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# Optimal Deployment of Container Weighing Equipment: Models and Properties

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**Abstract:** Container weighing is crucial to the safety of the shipping system and has garnered significant attention in the maritime industry. This research develops a container weighing optimization model and validates several propositions derived from this model. Then, a case study is conducted on ports along the Yangtze River, and the sensitivity analysis of the model is provided. We report the following findings. First, the model can be solved efficiently for large-scale optimization problems. Second, as the number of weighing machines increases, the container weighing mode changes—from selectively weighing containers at their origin ports, then weighing containers at their transshipment ports or destination ports, to all of the containers weighed at their origin ports. Third, in order to improve the safety benefits of weighing containers, port authorities can increase the weighing capacity of weighing machines. The research provides theoretical guidance for shipping system managers to design container weighing plans that enhance maritime safety.

**Keywords:** container weighing; maritime logistics; mathematical programming



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## 1. Introduction

Over the past five decades, technological advancements in container manufacturing and regulatory improvements have significantly enhanced the container shipping industry, establishing it as a crucial player in international freight transport [1]. Figure 1 depicts a bustling container port scene, highlighting the crucial role of container transportation within the global supply chain. Its prominence stems from cost-effectiveness, price stability, and improved safety. However, the maritime industry faces significant challenges, such as the risk of pandemic [2], the threat of piracy, global trade uncertainty, and the common practice among shippers of overloading containers to cut logistics costs. This tendency of carrying overweight containers not only compromises the structural integrity and safety during loading and stowage on vessels, but also poses severe risks. Improper stowage—such as placing a large number of heavy containers on one side of the vessel and lighter ones on the other—can cause an increased list, alter the vessel's metacentric height, and reduce stability, frequently leading to accidents.

At the same time, we need to realize that marine chokepoints are important to the international trade and the global economic security [3]. If an accident occurs on a ship due to overweight containers and results in the closure of a maritime chokepoint, there will be a significant impact on the international trade and the global economy. The adverse effects of overweight containers, coupled with the challenges in verifying accurate cargo weights, have escalated concerns about maritime safety, highlighting the urgent need for precise container weighing methods.

Moreover, transportation modes are now more interactive and collaborative than ever before [4], and the integration of container shipping with other modes of transport, known as intermodal transportation, has become increasingly vital for efficient global logistics [5].

Ensuring accurate container weights is essential not only for maritime safety but also for the seamless transfer of containers between ships, trucks, and trains, thereby enhancing the overall efficiency and reliability of the intermodal transport system.

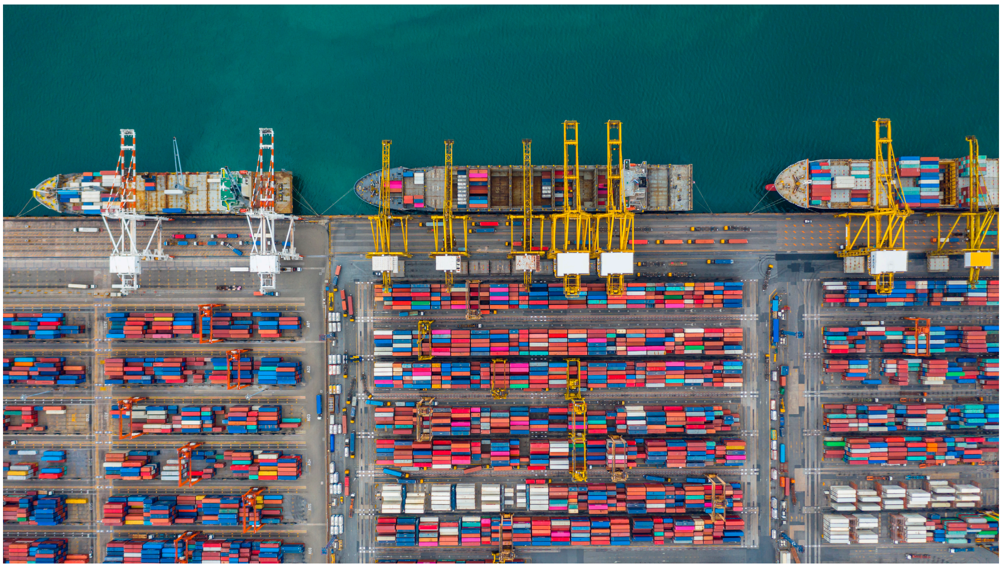

**Figure 1.** A busy port scene with containers.

Accurate weighing of containers significantly enhances the safety and stability of the shipping system. The earlier a container is weighed, the greater its contribution to the overall safety of the maritime system. Ideally, all of the containers should be weighed at their ports of origin. However, due to constraints such as the limited number and capacity of weighing machines, practical implementation necessitates optimization. This involves making strategic decisions about the number of weighing machines that each port should have and determining the optimal locations for weighing containers. The primary objective is to maximize the contribution of weighing containers to the safety of the maritime system, which we quantify in terms of "benefits." When containers are weighed at their origin ports, the benefits are maximized. Conversely, weighing containers at the destination ports results in the lowest benefits, leading to increased transportation risks due to the lack of weight measurements before the travel.

Despite the benefits, strategic planning for container weighing is under-researched. Based on the evidenced efficiency of advanced analytics in improving the efficiency and safety of transportation systems [6], this paper addresses the gap in container weighing by proposing an optimization model. This model aims to maximize the safety benefits of container weighing for the maritime system.

To address this gap, this paper proposes a container weighing scheme within the shipping system that aims to maximize the safety benefits of container weighing for the maritime system.

### *1.1. Literature Review*

Research topics related to container shipping primarily focus on the design of container shipping routes and the repositioning of empty containers. In the past few years, there has been a growing discussion about container weighing. In terms of container shipping route design problems, Rana and Vickson [7] developed a mathematical programming model to determine the optimal routing, container load, and service frequency for a chartered container ship. Maraš [8] conducted a detailed study of the routes taken by rented container ships or towed vessels in inland waterways. In addition, they found the best values for parameters that affect how efficiently these ships or tows are used, which impacts the commercial success of a container shipping business. Maraš et al. [9] conducted a study to optimize the shipping routes of barge container ships with the goal of maximizing the profit of a shipping company. This optimization determines the order in which the ships visit ports both upstream and downstream, as well as the number of loaded and empty containers transported between any two ports. Bian et al. [10] developed a mixed-integer nonlinear programming model to enhance the efficiency of container transport from Shanghai Port to inland river ports on the Yangtze River by minimizing total operating costs and optimizing feeder departure times. Zhou et al. [11] proposed a hub-and-spoke network for container shipping on inland waterways, using the river's branching structure like a tree. Feng et al. [12] examined the distribution of bulk cargo within a multimodal transportation network, considering inland waterway routes as well as containerization technology as transport modalities.

In terms of empty containers' repositioning, Shintani et al. [13] focused on empty containers' repositioning for the first time when designing a container shipping route. They formulated the problem as a two-stage task and developed an heuristic based on the genetic algorithm. Dong et al. [14] considered the problem of repositioning empty containers in multi-port, multi-vessel, and multi-voyage shipping systems characterized by dynamic, unpredictable, and uneven customer demand. Meng and Wang [15] suggested the problem of designing the network of liner shipping services that integrated both hub-and-spoke operations and multi-port calling strategies, along with the repositioning of empty containers. Brouer et al. [16] addressed the issue of cargo allocation while considering how empty containers must be repositioned and used the Dantzig–Wolfe principle to decompose an arc-flow problem formulation into path-flow problem formulations. Huang et al. [17] devised a multi-route strategy that took into account weekly scheduling, cargo transshipment, and transport duration while aiming to minimize costs.

In terms of container weighing, the importance of container weight measurement has been addressed in several safety regulations governing verified gross mass (VGM). Since about ten years ago, academics and stakeholders have tended to focus more on analyzing maritime transport risks and developing risk hierarchies [18,19]. While maritime safety has seen notable improvements from 2008 to 2018, as evidenced by lower average losses [20] and better identification of accident sources [21], a study in 2018 found that more than 60% of shipping accidents were recurring incidents [22]. Additionally, the shipping and port industries are confronting elevated risks stemming from environmental changes [23]. This context underscores the need for the progressively stricter regulatory framework imposed by the International Maritime Organization (IMO). The IMO has adopted about 50 conventions and protocols, as well as more than 1000 codes and recommendations [24]. Research by Wijnolst and Wergeland [25] has shown the positive impact of legislation on improving safety and environmental protection.

The IMO has addressed the safety of containerized maritime transport within its legal framework [26], specifically through the International Convention for the Safety of Life at Sea (SOLAS Convention) [27]. In September 2012, the 17th IMO DSC meeting [28] revised SOLAS VI/5 and introduced a draft amendment to SOLAS VI/2 (verification of the gross mass of a packed container) [29]. The amendment also requires cargo owners to ensure that the verified container weight is accurately documented in the transport papers. Subsequently, in 2014, the IMO added the requirement for weight verification of loaded containers prior to loading on board the ship to the SOLAS Convention, resulting in the SOLAS VI/2 (Cargo Information) Amendment (Revised SOLAS Convention), which came into effect on 1 July 2016, through a resolution adopted by the MSC 94th Meeting [30]. Some researchers have also analyzed the problem of overweight containers. A. Cristian [31] discussed the dangers associated with overweight containers and how this issue affects maritime safety. Rahmatika et al. [32] conducted a comprehensive assessment of the verified gross mass (VGM) implementation at Tg. Priok Port in 2017, utilizing a qualitative descriptive methodology. Their findings revealed significant disparities between the pre- and post-regulation periods, highlighting that the enforcement of VGM has notably influenced the port charges associated with shipping activities at Tg. Priok Port. Laurent

Fedi et al. [33] gathered input from 50 stakeholders, then performed a content analysis to inductively build a case study examining the influences of port community systems (PCS) on the implementation of the VGM regulations. Gujar and Tai [34] examined the legal implications of container security, with a particular focus on the issue of liability in the event of a security failure.

From reviewing these existing studies, it can be seen that optimizing the weighing plans of containers has not been studied. This involves making strategic decisions practically about how to allocate the number of weighing machines in each port and how to determine the optimal locations for weighing containers. Therefore, this paper aims to address this research gap by proposing a mathematical programming model that aims to maximize the safety benefits of weighing containers to the shipping system. To the best of our knowledge, this is the first study to present a mathematical programming model that optimizes container weighing decisions. We aim to maximize the benefits of weighing containers to the safety of the maritime system. Furthermore, we analyzed the properties of this model in detail and designed computational experiments to obtain managerial insights.

### 1.2. Contributions and Organization

The main contributions of this paper are the following. First, we model the practical container weighing problem for a shipping system as a mathematical programming model. In this model, we aim to maximize the safety benefits brought by weighing containers while considering multiple constraints such as the number of available weighing machines, demand on each route, and the capacity of weighing machines. Second, we analyze the properties of the established model; these properties are rigorously proved mathematically, enhancing the methodological contribution of this study. Third, we use a shipping system on the Yangtze River as a case study. We conduct computational experiments to investigate the influence of various parameters on the weighing plans, such as the number of ports considered in the system, the number of weighing machines, and the capacity of a weighing machine. These computational results verify the applicability and the effectiveness of the proposed model.

The remainder of this paper is organized as follows. Section 2 provides the problem description and formulates the container-weighing optimization model. Section 3 establishes properties of our proposed model and provides detailed rigorous proofs. Section 4 verifies the validity and applicability of the model with a concrete example in the Yangtze River and analyzes the sensitivity of parameters. Section 5 discusses our results, gives practical guidelines for port managers, and suggests future research directions.

## 2. Problem Description

Consider a maritime system (MS) that has $P$ ports, the set of which is $\mathcal{P} = \{1, 2, \ldots, P\}$, each indexed by $i$. Every week the MS needs to transfer containers between different ports. In most cases, containers are not shipped directly to their destinations, but are transshipped through intermediate ports before being delivered to their destinations. Assume that from the origin port $i$ to the destination port $j (i, j \in \mathcal{P}, i \neq j)$, the MS needs to transfer $N_{ij}$ containers in total. Furthermore, supposing that there is an intermediate port $k (k \in \mathcal{P} \backslash \{i, j\})$, we assume that the MS knows the number of containers to be transshipped at port $k$ from the origin port $i$ to the destination port $j$, denoted by $n_{ij}^k$, and the number of containers not to be transshipped, denoted by $\overline{n}_{ij}$. Therefore, we have $\sum_{k \in \mathcal{P} \backslash \{i,j\}} n_{ij}^k + \overline{n}_{ij} = N_{ij}, i \neq j \in \mathcal{P}$.

Accurately weighing containers helps improve the overall safety and stability of the shipping system. Enhancements to security and safety measures can increase the efficiency of supply chains [35]. Shipping is widely regarded as one of the riskiest activities, involving a broad range of potential hazards [33]. The earlier a container is weighed, the more it contributes to the overall safety of the maritime system, so our ideal system is that all of the containers are weighed at their origin ports. However, due to limited resources (number of weighing machines and their capacity), practical implementation of container weighing requires optimization. We need to make decisions about the number of weighing machines owned by each port as well as the location where each container should be weighed.

Assume that the MS has a total of $Q$ machines to weigh containers. The MS needs to decide how many weighing machines, denoted by $x_i$, to install in each port $i$ $(i \in \mathcal{P})$. The maximum number of containers that can be weighed per weighing machine per week is denoted as $M$. The MS needs to decide how many containers to be weighed at different ports. We assume that if a container is to be weighed, it is to be weighed only once. Specifically, it can be scheduled to be weighed at the origin port $i$, the intermediate port $k$, or the destination port $j$ $(i, j, k \in \mathcal{P}, i \neq j \neq k)$.

The objective of this problem is a little bit different from normal optimization problems whose optimization goals are measurable and concrete, such as distance and cost. Here, the optimization goal we are discussing is the safety of ships during navigation, which is abstract and difficult to measure.

The difficulty in measuring safety lies in the uncertainty of whether a ship is truly safe or whether an accident will occur. Before the International Maritime Organization (IMO) emphasized the issue of container weighing, many cargo ships did not weigh their containers, resulting in many overweight containers. This situation is dangerous and unsafe, but even so, a ship carrying overweight containers may not necessarily have an accident during navigation. Historically, only a few cargo ships have had accidents due to the overweight issue. Therefore, the occurrence of accidents is a probabilistic issue, and it is impossible to definitively determine whether a ship will have an accident or not, unlike the measurable distance or cost.

However, we can be certain that the safety level of a ship carrying overweight containers during navigation is lower than that of a ship without overweight containers (assuming other conditions are the same). To ensure the ship does not carry overweight containers, we can measure the weight of the containers and remove the overweight carriage as early as possible. The earlier the containers are weighed for this purpose, the shorter the distance that the ship will travel with overweight containers, and the safer the sailing process of the ship. This contribution to safety is what we call "benefit" in the paper.

Since safety is an abstract concept that is difficult to measure, we introduce some parameters, $\alpha$ and $\lambda$, to represent the relative benefits of weighing at different ports. These relative benefits do not represent absolute values but reflect the greater benefit (contribution to safety) of weighing containers at earlier ports. Using these parameters, we can optimize the weighing scheme to maximize the safety benefits brought by weighing. This is equivalent to minimizing the distance traveled with potentially overweight containers.

To explain this more clearly, we can further explore the specific meanings and roles of these parameters. If containers are weighed at their origin ports, we assume that each container can bring 1 unit of benefit; if they are weighed at their intermediate ports, we assume that each container can bring $\alpha$ units of benefit ($\alpha < 1$); and if they are weighed at their destination ports, we assume that each container can bring $\lambda$ units of benefit ($\lambda < \alpha < 1$). Here, we require that $\lambda < \alpha < 1$ because the later the container is weighed, the fewer the safety benefits will be to the shipping system.

The objective is to maximize the benefits of weighing containers to the safety of the maritime system. When all of the containers are weighed at their origin ports, the benefits are the highest. In contrast, when all of the containers are weighed at their destination ports, the benefits are the lowest.

If containers are weighed at the origin port $i$, we assume that each container can bring 1 unit of benefit; if they are weighed at the intermediate port $k$, we assume that each container can bring $\alpha$ units of benefit $(\alpha < 1)$; and if they are weighed at the destination port $j$, we assume that each container can bring $\lambda$ units of benefit $(\lambda < \alpha < 1)$. Here, we require that $\lambda < \alpha < 1$ because the later the container is weighed, the fewer the safety benefits will be to the shipping system.

For the containers that are to be transferred from the origin port $i$ to the destination $j$ via the intermediate port $k$ $(i, j, k \in \mathcal{P}, i \neq j \neq k)$, let $u_{ij}^k$ represent the number of containers weighed at the origin port $i$, $w_{ij}^k$ represent the number of containers weighed at the intermediate port $k$, and $v_{ij}^k$ represent the number of containers weighed at the destination port $j$. For the containers that are to be transferred directly from the origin port $i$ to the destination port $j$, let $\overline{u}_{ij}$ represent the number of containers weighed at the origin port $i$, and $\overline{v}_{ij}$ represent the number of containers weighed at the destination port $j$.

Now, the MS needs to decide $u_{ij}^k$, $w_{ij}^k$, $v_{ij}^k$, $\overline{u}_{ij}$, $\overline{v}_{ij}$, and $x_i$ to maximize the safety benefits from weighing containers in a week. Before constructing the mathematical model, we present all notations in Nomenclature section.

The model is described as follows:

$$\max \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1\cdot u_{ij}^k + \alpha\cdot w_{ij}^k + \lambda\cdot v_{ij}^k\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1\cdot\overline{u}_{ij} + \lambda\cdot\overline{v}_{ij}\right) \quad (1)$$

subject to

$$\sum_{i\in\mathcal{P}} x_i \le Q \quad (2)$$

$$u_{ij}^k + w_{ij}^k + v_{ij}^k \le n_{ij}^k, \forall i,j,k \in \mathcal{P}, i \ne j \ne k \quad (3)$$

$$\overline{u}_{ij} + \overline{v}_{ij} \le \overline{n}_{ij}, \forall i,j \in \mathcal{P}, i \ne j \quad (4)$$

$$\sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(u_{ij}^k + w_{jk}^i + v_{ji}^k\right) + \sum_{j\in\mathcal{P}\setminus\{i\}} \left(\overline{u}_{ij} + \overline{v}_{ji}\right) \le M * x_i, \forall i \in \mathcal{P} \quad (5)$$

$$x_i \in Z_+, \forall i \in \mathcal{P} \quad (6)$$

$$u_{ij}^k, w_{ij}^k, v_{ij}^k \ge 0, \forall i,j,k \in \mathcal{P}, i \ne j \ne k \quad (7)$$

$$\overline{u}_{ij}, \overline{v}_{ij} \ge 0, \forall i,j \in \mathcal{P}, i \ne j. \quad (8)$$

Objective function (1) aims to maximize the safety benefits of weighing all transported containers. It mainly consists of two parts, where the first part considers transshipped containers, and the second part considers containers not being transshipped. Constraint (2) indicates that the total number of weighing machines employed should be less than or equal to the number of available machines. Constraints (3) indicate that the sum of containers weighed at origin port $i$ or intermediate port $k$ or the destination port $j$ should be less than or equal to the total number of the containers transferred from port $i$ to port $j$ via port $k$ ($i,j,k \in \mathcal{P}, i \ne j \ne k$). Constraints (4) indicate that the sum of containers weighed at origin port $i$ or destination port $j$ should be less than or equal to the total number of the containers transported from port $i$ to port $j$ ($i,j \in \mathcal{P}, i \ne j$). Constraints (5) indicate that the sum of containers weighed at port $i$ should be less than or equal to the sum of weighing machines' capacity at port $i$. Constraints (6)–(8) define the domains of decision variables.

## 3. Mathematical Analyses

In this section, we present several propositions derived from the established model and provide detailed proofs for them. These propositions can help verify the effectiveness and practicality of the model.

**Proposition 1.** *If $Q$ is large enough, then any optimal solution shows that all containers are weighed at their origin ports.*

**Proof.** We first show that an upper bound of the optimal objective function value is $\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(\overline{n}_{ij} + \sum_{k\in\mathcal{P}} n_{ij}^k\right) = \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} N_{ij}$. To this end, we have

$$\begin{aligned}
&\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1\cdot u_{ij}^k + \alpha\cdot w_{ij}^k + \lambda\cdot v_{ij}^k\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1\cdot\overline{u}_{ij} + \lambda\cdot\overline{v}_{ij}\right) \\
&\le \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1\cdot u_{ij}^k + 1\cdot w_{ij}^k + 1\cdot v_{ij}^k\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1\cdot\overline{u}_{ij} + 1\cdot\overline{v}_{ij}\right) \\
&\le \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} n_{ij}^k + \overline{n}_{ij}\right) \\
&= \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} N_{ij}
\end{aligned}$$

where the first inequality holds because $\alpha < 1$ and $\lambda < 1$, and the second inequality holds because of Constraints (3)–(4). Next, if $Q$ is large enough, we can construct the following solution: $x_i^{\#} = \left\lceil \frac{\sum_{j\in\mathcal{P}\setminus\{i\}} N_{ij}}{M} \right\rceil$, $i \in \mathcal{P}$; $u_{ij}^{k\#} = n_{ij}^k$, $w_{ij}^{k\#} = 0$, $v_{ij}^{k\#} = 0$, $i,j,k \in \mathcal{P}, i \neq j \neq k$; $\overline{u}_{ij}^{\#} = \overline{n}_{ij}$, $\overline{v}_{ij}^{\#} = 0$, $i,j \in \mathcal{P}, i \neq j$. It is easy to see that this solution is feasible. Moreover, the objective function value of this solution is $\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} (\overline{n}_{ij} + \sum_{k\in\mathcal{P}\setminus\{i,j\}} n_{ij}^k) = \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\{i\}} N_{ij}$, which is equal to the upper bound. This shows that the optimal objective function value is $\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} N_{ij}$.

Finally, we prove that, by contradiction, any optimal solution shows that all containers are weighed at their origin ports. Suppose that there is an optimal solution denoted as ($x_i^{\&}$, $i \in \mathcal{P}$; $u_{ij}^{k\&}$, $w_{ij}^{k\&}$, $v_{ij}^{k\&}$, $i,j,k \in \mathcal{P}, i \neq j \neq k$; $\overline{u}_{ij}^{\&}$, $\overline{v}_{ij}^{\&}$, $i,j \in \mathcal{P}, i \neq j$) that does not weigh all containers at their origin ports; that is, there is an $(i', j')$ such that

$$\sum\nolimits_{k\in\mathcal{P}\setminus\{i',\, j'\}} u_{i'j'}^{k\&} + \overline{u}_{i'j'}^{\&} < N_{i'j'}. \tag{9}$$

Then, the objective function value of this solution is

$$\begin{aligned}
&\sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} \left(1 \cdot u_{ij}^{k\&} + \alpha\cdot w_{ij}^{k\&} + \lambda\cdot v_{ij}^{k\&}\right) + \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \left(1\cdot\overline{u}_{ij}^{\&} + \lambda\cdot\overline{v}_{ij}^{\&}\right) \\
&= \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \begin{bmatrix} \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} 1 \cdot u_{ij}^{k\&} + 1\cdot\overline{u}_{ij}^{\&}\right) \\ +\alpha \cdot \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} w_{ij}^{k\&}\right) + \lambda \cdot \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} v_{ij}^{k\&} + \overline{v}_{ij}^{\&}\right) \end{bmatrix} \\
&\le \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \begin{bmatrix} \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} 1 \cdot u_{ij}^{k\&} + 1\cdot\overline{u}_{ij}^{\&}\right) \\ +\alpha \cdot \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} \left(w_{ij}^{k\&} + v_{ij}^{k\&}\right) + \overline{v}_{ij}^{\&}\right) \end{bmatrix} \\
&\le \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \begin{bmatrix} \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} 1 \cdot u_{ij}^{k\&} + 1\cdot\overline{u}_{ij}^{\&}\right) \\ +\alpha \cdot \left(N_{ij} - \left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} u_{ij}^{k\&} + \overline{u}_{ij}^{\&}\right)\right) \end{bmatrix} \\
&= \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} \left[\alpha \cdot N_{ij} + (1-\alpha)\left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} u_{ij}^{k\&} + \overline{u}_{ij}^{\&}\right)\right] \\
&= \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\},(i,j)\neq(i',j')} \left[\alpha \cdot N_{ij} + (1-\alpha)\left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i,j\}} u_{ij}^{k\&} + \overline{u}_{ij}^{\&}\right)\right] \\
&+\alpha \cdot N_{i'j'} + (1-\alpha)\left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i',j'\}} u_{i'j'}^{k\&} + \overline{u}_{i'j'}^{\&}\right) \\
&\le \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\},(i,j)\neq(i',j')} \left[\alpha \cdot N_{ij} + (1-\alpha) \cdot N_{ij}\right] \\
&+\alpha \cdot N_{i'j'} + (1-\alpha)\left(\sum\nolimits_{k\in\mathcal{P}\setminus\{i',j'\}} u_{i'j'}^{k\&} + \overline{u}_{i'j'}^{\&}\right) \\
&< \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\},(i,j)\neq(i',j')} \left[\alpha \cdot N_{ij} + (1-\alpha) \cdot N_{ij}\right] + \alpha \cdot N_{i'j'} + (1-\alpha) \cdot N_{i'j'} \\
&= \sum\nolimits_{i\in\mathcal{P}} \sum\nolimits_{j\in\mathcal{P}\setminus\{i\}} N_{ij},
\end{aligned}$$

where the first inequality holds because $\lambda < \alpha$, the second inequality holds because of Constraints (3)–(4) and $\sum_{k\in\mathcal{P}} n_{ij}^k + \overline{n}_{ij} = N_{ij}$, $i,j \in \mathcal{P}, i \neq j$, the third inequality holds because of $\sum_{k\in\mathcal{P}\setminus\{i,j\}} u_{ij}^{k\&} + \overline{u}_{ij}^{\&} \le N_{ij}$, $i,j \in \mathcal{P}, i \neq j$, and the last inequality holds because of Equation (9). The objective function value of this solution is less than the optimal objective function value, implying that this solution is not optimal, which contradicts the assumption. Therefore, if $Q$ is large enough, then any optimal solution shows that all containers are weighed at their origin ports. This concludes the proof of the proposition. □

**Proposition 2.** *If $M$ is large enough, then the optimal solution may not show that all containers are weighed at their origin ports; that is, the optimal objective function value may be strictly less than $\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} N_{ij}$.*

**Proof.** We prove this proposition by finding an example that satisfies it. Assume that there are three ports 1, 2, and 3 and only one weighing machine, i.e., $Q = 1$. The MS needs to transfer 500 containers each from 1 and 3 to 2 and does not need to transship containers, i.e., $N_{12} = N_{32} = 500$, $N_{13} = N_{23} = N_{31} = N_{21} = 0$, $n_{12}^3 = n_{13}^2 = n_{23}^1 = n_{32}^1 = n_{31}^2 = n_{21}^3 = 0$, $\overline{n}_{13} = \overline{n}_{31} = 0$, $\overline{n}_{12} = 500$, and $\overline{n}_{32} = 500$. We further set $M = \infty$ and $\lambda = 0.8$. If all containers are weighed at their origin ports, the objective function value would be 1000, but after we solve the problem using the established mathematical model, we find that the optimal objective function value is 800. Note that in an optimal solution, the only weighing machine is put at port 2, i.e., $x_1^* = 0$, $x_2^* = 1$, and $x_3^* = 0$, and the other components in the optimal solution are obtained as follows:

$$u_{12}^{3*} = w_{12}^{3*} = v_{12}^{3*} = 0,\ \overline{u}_{12}^{*} = 0,\ \overline{v}_{12}^{*} = 500,$$
$$u_{32}^{1*} = w_{32}^{1*} = v_{32}^{1*} = 0,\ \overline{u}_{32}^{*} = 0,\ \overline{v}_{32}^{*} = 500.$$

This concludes the proof of the proposition. □

**Proposition 3.** *If an optimal solution has an intermediate port $\hat{i}$ weighing more than 0 containers, then all the containers starting from port $\hat{i}$ are weighed at port $\hat{i}$.*

**Proof.** We prove this proposition by contradiction. Suppose there is an optimal solution denoted as $\left(x_i^*, i \in \mathcal{P}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k \in \mathcal{P}, i \neq j \neq k; \overline{u}_{ij}^*, \overline{v}_{ij}^*, i,j \in \mathcal{P}, i \neq j\right)$ where port $\hat{i}$ weighs more than 0 container as an intermediate port, and some containers starting from port $\hat{i}$ are not weighed, i.e.,

$$\sum_{i \in \mathcal{P}\setminus\{\hat{i}\}} \sum_{j \in \mathcal{P}\setminus\{\hat{i},i\}} w_{i,j}^{\hat{i}*} > 0 \tag{10}$$

$$\sum_{j \in \mathcal{P}\setminus\{\hat{i}\}} \overline{u}_{\hat{i}j}^* + \sum_{j \in \mathcal{P}\setminus\{\hat{i}\}} \sum_{k \in \mathcal{P}\setminus\{\hat{i},j\}} u_{\hat{i}j}^{k*} < N_{\hat{i}j}^*. \tag{11}$$

Then, the objective function value of this solution is

$$\begin{aligned}
&\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1 \cdot u_{ij}^{k*} + \alpha \cdot w_{ij}^{k*} + \lambda \cdot v_{ij}^{k*}\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1 \cdot \overline{u}_{ij}^* + \lambda \cdot \overline{v}_{ij}^*\right) \\
= &\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} 1 \cdot u_{ij}^{k*} + 1 \cdot \overline{u}_{ij}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} w_{ij}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} v_{ij}^{k*} + \overline{v}_{ij}^*\right) \end{bmatrix}
\end{aligned}$$

$$= \begin{cases}
\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} 1 \cdot u_{ij}^{k*} + 1 \cdot \overline{u}_{ij}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} w_{ij}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} v_{ij}^{k*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{without } \hat{i}) \\
+ \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1 \cdot u_{\hat{i}j}^{k*} + \overline{u}_{\hat{i}j}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} w_{\hat{i}j}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} v_{\hat{i}j}^{k*} + \overline{v}_{\hat{i}j}^*\right) \end{bmatrix} (\text{start from } \hat{i}) \\
+ \sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} 1 \cdot u_{ij}^{\hat{i}*} + \overline{u}_{ij}^* \\ + \alpha \cdot w_{ij}^{\hat{i}*} + \lambda \cdot \left(v_{ij}^{\hat{i}*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{tranship at } \hat{i}) \\
+ \sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} 1 \cdot u_{i\hat{i}}^{k*} + 1 \cdot \overline{u}_{i\hat{i}}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} w_{i\hat{i}}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} v_{i\hat{i}}^{k*} + \overline{v}_{i\hat{i}}^*\right) \end{bmatrix} (\text{end at } \hat{i})
\end{cases}$$

$$< \begin{cases}
\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} 1 \cdot u_{ij}^{k*} + 1 \cdot \overline{u}_{ij}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} w_{ij}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} v_{ij}^{k*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{without } \hat{i}) \\
+ \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1 \cdot u_{\hat{i}j}^{k*} + 1 \cdot \overline{u}_{\hat{i}j}^*\right) + 1 \cdot 1 \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} w_{\hat{i}j}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} v_{\hat{i}j}^{k*} + \overline{v}_{\hat{i}j}^*\right) \end{bmatrix} (\text{start from } \hat{i}) \\
+ \sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} 1 \cdot u_{ij}^{\hat{i}*} + \overline{u}_{ij}^* \\ + \alpha \cdot \left(w_{ij}^{\hat{i}*} - 1\right) + \lambda \cdot \left(v_{ij}^{\hat{i}*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{tranship at } \hat{i}) \\
+ \sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} 1 \cdot u_{i\hat{i}}^{k*} + 1 \cdot \overline{u}_{i\hat{i}}^*\right) \\ + \alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} w_{i\hat{i}}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} v_{i\hat{i}}^{k*} + \overline{v}_{i\hat{i}}^*\right) \end{bmatrix} (\text{end at } \hat{i})
\end{cases}$$

Obviously, increasing the term $\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1\cdot u_{i\hat{j}}^{k*} + 1\cdot\overline{u}_{\hat{i}j}^{*}$ by one and simultaneously decreasing $w_{ij}^{\hat{i}*}$ by one ensures the feasibility of the solution. This adjustment reflects an additional container originating and weighed at port $\hat{i}$, offset by a reduction in containers that start from port $i$ and are transshipped at port $\hat{i}$ being weighed at the transshipped port $\hat{i}$; the solution is still feasible. The inequality holds because $\alpha < 1$; therefore, we see that the value of the original objective function is less than that of the new one, which contradicts with the assumption. This concludes the proof of the proposition. □

**Proposition 4.** *If an optimal solution* $(x_i^*,\ i\in\mathcal{P}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k\in\mathcal{P}, i\neq j\neq k; \overline{u}_{ij}^{*}, \overline{v}_{ij}^{*}, i, j\in\mathcal{P}, i\neq j)$ *has an intermediate port* $\hat{i}$ *weighing more than 0 containers, then it is possible that not all containers starting from port* $\hat{i}$ *are weighed at port* $\hat{i}$*. Mathematically, it is possible that there exist ports* $\hat{i}$*,* $\hat{k}$*, and* $\hat{j}$ *such that* $w_{\hat{k}\hat{j}}^{\hat{i}*} > 0$ *and* $\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}}\sum_{k\in\mathcal{P}\setminus\{\hat{i},j\}} u_{\hat{i}j}^{k*} + \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \overline{u}_{\hat{i}j}^{*} < \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} N_{\hat{i}j}$*.*

**Proof.** We prove this proposition by finding an example that satisfies it. Assume that $\mathcal{P} = \{1,2,3,4\}$, $Q = 2$, and $M = 10$. The MS needs to transfer 10 containers from port 2 to port 4 without transshipment, 1 container from port 1 to port 3 via 2, and 5 containers from port 4 to port 1 without transshipment, which can be described as

$$\begin{cases} Q = 2 \\ M = 10 \\ \alpha = 0.8 \\ \lambda = 0.4 \end{cases} \begin{cases} N_{24} = 10 \\ N_{13} = 1 \\ N_{41} = 5 \end{cases} \begin{cases} N_{12} = N_{14} = 0 \\ N_{21} = N_{23} = 0 \\ N_{31} = N_{32} = N_{34} = 0 \\ N_{42} = N_{43} = 0 \end{cases}$$

$$\begin{cases} n_{13}^{2} = 1 \\ n_{ij}^{k} = 0\,, (i,j,k) \neq (1,3,2) \\ i\neq j\neq k, i,j,k\in P \end{cases} \begin{cases} \overline{n}_{24} = 10 \\ \overline{n}_{41} = 5 \\ \overline{n}_{ij} = 0, (i,j) \neq \{(2,4),(4,1)\} \\ i\neq j, i,j\in P \end{cases}$$

Then, we solve the established model to obtain the optimal solution and the optimal objective function value. The optimal solution is $x_1^* = 0, x_2^* = 1, x_3^* = 0, x_4^* = 1, w_{13}^{2*} = 1$, $\overline{u}_{24}^{*} = 9$, $\overline{v}_{24}^{*} = 1$, $\overline{u}_{41}^{*} = 5$, and $w_{13}^{2*} = 1$ and all of the other decision variables are 0. Then, the objective function value is 15.2. This concludes the proof of the proposition. □

**Proposition 5.** *If an optimal solution has a destination port* $\hat{j}$ *weighing more than 0 containers, then all the containers starting from and transshipped at port* $\hat{j}$ *must have been weighed.*

**Proof.** Similar to the proof of Proposition 3, we can know that this proposition is true because of $\lambda < \alpha < 1$. □

**Proposition 6.** *If an optimal solution* $(x_i^*,\ i\in\mathcal{P}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k\in\mathcal{P}, i\neq j\neq k; \overline{u}_{ij}^{*}, \overline{v}_{ij}^{*}, i, j\in\mathcal{P}, i\neq j)$ *has a destination port* $\hat{j}$ *weighing more than 0 container, then it is possible that not all containers starting from or transshipped at port* $\hat{j}$ *are weighed at port* $\hat{j}$*.*

**Proof.** Similar to the proof of Proposition 4, we can find a solution that satisfies this proposition. One example is found and shown in Section 4.3.2. □

**Proposition 7.** *At port* $i$ $(i\in\mathcal{P})$*, the total number of containers starting from, transshipped at, and ending at port* $p$ *is denoted as* $T_i$*. There exists an optimal solution showing that at any port* $i$ $(i\in\mathcal{P})$*,* $x_i \leq \left\lceil \frac{T_i}{M} \right\rceil$*.*

**Proof.** Suppose that there is an optimal solution where at any port $i$, $x_i > \left\lceil \frac{T_i}{M} \right\rceil$, which is denoted as $\left(x_i^* > \left\lceil \frac{T_i}{M} \right\rceil, i \in \mathcal{P}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k \in \mathcal{P}, i \neq j \neq k; \overline{u}_{ij}^*, \overline{v}_{ij}^*, i,j \in \mathcal{P}, i \neq j\right)$. Then the objective function value of this solution is

$$\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1 \cdot u_{ij}^{k*} + \alpha \cdot w_{ij}^{k*} + \lambda \cdot v_{ij}^{k*}\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1 \cdot \overline{u}_{ij}^* + \lambda \cdot \overline{v}_{ij}^*\right).$$

When the decision variable $x_{\tilde{i}}^*$ of port $\tilde{i}$ ($\tilde{i} \in \mathcal{P}$) is gradually reduced to $\left\lceil \frac{T_{\tilde{i}}}{M} \right\rceil$, it can still be shown that the values of the other decision variables do not change, and we can obtain a new feasible solution that is $x_{\tilde{i}}^* = \left\lceil \frac{T_{\tilde{i}}}{M} \right\rceil; x_i^* > \left\lceil \frac{T_{\tilde{i}}}{M} \right\rceil, i \in \mathcal{P}\setminus\{\tilde{i}\}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k \in \mathcal{P}, i \neq j \neq k; \overline{u}_{ij}^*, \overline{v}_{ij}^*, i,j \in \mathcal{P}, i \neq j$.

Obviously, the objective function value of this solution does not change, indicating that this new solution is also an optimal solution. Subsequently, the decision variables $x_i^* \left(i \in \mathcal{P}\setminus\{\tilde{i}\}\right)$ for any other port $i$ are all reduced to $x_i = \left\lceil \frac{T_i}{M} \right\rceil$. This new solution shows that at any port $i (i \in \mathcal{P})$, $x_i \leq \left\lceil \frac{T_i}{M} \right\rceil$. This concludes the proof of the proposition. □

**Proposition 8.** *If there exists an optimal solution that shows that at a certain port $\hat{i}$, $x_{\hat{i}} \geq \left\lceil \frac{T_{\hat{i}}}{M} \right\rceil$, then all of the containers starting from $\hat{i}$ are weighed at $\hat{i}$.*

**Proof.** We prove this proposition by contradiction. Suppose there is an optimal solution $(x_i^*, i \in \mathcal{P}; u_{ij}^{k*}, w_{ij}^{k*}, v_{ij}^{k*}, i,j,k \in \mathcal{P}, i \neq j \neq k; \overline{u}_{ij}^*, \overline{v}_{ij}^*, i,j \in \mathcal{P}, i \neq j)$ such that at port $\hat{i}$, $x_{\hat{i}} \geq \left\lceil \frac{T_{\hat{i}}}{M} \right\rceil$ and some containers starting from port $\hat{i}$ are not weighed at $\hat{i}$, i.e.,

$$\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \overline{u}_{\hat{i}j}^* + \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{k\in\mathcal{P}\setminus\{\hat{i},j\}} u_{\hat{i}j}^{k*} < N_{\hat{i}j}^* \tag{12}$$

$$\sum_{j\in\mathcal{P}\{\hat{i}\}} \overline{v}_{\hat{i}j}^* + \sum_{j\in\mathcal{P}\{\hat{i}\}} \sum_{k\in\mathcal{P}\{\hat{i},j\}} \left(w_{\hat{i}j}^{k*} + v_{\hat{i}j}^{k*}\right) > 0. \tag{13}$$

Then the objective function value of this solution is

$$\begin{aligned}
&\sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \sum_{k\in\mathcal{P}\setminus\{i,j\}} \left(1 \cdot u_{ij}^{k*} + \alpha \cdot w_{ij}^{k*} + \lambda \cdot v_{ij}^{k*}\right) + \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \left(1 \cdot \overline{u}_{ij}^* + \lambda \cdot \overline{v}_{ij}^*\right) \\
&= \sum_{i\in\mathcal{P}} \sum_{j\in\mathcal{P}\setminus\{i\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} 1 \cdot u_{ij}^{k*} + 1 \cdot \overline{u}_{ij}^*\right) \\ +\alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} w_{ij}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j\}} v_{ij}^{k*} + \overline{v}_{ij}^*\right) \end{bmatrix}
\end{aligned}$$

$$= \begin{cases}
\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} 1 \cdot u_{ij}^{k*} + 1 \cdot \overline{u}_{ij}^*\right) \\ +\alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} w_{ij}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} v_{ij}^{k*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{without } \hat{i}) \\
+\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1 \cdot u_{\hat{i}j}^{k*} + \overline{u}_{\hat{i}j}^*\right) \\ +\alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} w_{\hat{i}j}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} v_{\hat{i}j}^{k*} + \overline{v}_{\hat{i}j}^*\right) \end{bmatrix} (\text{start from } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \begin{bmatrix} 1 \cdot u_{ij}^{\hat{i}*} + \overline{u}_{ij}^* \\ +\alpha \cdot w_{ij}^{\hat{i}*} + \lambda \cdot \left(v_{ij}^{\hat{i}*} + \overline{v}_{ij}^*\right) \end{bmatrix} (\text{transship at } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \begin{bmatrix} \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} 1 \cdot u_{i\hat{i}}^{k*} + 1 \cdot \overline{u}_{i\hat{i}}^*\right) \\ +\alpha \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} w_{i\hat{i}}^{k*}\right) + \lambda \cdot \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} v_{i\hat{i}}^{k*} + \overline{v}_{i\hat{i}}^*\right) \end{bmatrix} (\text{end at } \hat{i})
\end{cases}$$

$$
< \left\{ \begin{array}{l}
\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} 1\cdot u_{ij}^{k*} + 1\cdot\overline{u}_{ij}^{*}\right) \\ +\alpha\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} w_{ij}^{k*}\right) + \lambda\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} v_{ij}^{k*} + \overline{v}_{ij}^{*}\right) \end{array} \right] (\text{without } \hat{i}) \\
+\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1\cdot u_{\hat{i}j}^{k*} + 1\cdot\overline{u}_{\hat{i}j}^{*}\right) \\ +\alpha\cdot\left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} w_{\hat{i}j}^{k*} + \sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}}\left(v_{\hat{i}j}^{k*} + \overline{v}_{\hat{i}j}^{*}\right)\right) \end{array} \right] (\text{start from } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \left[ \begin{array}{c} 1\cdot u_{ij}^{\hat{i}*} + \overline{u}_{ij}^{*} \\ +\alpha\cdot\left(w_{ij}^{\hat{i}*} - 1\right) + \lambda\cdot\left(v_{ij}^{\hat{i}*} + \overline{v}_{ij}^{*}\right) \end{array} \right] (\text{tranship at } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} 1\cdot u_{i\hat{i}}^{k*} + 1\cdot\overline{u}_{i\hat{i}}^{*}\right) \\ +\alpha\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} w_{i\hat{i}}^{k*}\right) + \lambda\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} v_{i\hat{i}}^{k*} + \overline{v}_{i\hat{i}}^{*}\right) \end{array} \right] (\text{end at } \hat{i})
\end{array} \right.
$$

$$
< \left\{ \begin{array}{l}
\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} 1\cdot u_{ij}^{k*} + 1\cdot\overline{u}_{ij}^{*}\right) \\ +\alpha\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} w_{ij}^{k*}\right) + \lambda\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,j,\hat{i}\}} v_{ij}^{k*} + \overline{v}_{ij}^{*}\right) \end{array} \right] (\text{without } \hat{i}) \\
+\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1\cdot u_{\hat{i}j}^{k*} + 1\cdot\overline{u}_{\hat{i}j}^{*}\right) + 1\cdot 1 \\ +\alpha\cdot\left(\left(\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} w_{\hat{i}j}^{k*} + \sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}}\left(v_{\hat{i}j}^{k*} + \overline{v}_{\hat{i}j}^{*}\right)\right) - 1\right) \end{array} \right] (\text{start from } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{j\in\mathcal{P}\setminus\{i,\hat{i}\}} \left[ \begin{array}{c} 1\cdot u_{ij}^{\hat{i}*} + \overline{u}_{ij}^{*} \\ +\alpha\cdot\left(w_{ij}^{\hat{i}*} - 1\right) + \lambda\cdot\left(v_{ij}^{\hat{i}*} + \overline{v}_{ij}^{*}\right) \end{array} \right] (\text{tranship at } \hat{i}) \\
+\sum_{i\in\mathcal{P}\setminus\{\hat{i}\}} \left[ \begin{array}{c} \left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} 1\cdot u_{i\hat{i}}^{k*} + 1\cdot\overline{u}_{i\hat{i}}^{*}\right) \\ +\alpha\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} w_{i\hat{i}}^{k*}\right) + \lambda\cdot\left(\sum_{k\in\mathcal{P}\setminus\{i,\hat{i}\}} v_{i\hat{i}}^{k*} + \overline{v}_{i\hat{i}}^{*}\right) \end{array} \right] (\text{end at } \hat{i}).
\end{array} \right.
$$

Obviously, when adding 1 to $\sum_{k\in\mathcal{P}\setminus\{j,\hat{i}\}} 1\cdot u_{\hat{i}j}^{k*} + 1\cdot\overline{u}_{\hat{i}j}^{*}$ and subtracting 1 from $\sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \overline{v}_{\hat{i}j}^{*} + \sum_{j\in\mathcal{P}\setminus\{\hat{i}\}} \sum_{k\in\mathcal{P}\setminus\{\hat{i},j\}} (w_{\hat{i}j}^{k*} + v_{\hat{i}j}^{k*})$, the solution is still feasible. The first inequality holds because $\lambda < \alpha$, and the second inequality holds because $\alpha < 1$. We further see that the value of original objective function is less than that of the new one. Therefore, this contradicts with the assumption. This concludes the proof of the proposition. □

**Proposition 9.** *If a certain port $\hat{i}$ only acts as the destination port without any containers starting from or being transshipped there, in an optimal solution, we may still weigh containers at this port.*

**Proof.** We can prove it by finding an example that satisfies this proposition. Assume that there are 11 ports and only one weighing machine ($Q = 1$), $\hat{i} = 11$, and every port except $\hat{i}$ needs to transfer one container directly to $\hat{i}$, i.e.,

$$\overline{n}_{j\hat{i}} = 1, j \in P\setminus\{\hat{i}\}.$$

We further assume that $N_{ij} = 0, i, j \in P\setminus\{\hat{i}\}, i \neq j$; $N_{j\hat{i}} = 1, j \in P\setminus\{\hat{i}\}$; $0.1 < \lambda < 1$; and $M = 10$. Then, the optimal solution is that all the containers are weighed at port $\hat{i}$. This concludes the proof of the proposition. □

## 4. Model Verification and Analysis

In this section, we use a case study to evaluate the performance of the constructed model. The experiments are conducted on a laptop computer equipped with Apple M2 Pro CPU and 16 GB of RAM, and mathematical models are solved by GUROBI Optimizer via Python 3.8.0.

### *4.1. Experiment Settings*

We select 10 ports along the Yangtze River to test the performance of the model.

The numbers of containers $n_{ij}^{k}$ and $\overline{n}_{ij}$ (TEU) transported from port $i$ to port $j$ per week are randomly generated from a uniform distribution ranging from 1000 to 1500 TEUs. For the total number of weighing machines, we set $Q = 400$. For the maximum weighing capability per machine per week, we set $M = 1000$.

From a safety perspective, the earlier a container is weighed, the safer it is, which means that the benefits to the shipping system diminish when weighing is conducted later. Consequently, for the benefit when a container is weighed at the intermediate port, we set $\alpha = 0.8$; for the benefit when a container is weighed at the destination port, we set $\lambda = 0.6$.

### 4.2. Basic Results of Container Weighing and Allocation of Weighing Machines

The optimal value of the objective function under basic settings is 400,000. The details of the optimal values of the decision variables and the analysis of results are presented in Table 1 and Figure 2, respectively.

**Table 1.** The number of weighing machines allocated to each port $x_i$.

| **Port index** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_i$ | 17 | 46 | 43 | 46 | 42 | 46 | 42 | 43 | 44 | 31 |

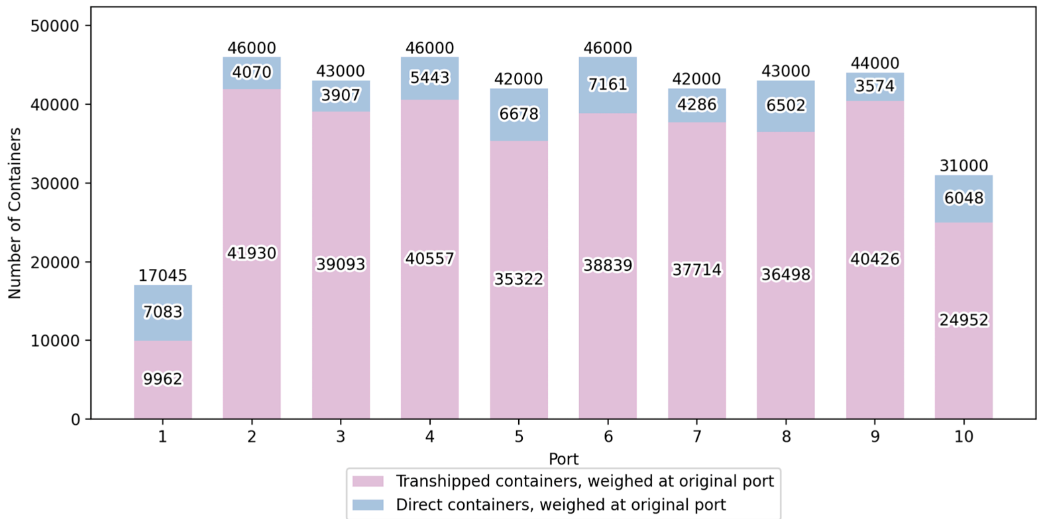


**Figure 2.** Results of the basic setting.

### 4.3. Sensitivity Analysis

In this subsection, we investigate the influence of three key factors—the number of ports, the number of weighing machines, and the capacity of one weighing machine—on solution results.

#### 4.3.1. The Impact of Port Number on Solution Efficiency

In our benchmark experiment, we assume that there are 10 ports. Generally, the larger the model scale is, the lower the solution efficiency will be. In order to check the efficiency of the model, we set the number of ports from 10 to 20 with 1 as the interval to examine the solution time for each problem. The solution time is shown in Table 2.

**Table 2.** Solution time (ms) under different number of ports.

| **Port index** | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Solution time** | 156 | 194 | 236 | 307 | 412 | 527 | 711 | 908 | 1210 | 1540 | 1460 |

We can see that the solution time becomes gradually larger with the increase of the number of ports, but the change is very small. Thus, we can conclude that our model can be solved efficiently even with the increase in the port number.

#### 4.3.2. The Impact of the Number of Weighing Machines

In this analysis, the baseline scenario involves 400 weighing machines. As the number of machines increases to 1013, there is a linear growth in the optimal objective function value, with a slope of 1000. This reflects the maximum weekly weighing capacity ($M$) of one machine, confirming that during this period, all machines are fully utilized to weigh containers at their origin ports. However, between 1014 and 1023 machines, the growth rate of the objective function value decelerates, suggesting that some machines are now weighing containers at transshipment or destination ports. Upon reaching 1024 machines, the objective function value stabilizes, indicating that the capacity to weigh all containers has been met. Figure 3 illustrates these dynamics.

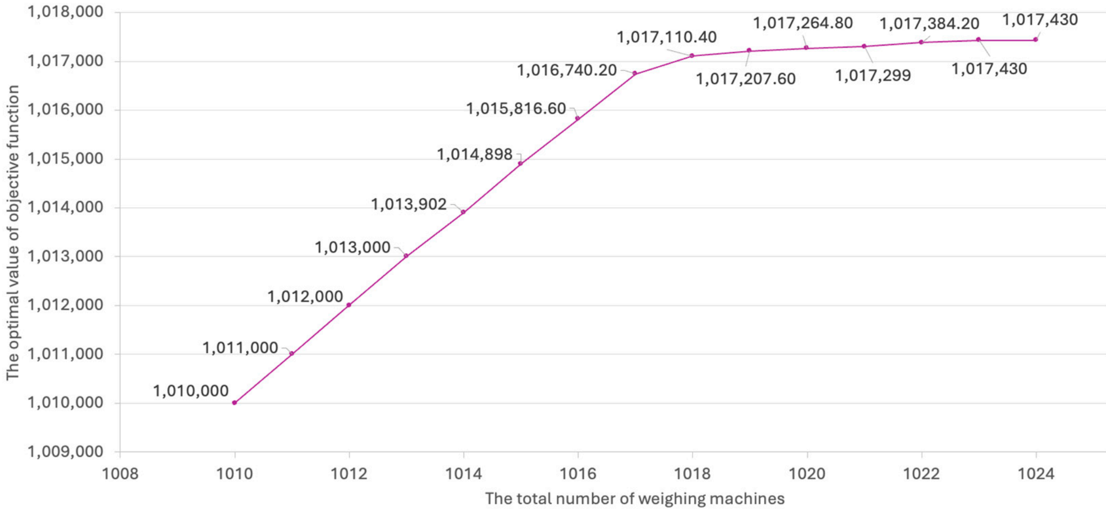


**Figure 3.** The optimal objective function value under different numbers of weighing machines.

Table 3 and Figure 4 illustrate the distribution of weighing machines and containers across various ports when the total number of machines is 1014. At this juncture, the data indicate that containers begin to be weighed at their transshipment location, specifically at port 4. Consequently, the value of the optimal objective function is recorded at 1,013,902.

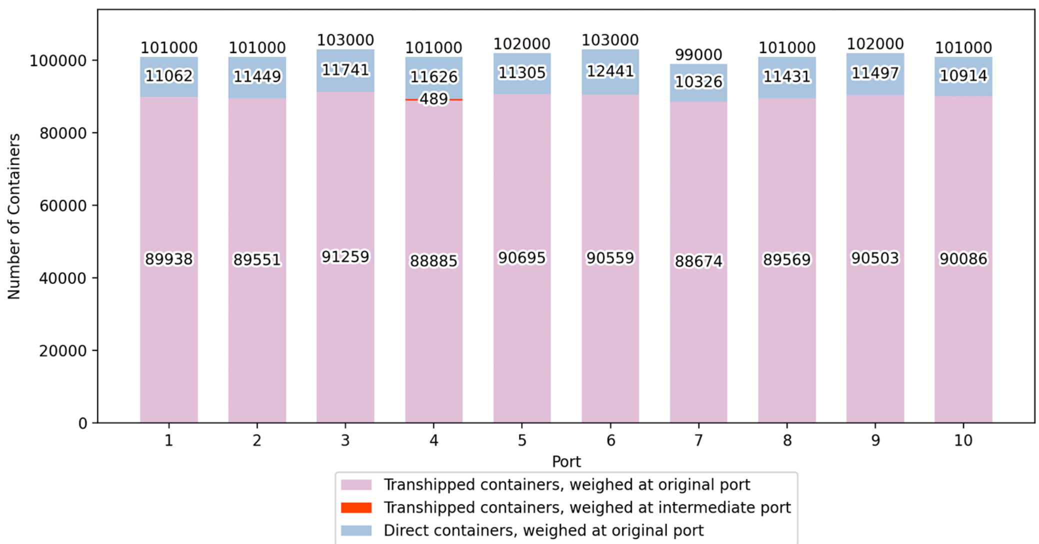


**Figure 4.** Container weighing scheme with 1014 weighing machines.

**Table 3.** The number of weighing machines allocated to each port $x_i$ when $Q = 1014$.

| Port index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_i$ | 101 | 101 | 103 | 101 | 102 | 103 | 99 | 101 | 102 | 101 |

Table 4 and Figure 5 present the allocation of weighing machines and containers across each port, when there is a total of 1018 machines. Notably, several containers are weighed at their intermediate ports: 386 containers are weighed at their transshipment port 1, with 76, 404, and 97 containers weighed at ports 4, 8, and 10, respectively. Additionally, an interesting observation is that while 219 containers destined for port 3 are weighed there, surpassing the initial count of zero, containers transshipped at port 3 are not weighed at this location. This serves as empirical validation of Proposition 6.

**Table 4.** The number of weighing machines allocated to each port $x_i$ when $Q = 1018$.

| Port Index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_i$ | 102 | 101 | 104 | 101 | 102 | 103 | 99 | 102 | 102 | 102 |

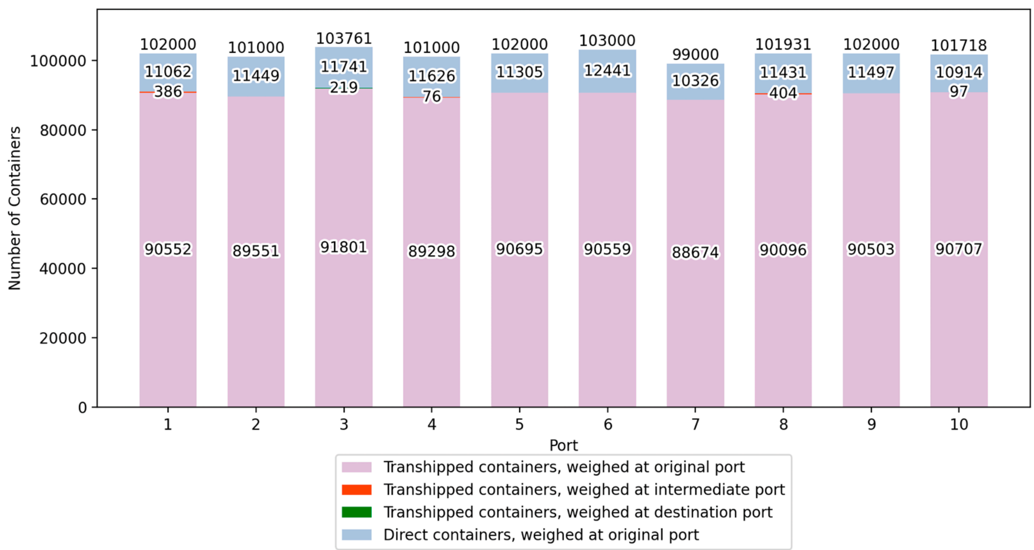


**Figure 5.** Container weighing scheme with 1018 weighing machines.

Table 5 and Figure 6 display the distribution of weighing machines and containers across ports when the total number of machines reaches 1024. Notably, even with an increase from 1023 to 1024 machines, all containers remain to be weighed at their respective ports of origin. This observation confirms that all containers have been weighed, in alignment with Proposition 1. Subsequently, further additions of weighing machines do not alter the objective function value, which remains fixed at 1,017,430.

**Table 5.** The number of weighing machines allocated to each port $x_i$ when $Q = 1024$.

| Port index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $x_i$ | 103 | 102 | 104 | 101 | 103 | 104 | 100 | 102 | 103 | 102 |

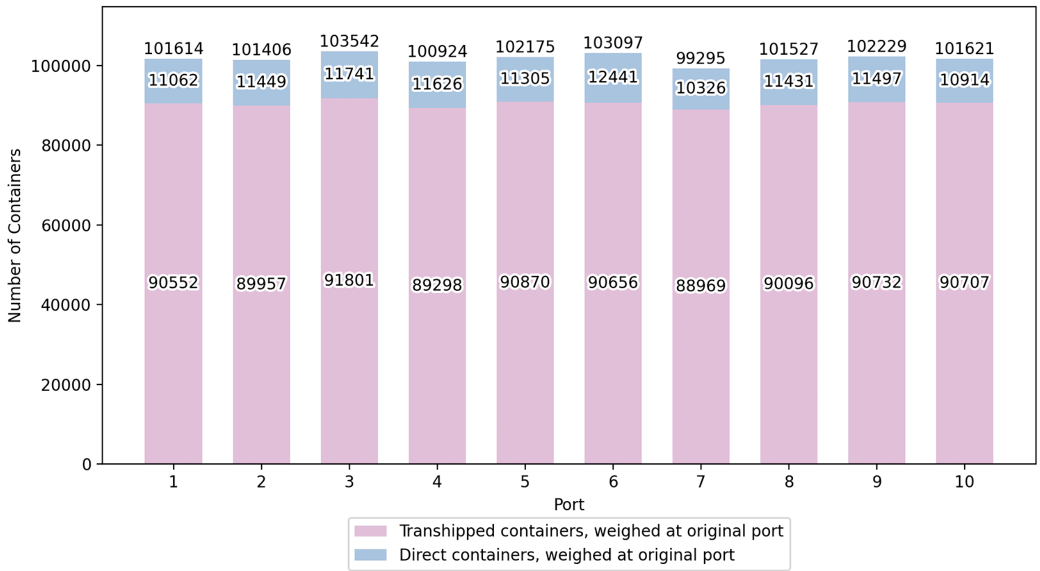


**Figure 6.** Container weighing scheme with 1024 weighing machines.

Through a sensitivity analysis of the number of weighing machines, we determine the relationship between the changes in the number of machines and the corresponding shifts in the value of the objective function. This analysis has substantiated several propositions discussed earlier, illustrating examples of these relationships. Additionally, the analysis provides a visual representation of the variations in the number of weighing machines and the corresponding fluctuations in the number of containers weighed at each port.

#### 4.3.3. The Impact of Weighing Machine Capacity

In our benchmark experiment, each weighing machine initially has a capacity of 1000. As shown in Figure 7, as this capacity increases to 2523, the optimal objective function value exhibits a positive correlation, with a slope of 400, reflecting the constant number of weighing machines ($Q = 400$). This indicates full utilization of all machines for weighing containers at their originating ports during this period. However, as the capacity extends from 2523 to 2578, the growth rate in the objective function value diminishes, suggesting that machines are increasingly used to weigh containers at transshipment or destination ports. Upon reaching a capacity of 2579, the objective function value stabilizes, indicating that the capacity suffices to weigh all containers.

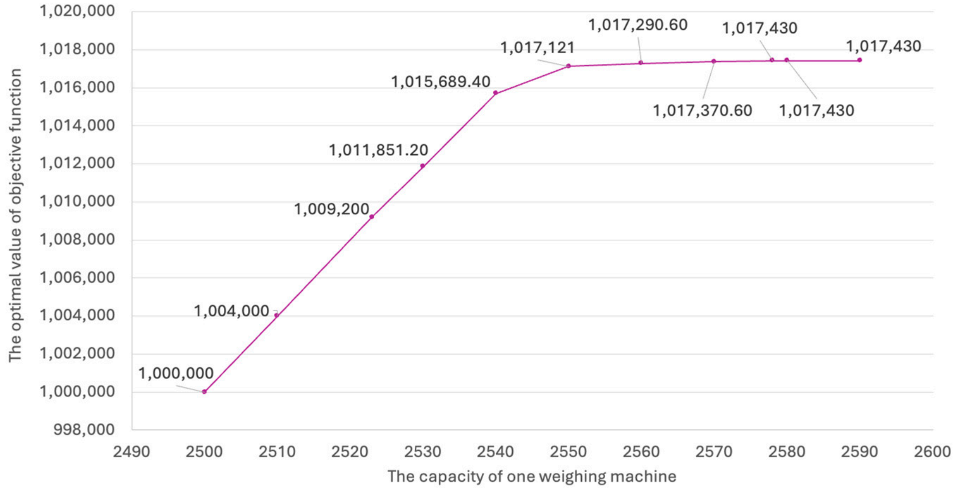


**Figure 7.** The optimal objective function value with respect to the increase of weighing machine capacity.

## 5. Discussion

In this paper, we introduce a container weighing optimization model and validate several propositions derived from this model. A case study conducted on ports along the Yangtze River, coupled with sensitivity analyses of the model, yields the following conclusions, along with practical guidelines for port managers to directly improve operations in the discussed areas.

First, the model proves capable of addressing large-scale port container optimization challenges, offering practical solutions to real-world problems. Port managers can use this model to develop more effective container weighing strategies, thereby enhancing overall operational efficiency.

Second, as the number of weighing machines increases, the strategy for container weighing transitions. When the number of machines is insufficient, only a subset of containers is weighed, but these are all weighed at their origin ports. In this scenario, port managers should prioritize weighing containers that have a significant impact on the total weight. When there are more weighing machines available, more containers can be weighed, but some of them are weighed at transshipment or destination ports. In this case, port managers should strategically allocate weighing locations to ensure that transshipment weighing spots are as close as possible to the destination ports. Ultimately, sufficient machine availability ensures that all of the containers can be weighed at their origin ports. In this context, port managers should focus on maintaining and scheduling weighing equipment to ensure that the weighing process remains efficient and stable.

Third, ports can increase the capacity of each weighing machine, which allows for an increased number of containers to be weighed. When the capacity of the machine is increased, the objective function value increases in a fashion similar to that when the number of machines is increased. Specifically, the objective function value transitions from steady to decelerating before stabilizing. Port managers can increase weighing capacity through upgrading weighing technology, enhancing personnel training, and implementing intelligent management. By implementing these measures, port managers can effectively increase the capacity and accuracy of container weighing machines, thereby improving port operations and enhancing overall competitiveness.

The mathematical model and methodology presented in this paper are not limited to the Yangtze River. The same methods can be applied to any other region by adjusting the relevant parameters in the model. For example, the importance of weighing containers in a certain place can be changed. This allows applying the model in practice across multiple scenarios.

One direction for future research is to design a two-stage stochastic optimization problem with a stochastic demand for containers. Another direction involves determining the model's decisions with the actual historical demand data. Through this study, the model could be refined further, and the inefficiencies of the real maritime system could be diminished.

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## Nomenclature

| | |
|---|---|
| Parameter | |
| $\mathcal{P}$ | The set of ports. |
| $i$ | The index of port, where $i \in \{1, 2, \ldots, P\}$. |
| $\alpha$ | The benefit when a container is weighed at the intermediate port. |
| $\lambda$ | The benefit when a container is weighed at the destination port. |
| $Q$ | The total number of machines used to weigh containers. |
| $N_{ij}$ | The total number of containers transferred from port $i$ to port $j$. |
| $n_{ij}^k$ | The number of containers transferred from port $i$ to port $j$ via port $k$. |
| $\overline{n}_{ij}$ | The number of containers transferred from port $i$ to port $j$ without being transshipped. |
| $M$ | The maximum number of containers to weigh per machine per week. |
| Decision variable | |
| $x_i$ | The number of weighing machines in each port $i$. |
| $u_{ij}^k$ | The number of containers weighed in origin port $i$ transferred from port $i$ to port $j$ via port $k$. |
| $w_{ij}^k$ | The number of containers weighed in the intermediate port $k$ transferred from port $i$ to port $j$ via port $k$. |
| $v_{ij}^k$ | The number of containers weighed in the destination port $j$ transferred from port $i$ to port $j$ via port $k$. |
| $\overline{u}_{ij}$ | The number of containers weighed at the origin $i$ transferred from port $i$ to port $j$. |
| $\overline{v}_{ij}$ | The number of containers weighed at the destination $j$ transferred from port $i$ to port $j$. |

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