirements for the brute force test.

Test	Test Specifics	σ_ψ, σ_z [mgon]	σ_d [mm]	LSEE Requirement [mm]
1.1	High accuracy requirement	1	2 + 2 ppm	0.6
1.2	Low accuracy requirement	1	2 + 2 ppm	1.1
1.3	Accuracy of angle measurements predominates	0.3	3 + 2 ppm	0.6
1.4	Accuracy of distance measurements predominates	1.5	0.5 + 1 ppm	0.8

(2) Large-scale comparison of optimization algorithms on all networks

The brute force comparison (A1–A4) was, due to the computational demands, only possible for the simplest (square-like) network and a limited number of network variants (50). Hence, a mutual comparison of all algorithms for all networks with a large number of network variants (500) against the absolute objective optimum was not possible. Instead, the result achieved by the algorithm showed the best performance for the network variant and a combination of the parameters served as the reference (100%) for the particular combination of parameters.

The reader can note that accuracy requirements differ among networks. This was estimated in view of the network size (maximum distance between points)—with growing network size, the absolute maximum accuracy is obviously decreasing (considering the same number of measurements) and to improve it further, the number of repetitions would have to grow. However, if the number of repetitions is increased to above a limit value, its further increase would cease to add sufficient information. For this reason, we proposed the required accuracies to ensure that no more than three measurements of the same parameter are needed, considering the likely practical requirements on geodetic accuracy for respective networks. The total station accuracies (σ_ψ, σ_z, σ_d) and required resulting accuracies of the network are shown in Table 3.

Table 3. Expected total station accuracies (σ_ψ, σ_z, σ_d) and required resulting accuracies (LSEE—longest semiaxis of the error ellipsoid) for the tests of individual networks.

Test	Tested Network	σ_ψ, σ_z [mgon]	σ_d [mm]	LSEE Requirement [mm]
2.1	Square-like network	1	2 + 2 ppm	0.75
2.2	Bridge network	1	2 + 2 ppm	1.0
2.3	Triangular network	0.6	2 + 2 ppm	1.0
2.4	Building network	0.6	1.5 + 2 ppm	1.0

(3) Comparison of optimization strategies when changing the required accuracies for individual networks

In this test, the results of individual optimization strategies when changing the required accuracy are evaluated (LSEE, changes in 20 steps for 50 network variants for each network; see Table 4).

Table 4. Parameters for testing of changes in required accuracies of individual networks.

Test	Tested Network	σ_ψ, σ_z [mgon]	σ_d [mm]	LSEE Requirement [mm]
3.1	Square-like network	1	2 + 2 ppm	0.5 to 1. 5
3.2	Bridge network	1	2 + 2 ppm	0.5 to 1.5
3.3	Triangular network	1	2 + 2 ppm	1.0 to 2.0
3.4	Building network	1	2 + 2 ppm	1.0 to 2.0

(4) Comparison of the performance of optimization strategies with changes in total station parameters

In this test, the results of individual optimization strategies with the changes in the accuracy of the total station (or, rather, the combination of direction/angle and distance measurement accuracies) are evaluated. The horizontal direction and zenith angle measurement accuracies gradually decrease while the accuracy of distance measurement increases, thus gradually reducing the contribution of the angles and increasing the contribution of distance measurements. The accuracy of all three parameters is always changed simultaneously, and the accuracy of both angle measurements is always changed by the same step (20 steps in all). The test parameters are detailed in Table 5.

Table 5. Network parameters and accuracy requirements for testing of changing angle and distance measurement accuracies.

Test	Tested Network	σ_ψ, σ_z [mgon]	σ_d [mm]	LSEE Requirement [mm]
4.1	Square-like network	0.1 to 1.1	2.5 + 2.5 ppm to 0.5 + 0.5 ppm	0.6
4.2	Bridge network	0.1 to 1.1	2.5 + 2.5 ppm to 0.5 + 0.5 ppm	0.8
4.3	Triangular network	0.3 to 1.8	2.5 + 2.5 ppm to 0.5 + 0.5 ppm	1
4.4	Building network	0.3 to 1.8	2.5 + 2.5 ppm to 0.5 + 0.5 ppm	1

3. Results

3.1. Comparison of the Algorithms with Brute force Optimization

Table 6 shows the results of the absolute performance of the individual algorithms compared to the results of the brute force solution (i.e., compared to the absolute optimum) for the square-like network.

Table 6. Comparison of the results of optimization algorithms and brute-force-acquired absolute optimum for the square-like network [%].

Test	Criterion	A1	A2	A3	A4	A_{OPT}
1.1—High accuracy required	MNP	104	110	102	108	101
	ORD	64	24	78	26	96
1.2—Low accuracy required	MNP	119	107	106	104	100
	ORD	28	72	56	72	100
1.3—Direction/angles more accurate than distances	MNP	107	110	102	106	101
	ORD	48	32	74	40	94
1.4—Distances more accurate than directions/angles	MNP	105	109	105	107	102
	ORD	52	26	42	32	82
Average results	MNP	109	109	104	106	101
	ORD	48	39	63	43	93

MNP—Mean normalized (to the brute-force-acquired optimum) performance of the algorithm; ORD—percentage of successful detection of the optimal result.

When higher accuracy was required (Test 1.1), the optimal result was detected by the combined algorithm AOPT in 96% of network variants, with the A3 algorithm performing the best in both evaluated parameters (MNP and ORD). When reducing the accuracy requirement, the combined A_{OPT} detected the optimal solution in all network variants, albeit none of the individual algorithms turned out to be superior to the rest. Very similar results were found for the remaining two tests that evaluated the influence of the better or worse accuracy of direction/angle and distance measurement. It is important to note that although the optimum result was achieved by the A_{OPT} algorithm in "only" 94% and 82% of network variants, respectively, the difference between the result determined by the combined algorithm and the true optimum was negligible.

The overall average result further underlines the benefits of our approach combining all four algorithms for the optimization—while none of the algorithms was capable of detecting the optimum result in more than 63% of network variants, their combination was able to do so in 93% of variants, with on average only 1% of measurements above the optimal solution required. More detailed results are given in Table A2 (Appendix C).

3.2. Comparison of the Performance of the Algorithm between Networks

In this set of tests, we did not use brute force to find the absolute optimum as, considering the numbers of points and variants, it would have been too computationally demanding. Hence, in this and the following tests, the individual algorithms will only be mutually compared relative to the results of the combined A_{OPT} (which, principally, has to always provide the best results) for the individual test and network variant. Thus, the results shown in Table 7 indicate which of the individual algorithms (A1–A4) performed the best and the worst.

Table 7 confirms the conclusion implied by the testing of the square-like network using brute force—the performance of none of the individual algorithms could be considered satisfactory as none of them (perhaps with the exception of the two simplest networks—square-like and triangular, where the A4 algorithm performed very well) consistently provided the best results. More detailed results are given in Table A3 (Appendix C).

Table 7. Comparison of optimization algorithms on individual networks (in [%]).

Test	Criterion	A1	A2	A3	A4	A_{OPT}
2.1—Square-like network	MNP ORD	114 33	122 33	108 47	101 92	100 100
2.2—Bridge network	MNP ORD	105 67	122 5	115 38	119 10	100 100
2.3—Triangular network	MNP ORD	117 10	113 24	112 12	101 87	100 100
2.4—Building network	MNP ORD	103 73	138 0	112 30	134 0	100 100
Average results	MNP ORD	110 46	124 16	112 31	114 47	100 100

3.3. Comparison of the Algorithms when Testing Changes in Required Resulting Accuracies (LSEE) of Individual Networks

The mean performances of each algorithm over all 20 steps of required accuracy are shown in Table 8. At first sight, it appears that the simplest algorithm (A4) generally performs the best, and if using only a single strategy, it would probably be the strategy of choice. However, the fact that the mean values are over 100% indicates that there are quite a few occasions where one of the other algorithms performs better.

Table 8. The mean performance of individual algorithms over the entire range of required accuracies (showing MNP%).

Network	A1	A2	A3	A4
3.1—Square-like network	125	118	111	106
3.2—Bridge network	124	132	119	106
3.3—Triangular network	120	126	124	102
3.4—Building network	123	138	107	108
Mean MNP	123	129	115	105

To be able to investigate this issue further, the results are also presented in the form of graphs (Figure 4). The horizontal axis shows the accuracy requirement (LSEE) for the particular network, while the vertical axis shows the MNP.

As revealed in the previous tests, none of the proposed algorithms was capable of consistently producing the best results across networks and accuracies. Generally, we can observe that the algorithms with initial network configuration perform better at higher accuracy requirements; the opposite is typically true when relatively lower accuracies of 1.5 mm and more are required. The only exception can be observed in the triangular network for the A4 algorithm, which performed best across the entire accuracy range (although there were occasions where other algorithms performed better as obvious from the fact that the line does not copy the horizontal axis). More detailed results are given in Table A3 (Appendix C).

3.4. Comparison of Individual Methods When Changing Total Station Accuraccy Parameters

In this batch of tests, the combination of direction/angle and distance measurement accuracies changing in opposite directions (i.e., the accuracy of direction/angle measurements decreasing along with an increasing accuracy of distance measurement) were tested.

Similar to the previous batch of tests, we can see in Table 9 that even here, the A4 algorithm generally performs the best across networks and measurement accuracies. However, just like in the previous section, the depiction of the results in Figure 5 reveals the differences among algorithms at various combinations of the total station accuracies for individual networks. As in the previous test, A4 would be the algorithm of choice if only one strategy was allowed; however, the entire battery is bound to yield overall superior results (note that the outcome of the entire battery of algorithms, i.e., of the A_{OPT}, serves as the reference, i.e., always yields 100%). It is also worth noting that the vertical axes are scaled differently—noting this for the bridge network, we can see that the algorithms differed by more than 200% in the necessary numbers of measurements for some combinations of total station parameters. More detailed results are given in Table A3 (Appendix C).

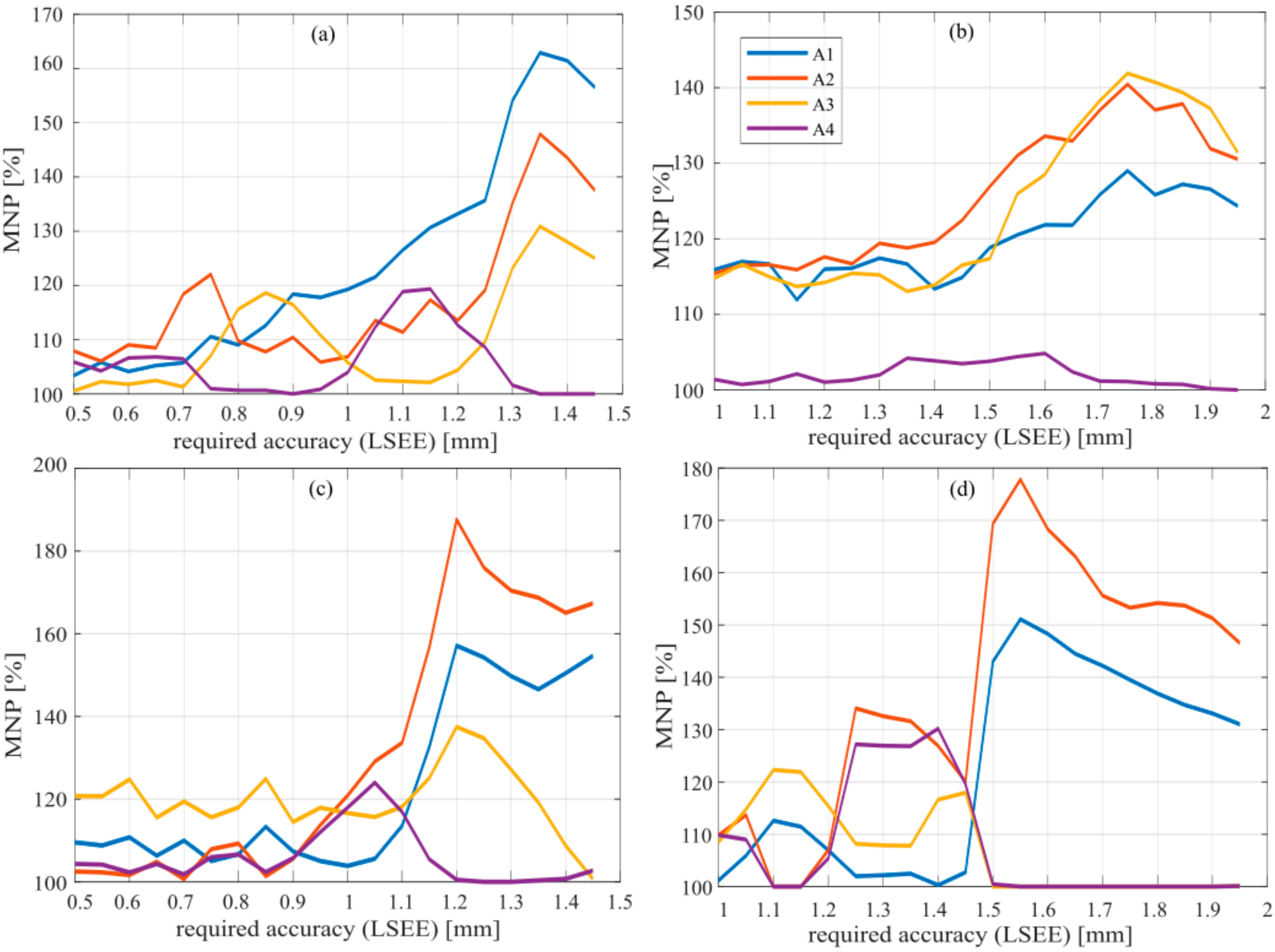

Figure 4. Changes in the performance (mean normalized performance, MNP) of individual algorithms as a function of the required accuracy for (**a**) the square-like network, (**b**) triangular network, (**c**) bridge network, and (**d**) building network.

Table 9. The mean performance of individual algorithms over the entire range of the combinations of total station accuracies (showing MNP%).

Network	A1	A2	A3	A4
4.1—Square-like network	115	117	111	102
4.2—Bridge network	143	185	116	108
4.3—Triangular network	112	112	112	103
4.4—Building network	109	121	110	114
Mean MNP	120	134	112	107

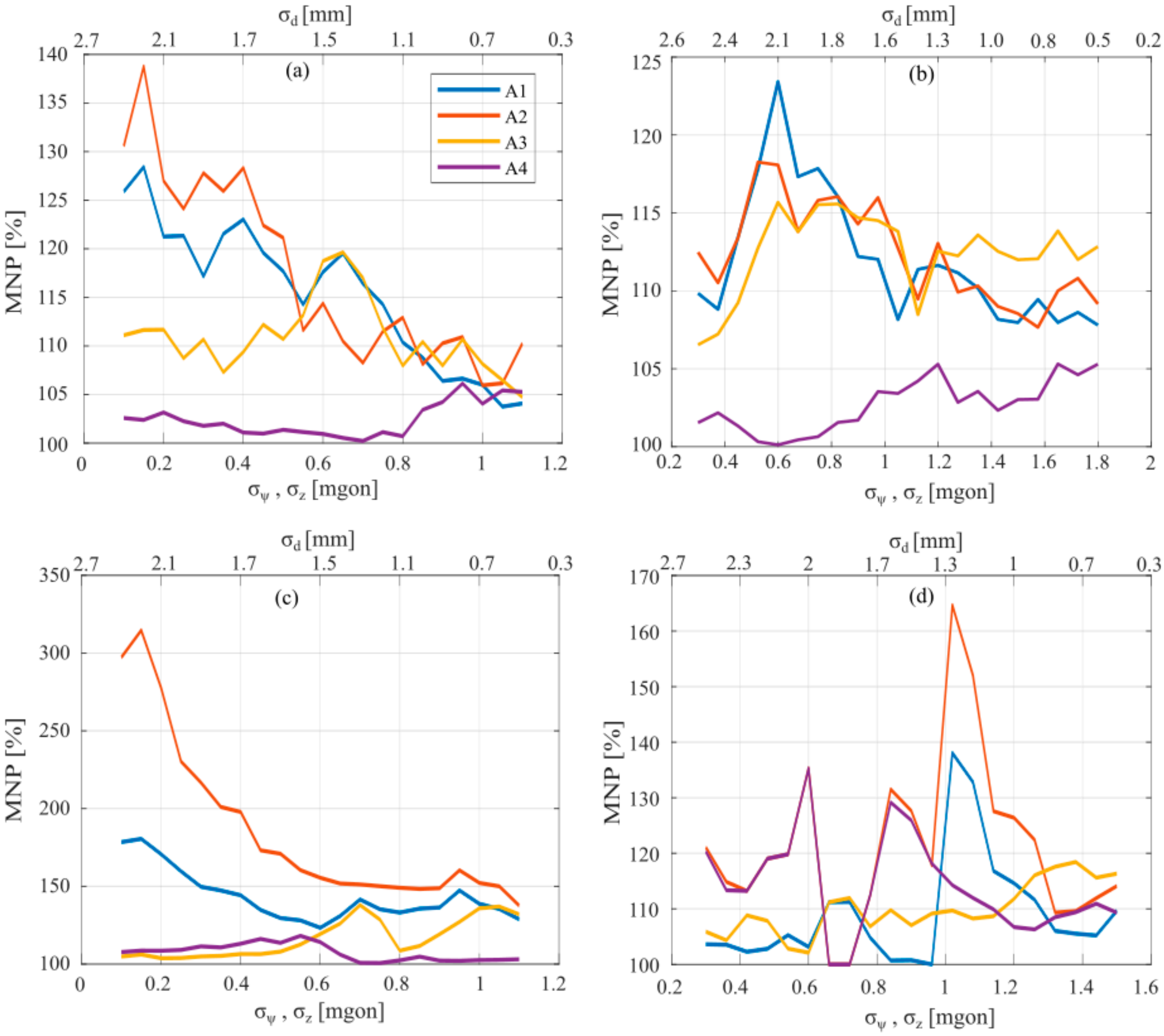

Figure 5. Changes in the performance (mean normalized performance, MNP) of individual algorithms as a function of the total station accuracy for (**a**) the square-like network, (**b**) triangular network, (**c**) bridge network, and (**d**) building network.

4. Discussion

As we have mentioned in the Introduction, most studies published on the topic of geodetic network optimization, although mathematically often correct, do not provide results that could be directly applied in the geodetic practice as they do not take into account the aforementioned basic principles of geodetic practice. Kuang studied the designs of both the first and second orders [15]. He simultaneously optimized the position of both points and weights of the measurements, combining the accuracy and reliability criteria. In addition to this, he introduced a new criterion—sensitivity—describing the ability to detect the displacements occurring between stages of measurement. However, the results of the presented simulated examples were not applicable in practice; similarly to Berne and Baselga [11], the network points converged to predefined borders or were often in impossible locations (e.g., high above the ground or below the terrain). When applying these theoretical findings to the monitoring of an existing dam (employing SOD), the optimization led to a significant reduction in the measurement weights (representing repetitions) while maintaining the required accuracy. Unfortunately, the use of continuous values for weights is suboptimal as when calculating the required numbers of measurements, the calculation returns non-integer numbers. Despite these shortcomings, Abdallah and Wang followed up on this work [13], using FOD for the optimization of a geodetic network of a mine. Again, however, the optimized numbers ended up on the predefined borders.

Amiri-Seemkooei worked on SOD in several papers. In [16], he and his team optimized the weights in the geodetic network with an objective function for reliability. In their follow-up work [17], they optimized the network in a way ensuring that all measurements contributed equally to the network accuracy. Both studies work with the measurement weights, which are not directly applicable to real-world measurements. Yetkin et al. [18] used a heuristic method of particle swarm optimization (see [19] for details) for the second-order optimization of network measurements using global navigation satellite systems (GNSSs). In that study, the work with weights may make more sense than when the total station is used for measurement as the weights of measurement change almost continuously with the observation duration.

In this paper, we have proposed and tested a practically applicable automated method for SOD geodetic network optimization. The proposed method consists of a battery of algorithms that are applied in parallel and, subsequently, the best one is automatically selected. The only inputs for this method comprise the approximate coordinates of the points of the geodetic network, the information about what measurements can be taken in reality, the accuracy of the total station, and the required resulting accuracy of the geodetic network. In this context, we define the optimal network as one with all points measured with an accuracy parameter equal to or better than a set value obtained using the minimum effort (i.e., number of measurements). The ease of practical applicability is especially given by the use of integer numbers of measurements and by considering the measurements taken at the same moment using the total station as a single group of measurements (which exactly corresponds to the practical terrain measurements) and by having all measurements from a single standpoint taken for an equal number of times. We have based this algorithm on the maximum accuracy increment method, previously proposed by our group for the optimization of geometric leveling [34] or GNSS measurements [37].

The comparison of the results of our optimization with the absolute optima determined using a brute force approach in a simple (square-like) network revealed that the proposed method combining the four algorithms and by always selecting the best performing one yielded almost perfect results, i.e., detected the optimal solution in a vast majority of cases (93%). Although this means that in 7% of the network variants, the combined algorithm did not select the optimal solution, the mean difference in the number of measurements from the optimum was negligible—as low as 1%.

For the other networks and analyzed parameters, we were unable to calculate the absolute optima using the brute force method due to computational demands, which can be considered a limitation of the study. However, considering the excellent agreement between the results of the brute force solution and the A_{OPT} algorithm (i.e., selection of the best result from all four algorithms for each network variant) in the square-like network, we can reasonably assume that the combined algorithm will perform close to the optimum for the other networks as well. Importantly, we do not claim that we have achieved the optimal results in each network/calculation variant in Sections 3.2–3.4. Rather, we demonstrate that none of the four partial algorithms A1–A4 universally performs the best and can be reasonably used for network optimization on its own. On the other hand, the additional step of selecting the best-performing algorithm out of all four in each calculation variant appears to provide results that are sufficiently close to the real-life optimum to be applicable in practice. The ease of use and low computational demand of this method are additional major benefits of our approach.

5. Conclusions

In this paper, we have proposed and tested an automated method for geodetic network optimization that is fully applicable in geodetic practice. This method is based on combining four distinct optimization algorithms calculated in parallel, the best of which (i.e., the one meeting accuracy requirements for all points of the geodetic network with the minimum number of necessary measurements) is automatically selected. The testing of this algorithm on four networks with 50–500 network variants confirmed excellent optimization results of the combined algorithm, while none of the partial algorithms was superior to a degree that could justify its use as the only algorithm. The ease of practical applicability is especially given by the use of integer numbers of measurements and by considering the measurements taken at the same moment using the total station as a single group of measurements (which exactly corresponds to the practice of terrain measurements).

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Appendix A. The Principal Coordinates of the Tested Geodetic Networks

Table A1. The principal coordinates of the tested geodetic networks (in local coordinate system).

Network	Point No.	Y [m]	X [m]	Z [m]
Square-like network	1	100.2	150.1	100.0
	2	200.1	100.5	110.2
	3	250.7	200.2	105.3
	4	150.3	250.8	115.8
Bridge network	1	1080.0	5200.0	365.0
	2	1042.0	5192.0	341.0
	3	1078.0	5168.0	342.0
	4	1040.0	5150.0	368.0
	5	1133.0	5194.0	355.0
	6	1037.0	5275.0	375.0
	7	1065.0	5102.0	371.0
	8	1001.0	5164.0	338.0
Triangular network	1	1444.0	5395.0	371.0
	2	1397.0	5320.0	368.0
	3	1321.0	5339.0	367.1
	4	1290.0	5202.0	361.0
	5	1199.0	5255.0	355.0
	6	1161.0	5104.0	348.0
	7	1050.0	5133.0	341.0
	8	1000.0	5000.0	333.0
Building network	1	1190.0	5010.0	227.5
	2	1020.0	5150.0	226.0
	3	1070.0	5150.0	225.0
	4	1130.0	5150.0	229.0
	5	1180.0	5150.0	233.0
	6	1180.0	5110.0	245.0
	7	1189.0	5060.0	231.0
	8	1130.0	5020.0	230.0
	9	1070.0	5020.0	222.0
	10	1010.0	5020.0	226.5
	11	1131.0	5050.0	218.0
	12	1132.0	5120.0	219.0
	13	1040.0	5080.0	220.0

Appendix B. Basic Algorithm in Pseudocode

Appendix B.1. Variable Definition

A	Jacobi matrix (design matrix) of optimized network
P	Weight matrix of measurement
B	Matrix of additional conditions for free network solution
N	Matrix contains number of repetitions of measurement (starting on number close to zero)
target_precision	Threshold for optimization, often LSEE
measurement_group	Vector defining group of measurements with same number of repetition (whole station measurement)

Appendix B.2. Initialization of Number of Repetitions

Calculation of weight matrix for exact number of repetitions
PP = N*P
Solution of least square method
Normal_equation = [A′*PP*A, B; B′, zeros(4,4)]
Calculation of covariance matrix
EX_0 = inv(Normal_equation);
Calculation accuracy characteristic K (LSEE) for each point
LSEE_0 = function evaluation (EX_0)

Appendix B.3. Greedy Algorithm

Algorithm A1 Greedy algorithm

```
while max(LSEE_0) > target precision
    Finding which group of measurement would result in the most benefit for the point with the lowest LSEE
    N2 = N;
    Calculation of LSM and LSEE from covariance matrix
    PP = N2*P;
    Normal_equation = [A′*PP*A, B; B′, zeros(4,4)];
    EX_0 = inv(Normal_equation);
    LSEE_0 = evaluation (Ex)
    In the cycle, the number of repetitions of every group of measurement is increased by 1 and LSEE is calculated
    for k = 1:number of groups in measurement_group
        indexing k-th group of measurement in matrixes N, A,. . .
        index(k);
        calculation of LSM and LSEE from covariance matrix
        N2(index) = N2(index)+1;
        PP = N2*P
        Normal_equation = [A′*PP*A, B; B′, zeros(4,4)]
        EX = inv(Normal_equation);
        LSEE = evaluation (Ex)
        vector of differences of LSEE for each point
        LSEE_difference=LSEE − LSEE_0
        differences for each group of measurement increasing
        LSEE_difference_group(k,:)=LSEE_difference
    end for
    Evaluation of which group of measurement had the biggest benefit for point with worse LSEE
    finding the worst point with index i
    i = max_K_point_index (LSEE_0);
    finding group of measurement with maximal benefit for worst point i, index-ing this group by i
    i2 = max_benefit_index (LSEE_difference_group(:,i));
    Increasing the number of repetitions with the biggest benefit
    N(i2) = N(i2)+1;
end while
```

Appendix B.4. Elimination of Unnecessary Measurement Groups

Algorithm A2 Elimination algorithm

```
In this case, groups_of_measurement values are all observation parameters (slope distance, zenith angle and horizontal angle from one point to another)
Number of repetitions in N starting with the values from previous algorithm calculation of LSM and LSEE from covariance matrix
PP = N * P;
Normal_equation = [A'*PP*A, B; B', zeros(4,4)];
EX_0 = inv(Normal_equation);
LSEE_0 = evaluation (Ex);
eliminating redundant groups of measurement until the target precision (LSEE) is fulfilled
do
    N2 = N;
    Searching which group of measurement has the smallest influence on the worse LSEE in the network
    for k = 1:number of groups in measurement_group
        indexing k-th group of measurement in matrixes N, A,. . .
        index(k);
        decreasing the number of repetitions of k-th group to zero
        N2(index)=zeros;
        calculation of LSM and LSEE from covariance matrix
        PP = N2*P;
        Normal_equation = [A'*PP*A, B; B', zeros(4,4)]
        EX = inv(Normal_equation);
        LSEE = evaluation (Ex);
        N2=N;
        Finding the point with the worse LSEE for each eliminated group of measurement
        Worse_LSEE(k) = max(LSEE);
    end for
    finding which group of measurement has the smallest influence on the worse LSEE in the network
    best_worse_LSEE = min(Worse_LSEE)
    decision whether the determine group of measurement can be eliminated
    if best_worse_LSEE ≤ target_precision
        eliminate the group of measurement with smallest influence on LSEE
        N(group of measurement with smallest influence on LSEE) = zeros
    else
        calculation of LSM and LSEE from covariance matrix
        PP = N2*P;
        Normal_equation = [A'*PP*A, B; B', zeros(4,4)];
        EX = inv(Normal_equation);
        LSEE = evaluation (EX)
    end if
end do
end algorithm
```

Appendix C. Detailed Test 1 and Test 2 Results

Table A2. Comparison of the results of optimization algorithms and brute-force-acquired absolute optimum for the square-like network [%]—Test 1.

Test	Criterion	A1	A2	A3	A4	A_{OPT}
1.1—High accuracy required	MNP	104	110	102	108	101
	MIN	100	100	100	100	100
	MAX	133	131	113	125	107
	STD	8	8	4	6	1
	ORD	64	24	78	26	96
1.2—Low accuracy required	MNP	119	107	106	104	100
	MIN	100	100	100	100	100
	MAX	163	188	125	114	100
	STD	17	16	7	6	0
	ORD	28	72	56	72	100

Table A2. *Cont.*

Test	Criterion	A1	A2	A3	A4	A_{OPT}
1.3—Direction/angles more accurate than distances	MNP	107	110	102	106	101
	MIN	100	100	100	100	100
	MAX	133	156	113	129	107
	STD	9	11	4	6	2
	ORD	48	32	74	40	94
1.4—Distances more accurate than directions/angles	MNP	105	109	105	107	102
	MIN	100	100	100	100	100
	MAX	128	120	125	120	117
	STD	6	7	6	6	4
	ORD	52	26	42	32	82
Average results	MNP	109	109	104	106	101
	MIN	100	100	100	100	100
	MAX	139	149	119	122	108
	STD	11	11	5	6	2
	ORD	48	39	63	43	93

MNP—Mean normalized (to the brute-force-acquired optimum) performance of the algorithm; ORD—percentage of successful detection of the optimal result; MIN—the most optimal result achieved; MAX—the least optimal result achieved; STD—standard deviation of the results.

Table A3. Comparison of optimization algorithms on individual networks (in [%])—Test 2.

Test	Criterion	A1	A2	A3	A4	A_{OPT}
2.1—Square-like network	MNP	114	122	108	101	100
	MIN	100	100	100	100	100
	MAX	170	160	133	133	100
	STD	14	19	9	6	0
2.2—Bridge network	MNP	105	122	115	119	100
	MIN	100	100	100	100	100
	MAX	188	194	167	150	100
	STD	8	9	18	9	0
2.3—Triangular network	MNP	117	113	112	101	100
	MIN	100	100	100	100	100
	MAX	174	181	138	111	100
	STD	13	12	7	2	0
2.4—Building network	MNP	103	138	112	134	100
	MIN	100	105	100	100	100
	MAX	168	180	135	167	100
	STD	6	10	9	13	0
Average results	MNP	110	124	112	114	100
	MIN	100	101	100	100	100
	MAX	175	179	143	140	100
	STD	11	13	12	9	0

MNP—Mean normalized (to the brute-force-acquired optimum) performance of the algorithm; MIN—the most optimal result achieved; MAX—the least optimal result achieved; STD—standard deviation of the results.

Table A4. Comparison of optimization algorithms on individual networks (in [%])—Test 3.

Test	Criterion	A1	A2	A3	A4
3.1—Square-like network	MNP	125	118	111	106
	MIN	100	100	100	100
	MAX	450	500	150	160
	STD	32	35	8	6
3.2—Bridge network	MNP	124	132	119	106
	MIN	100	100	100	100
	MAX	414	491	167	147
	STD	25	40	18	7

Table A4. *Cont.*

Test	Criterion	A1	A2	A3	A4
3.3—Triangular network	MNP	120	126	124	102
	MIN	100	100	100	100
	MAX	193	236	193	133
	STD	17	24	20	5
3.4—Building network	MNP	123	138	107	108
	MIN	100	100	100	100
	MAX	360	350	135	142
	STD	24	25	6	5
Average results	MNP	123	129	115	106
	MIN	100	100	100	100
	MAX	354	394	161	146
	STD	25	32	14	6

MNP—Mean normalized (to the brute-force-acquired optimum) performance of the algorithm; MIN—the most optimal result achieved; MAX—the least optimal result achieved; STD—standard deviation of the results.

Table A5. Comparison of optimization algorithms on individual networks (in [%])—Test 4.

Test	Criterion	A1	A2	A3	A4
4.1—Square-like network	MNP	115	117	111	102
	MIN	100	100	100	100
	MAX	463	650	150	133
	STD	20	27	11	5
4.2—Bridge network	MNP	143	185	116	108
	MIN	100	100	100	100
	MAX	407	771	200	157
	STD	42	87	18	11
4.3—Triangular network	MNP	112	112	112	103
	MIN	100	100	100	100
	MAX	161	169	158	143
	STD	10	13	9	6
4.4—Building network	MNP	109	121	110	114
	MIN	100	100	100	100
	MAX	300	475	158	155
	STD	23	34	11	8
Average results	MNP	120	134	112	107
	MIN	100	100	100	100
	MAX	333	516	167	147
	STD	26	49	13	8

MNP—Mean normalized (to the brute-force-acquired optimum) performance of the algorithm; MIN—the most optimal result achieved; MAX—the least optimal result achieved; STD—standard deviation of the results.

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