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Optimizing Rural Highway Maintenance Scheme with Mathematical Programming

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Abstract: Maintaining rural highways is crucial in ensuring the reliability and efficiency of transportation infrastructure in modern rural areas. Rural highways often suffer heavy traffic from logistics and regular transportation users. The efficient management of these roads is essential to avoid issues like traffic bottlenecks, fuel consumption, and environmental problems. Traditional maintenance approaches focus on cost reduction, which can lead to adverse effects such as network congestion and environmental damage. To address these challenges, this study proposes a bi-level mathematical programming model aiming at optimizing rural highway maintenance. This model balances maintenance costs, network congestion, system fuel consumption, and environmental impacts. By transforming the bi-level model into a single-level mixed-integer linear programming model, the study enhances the computational feasibility, enabling practical implementation using commercial solvers. The model's effectiveness is validated through numerical examples, providing insights for the development of optimal maintenance schedules that minimize externality costs while adhering to financial constraints and operational guidelines, providing a valuable addition to the road engineer's toolbox.

Keywords: rural highway maintenance; bi-level mathematical programming; network optimization; user equilibrium

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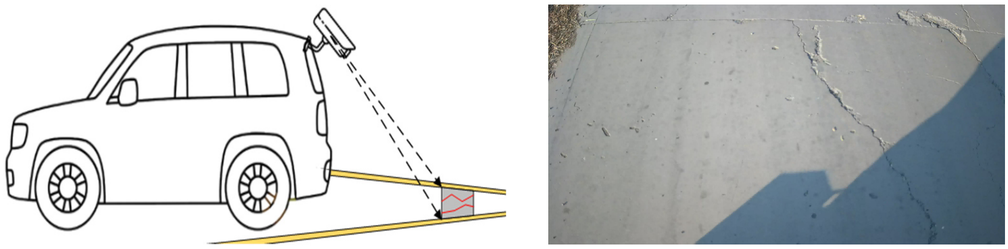
1. Introduction

Proper highway maintenance is essential in enhancing the longevity and serviceability of transportation infrastructure, which in turn is a key component of sustainable and resilient transportation systems. By addressing issues such as pavement degradation and safety hazards, regular maintenance not only extends the life cycle of roads but also contributes to reduced environmental impacts, minimized traffic disruptions, and improved travel efficiency. Road management agencies need to spend significant amounts of money every year on pavement management within the highway network. At present, transportation infrastructure construction in many countries is being gradually completed. The rural roads built on a large scale during the 21st century are about to enter a periodic maintenance peak, necessitating concentrated maintenance work [1]. These road networks are typically vast, making the optimization of maintenance strategies complex. The traditional methods used in the last century have employed single-objective optimization, such as pavement management systems (PMS), aiming primarily at reducing institutional costs. However, these methods often lead to serious traffic disruption, increased vehicle delays, and higher carbon emissions [2,3]. These phenomena all signify an increase in the external costs of road transportation, leading to an escalation in the life cycle costs of road transportation.

Concurrently, they also amplify the difficulty of cost estimation and reduce the operational efficiency of roads and may potentially influence the formulation and effectiveness of traffic policies [4–6]. In light of these issues, highway management agencies increasingly need to adopt advances in operations research and balance multiple decision-making objectives to make wise decisions. This requires consideration of the integrity of the road network and the impact of various types of highway maintenance plans on the overall traffic flow. Figure 1 illustrates a typical form of pavement damage and the pavement management system that detects it. The detection of pavement damage is a mature field, but developing an optimal maintenance scheme that minimizes the delay costs is still a major challenge.



(**a**) (**b**)

Figure 1. (**a**) A pavement management system that detects pavement damage. (**b**) A typical rural pavement with crack damage.

Current research focuses on minimizing institutional maintenance costs and maximizing the pavement quality. Multiple optimization goals are involved in road maintenance, and pavement management systems have employed multi-criteria optimization to incorporate various objectives simultaneously [7–10]. Topics include the utilization of road networks [11]; the road infrastructure quality (e.g., pavement deterioration uncertainty) [12,13]; budgeting and investment, where the budget is also often linked to sustainability goals [14,15]; vulnerable transport infrastructure [16]; road traffic control [17]; interconnection limits [18]; sustainable pavement maintenance (e.g., economy, energy, and the environment) [19–21]; greenhouse gas emissions [22]; interdisciplinary tasks, such as the integration and application of computer technology [23–25]; separating trucks from private vehicles [26–28]; and the impacts of the climate and seasons [29,30]. These are commonly covered in research on highway maintenance. However, the calculation of the comprehensive cost of rural road network maintenance often ignores the user costs [9].

The World Bank defines the benefits of road projects as reducing vehicle operating costs, reducing user time costs, and reducing traffic accidents [31]. Rural transportation networks are systems of transport infrastructure consisting of a series of roads and streets located in rural areas. These networks typically serve rural communities, connecting towns, villages, farmland, and remote areas, and provide essential transport services to local residents, while also supporting agro-logistics, tourism, and other economic activities. As rural areas grow, the complexity and scale of rural transportation networks increase. Many tourists and road users rely on these networks for regular service. Therefore, an irrational highway repair strategy could lead to numerous traffic bottlenecks, which would be detrimental to system users. Additionally, one of the key factors limiting the highway maintenance timetable is the availability of funds. The operators of rural transportation systems must create optimal maintenance schedules within a limited budget, necessitating greater adaptability in mode selection.

To address this problem, we propose using bi-level mathematical programming. At the lower level, the traffic flow generated by the maintenance scheme is determined, while, at the higher level, the road segment combination to be maintained is decided. This issue has been studied extensively and is known as the Discrete Network Design Problem (DNDP). The DNDP is an NP-hard problem. To balance the computing speed and precision, several methods have been developed [32–36]. Although large-scale networks present a well-known NP-hard challenge, small-scale networks can still be solved. Furthermore, by incorporating binary variables, we reformulate the proposed bi-level mathematical programming into a single-level mathematical programming problem. This allows the DNDP to be solved using effective solvers like Gurobi (https://www.gurobi.com/) and CPLEX (https://www.ibm.com/products/ilog-cplex-optimization-studio).

We present a reliable model for the optimization of the rural transportation system. Rural road network administrators can use this model to determine the number of links that they wish to maintain concurrently while adhering to specific constraints, including financial plans, maintenance schedules, and other guidelines. This model enables the development of policies that minimize the total general cost of the rural transportation system. The following summarizes the primary findings of this study:

1. We suggest using bi-level mathematical programming to optimize maintenance projects for rural roads, aiming to reduce the total costs and disruptions to traffic;
2. By incorporating binary variables, we transform the proposed bi-level mathematical programming into a single-level mathematical programming problem, allowing solvers like Gurobi and CPLEX to be used to solve the DNDP;
3. We validate our proposed models on four benchmark example networks—the Eastern Massachusetts network, the Sioux Falls network, the Nguyen–Dupuis network, and the Braess network—providing practical insights into model implementation.

The remainder of this paper is organized as follows. In the second section, we present a mathematical model for the DNDP at two levels. In Section 3, we reformulate the suggested bi-level mathematical model into a single-level form that is amenable to commercial solvers. In Section 4, we validate the proposed model on four benchmark networks: the Eastern Massachusetts network, the Sioux Falls network, the Braess network, and the Nguyen–Dupuis network. Finally, we conclude in Section 5.

2. A Bi-Level Mathematical Programming Model

In this section, we delve into the intricacies of developing a bi-level mathematical programming model tailored specifically to rural highway maintenance management. Before presenting the model itself, we begin by defining a comprehensive set of notations that will be utilized throughout this work. These notations serve as the foundation upon which our model is built, ensuring clarity and consistency in our approach.

A the set of all road segments within the rural road network;

c_p^w the travel time of the route p between OD pair $w \in W$;

f_p^w the flow of the route p between OD pair $w \in W$;

$\mathbf{f}$ the set of route flows, i.e., $\mathbf{f} = [f_p^w],\ \forall p \in R_w,\ w \in W$;

q^w the travel demand between OD pair $w \in W$;

R_w the set of all routes between OD pair $w \in W$;

t_a the travel time on the road segment a;

$\mathbf{u}$ the set of network design decision variables u_a;

u_a the binary variable that specifies whether the road segment must be preserved—if the road segment must be preserved, $u_a = 1$; otherwise, $u_a = 0$;

N the set of all nodes in the rural road network;

V the number of road segments that are maintained simultaneously;

$\mathbf{x}$ a vector that is defined as $\mathbf{x} = [x_a],\ \forall a \in A$;

$\boldsymbol{\delta}$ the road route incidence matrix, $\boldsymbol{\delta} = [\delta_{a,p}^w],\ \forall w \in W,\ a \in A,\ p \in R_w$, where $\delta_{a,p}^w = 1$ if road segment a belongs to route p between OD pair w, and $\delta_{a,p}^w = 0$; otherwise, $\delta_{a,p}^w = 1$.

x_a the traffic flow on the road segment $a \in A$;

Let us consider a rural transportation network $G = (N, A)$, in which the sets of nodes and road segments are represented by N and A, respectively. The Bureau of Public Road (BPR) function, a well-established model in transportation engineering, is utilized to determine the journey time on each road segment. We give the BPR function as follows:

$$t_a = t_{a,0}[1 + b(x_a/y_a)^m] + u_a E,\ a \in A \tag{1}$$

where x_a represents the flow on road segment a; y_a denotes the capacity of road segment a; E is the maintenance cost of the road segments, which is chosen to be a very large number, signifying that no vehicles are allowed to travel on sections of the road that are currently under maintenance; $t_{a,0}$ is the free-flow travel time of road segment a. The parameters b and m can be adjusted based on particular circumstances, although they are typically set to 1 and 4, respectively.

In this section, we propose a bi-level approach to optimize the road maintenance scheme, balancing costs, congestion, fuel consumption, and environmental impacts. The bi-level model is structured as follows.

Upper level:

$$\min_{\mathbf{u},\mathbf{x}} \sum_{a \in A} x_a t_a \tag{2}$$

subject to

$$\begin{cases} \sum\limits_{a \in A'} u_a \le V \\ u_a = 0,\ a \in A \setminus A' \\ u_a \in \{0,1\},\ a \in A' \end{cases} \tag{3}$$

The upper level determines the combination of road segments to be maintained, focusing on minimizing the overall travel time across the network. Equation (2) serves as the objective function, aiming to minimize the total travel time of the entire rural transportation network. This objective is subject to Constraint (3), which limits the number of road segments that can be maintained simultaneously.

Constraint (3) ensures that the sum of the binary variables u_a does not exceed the maximum number of road segments that can be simultaneously maintained at any given time, i.e., V.

Lower level:

$$\min \sum_{a \in A} \int_0^{x_a} t_a(x)dx \tag{4}$$

subject to

$$\sum_{p \in R_w} f_p^w = q_w,\ \forall w \in W \tag{5}$$

$$x_a = \sum_{w \in W} \sum_{p \in R_w} \delta_{a,p}^w f_p^w,\ \forall a \in A \tag{6}$$

$$f_p^w \ge 0,\ \forall p \in R_w,\ \forall w \in W \tag{7}$$

The lower level manages the traffic flow resulting from the maintenance schemes, ensuring the optimal distribution of traffic and minimizing delays. Equations (4)–(7) constitute the lower-level model, which is the well-known user equilibrium (UE) model. This model predicts the traffic flow pattern of the entire network under the given road maintenance scheme. At the user equilibrium, each user attempts to choose the optimal route to their destination, ultimately leading to a situation where the travel cost for each route is equal due to the Nash equilibrium principle [37].

The user equilibrium state is characterized by the fact that no individual user can reduce their travel expenses by switching to a different route. This state is achieved as users continuously adjust their routing choices in response to changes in the network conditions, ultimately converging to a stable state. However, it is important to note that the user equilibrium state is not necessarily unique and can be influenced by various factors, such as the network topology, the travel demand, and the distribution of user knowledge and awareness.

In the context of rural road networks, the scale of the network and the large number of users can lead to complex traffic conditions at the user equilibrium state. This complexity is further compounded by the presence of paradoxes, such as the Braess paradox, which suggests that adding capacity to a network can sometimes result in longer travel times for users. Therefore, the goal of our proposed programming paradigm is to minimize any potential delays caused by maintenance activities while considering the broader implications for the network as a whole.

The bi-level model presented above fits within the framework of Stackelberg games [38], where decisions made at the upper level (e.g., maintenance schedule) influence the behavior and outcomes at the lower level (e.g., traffic flow patterns). However, bi-level models are inherently complex and are known to be NP-hard, meaning that finding optimal solutions for large-scale problems within a reasonable time frame is generally not feasible. To address this challenge, we reformulate the bi-level model into a single-level mathematical programming model that can be effectively solved by commercial solvers. This reformulation involves the introduction of binary variables and linearization techniques, which transform the problem into a mixed-integer linear programming (MILP) problem.

3. A Single-Level Mathematical Programming Model

In the previous section, we discussed the development of a bi-level mathematical programming model tailored specifically to rural highway maintenance management. However, due to the inherent complexity and NP-hardness of bi-level models, solving them for large-scale problems within a reasonable time frame is generally not feasible. To address this challenge, we reformulate the bi-level model into a single-level mathematical programming model that can be effectively solved by commercial solvers. This reformulation involves the introduction of binary variables and linearization techniques, which transform the problem into a mixed-integer linear programming (MILP) problem.

The cost of the road segments that a route passes through is added to form a route cost function:

$$c_p^w = \sum_{a \in A} \delta_{a,p}^w t_a,\ p \in R_w,\ w \in W \tag{8}$$

where $\delta_{a,p}^w = 1$ if route p travels through the road segment a; otherwise, $\delta_{a,p}^w = 0$.

As per the traditional illustration of the user equilibrium (UE) principle in traffic assignment problems (TAPs) [39], we are aware that the system of inequalities can be derived from the lower-level model:

$$\begin{cases} L \cdot \sigma_p^w + \varepsilon \le f_p^w \le M \cdot (1 - \sigma_p^w) \\ L \cdot \sigma_p^w \le c_p^w - \pi^w \le M \cdot \sigma_p^w \\ c_p^w - \pi^w \ge 0 \\ \sigma_p^w \in \{0,1\} \\ \forall p \in R_w,\ w \in W \end{cases} \tag{9}$$

where L and M are parameters that are small and large enough, respectively; the variable σ_p^w is a binary decision variable that is used for reformulation.

However, due to the non-linear variable c_p^w, the above system of inequalities (9) transforms the mathematical programming into mixed integer non-linear programming (MINLP), which is still challenging to solve. Observing that c_p^w is the only non-linear variable in the above model, we use a linearization approach to divide the feasible domains of c_p^w into many small areas. In every area, the variable becomes a linear one. In other words, the system of inequalities (9) is linearized as follows:

$$\begin{cases} L \cdot \zeta_{a,n} \le x_a - K_{a,n} \le U \cdot (1 - \zeta_{a,n}) - \varepsilon \\ \zeta_{a,n} = \zeta_{a,n+1} - \zeta_{a,n} \\ L \cdot (1 - \zeta_{a,n}) \le t_a - a_{a,n} x_a \le U \cdot (1 - \zeta_{a,n}) \\ \zeta_{a,n} \in \{0,1\} \end{cases} \tag{9a}$$

where $K_{a,n}$ is a known coefficient, since it is pre-specified; the variable $\zeta_{a,n}$ is a binary decision variable that is used for reformulation.

Thus, the revised single-level model is provided by

$$\min Z = \sum_{a \in A} x_a t_a \tag{10}$$

$$\begin{cases} \sum\limits_{a \in A'} u_a \le V \\ u_a = 0,\ a \in A \setminus A' \\ u_a \in \{0,1\},\ a \in A' \end{cases} \tag{11}$$

$$\begin{cases} L \cdot \zeta_{a,n} \le x_a - K_{a,n} \le U \cdot (1 - \zeta_{a,n}) - \varepsilon \\ \zeta_{a,n} = \zeta_{a,n+1} - \zeta_{a,n} \\ L \cdot (1 - \zeta_{a,n}) \le t_a - a_{a,n} x_a \le U \cdot (1 - \zeta_{a,n}) \\ \zeta_{a,n} \in \{0,1\} \end{cases} \tag{12}$$

$$\sum_{p \in R_w} f_p^w = q_w,\ \forall w \in W \tag{13}$$

$$x_a = \sum_{w \in W} \sum_{p \in R_w} \delta_{a,p}^w f_p^w,\ \forall a \in A \tag{14}$$

$$f_p^w \ge 0,\ \forall p \in R_w,\ \forall w \in W \tag{15}$$

The NP-hard bi-level model, which is known for its complexity and challenging nature, has been successfully transformed into a mixed-integer linear programming (MILP) problem through the application of the model discussed earlier. This transformation is crucial as it enables the problem to be approached and solved by a wide range of commercial solvers, thereby facilitating its practical implementation.

However, it is important to note that the linearization technique employed in this transformation also has a considerable effect on the computational efficiency. While it simplifies the problem and makes it more tractable for MILP solvers, it also introduces an additional computational overhead. The literature on this topic—specifically reference [32]—provides detailed information and mathematical proof regarding the implications of linearization on computational efficiency.

Furthermore, if a high-resolution linear solution is sought, the computational time required to solve the problem can increase significantly. This is due to the increased complexity introduced by the higher resolution requirement, which in turn demands more computational resources. The pursuit of high-resolution linear solutions, while offering greater precision, escalates the computational demands of our model. The increased complexity, necessitated by a finer resolution, leads to a significant rise in the computational time required to solve the problem, highlighting the trade-off between the solution accuracy and computational efficiency.

Fortunately, the model size that we are concerned with for the maintenance scheme of rural roads is generally not very large. This aspect allows us to effectively solve the transformed MILP problem using a CPLEX solver, which is widely recognized for its robust performance and efficiency in handling large-scale optimization problems.

In the upcoming section, we will demonstrate the applicability of the proposed approach to the road network maintenance scheme issue for four well-known road networks, i.e., the Nguyen–Dupuis road network, the Braess road network, the Eastern Massachusetts network, and the Sioux Falls network. Through this demonstration, we aim to showcase the practical utility of our model and its ability to provide effective solutions to real-world transportation challenges.

4. Numerical Examples

The effectiveness of the proposed rural highway maintenance management approach is demonstrated using four well-known transportation networks, the Nguyen–Dupuis network, the Braess network, the Eastern Massachusetts network, and the Sioux Falls network, of which specific details can be found in reference [5]. The experiments are conducted on an Intel (R) Core (TM) i7 CPU running Windows 11 with 8 GB of RAM.

4.1. Nguyen–Dupius Network

The Nguyen–Dupuis network, as shown in Figure 2, serves as our first case study. The free-flow travel time and capacities of the road segments in the Nguyen–Dupuis network are presented in Table 1. To validate our approach, we assume that three road segments—the red segments 5–9, 7–8, and 9–10—require maintenance. Given that only one road segment can be maintained at each stage, we solve the single-level model using the CPLEX solver.

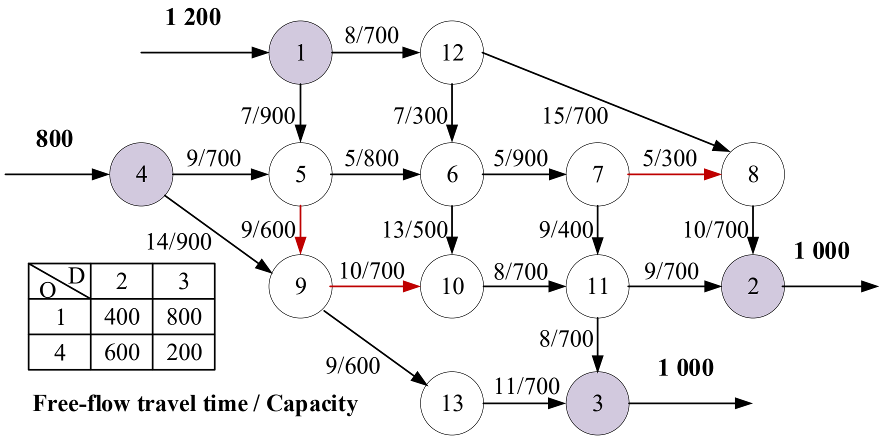


Figure 2. Nguyen–Dupuis network with its OD pairs and capacities.

Table 1. Free-flow travel times and capacities of the road segments in the Nguyen–Dupuis network.

Road Segment	Road Segment Serial	c_a veh/h	$t_{a,0}$	Road Segment	Road Segment Serial	c_a veh/h	$t_{a,0}$
1–5	1	900	7	8–2	11	700	10
1–12	2	700	8	9–10	12	700	10
4–5	3	700	9	9–13	13	600	9
4–9	4	900	14	10–11	14	700	8
5–6	5	800	5	11–2	15	700	9
5–9	6	600	9	11–3	16	700	8
6–7	7	900	5	12–6	17	300	7
6–10	8	500	13	12–8	18	700	15
7–8	9	300	5	13–3	19	700	11
7–11	10	400	9				

Table 1 provides the free-flow travel times and capacities of the road segments in the Nguyen–Dupuis network. The results are summarized in Table 2, which presents the feasible solutions for the first stage of the optimal maintenance scheme. The third column in Table 2 displays the expenses of the maintenance plans, indicating that the third scheme has the lowest cost. Therefore, we recommend maintaining road segments (5–9) first and then continuing with the suggested approach until all three road segments are included in the maintenance plan. The overall cost of the recommended maintenance plan is shown in Table 3.

Table 2. Feasible solutions for the first stage of the optimal maintenance scheme.

Scheme	Road Segment	Cost
1	(9–10)	9009
2	(7–8)	5766
3	(5–9)	2325

Table 3. Optimal maintenance scheme of the Nguyen–Dupuis network.

Stage	Road Segment	Cost of the Stage	Cost in Total
1	(5–9)	2325	2325
2	(7–8)	5766	8091
3	(9–10)	9009	17,101

4.2. Braess Network

In this section, we conduct a sensitivity analysis using the Braess network to further explore the implications of our approach. The network topology and BPR functions of the road segments are depicted in Figure 3. We maintain the red road segments (1–2), (2–3), and (2–4) in this network.

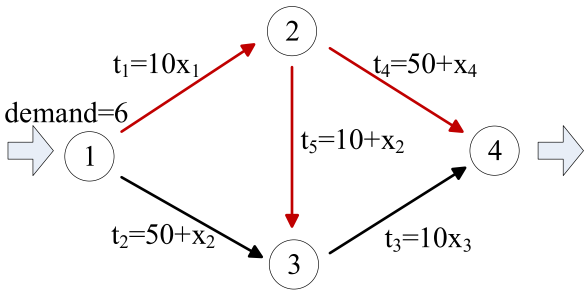


Figure 3. Braess network with the BPR functions of its road segments.

An understanding of the ways in which fluctuations in the travel demand can affect the maintenance schedule is sought. To address this question, we allow the trip demand, which was described as demand = 6 by Braess in 1964 [40], to vary from 1 to 12. The outcomes of the maintenance strategies are presented in Table 4.

Table 4. Optimal maintenance scheme of the Braess network.

Travel Demand	Maintenance Scheme
1	(2–3), (1–2), and (2–4)
2	(2–3), (1–2), and (2–4)
3	(2–3), (1–2), and (2–4)
4	(2–3), (1–2), and (2–4)
5	(1–2), (2–3), and (2–4)
6	(1–2), (2–3), and (2–4)
7	(1–2), (2–3), and (2–4)
8	(1–2), (2–3), and (2–4)
9	(1–2), (2–3), and (2–4)
10	(1–2), (2–3), and (2–4)
11	(2–4), (2–3), and (1–2)
12	(2–4), (2–3), and (1–2)

Table 4 demonstrates that the optimal maintenance plan changes as the travel demand varies. The overall maintenance cost for each maintenance scheme is plotted in Figure 4, showing that the cost rises significantly faster as the travel demand increases linearly. It is found that the total maintenance costs of the scenarios where demand = 5 and demand = 11 slightly drop, as circled in Figure 4. This is because the optimal maintenance schemes of these two scenarios change (refer to Rows 5 and 11 in Table 4). This finding emphasizes the importance of an optimal maintenance plan that minimizes the total maintenance cost for busier rural road networks.

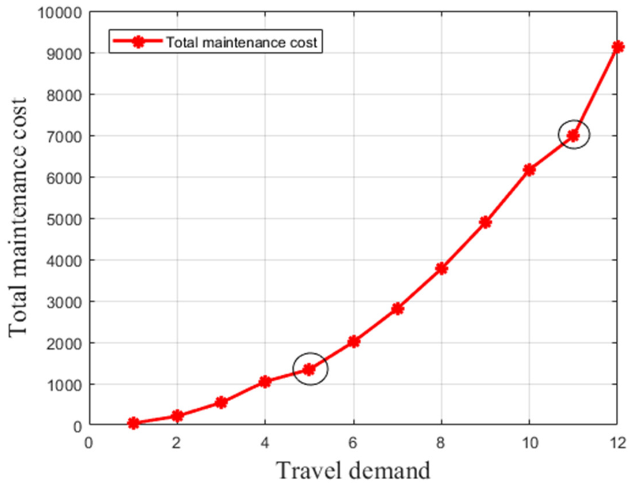


Figure 4. Entire cost of maintenance for each maintenance scheme.

4.3. Eastern Massachusetts Network

Finally, we investigate the computational feasibility of our rural road maintenance model using the Eastern Massachusetts network, a much larger-scale network. We also perform a sensitivity analysis to examine the relationship between the number of country roads and the computing time under a 24 h computational time limitation.

The network topology of Eastern Massachusetts is shown in Figure 5, with the rural road segments highlighted. This network has a total of 74 nodes and 258 road segments. The brown circles in Figure 5 denote the nodes of the network with the nodes numbered. The nodes in orange circles belong to a zone with the same travel demand level. Ten of these segments are randomly selected for maintenance. We test the computing feasibility of the proposed method by varying the number of maintenance road segments. The input is the number of maintenance segments. The output is the computing time. The test results are presented in Table 5.

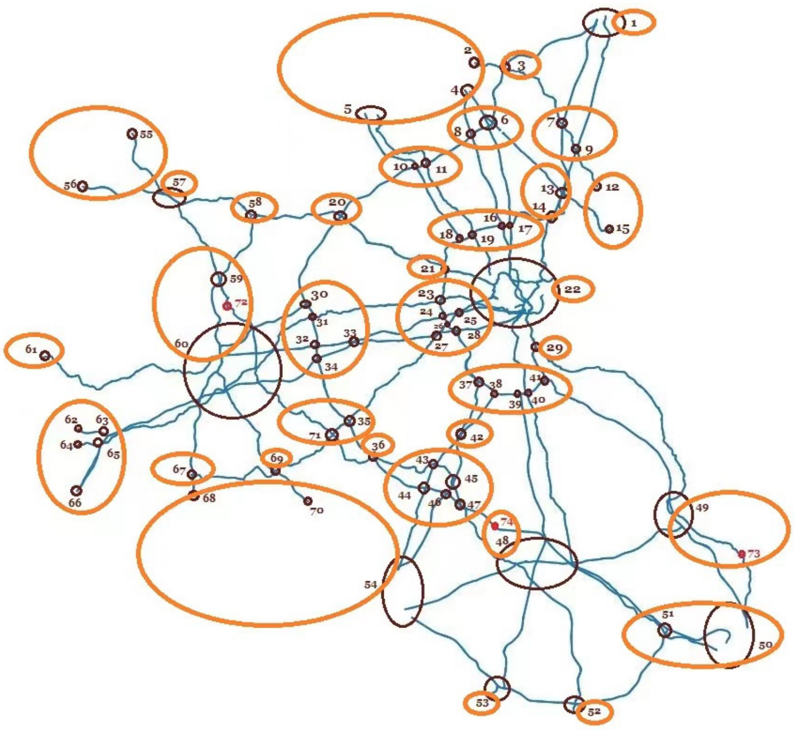


Figure 5. Eastern Massachusetts network with its rural road segments.

Table 5. Sensitivity analysis results using the Eastern Massachusetts network.

Number of Maintenance Road Segments	Computing Hours	Computing Feasibility in 24 h
1	0.02	Yes
2	0.27	Yes
3	1.24	Yes
4	3.67	Yes
5	8.49	Yes
6	16.86	Yes
7	30.10	No
8	49.73	No
9	77.44	No
10	115.09	No

Table 5 and Figure 6 show the relationship between the number of road segments and the computing hours required to solve the problem. The results indicate that the computing time grows exponentially with the number of maintenance road segments.

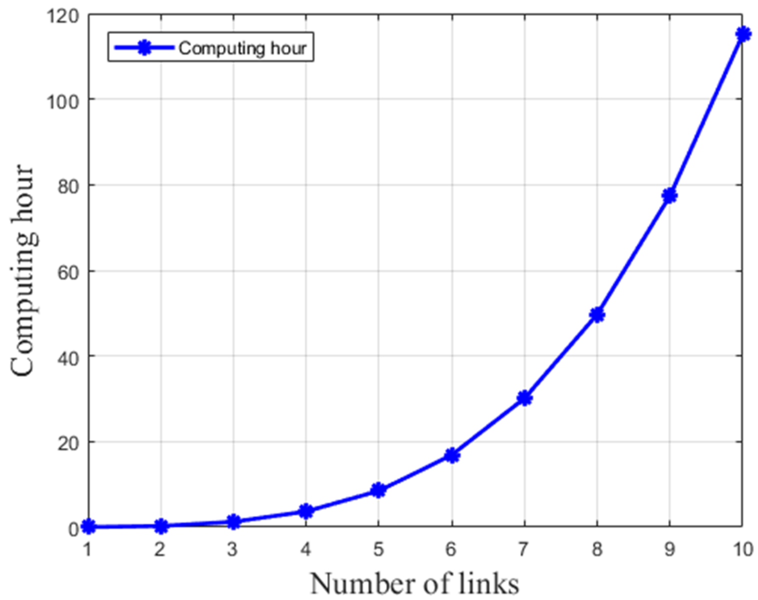


Figure 6. Computing hours against the number of maintenance road segments in the Eastern Massachusetts network.

4.4. Sioux Falls Network

The Sioux Falls network, a standard testbed in transportation analysis [41], represents the road infrastructure of Sioux Falls, South Dakota, and is employed in our study to further validate the scalability and computational efficiency of our proposed model. We also conduct a sensitivity analysis on the computing time against the scales of a test network. The test network used here is the well-known Sioux Falls network, which refers to the largest city in the state of South Dakota in the USA. The topology of this network is given in Figure 7.

We achieve this by partially blocking the OD pairs and links, from 10% to 90%, of the Eastern Massachusetts network. Figure 8a illustrates an example in which 50% of the OD pairs and links are blocked in the network. One should note that the blocking of the network is not a random operation. The blocked OD pairs and links are carefully chosen because the connectivity of the network must be guaranteed. The larger the partition is, the larger computational resources needed. Figure 8b shows the computing time with respect to the scales of the tested network.

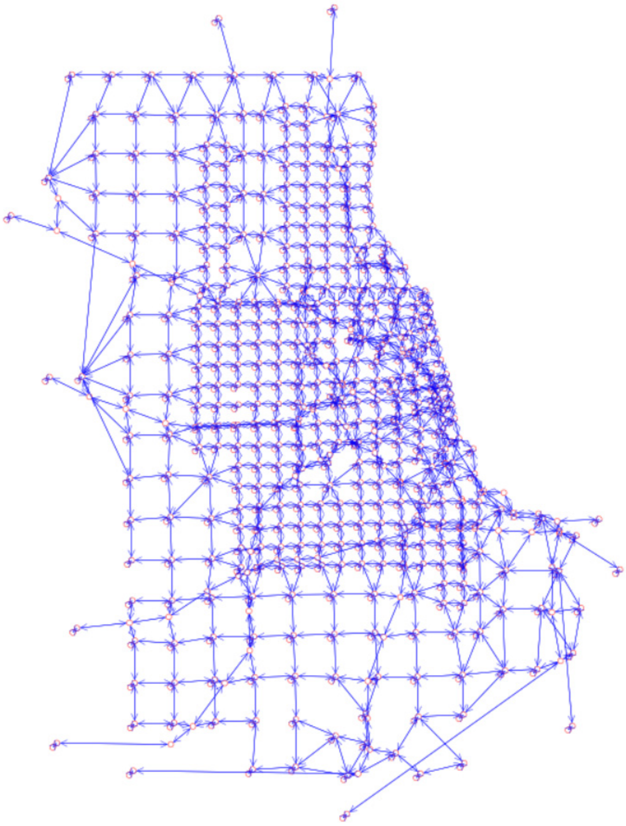
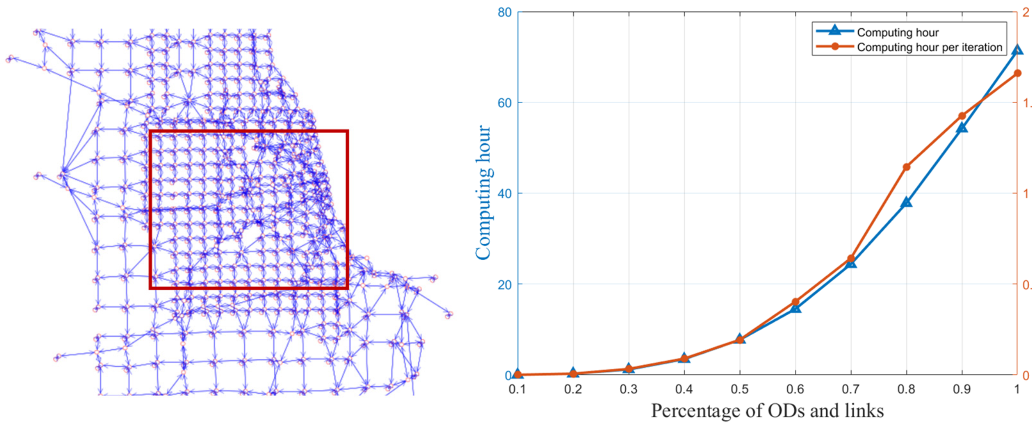


Figure 7. Sioux Falls network with its rural road segments.



(**a**) (**b**)

Figure 8. (**a**) An example in which 50% of the ODs and links, displayed by a red box, are selected. (**b**) The computing hours and the computing hours per iteration with respect to the percentage of the selected ODs and links.

The above results highlight the importance of considering travel demand fluctuations and the scalability of the model for different network sizes.

5. Findings and Research Extensions

This section expands on the previous findings and discusses their implications in the context of current research and the state of rural highway maintenance.

5.1. Model Advancements and Implications

Bi-Level to Single-Level Transformation: The key innovation of this study lies in the transformation of the complex bi-level model into a more tractable single-level MILP problem. This allows for efficient solution using commercial solvers like Gurobi and CPLEX, opening the door to practical implementation in real-world scenarios.

Cost Optimization with User Costs: Unlike traditional maintenance approaches that focus solely on institutional costs, this model incorporates the user costs associated with travel delays and fuel consumption. This holistic perspective leads to more efficient maintenance plans that minimize the total costs, including both direct and indirect impacts on road users.

Sensitivity Analysis: The sensitivity analysis conducted on various network scenarios reveals important insights. The relationship between the travel demand and maintenance cost highlights the need for adaptable strategies that can respond to changing traffic patterns. Additionally, the exponential growth in the computing time with the increasing network size emphasizes the need for efficient computational methods, potentially through decentralized maintenance management.

5.2. Comparison with Existing Research

This study builds upon existing research in the field of rural highway maintenance and optimization. Previous studies have primarily focused on single-objective optimization, such as pavement management systems (PMS) aiming to minimize institutional costs. While these approaches have been successful in reducing maintenance expenses, they often overlook the negative externalities imposed on road users, such as traffic congestion and increased travel times. This study addresses this gap by incorporating user costs into the optimization framework, resulting in more comprehensive and socially responsible maintenance strategies.

5.3. Limitations and Future Research Directions

While the proposed model offers valuable insights, there are several limitations and opportunities for future research:

Non-Synchronous Maintenance Periods: The current model assumes synchronous maintenance periods, where all selected road segments are maintained simultaneously. Exploring non-synchronous maintenance schedules could potentially lead to more efficient solutions, although it would introduce additional complexity.

Dynamic Traffic Demand: The model assumes a fixed traffic demand, whereas real-world traffic patterns can fluctuate significantly. Incorporating dynamic traffic demand models could enhance the realism and effectiveness of the maintenance strategies.

Environmental Impact: While the model considers fuel consumption as an indirect measure of the environmental impact, a more comprehensive analysis incorporating other environmental factors, such as emissions and noise pollution, would be beneficial.

Multi-Modal Transportation Systems: The current model focuses on rural highway networks. Future research could explore the extension of the model to multi-modal transportation systems, incorporating different modes of transportation and their interactions.

6. Conclusions

This study presents a novel approach to optimizing rural highway maintenance using bi-level mathematical programming. By considering both institutional and user costs, the proposed model aims to minimize the total costs while ensuring minimal disruption to the traffic flow. The transformation of the bi-level model into a single-level MILP problem facilitates practical implementation using commercial solvers, making the model accessible for real-world applications.

The numerical examples demonstrate the effectiveness of the proposed approach in various network scenarios, providing valuable insights into the development of optimal maintenance strategies. The sensitivity analysis further highlights the importance of considering travel demand fluctuations and the scalability of the model for different network sizes.

While the study presents valuable findings, further research is needed to explore the potential of non-synchronous maintenance schedules, dynamic traffic demands, and the integration of environmental and multi-modal transportation considerations. Overall, this study contributes to the field of rural highway maintenance by providing a comprehensive and practical framework for the optimization of maintenance strategies while minimizing the total costs.

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