S1. Quadrant based Average Velocity

This analysis provides an average velocity based upon the maximum central velocity (Vmax) by double integration as follows:

$$\frac{V}{4} = V_{max} \int_0^b \int_0^a \left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\right] dxdy$$

$$= V_{max} \int_0^b \left| x - \frac{x^3}{3a^2} - \frac{y^2 x}{b^2} + \frac{x^2}{2a}\left(\frac{y}{b}\right) \right|_0^a dy$$

$$= V_{max} \int_0^b \left| a - \frac{a^3}{3a^2} - \frac{ay^2}{b^2} + \frac{a^2}{2a}\left(\frac{y}{b}\right) \right| dy$$

$$= V_{max} \int_0^b \left(\frac{2a}{3} - \frac{ay^2}{b^2} + \frac{ay}{2b}\right) dy$$

$$= V_{max} \left| \frac{2ay}{3} - \frac{ay^3}{3b^2} + \frac{ay^2}{4b} \right|_0^b dy$$

$$= V_{max} \left(\frac{2ab}{3} - \frac{ab^3}{3b^2} + \frac{ab}{4}\right)$$

$$V = 4V_{max} \left(\frac{11ab}{12} - \frac{ab}{3}\right)$$

$$= V_{max}\, ab \left(\frac{11}{3} - \frac{4}{3}\right)$$

$$V = \frac{7}{3}\, ab\, V_{max}$$

$$(2a * 2b)\bar{V} = \frac{7}{3}\, ab\, V_{max}$$

$$\bar{V} = \frac{7}{12} V_{max}$$

S2. Navier-Stokes equations

The two N-S equations are [1, 2]:

1) Mass conservation (continuity equation), which states that the mass flow difference between the inlet and outlet is zero:

$$\frac{\partial \rho}{\partial t} + \rho \left(\nabla \cdot \vec{V}\right) = 0$$

where ρ is the density of the fluid and $\vec{V}$ is the fluid velocity vector.

2) Momentum conservation:

$$\rho \frac{\partial V}{\partial t} = \rho g - \nabla p + \mu \nabla^2 V$$

where p is the fluid pressure, ρg the gravitational force, μ the kinematic viscosity of the fluid, V the fluid velocity, vector and ∇^2 the Laplacian operator.

S3. Flow through porous media (facemask)

To simulate the turbulent fluid flow resistance of the mask, the following equations have been used [3]:

$$\frac{\Delta p}{d_f} = -D\, \mu_f U_f - 0.5\, I\, \rho_f \left|U_f\right|^2$$

where D is the Darcy viscous coefficient, (*I*) is the inertial coefficients, μ_f, ρ_f and U_f are the viscosity, density, and velocity of the fluid, respectively. The (*I*) coefficient can be estimated as follows [4, 5]:

$$I = \frac{1}{1.28^2} \frac{1}{0.5\, d_f}$$

where d_f is the fibre diameter of the mask. When considering a face mask, D can be calculated as follows [3]:

$$D = \frac{64\, \xi^{1.5}\left(1 + 56\, \xi^3\right)}{{d_{p_c}}^2}$$

where ξ is the packing fibres density and d_{p_c} is the particles diameter.

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