Review

Lattice Structures Built with Different Polygon Hollow Shapes: A Review on Their Analytical Modelling and Engineering Applications

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Abstract: Lattice structures are useful in the aerospace, automotive, infrastructural, and medical fields due to the way they incorporate a lightweight design and good mechanical properties, because of their hollow shapes. This review paper documents work carried out using various analytical models for lattice structures designed with different polygon hollow shapes, for loading in the in-plane and out-of-plane directions, in order to advise their ranking in terms of mechanical behaviour. A primer on lattice structures and polygon hollow shapes is first provided. This is followed by a review of relevant analytical models applied to lattice structures with various polygon hollow shapes that are available in the literature, and then a ranking of the polygon hollow structures in terms of their mechanical properties is performed. Following on from this, a review of the mechanical properties of polygon hollow structures is given. Engineering applications of different polygon hollow structures are then identified. A next-generation structural optimisation and design guide is then highlighted, and some of the primary prospective areas to be focused on when designing lattice parts are pointed out. The last section highlights current challenges, as well as recommendations for extending the use of design for the additive manufacturing of lattice parts.

Keywords: lattice structures; polygon hollow shapes; analytical modelling; engineering applications

1. Introduction

Lattice structures have attracted significant interest in various engineering sectors because of their high specific stiffness [1–4], high specific strength [1,2,5–8], high energy absorption [2,5,7,8], fracture toughness [1,3,7,8], and peculiar lightweight designs [7,8]. Lattice designs are three-dimensional structures built up of a series of interconnecting beams, struts, or Triply Periodic Minimal Surfaces (TPMSs) [7]. They are typically used to build lightweight [8] and robust structures [7,8] in engineering applications. Taking advantage of polygonal hollow shapes in lattice structures gives an added level of design flexibility [7,9]. Hollow polygonal shapes, including hexagons, triangles, rectangles/squares, and circles, provide numerous benefits [7] in lattice structures. These designs have the capacity to improve the distribution of the load [2], stiffness [7], and efficient use of materials [8].

Lattice structures attract significant interest for research in structural engineering, with their importance arising from numerous primary factors, such as their good strength, lightness, and customisability [7,8]. Lightweight structures formed from lattice structures are in substantial demand in an era where sustainability and resource efficiency are key [8,10]. Lattice structures, particularly those built with polygonal hollow shapes, offer the rare opportunity of saving materials while maintaining the structural integrity of components [7]. Furthermore, lattice structures are ideal for use in a number of industries, including the

aerospace, automotive, civil engineering, and biomedical industries, because of their inherent strength-to-weight ratio [11–15]. Engineers optimise the performance of their designs by analysing the use of different polygonal hollow shapes in lattice structures [10,14,15]. The process of designing and manufacturing lattice structures has also undergone radical changes as a result of emerging new materials and manufacturing technologies such as additive manufacturing (AM) [10–15]. These developments have made it possible to build complex lattice structures using different polygonal hollow shapes [7,10,15]. Additionally, lattice structures have applications that go beyond typical structural engineering, such as transferring heat, damping for vibrations, and acoustics [8,12,15]. A review of their application in a number of fields provides cross-disciplinary insights from materials science, mechanical engineering, structural engineering, aerospace engineering, biomechanics, and medical engineering, as well as energy and environmental engineering, AM, computer-aided design (CAD) and computational modelling, and robotics and automation. Comprehending the analytical models for lattice structures built with different polygonal hollow shapes is critical for the optimisation of their design and prediction of their behaviour under different loads and boundary conditions [7]. Given the global trend towards reducing carbon footprints and energy consumption, lattice structures offer an eco-friendly alternative by minimising material use and allowing energy-efficient manufacturing [8,10–12,14,15]. The research area of lattice structures is constantly changing with ongoing advancements and breakthroughs, such as the recently reported work integrating metamaterials into lattice parts [10]. This review attempts to shed light on present trends in structural engineering related to lattice structures and prospective future developments.

This review article first provides an overview of lattice structures and the polygon hollow shapes that are used in them. This is followed by a section that examines current as well as relevant principles and methods for modelling lattice structures. This section additionally identifies current and relevant equations and/or mathematical representations for modelling the behaviour of lattice structures with different polygonal hollow shapes, as well as the effect of the various polygonal hollow shapes on the analytical models. Following this, a section is introduced that highlights mechanical properties of lattice structures built with polygon hollow shapes. A subsequent section focuses on contemporary engineering applications of lattice structures built using different polygon hollow shapes. This section additionally explores how the different topologies of these structures affect their applications. Another section provides an overview, highlighting the next-generation methods of structural optimisation, and a design guide for improving the mechanical performance of lattice structures follows. The last section highlights the current challenges and prospects in extending the application of design for the AM of lattice parts.

2. An Overview of Lattice Structures Built from Polygonal Hollow Shapes

This section gives a brief review of lattice architectures built with different polygon hollow shapes. This is conducted to draw attention to often-used polygon hollow shapes in the design of lattice structures.

Lattice structures that incorporate polygonal hollow shapes do so by building a configuration of interconnecting beams or struts structured in a grid-like pattern. These polygonal hollow shapes, which may comprise triangles, hexagons, rectangle/squares, and circles or other types of polygons, are used as the basic building blocks of lattice structures. The strut-and-node structures differ from the polygonal hollow cells that are the subject of this paper in that the latter have walls, while the others do not. Figure 1 shows polygon hollow shapes typically used in engineering applications [16,17].
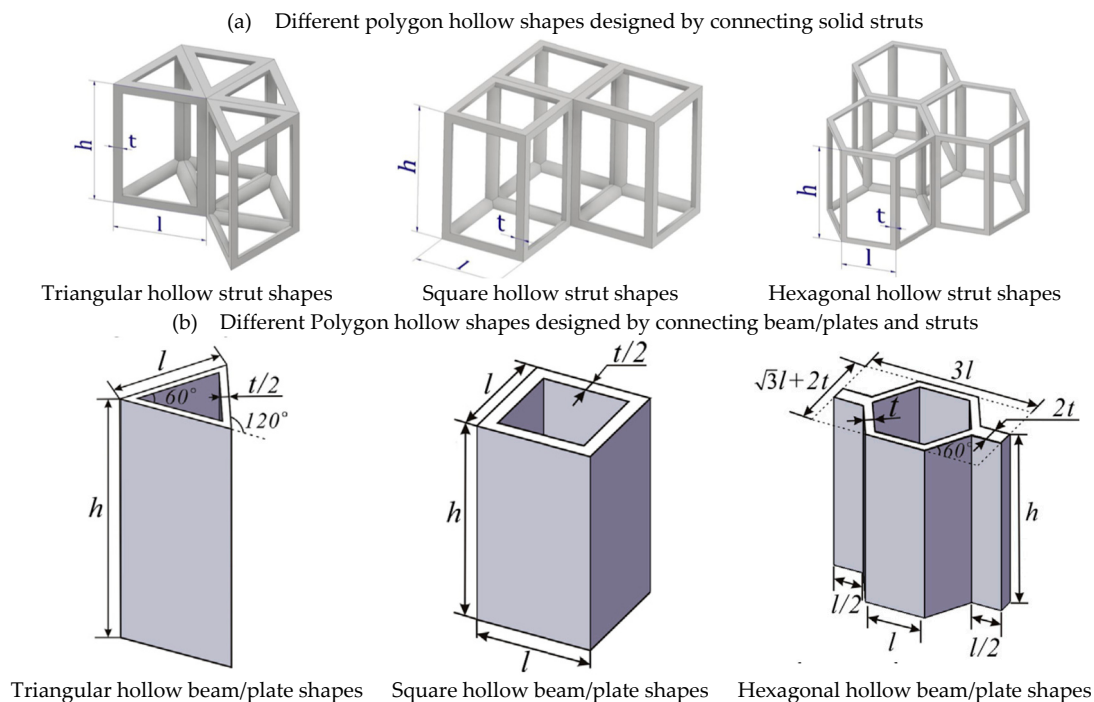
(a) Different polygon hollow shapes designed by connecting solid struts

Triangular hollow strut shapes   Square hollow strut shapes   Hexagonal hollow strut shapes

(b) Different Polygon hollow shapes designed by connecting beam/plates and struts

Triangular hollow beam/plate shapes   Square hollow beam/plate shapes   Hexagonal hollow beam/plate shapes

Figure 1. Typical polygon hollow shapes built using (**a**) struts [16] and (**b**) plates/beams [17].

Figure 1a shows struts being used as the basic building blocks for polygonal hollow structures, whereas Figure 1b displays beams or plates as the alternative basic building blocks. Given that such polygon hollow structures can be built with either one of the two types of basic building blocks, it is imperative to make an informed decision on the best choice using analytical models that describe their respective behaviour [17,18]. As a result, it is crucial to conduct a comparative examination of the behaviour of polygon hollow shapes built using the two building blocks in order to rank them with regard to their mechanical efficiency. Additionally, for lattice structures with the same dimensions, it is important to determine the difference in the amount of material used to design these structures based on the two types of building blocks [18]. Polygon hollow structures built with struts are predicted to use less material than the ones built with beams or plates [16,18]. This is due to the fact that the latter type of building blocks requires the use of more material as compared to adopting struts [17].

In its entirety, selecting between struts and beams in the design and manufacturing of polygon-based lattice structures is determined by the engineering application's particular requirements [18]. When simple, straight-line load pathways are preferred and efficiency in the usage of materials and axial stiffness are crucial, struts are typically used [16]. Beams, in contrast, provide a higher degree of flexibility in terms of how to apply loads and have the capacity to carry loads with more complex distributions [17]. However, their manufacturing could prove more challenging in terms of the selection of materials, speed of manufacturing, and scaling of parts, in addition to requiring more material [15]. The choice between struts or beams/plates must therefore be based on a careful analysis of the primary structural specifications, cost, and desired design outcomes of a project.

3. Analytical Models of Lattice Structures Built with Polygon Hollow Shapes

Analytical modelling of lattice structures constructed with polygonal hollow shapes requires the development of mathematical representations and equations to describe the geometrical and mechanical properties of these structures. This modelling approach is critical for predicting how these structures respond to different loads and also assists engineers and designers in optimising their structural designs. The next sub-section

contains a review of several pertinent analytical models available in the literature for polygon hollow structures, built with plates as the basic building blocks.

3.1. Geometry Representation Models of Selected Polygon Hollow Structures

Geometrical representations of four polygon unit cell shapes, including the hexagon, triangle, square, and circle, are adopted for determining the relevant geometrical parameters used in describing the various lattice designs. Figure 2 shows one of the most commonly used polygonal shapes and its geometrical parameters, the hexagonal cell inspired by the honeycomb [17].
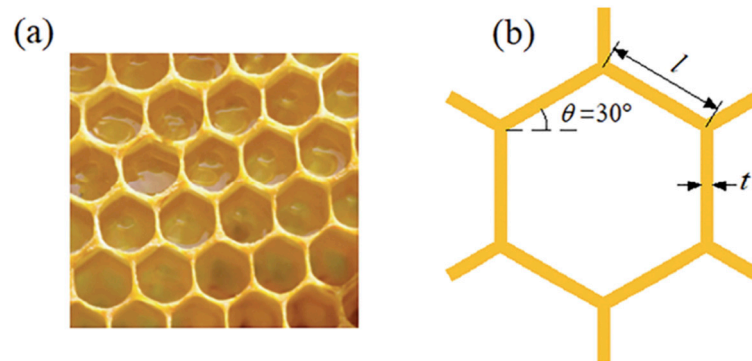

Figure 2. (**a**) Natural beehive honeycomb and (**b**) regular hexagonal geometry of a honeycomb [17].

These geometrical parameters are modelled and represented in the form of density equations for the four polygonal hollow shapes of the hexagon, triangle, square, and circle, as is illustrated by the following equations [18].

$$\frac{\rho_h^*}{\rho_s} = \frac{2}{\sqrt{3}} \frac{t}{L} \tag{1}$$

$$\frac{\rho_t^*}{\rho_s} = 2\sqrt{3} \frac{t}{L} \tag{2}$$

$$\frac{\rho_s^*}{\rho_s} = 2\frac{t}{L} \tag{3}$$

$$\frac{\rho_c^*}{\rho_s} = \frac{4}{\sqrt{3}} \frac{t}{L} \left[\frac{1}{2} + \left(\frac{2\pi}{3} - 1 \right) \frac{R}{L} \right] \tag{4}$$

where the symbols t and L, R, and ρ^* and ρ_s represent the wall thickness and length of the polygon hollow walls, radius of a unit circular cell, and density of the polygon hollow structure and of the solid material for the cell wall, respectively. The parameters ρ_h^*, ρ_t^*, ρ_s^*, and ρ_c^* denote the densities of regular hexagonal, triangular, square, and circular structures, respectively. Equations (1)–(4) show that the relative densities of each of the four polygon hollow cells are related to their ratios of wall thickness t to wall length L. These ratios have a significant impact on the stiffness of polygonal hollow structures. When the thickness (t) is greater than the overall dimension (L), the structure is more resilient to deformation. The material is distributed over a larger cross-sectional area in this particular case, resulting in an improved stiffness. Structures are referred to as thin-walled when their thickness is ten times or less smaller than the overall dimension L. Structures with thin walls are more prone to buckling deflection and are less resilient. Their stiffness is lower given the fact that buckling deflection will precede direct deformation for such thin cross-sections. In investigating all four polygon designs discussed in this review, consistency is guaranteed by considering t/L ratios lesser or equal to 0.1.

Numerous studies [17–22] have, however, ignored analytical modelling of the effects of the connections at the nodes of polygonal hollow structures on their stiffness. This is

a concern because numerous studies [17,23–26] suggest that vertices are highly stressed regions, which, thus, are prone to failing first under applied loads. This challenge is currently the focus of ongoing research by the authors.

3.2. Analytical Models for Predicting the Behaviour of Selected Polygon Hollow Structures

Analytical models predicting the structural behaviour of lattice structures are available in the literature for in-plane and out-of-plane loads [18–26]. The in-plane and out-of-plane loads for the analytical models derived for polygon hollow structures based on plates/beams are depicted in Figure 3.
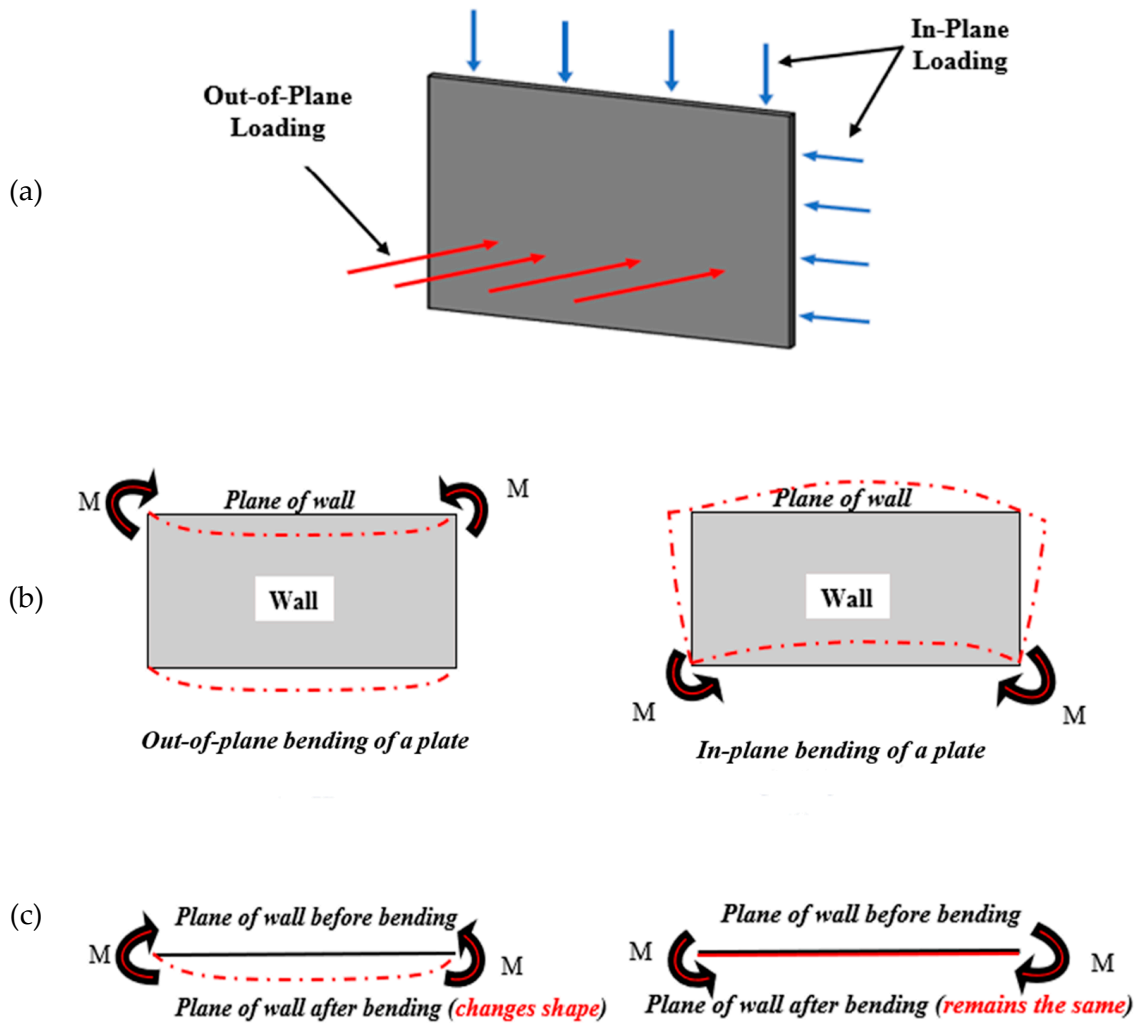
Figure 3. Shows (**a**) in-plane and out-of-plane loading of a plate and (**b**) out-of-plane bending on the left and in-plane bending on the right and (**c**) planes of walls before and after bending.

Analytical models for polygon structures, such as hexagonal hollow shapes built with plates or beams, have been developed using tensile or compressive loads, which leads to respective tensile or compressive deformation behaviour [18,19]. These two deformation behaviours have been investigated in numerous studies, for three mutually orthogonal directions [18]. Figure 4 illustrates the three mutually orthogonal directions used in studying the deformation behaviour of polygonal structures, in this case a hexagonal hollow structure.
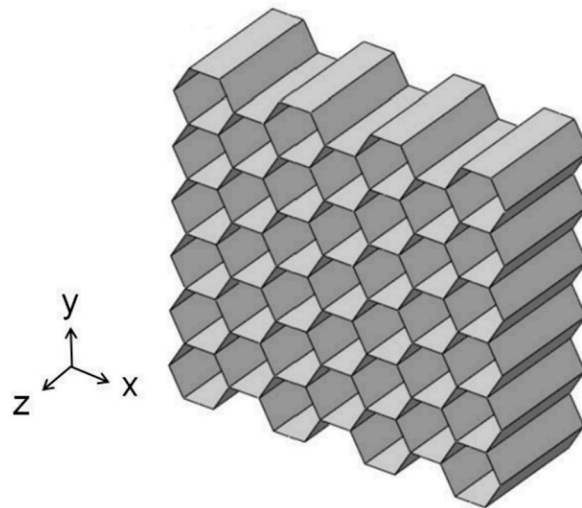
Figure 4. A typical hexagonal hollow structure showing all three mutually orthogonal directions in which the behaviour of polygonal hollow structures is analysed.

Currently, the analytical models for the deformation behaviour of polygonal structures that are available in the literature only describe the bending stiffness of these hollow structures with respect to their elastic moduli [17,18,20] and predict their load-bearing capacity with reference to the yield stress of their wall-material [18,20]. Polygonal structures are particularly useful in applications for high-energy absorption [2,5,7,8,13–17,21–25]. Given the fact that there are no specific models for the absorption mechanisms of energy for polygon hollow structures, this poses a challenge in accurately predicting their related behaviour. Presently, the analytical models that are available in the literature for use in predicting the absorption mechanisms of energy for polygon hollow structures are predicated on general mathematical equations that describe the strain energies caused by bending, tensile, and shear loading [7,17–19]. A supposition is being investigated by the authors in ongoing work that the two-stage stress–strain curve typically used for predicting the behaviour of cellular/lattice structures is not accurate as it does not represent all the deformation mechanisms taking place in a lattice structure. This presents a clear limitation with regard to having specific analytical models for describing the energy-absorption behaviour of polygon structures. Therefore, further investigation is required to address these gaps in knowledge with a view to building more accurate analytical models to describe the deformation behaviour of polygon hollow structures. For particularly porous materials such as polygon hollow structures, the Gibson–Ashby model is typically employed to relate their stiffness and yield strength to density [20].

3.2.1. The Gibson–Ashby Model for a Regular Hexagonal Hollow Cell

Figure 5 shows a unit planar hexagonal hollow cell with two mutually orthogonal planar in-plane directions of loading indicated. Equation (5) represents the analytical model describing the behaviour of a planar hexagonal polygon structure when subjected to loads in the in-plane directions [18,20].

The stiffness of the hexagonal hollow cell is seen from Equation (5) to be related directly to the third power of the ratio (t/L), which from Equation (1) implies a direct relationship to the third power of the relative density of the cell [18].

$$E_y^* = E_x^* = \frac{4}{\sqrt{3}}\, E_s \left(\frac{t}{L}\right)^3 \tag{5}$$

The symbols E^* and E_s in this equation represent the elastic modulus of the polygon hollow structure and that of the solid material of the polygon walls, respectively. Subscripts y and x in the parameter E^* represent the y- and x-directions, respectively. It is garnered

from Equation (5) that the stiffness of the structure with respect to its elastic modulus grows with an exponential of order 3 of the ratio t/L for a hexagonal hollow structure loaded in the two planar in-plane directions, individually.
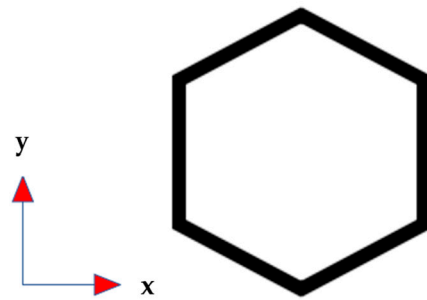

Figure 5. A planar unit hexagonal cell showing two mutually orthogonal planar in-plane directions of loading.

3.2.2. The Gibson–Ashby Model of an Equilateral Triangular Hollow Cell

Figure 6 shows a unit planar triangular cell highlighting the directions for the planar in-plane loading of an equilateral triangular hollow cell. Equation (6) represents the analytical model describing the behaviour of a planar triangular polygon structure when subjected to loads in the in-plane directions [18,20].
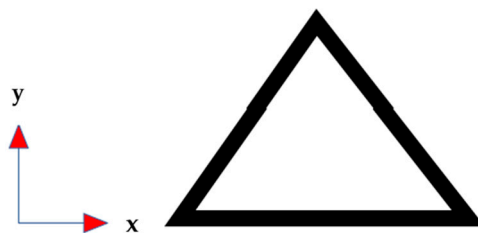

Figure 6. A planar unit triangular hollow cell showing two mutually orthogonal planar in-plane directions for loading.

For this triangular hollow structure, the Equation (6) represents stiffness in each of the two planar in-plane directions [18].

$$E_y^* = E_x^* = 1.15E_s\left(\frac{t}{L}\right) \tag{6}$$

It is observed that the stiffness of the triangular hollow cell is directly related to its relative density, which from Equation (2) is known to be equal to the t/L ratio. This difference in the relationship of stiffness to the t/L ratio between a hexagonal hollow cell and a triangular hollow cell is because the triangular hollow cell deforms primarily through the axial deformation of the cell walls, as opposed to the hexagonal hollow cells, which deform predominantly through the bending of the cell walls [17,18,23]. Equations (5) and (6) show that the triangular hollow polygon cell is less stiff than the hexagonal hollow cell under planar in-plane loading by a factor of 2.

3.2.3. The Gibson–Ashby Model of a Square Hollow Cell

Gibson and Ashby [18] suggested that the stiffness behaviour of a square hollow structure was comparable to that of a triangular hollow structure built using the same relative density. In addition, square hollow structures are designed for particularly anisotropic structural applications [17]. Inherently, the square hollow structure consists of two mutually parallel stiff walls and another two mutually parallel walls that are perpendicular to the first set of walls, which are particularly prone to transverse bending deflections when

subjected to planar in-plane loading [18]. A unit planar square cell showing the planar in-plane directions of loading for a square hollow cell is shown in Figure 7.
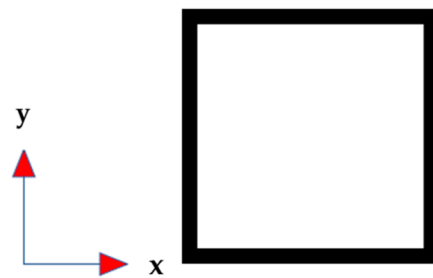

Figure 7. A planar unit square hollow cell showing two mutually orthogonal planar in-plane directions of loading.

Equation (7) represents the stiffness behaviour in either of the two planar in-plane directions of the square hollow cell [18]:

$$E_y^* = E_x^* = E_s\left(\frac{t}{L}\right) \tag{7}$$

The ratio of the stiffnesses of the triangular and square hollow cells is seen from Equations (6) and (7) to be 1.15, with both stiffnesses being directly related to the relative density. Therefore, the triangular hollow cell ranks higher than that of the square hollow cell by a factor of this ratio of 1.15, with reference to transverse stiffness. However, longitudinal deformation was observed to be more prevalent in the triangular hollow cell than in the square hollow cell [18]. Gibson and Ashby [18] determined that the triangular hollow cell underwent less than 2% deformation due to bending loads and that more than 90% of its primary deformation was a result of longitudinal loads. They further found that the square hollow cell deformed by close to 13% due to bending loads and that approximately 80% of its deformation was caused by longitudinal loads. Stiffness was determined by these two authors to be critical for transverse loads.

3.2.4. The Gibson–Ashby Model of a Circular Hollow Cell

A circular unit cell illustrating the two mutually orthogonal planar in-plane directions of loading for a circular hollow cell is shown in Figure 8.
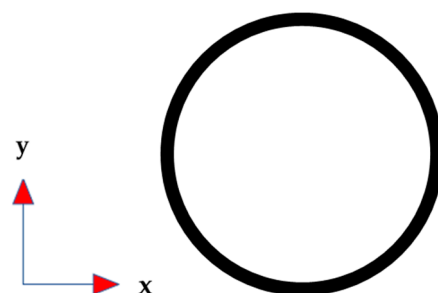

Figure 8. A planar circular unit hollow cell showing two mutually orthogonal planar in-plane directions of loading.

Equation (8) represents stiffness in either of one of the two planar in-plane directions [18]:

$$E_y^* = E_x^* = \frac{8.329}{(1 - \nu^2)}\left(\frac{t}{R}\right)^3 E_s \tag{8}$$

where symbol ν represents the Poisson's ratio of the walls of the hollow cell. The circular hollow cell under planar in-plane compression loads deforms due to the contraction of

the circular unit cells in the direction of the applied load, as well as stretching in the perpendicular direction, the direction without an applied load [2,8,17,18]. Therefore, the loading and perpendicular directions of the circular hollow structure experience a reduction and increase in radius, respectively, upon the application of the load. This, converts the circular cell into an ellipse [18]. From a comparison of Equations (5)–(8), it is evident that the circular hollow structure ranks first with regard to its transverse stiffness for values of the radius (R) of the cell equal to the lengths (L) of the other types of polygon hollow cells considered here. This is because, compared to polygon structures with corners, circular shapes transfer stress more uniformly. Stress in a circular hollow design tends to be distributed uniformly along its circumference, resulting in an efficient load transfer [2,8]. Stress concentrations are often observed at the vertices of polygonal structures with non-circular unit cells, which in turn serve as weak points. Given that circular designs do not have corners, there is a reduced possibility of stress concentrations, leading to them being more resilient against localised deformation and deflection [8].

It is clear from the analytical models reviewed here that the order of the ranking of the four polygonal hollow cells goes from circular to hexagonal, triangular, and square cells, in decreasing order of transverse stiffness.

Analytical models describing the load-bearing capacity of three-dimensional (3D) polygon hollow structures loaded in the cell walls' in-plane directions are lacking in the available literature. Presently available analytical models only describe the load-bearing capability of 3D polygon hollow structures for loads applied in the cell wall out-of-plane direction [2,4,7,8,16–19,23,26]. This is due to the fact that all the identified studies [17,27–35] investigating the crashworthiness of polygonal hollow structures are based on loading in the cell wall out-of-plane direction, as a result of its high capacity to absorb strain energy compared to loading in the cell wall in-plane directions. Therefore, there is a need to develop analytical models that describe the load-bearing capability of polygon hollow structures loaded along the cell wall in-plane direction.

Gibson and Ashby [18] highlighted the fact that the cell wall in-plane loading of polygon structures discussed here could be simplified and represented by a rectangular plate or beam model subjected to either direct or buckling loads based on the t/L ratio that was used. They also highlighted the fact that the cell wall in-plane loading of polygon structures resulted in relatively smaller changes in the load-bearing capacity and stiffness, as the cell wall has insignificant in-plane anisotropy compared to the out-of-plane direction. Figure 9 shows a hexagonal structure subjected to a cell wall in-plane load [36].

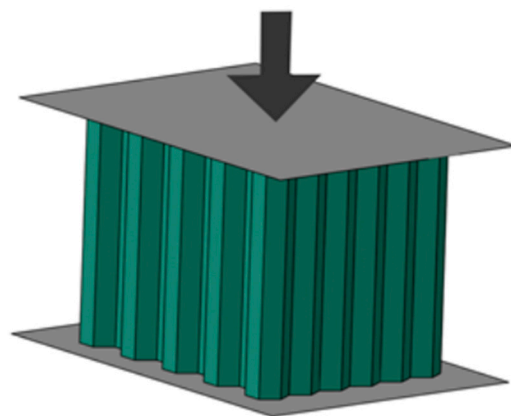**Figure 9.** A hexagonal structure under planar in-plane loading [36].

Predicting the deformation behaviour of polygon structures subjected to cell wall in-plane loads accurately depends on the cell wall in-plane geometrical properties [17]. Compared to the cell wall out-of-plane loading of polygon structures, the cell walls of these structures exhibit a higher stiffness as well as a higher load-bearing capacity along the

in-plane direction. The cell walls in in-plane loading predominantly deform due to stretching, compression, and buckling behaviour instead of the transverse bending deflections observed when the structures are loaded in the cell wall out-of-plane direction [18].

To determine the cell wall in-plane behaviour, the four polygonal hollow cells discussed here are extruded in the in-plane direction to generate 3D cells. To determine the compressive strength related to the elastic buckling behaviour, Gibson and Ashby [18] developed an analytical model representing the compression strength or crucial buckling stress $(\sigma_{el}^*)_3$ of a hexagonal unit cell subjected to cell wall in-plane compression loading given in Equation (9):

$$(\sigma_{el}^*)_3 = 5.2E_s\left(\frac{t}{L}\right)^3 \tag{9}$$

The resulting compressive strength was determined to be 20 times greater than the compressive strength calculated for cell wall out-of-plane loading [18]. This suggests a possibility of using hexagonal hollow structures in crashworthy applications for cell wall in-plane loads instead of cell wall out-of-plane loads for materials with lower stiffnesses that those presently used for their out-of-plane crashworthiness.

Currently, available analytical models face the limitation of being based on unit cells rather than the overall structure [7,17–19,23,26]. This approach fails to take account of the influence of connectivity of the unit cells in a structure and therefore cannot accurately predict how the final polygon structure responds to applied loads. For one, the unit cells are connected at the vertices, which are designated as high-stress concentration regions and are prone to failure under lower applied loads [17–19,23]. However, the vertices act to stiffen lattice structures [17,19]. Therefore, there is a need to build analytical models that account for the whole structure's configuration and to introduce the mechanics related to the connectivity of cells at the vertices. Furthermore, the analytical models for polygonal shapes under cell wall in-plane loading are only available in the literature for hexagon hollow structures. Therefore, analytical models should be developed for the other polygonal shapes in order to accurately predict their behaviour rather than having to estimate their response based solely on the available model for the hexagonal hollow structure.

The current analytical models of polygon hollow structures in the literature are typically based on the four basic polygon forms, namely the hexagonal, square, triangular, and circular cells presented here. Thus, this review on analytical modelling is limited to the foregoing four shapes. Other types of polygon hollow shapes that could be adopted in building lattice structures include pentagonal, octagonal, dodecagonal, irregular, mixed, kagome, and rhombic shapes, despite the lack of knowledge of their corresponding analytical models. Figure 10 shows these other types of polygonal shapes that could potentially be used for designing hollow lattice parts.
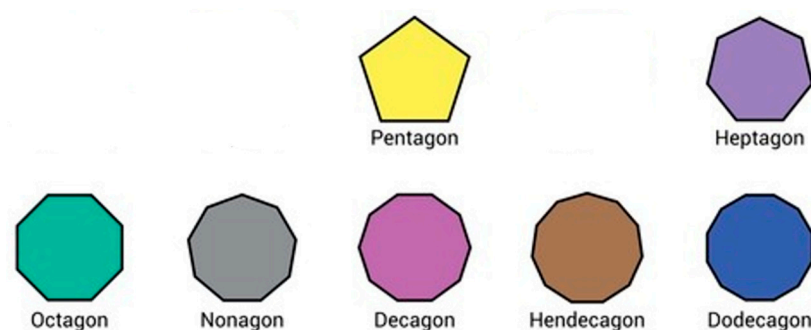

Figure 10. Other types of polygonal shapes that could potentially be used in generating hollow lattice designs for different engineering applications.

However, numerous research [5,7,13,16–19,22,37] has been conducted using other types of lattice shapes excluding the polygonal ones, such as the strut-based, skeletal

TPMS-based, and sheet TPMS-based geometries. Figure 11 shows examples of other types of hollow structures adopted in numerous studies [13,16,17].
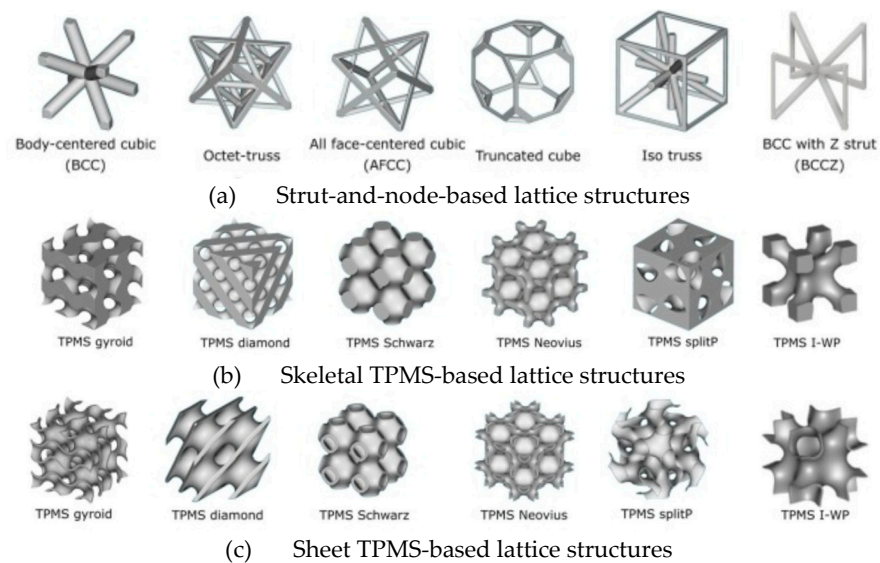

(a)   Strut-and-node-based lattice structures

(b)   Skeletal TPMS-based lattice structures

(c)   Sheet TPMS-based lattice structures

Figure 11. Other types of hollow structures adopted in engineering applications [13,16,17].

4. Mechanical Properties of Lattice Structures Built with Polygonal Hollow Shapes

A growing number of engineering and manufacturing applications today take advantage of lattice structures built with polygonal hollow shapes because of their lightweight design [1,2,4,7,15,17,24–27,29–36]. Multiple materials, such as composites, metals, and polymers, are available to build these lattice structures [15,37,38]. The shape of the polygons [17,18], the material used [15,37,38], the manufacturing method adopted [12–14,38], and the overall geometry of the lattice design [17] have a bearing on the mechanical properties of such lattice structures.

The degree of porosity inherent in polygon structures is influenced by the density of the lattice structure, which is determined by the lattice design and the size of the polygonal hollow shape used [2,13,17]. Porosity in this case refers to the volume of the hollow space against the volume of the entire structure. A higher porosity lowers the overall weight of the structure but could potentially reduce its structural integrity [2,8,17]. To repair human bone defects, Wang et al. [39] built four Ti6Al4V scaffolds inspired by honeycomb designs. They observed that increasing the degree of porosity inherent in lattice designs lowered the stress shielding between the bone and lattice implant. This was because the elastic modulus of the implant was lowered to match that of the bone by increasing the volume of pores in honeycomb scaffolds.

The load-bearing capacity of a polygon structure is influenced by the choice of materials, the geometry of the lattice structure, and the loading conditions. The strength-to-weight ratio can be optimised through the careful design of these factors. For modelling and predicting structural behaviour under different loading conditions, finite element analysis (FEA) is often adopted [40–43]. For modelling the behaviour of lattice structures, FEA requires significant quantities of computational power. This is attributable to the high structural complexity of lattice structures. As a result, methods of simplification and advanced strategies for modelling are necessary to address this challenge. Zhang et al. [17] reviewed different geometrical designs of lattice structures and observed that they typically show great compressive strength. However, they also highlighted the fact that particular geometries of structures and chosen materials had significant effects on their compressive strength. Benedetti et al. [16] reviewed the properties of tensile strength of lattice structures and observed that they presented a challenge, as tensile loads were often transferred by the struts, which were designated as thin members that were then liable to fail by buckling at low applied loads. The tensile strength of the lattice structures is often influenced by

the polygonal shapes used to build them; therefore, different morphologies of polygon structures are likely to differ in terms of their mechanical properties [1,7,8,13,18]. In impact or crash conditions, the energy absorption capability of a lattice structure is critical. Numerous studies [17,31,44,45] have highlighted the fact that the integration of the material properties and lattice design have a substantial impact on the structure's capacity to absorb and disperse energy. Additionally, lattice structures are typically used in applications that require cyclic loading [13,16,46,47]. Their fatigue resistance is determined by the choice of materials, architecture, and manufacturing standards [13,16]. It is clear from this that a substantial fatigue life requires an appropriate engineering design, manufacturing methods, and choice of materials.

Under compressive loads, numerous studies [8,13,16–18] showed that lattice structures were prone to buckling. For polygonal hollow structures, the critical buckling loads were observed to be influenced by the shape and size, strut thickness, and material properties of the specific polygon used [8,17,18]. The resistance of a polygon hollow structure to the propagation of cracks was also noted to be critical for its endurance. It was noted that the fracture toughness of the hollow structure was also influenced by the choice of materials and the degree of imperfections or notches inherent in the structure [3]. Heat exchangers and other applications requiring thermal properties were observed to often use lattice architectures [13,48–50]. The heat transfer and thermal conductivity of lattice structures are both known to be affected by the shape and configuration of polygonal structures [17].

Polygon hollow structures are capable of damping vibrations at different levels, determined by the choice of lattice design and material properties [13,51]. This could prove useful in applications where controlling vibration is critical. In addition, the processes of manufacturing polygon hollow structures, such as AM (3D printing) or conventional machining, could have a bearing on their mechanical properties [11–15,19]. Thus, it is known that complex lattice structures that are difficult to fabricate using conventional techniques are now attainable using AM technologies [15,19,38].

When designing lattice structures using polygonal hollow shapes, it is important to strike a balance between the structural performance and reduction in weight. Computer-aided design (CAD) and simulation technologies can assist in optimising the design to meet specific mechanical requirements for a specific use. Additionally, experimental testing and validation are often required to confirm whether the lattice structure behaves as envisioned through design in general engineering applications. Cutting-edge manufacturing technologies such as AM are strongly recommended for incorporation into the design and manufacturing procedure of lattice structures [8–10].

5. Engineering Applications of Lattice Structures Built with Polygon Hollow Shapes

Recent advances in polygon lattice part design and fabrication have had a significant impact on engineering applications related to the aerospace, automotive, and medical engineering industries [8,10,14–17,36,39,45,46]. Additionally, the good mechanical properties of polygon hollow parts have assisted in addressing some of the primary gaps related to the manufacturability of complicated engineering structures [10,17,45,46]. The engineering application of polygon hollow parts is predicated on their mechanical properties [17,18]. At macroscales, polygon hollow structures are used for numerous applications, such as absorbing strain energies, air circulation, thermal management, noise reduction, the transmission of light, and magnetic shielding [17]. A number of advanced manufacturing technologies have also emerged over the past three decades and have extended the application to structures of lower scales. The lower scale structures include micro- and nano-built polygon hollow parts, as well as their evolutional architectures [17,18].

5.1. Applications in Aerospace Industries

Polygonal lattice designs are advantageous for aerospace uses because of their high strength-to-weight ratios. These structures are capable of reducing the overall weight of aircraft, spacecraft, and drone parts, such as wing ribs, landing gear parts, and fuse-

lage frames, while preserving their structural integrity [17,52]. This in turn, reduces the consumption of fuel and also improves the mechanical performance [8,17]. Lightweight structures play a crucial role in the design of satellites and spacecrafts to reduce costs associated with launches as well as improve the success rate of missions. Polygon hollow structures are often used to reduce the weight in satellite structures, antenna supports, and other parts [8,53]. The structures are additionally used in the design of rocket parts, such as the rocket body and engine parts. Their lightweight characteristics assist in improving the cargo capacity and mechanical performance [8,53,54]. Heat management is also crucial in spacecraft and aeronautical applications. Lattice parts with polygon hollow shapes are typically built with channels for the passage of coolant or heat transfer fluids, making them therefore ideal for lightweight, high-efficiency heat exchangers [48,49]. Polygon hollow structures are used for building radiation barriers, which act as protective covers for antennas and sensors on aircraft and spacecraft. These structures offer electromagnetic transparency whilst remaining lightweight [48]. Furthermore, antennas on aircraft and spacecraft are supported by lightweight lattice structures, allowing the transmission of signals and reception while minimising weight and aerodynamic effects [55].

Solar arrays for spacecraft and satellites are readily designed using lattice architectures. These structures support solar panels while minimising the weight, resulting in an improved power generation in space [56–58]. In addition, lightweight polygon hollow structures are particularly useful for unmanned aerial vehicles (UAVs), wherein a balance of the structural integrity and weight is crucial for increased flight periods and the performance of missions [57]. These structures are also used in composite materials, particularly for aerospace uses that enhance the structural performance while retaining the lightweight property [56]. Integrating AM methods, such as selective laser sintering (SLS) and fused deposition modelling (FDM), into the design of lattice parts allows for the manufacture of complicated polygon hollow structures that can be tailored to specific applications in the aerospace sector [12,59].

5.2. Applications in the Automotive Industries

To reduce the overall weight while still maintaining the structural integrity, lattice structures composed of polygon hollow shapes are used in the design of chassis and frames of vehicles [17]. As a result, the fuel efficiency and overall performance are improved. In addition, lightweight polygon structures are built into body panels to maintain the structural integrity while lowering the weight of vehicles [8,10,60]. During collisions, lattice structures with polygonal hollow shapes are meant to absorb and disperse the impact energy more effectively. This enhances safety in vehicles by reducing the magnitude of force transmitted to passengers. Bumpers with lattice structures offer impact resistance, as well as the absorption of energy, thus increasing safety in collisions at low speeds [53,60,61]. The geometric shapes of lattice structures are chosen to allow for a good absorption of energy during crashes, and such structures are best suited for parts that receive impact, such as front and rear crash zones [17,53,61]. Integrating polygon hollow structures into particular regions of a vehicle, such as crash boxes, assists in controlling and distributing energy during a collision, thus leading to an enhanced crashworthiness [17,31,61].

Seat frames are designed with lattice structures, allowing for a balance of strength and reduction in weight. This improves the fuel efficiency and overall performance of vehicles [62]. Integrating lattice structures into interior parts, including dashboards and trim panels, also assists in reducing the weight of non-structural parts while preserving the safety or aesthetic appeal [63]. In the design and manufacture of control arms for suspension systems, lattice structures offer the required stiffness and strength while maintaining the weight at a minimum [17,62]. Furthermore, lightweight lattice structures are typically used in the design of springs, reducing unsprung material, and optimising the control of vehicles and comfort of the passenger [62].

Lattice structures are useful for the design of heat-dissipating parts, such as engine mountings and heat shields, where an efficient heat transfer is crucial for maintaining ideal

operating temperatures [62,64]. Lattice structures are used in the design of grilles and air intakes as well, to preserve the structural integrity whilst optimising the aerodynamics, reducing the drag, and improving the fuel efficiency [62]. The incorporation of lattice parts within designs for wheel rims reduces the weight whilst maintaining the strength, improving the performance and control in vehicles [17]. In order to integrate considerations of weight with structural requirements, lattice structures are also used in the design of parts within electric vehicle (EV) battery packs [65].

5.3. Applications in Medical Industries

Lattice structures constructed out of polygonal hollow shapes are capable of mimicking the structure of bone [8]. These structures are adopted for bone implants due to the fact that they are more effective in integrating with natural bone tissue and encourage the regeneration and healing processes of tissues [8,17]. Such structures are additionally used for building scaffolds for the replacement or repair of soft tissue, as well as providing structural support for the growth of cells and regeneration of tissues [16,17,20]. Moreover, the porous nature of polygon hollow structures facilitates precise control over the distribution of drugs as well. Different drugs or chemical compounds are incorporated into the design of lattices, and their release is controlled by the design of the structures. This provides a more targeted and carefully controlled delivery method for drugs [8,16]. For procedures in medical imaging, polygon hollow structures are often used as phantoms, whereby they mimic the structure of human tissues, thus assisting in the calibration as well as validation of imaging devices, such as magnetic resonance imaging (MRI) and computed tomography (CT) scanners [66].

Lattice-structured tools and devices are lightweight but rigid and are therefore ideal for surgical procedures [17]. They can be tailored by being manufactured hard in particular regions whilst maintaining flexibility in other regions, which allows for precise as well as reduced invasive operations [8,16]. Lattice structures are implemented for generating complex three-dimensional configurations for cell cultures that mimic the natural structures of tissues or organs. This is particularly useful in the development of organ-on-a-chip systems or bioreactors for the testing of drugs and tissue engineering [67]. In addition, custom lattice-structured prostheses offer the advantage of conforming and therefore are fitted to a human's exact anatomy. The strength, weight, and flexibility of these structures can be optimised, resulting in more beneficial and functional prosthetic devices [67]. Dental implants like prostheses make use of lattice designs. They are capable of increasing osseointegration and the jaw-bone strength, improving the mechanical performance and life expectancy of dental implants [17,67].

It is deduced from this section of this paper that lattice structures built of polygon hollow shapes are highly adaptable and can be precisely tailored to meet specific engineering applications. Moreover, computer-aided design (CAD) software and contemporary manufacturing methods such as 3D printing have simplified the design and fabrication of these structures. Given their lightweight nature as well as their good strength-to-weight ratio, lattice structures are particularly advantageous for applications necessitating both structural integrity and weight reduction.

6. Next-Generation Structural Optimisation and Design Guide for Lattice Structures

The next generation of structural optimisation and design principles for lattice structures is expected to incorporate a number of breakthroughs and trends in response to changing technology, materials, and engineering requirements. Some prospective paths for the development of lattice structures are discussed in the following paragraphs.

Through the advancement of metamaterials and engineered composites, lattice structures are expected to be designed with materials that have specific properties, such as variable stiffness or properties that can dynamically change based on external influences, such as stress or temperature [68–70]. Using auxetic metamaterials in lattice structures provides varying stiffnesses. Auxetic materials have a negative Poisson's ratio and expand

in numerous directions when stretched. Engineers incorporate these materials to build lattice structures that dynamically vary their stiffness depending on the applied load. As an illustration, in the field of sports equipment, a tennis racket could potentially be designed with lattice structures that use auxetic materials. During a robust swing, the lattice structure customises to the amount of load applied, optimising the racket's stiffness for the maximum energy transfer. Additionally, new lattice designs could incorporate bio-inspired materials with exceptional strength-to-weight ratios, mimicking the efficiency seen in structures such as bone or spider silk [17]. Lattice parts could be manufactured using synthetic materials that mimic specific properties of spider silk, such as tensile strength and elasticity. Spider silk is recognised for its remarkable strength and elasticity. Mimicking the molecular structure of the proteins of spider silk in synthetic materials could yield lattices that are both strong and resilient. These structures potentially have uses in a variety of fields, including aerospace engineering as well as construction. Machine learning and artificial intelligence (AI) are coming into use to automate and optimise the design process, allowing for the rapid production and testing of multiple design alterations. The AI and machine learning techniques are expected to be implemented more often to optimise lattice structures by analysing huge amounts of data and iteratively improving designs, resulting in more efficient designs [71,72]. Artificial intelligence (AI) algorithms can respond to real-time feedback during the manufacturing and testing processes, by incorporating this knowledge into subsequent design iterations. As lattice structures are built and tested, AI systems are able to gather information about the designs' actual performance and behaviour. The algorithms use this feedback to iteratively improve future designs, learning via both successes and failures. This adaptive learning strategy speeds up the optimisation of lattice designs by using real-world performance data. Furthermore, improving predictive capabilities by using digital twin technologies to simulate and analyse the functionality of lattice structures in real-world engineering conditions is anticipated as well [73]. Using contemporary 3D modelling and simulation software to generate digital replicas of actual lattice parts, digital twin technologies allow for the generation of computer-generated alternatives that mirror actual lattice designs, capturing their geometry, material properties, and behaviour under various loading conditions. Quantum computing is expected to come into play for use in highly complicated simulations and optimisations, thus allowing for the design of traditionally unattainable lattice structures [74]. Engineers must also interact with modern design software and AI to create novel and efficient lattice structure designs [72]. In addition, engineers will ultimately be allowed to enter more complicated and specific design constraints as well as performance goals into automated generative design procedures, and the modelling packages will suggest optimised lattice designs [10]. Lattice structures that are tailored to certain performance parameters, such as the maximum strength-to-weight ratio, can be created. Generative design techniques, which are driven by complex algorithms, would not only explore a wide range of design choices but additionally pick and recommend lattice structures that are optimised based on the provided constraints and goals. This optimisation could involve the material usage, manufacturability, and overall performance, resulting in highly efficient lattice designs.

Multi-material AM technologies are emerging that enable the printing of lattice structures with graded materials and different properties within the same structure [8]. Techniques such as multi-material 3D printing and Digital Light Processing (DLP) with numerous resin vats can be used. These methods allow the simultaneous use of various materials during the printing process. Multi-material 3D printing, for example, allows for the layer-by-layer deposition of different materials, resulting in it being easier to build complex structures with multiple material compositions. Improved methods of AM, such as nanoscale printing, have the capacity to support the manufacture of complicated lattice structures at extremely small dimensional scales and thus serve to update industries such as microelectronics and the manufacture of medical devices [17]. Nanoscale 3D printing uses techniques such as two-photon polymerisation and direct laser writing. Current 3D printing processes could be limited in their ability to produce high-resolution structures at the

nanoscale. Improved methods, such as two-photon polymerisation or direct laser writing, provide an improved precision and resolution, allowing for the creation of complex lattice structures at levels that cannot be achieved with conventional manufacturing procedures.

Material advancements could also allow lattice structures to self-assemble or repair damage, providing a greater resilience and life expectancy [9,12]. Materials with shape-memory capabilities are capable of self-assembling into lattice forms. Shape-memory materials can change their shape in response to external stimuli, such as temperature or stress. By incorporating these materials into lattice designs, they could self-assemble into predetermined configurations, improving the manufacturing effectiveness and facilitating the building of complicated structures, without the need for external assembly procedures. Additionally, lattice parts that offer different functions, such as structural support and energy storage or sensing capabilities, could become more prevalent. The integration of sensors into lattice structures is expected to allow for the real-time monitoring of stress and strain, allowing responses that adapt to changing conditions or loads. This will allow the growth of design principles that take account of dynamic loading conditions, which is particularly crucial for applications in aerospace, automotive, and civil engineering [1,2,17].

Methods of topological optimisation are changing as well to take advantage of more complex constraints and allow for the generation of lattice designs that are tailored to specific functions [10], considering thermal, acoustic, and multifunctional constraints in addition to structural requirements. Topological optimisation has generally focused on structural challenges. However, emerging approaches now include a broader set of constraints, allowing engineers to optimise lattice designs for heat dissipation, sound absorption, and other particular functional requirements.

A focus on concepts of sustainable design should also result in the manufacturing of lattice structures constructed using eco-friendly and recyclable materials. Circular design principles, such as designing for disassembly and reusability, are expected to reduce waste and encourage a more sustainable lifecycle for these structures [17]. Bio-based polymers, recycled metals, or sustainable composites are used to produce lattice structures. Sustainable design strategies include using materials that have a lesser environmental impact. For lattice structures, this could include choosing materials produced from renewable sources, recycled content, or those with a lower environmental imprint. Bio-based polymers, for example, can be used to produce functional and eco-friendly lattice structures.

The next generation of the design and optimisation of lattice structures is anticipated to be highly dynamic, driven by emerging technology and the growing demand for robust, lightweight, recyclable, and efficient structures in a number of industries. This will lead to more environmentally friendly and advanced cutting-edge methods and technology.

7. Challenges and Future Prospects in Design for Additive Manufacturing of Lattice Structures

Design for the additive manufacturing (DfAM) of lattice parts holds enormous promise for the aerospace, automotive, and medical engineering industries, but it is also facing a number of challenges as well, which raises prospects for the future to consider.

7.1. Challenges Related to Design for Additive Manufacturing of Lattice Structues

Given the complexity of the geometry, designing intricate lattice parts can be challenging. This could result in manufacturing difficulties related to the support structure requirements, printability, and post-processing constraints. AM often results in rough surfaces that require post-processing in order to improve the surface design and mechanical properties. Post-processing methods for lattice parts tend to be challenging to optimise [1,2,6–8,13,14,17]. For instance, challenges associated with removing support structures without damaging delicate lattice features. After printing, lattice parts typically call for post-processing to remove support structures. The delicate nature of lattice features makes this process challenging, as excessive force or the use of improper techniques could damage the structure. Addressing these post-processing limitations may require careful

study and manual labour. Furthermore, identifying and developing materials that are equally good for AM and with the capacity to attain the required mechanical properties of lattice designs is presently a significant challenge as well [13], for example, developing polymers or metal alloys that perform well during the layer-by-layer deposition process in 3D printing. Some materials might not be naturally compatible with specific additive manufacturing procedures. Certain polymers or metal alloys, for example, can present challenges during the printing process due to their melting viscosity, cooling rates, or adhesion, compromising the overall quality and structural integrity of lattice structures. The current methods for optimising lattice structures are not capable of taking advantage of AM's capacity. These methods fail to take full advantage of multiple objectives, as well as other aspects, such as support structure requirements, layer adhesion, and minimising the manufacturing time while maximising the structural performance. Adapting designs for different scales and resolutions in AM technologies, as well as guaranteeing the structure's integrity and performance over different sizes, is presently challenging [5,11,16,38,47,55,56]. In a lattice design, weight reduction is balanced with the mechanical strength and thermal conductivity. Traditional optimisation methods could zero in on a particular objective, such as minimising the material usage in lightweight structures. However, in additive manufacturing, where complex lattice geometries are feasible, it is possible to optimise multiple targets at the same time, such as obtaining lightweight designs with precise mechanical and thermal properties.

Maintaining consistency and quality control in AM is difficult, particularly for lattice parts requiring a specific degree of porosity inherent in the structure and geometrical accuracy [11,16,38,55]. In addition, building advanced multi-material and multi-color printing technologies in order to attain adaptable and useful lattice structures is also difficult. To ensure that the planned lattice parts meet performance and safety requirements that could vary from typical solid structures, advanced validation procedures are required. These methods are presently challenged when it comes to predicting the behaviour of resulting lattice designs accurately [11,16,17], for instance, constructing lattice parts with a specific degree of porosity for use in bone implants. Particular lattice parts, such as those used in medical implants, could require an accurate degree of porosity for biological integration. Maintaining consistency in attaining the correct porosity over multiple prints while adhering to quality control is often difficult due to differences in material characteristics, printing conditions, and post-processing steps.

7.2. Future Prospects in Design for Additive Manufacturing of Lattice Structures

Ongoing developments in designing tools ideally suited to lattice structures could assist in making complex designs simpler and more accurate to generate. Engineers will be able to rapidly generate complex lattice parts tailored for AM because of the ongoing developments of contemporary design software packages. Additionally, the research and development of new materials designed for AM in particular, with customised characteristics, lightweight properties, and structural integrity, is anticipated to improve the functionality of lattice parts [10,11], for instance, the introduction of CAD (computer-aided design) plugins or specialised design algorithms that make it easier to create lattice geometries. Current advancements in design tools aim to develop capabilities and plugins for CAD software that cater specifically to lattice designs. These tools could produce complex lattice designs automatically, giving engineers efficient and user-friendly interfaces for exploring, modifying, and generating complex lattice geometries.

Further advances in methods of topology optimisation are expected to assist in the generation of lattice designs that fully use the design freedom offered by AM machines [8,11,71,72]. Future work consists of AI-driven design procedures that automate and optimise lattice structures for specific performance criteria while simultaneously taking multiple variables into account all at the same time [71,72]. The generation of lattice structures for medical implants that are optimised for biocompatibility and load-bearing capacity is one such example. Future advances in AI-driven design techniques could auto-

mate the building of lattice designs matched to particular performance requirements. For instance, in the medical industry, an AI-driven design strategy could generate lattice structures for implants with optimised properties for biocompatibility, load-bearing capacity, and additional characteristics relevant to the medical application. Moreover, digital twin technology, in conjunction with upgraded simulation, is expected to play a significant part in predicting the behaviour of lattice designs during and after AM and in this way assist in the performance analysis and quality control of the resulting parts [71], for instance, the use digital twin technology to compare the virtual models of lattice structures to actual 3D scan data of manufactured products for the reason of quality control. Digital twins allow for a direct comparison of the virtual and physical representations of lattice structures. By integrating 3D scan data from the actual product into the digital twin, methods of quality control can detect differences between the intended design and the manufactured part, thereby ensuring quality and consistency.

Lattice designs are predicted to be used in structures that have different functions, such as embedded sensors for real-time monitoring, conduits, and advanced metamaterials [68–70], for instance, designing lattice structures within aircraft parts that integrate functions such as fuel conduits or vibration dampening. Lattice designs are often used in aeronautical applications when a reduction in weight is critical. Beyond structural support, lattice structures can incorporate additional functions, such as conduits for fuel or hydraulic fluids, which improves the overall efficiency of aircraft. This multifunctional strategy expands the applications of lattice structures in aerospace engineering. Collaboration across the globe should prove crucial in order to facilitate the sharing of knowledge and best practices and in overcoming global challenges as the field of AM lattice designs grows [17]. This can be achieved through the creation of online platforms or forums for researchers and engineers worldwide to communicate research papers, methodologies, and insights learned in AM lattice design. The setting up of websites for sharing knowledge will allow experts to share their experiences, successes, and challenges with AM lattice designs. This collective exchange of information will assist in identifying best practices, efficient design strategies, and insights gained, thus allowing a community-driven strategy for advancement. In addition, mimicking the design concepts observed in biological structures, such as trabecular bones or honeycombs, should provide new alternative designs for lattice parts [17]. This can be achieved through the use of optimisation algorithms inspired by natural phenomena, such as genetic algorithms or swarm intelligence, to improve lattice design. Nature-inspired optimisation algorithms are designed to replicate natural problem-solving processes. Applying these methods to lattice design could result in novel geometries that are optimised according to particular performance criteria. For instance, evolutionary algorithms can iteratively evolve lattice structures to attain desired characteristics, resulting in alternative designs that are challenging to envision using traditional methods.

The use of DfAM for lattice structures offers a promising future, with the capacity to change industries such as aerospace, automotive, and medical engineering. Addressing the existing challenges and exploring prospects in this respect should pave the path for new and efficient designs that take full advantage of the capacities of AM.

8. Summary of the Main Findings in this Paper

This section serves the purpose of providing a quick overview for readers. It underscores the significance of the material presented in this review paper. Table 1 summarises the primary findings of this review on the design and application of polygon hollow structures in various fields of engineering.

Table 1. Summary of the main results of the review sections on the design and application of polygonal lattice structures.

Section of Review	Main Findings
■ An overview of lattice structures built from polygonal hollow shapes	● The choice as to whether to use struts or beams/plates for building polygonal lattice structures is based on an in-depth analysis of the project's primary structural specifications, cost, and desired design implications.
■ Analytical models of lattice structures built with polygon hollow shapes	● Numerous research works [17–22] have overlooked analytical modelling of the effects of polygonal hollow structures' node connections on stiffness. This is a worry because multiple studies [17,23–26] indicate that vertices are highly stressed locations and so are more likely to fail first under applied loads. ● Currently, the analytical models available in the literature for predicting the energy absorption mechanisms of polygon hollow structures are based on generic mathematical equations that characterise the strain energies induced by bending, tensile, and shear loading [7,17–19]. ● The authors are presently investigating limitations of two-stage stress–strain curves commonly used to predict the behaviour of cellular/lattice structures as they do not include all of the deformation mechanisms that occur in lattice structures. ● According to the analytical models described here, the four polygonal hollow cells are ranked in decreasing order of transverse stiffness, from circular to hexagonal, triangular, and square. ● The present literature lacks analytical models that describe the load-bearing capability of three-dimensional (3D) polygon hollow constructions loaded in the cell walls' in-plane directions. ● Currently, available analytical models have the drawback of being based on unit cells rather than the overall structure [7,17–19,23,26]. This approach ignores the influence of the unit cell connection in a structure and so cannot accurately anticipate how the final polygon structure will behave to applied loads. ● The only analytical models for polygonal shapes under cell wall in-plane loading available in the literature are for hexagonal hollow structures. As a result, analytical models for the other polygonal forms should be constructed to accurately predict their behaviour, rather than relying exclusively on the available model for the hexagonal hollow structure.
■ Mechanical properties of lattice structures built with polygonal hollow shapes	● When building lattice structures using polygonal hollow shapes, it is critical to strike the right balance between the structural performance and weight reduction. ● Computer-aided design (CAD) and simulation technologies can help optimise designs to fulfil specific mechanical criteria for a given application. ● In general, engineering applications, experimental testing, and validation are frequently required to confirm whether the lattice structure performs as designed. ● Cutting-edge manufacturing methods, such as AM, are strongly recommended for incorporation into the design and manufacturing procedure of lattice systems.
■ Engineering applications of lattice structures built with polygon hollow shapes	● Lattice structures made of polygon hollow shapes are highly adaptable and can be accurately tailored to satisfy specific engineering applications. ● Given their lightweight nature and high strength-to-weight ratio, lattice structures are particularly useful for applications that require both structural integrity and weight reduction.
■ Next-generation structural optimisation and design guide for lattice structures	● The next generation of lattice structure design and optimisation is expected to be highly dynamic, driven by evolving technologies and the growing demand for strong, lightweight, recyclable, and efficient structures across a wide range of industries. This will result in more ecologically friendly and novel procedures and technology.
■ Challenges and future prospects in design for the additive manufacturing of lattice structures	● The use of DfAM for lattice structures has a promising future, with the potential to transform industries such as aerospace, automotive, and medical engineering. Addressing current challenges and investigating opportunities in this area can pave the way for new and efficient designs that make full use of AM capacities.

9. Conclusions

Polygon-based lattice structures are typically designed and manufactured using struts or beams. The mechanical properties of lattice parts are different for varying designs and configurations. The planar in-plane loaded circular hollow structure ranks highest in terms of the transverse stiffness compared to those built using hexagonal, triangular, and square cells. Polygon lattice structures are predominantly used in applications requiring a high absorption of strain energy. This is associated with their capability in absorbing high strain energies for loading in the planar out-of-plane directions. The higher values of stiffness of lattice structures in the planar in-plane directions create possibilities of using them this way for crashworthiness using materials with lower values of stiffness. The analytical models adopted for describing the load-bearing capability of polygon lattice parts loaded in the planar in-plane directions are observed to be limited to general mathematical representations of strain energies. Additionally, the analytical models are lacking with regard to accounting for the effect of unit cell connectivity or the way in which the polygon lattice architecture functions. For loading polygon hollow parts in the planar out-of-plane direction, analytical models in the literature are observed to be limited to the hexagonal shape only. This creates space to develop models based on other types of polygon structures. Lattice structures built using polygon shapes are highly adaptable and tailored for different applications in the aerospace, medical, and automotive industries. Lattice structure design and optimisation is expected to be more dynamic, as a result of technological advances such as AM and the rising need for robust, lightweight, reusable, and effective structures in different engineering industries.

Data Availability Statement: The raw data supporting the conclusions of this article will be made available by the authors on request.

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