Supplementary Materials: Validation of a Mathematical Model for Green Algae (*Raphidocelis Subcapitata*) Growth and Implications for a Coupled Dynamical System With *Daphnia Magna*

Michael Stemkovski, Robert Baraldi, Kevin B. Flores and H.T. Banks

1. Residual Plots

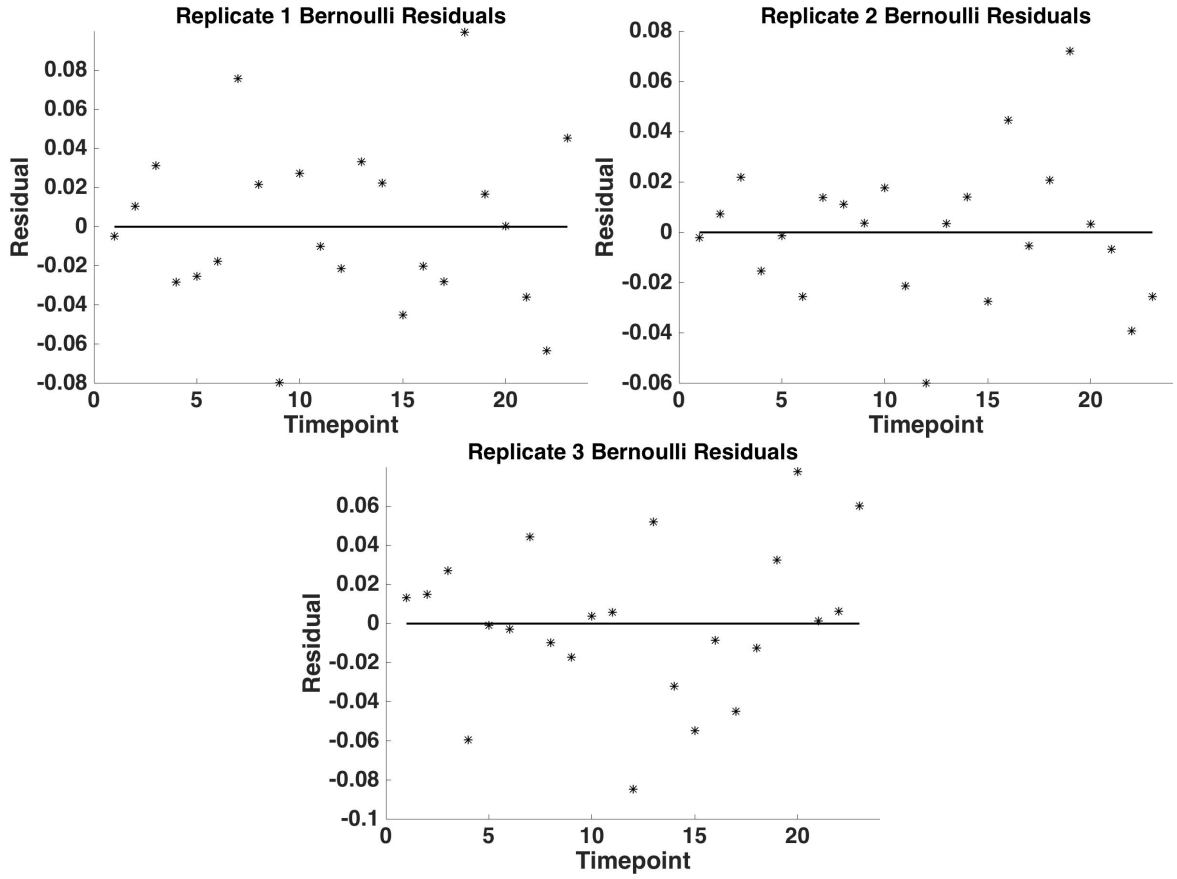

Figure S1. Residual plots for the Bernoulli curve for the three replicates of the data. Replicate one is on top and three is on the bottom.

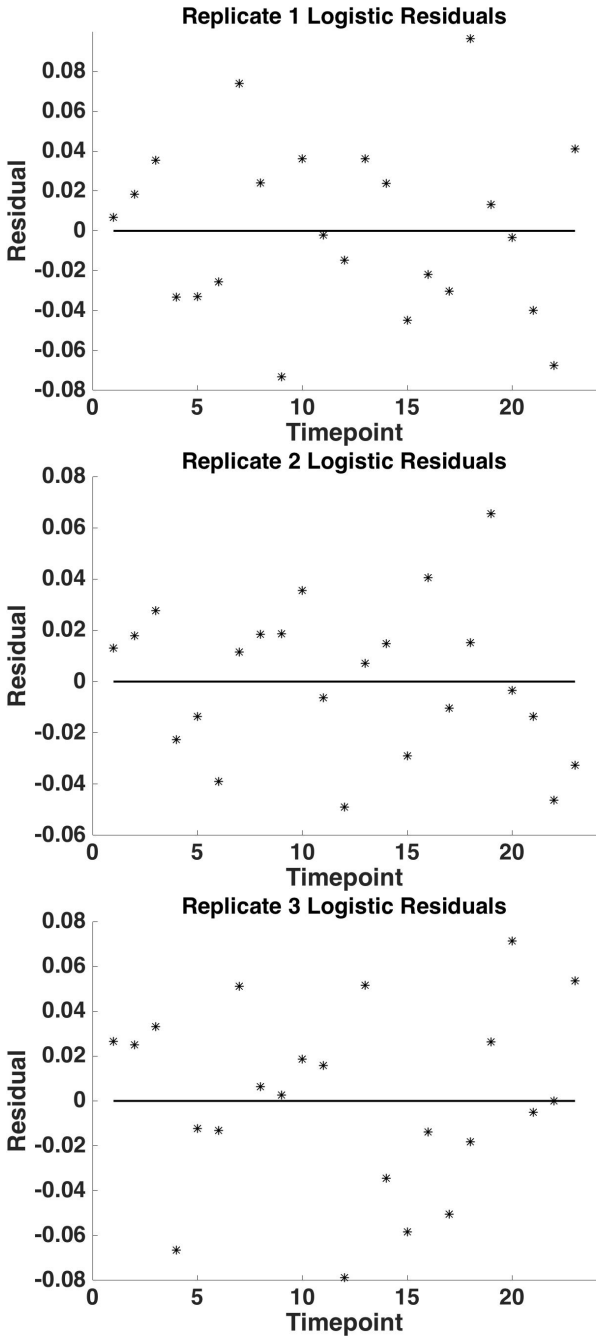

Figure S2. Residual plots for the Logistic curve for the three replicates of the data. Replicate one is on top and three is on the bottom.

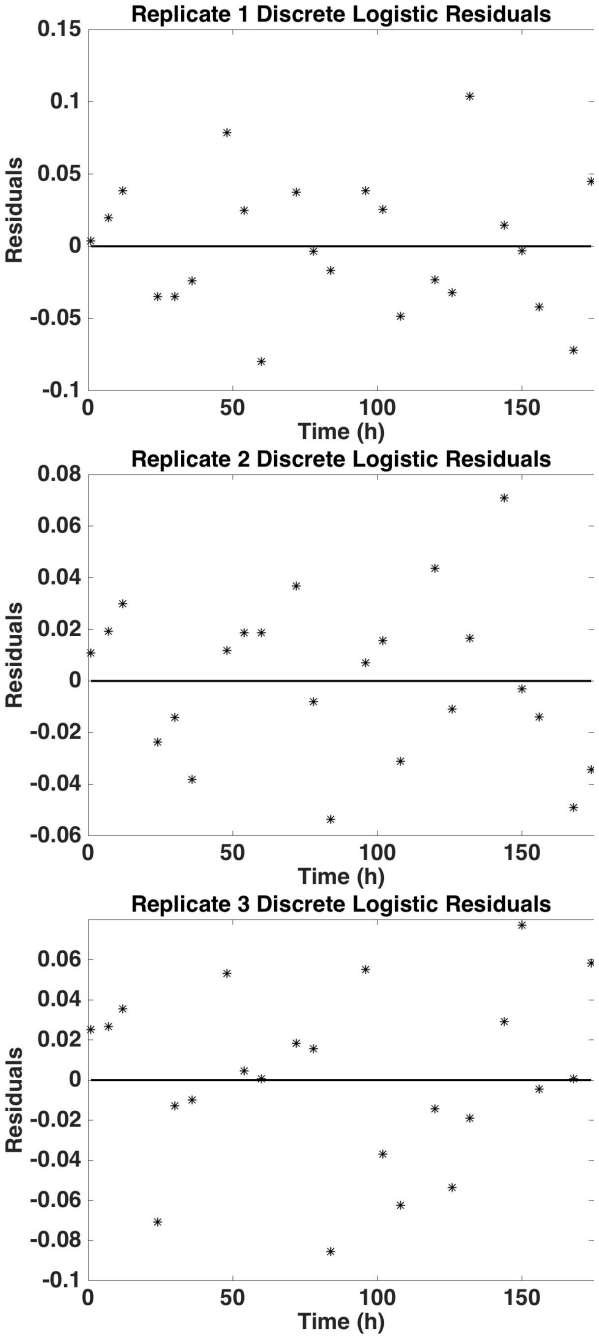

Figure S3. Residual plots for the discrete Euler-method logistic (DEL) curve for the three replicates of the data. Replicate one is on top and three is on the bottom.

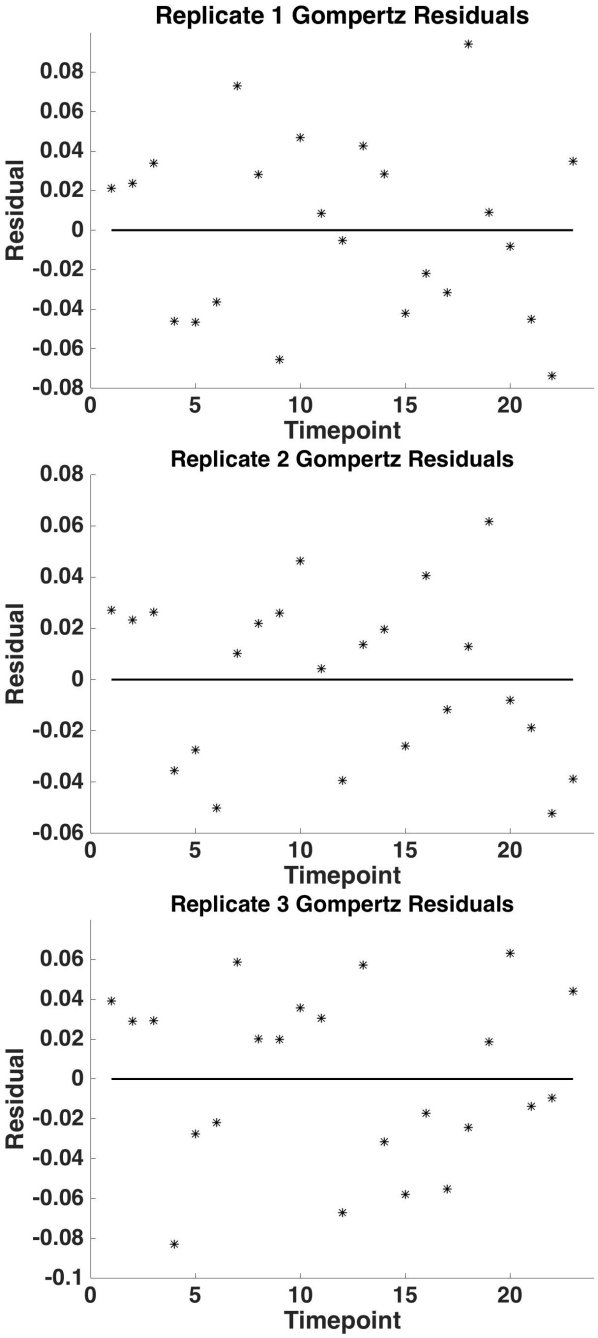

Figure S4. Residual plots for the Gompertz curve for the three replicates of the data. Replicate one is on top and three is on the bottom.

2. Bootstrapping Results: Initial Condition

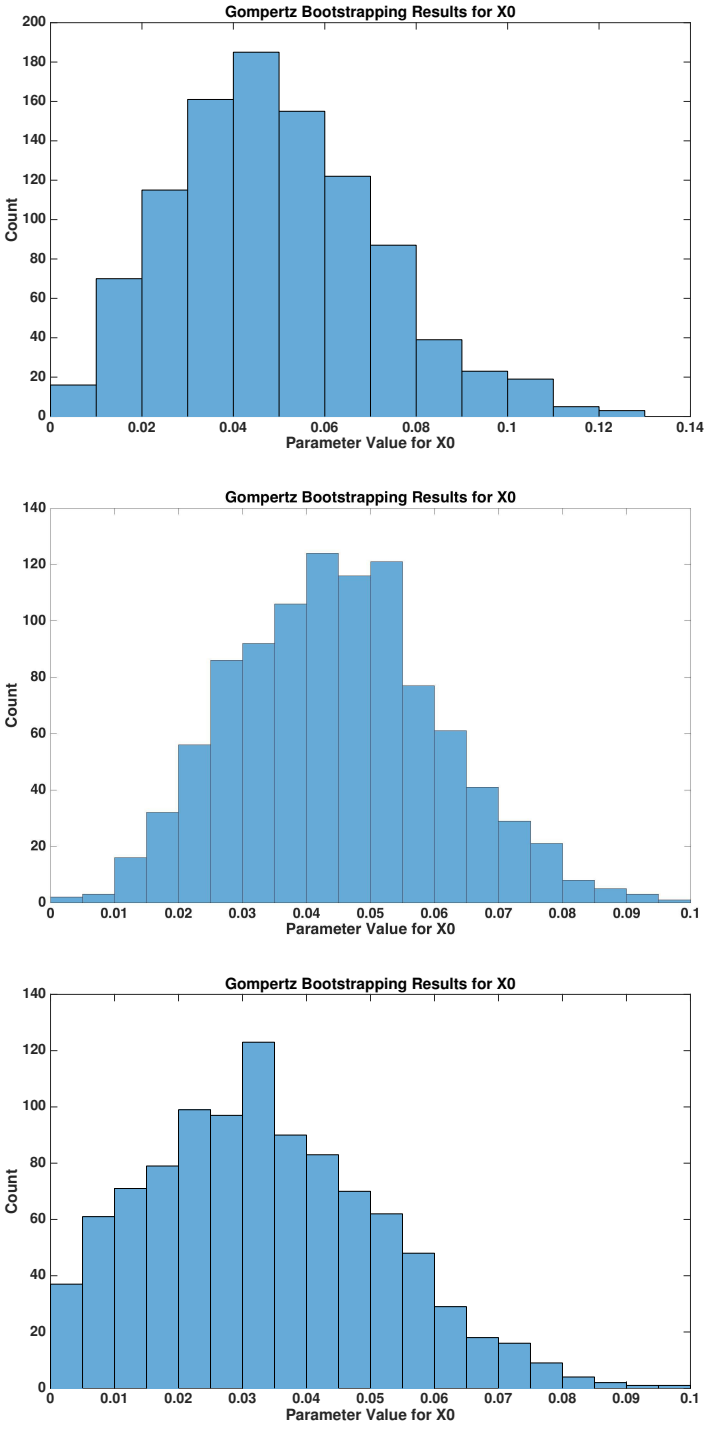

Figure S5. Plots of the Gompertz curve X_0 parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

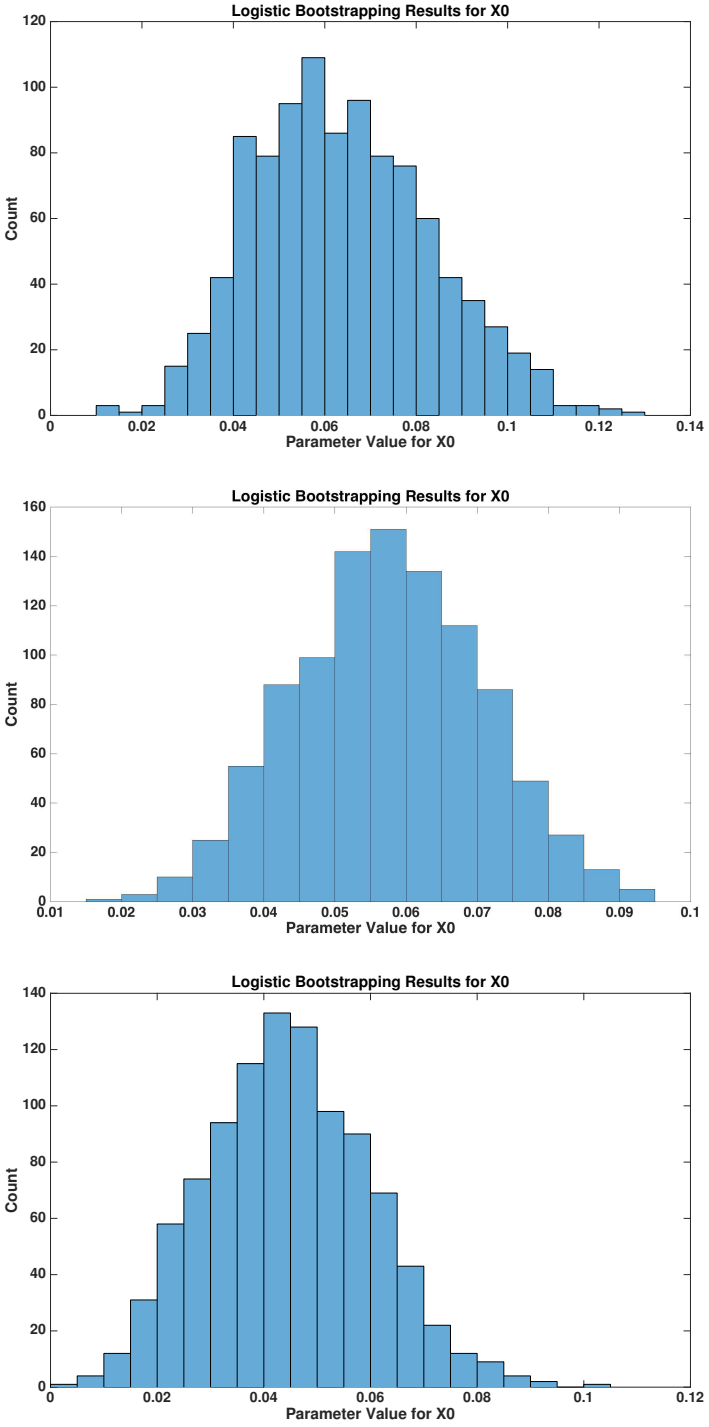

Figure S6. Plots of the Logistic curve X_0 parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

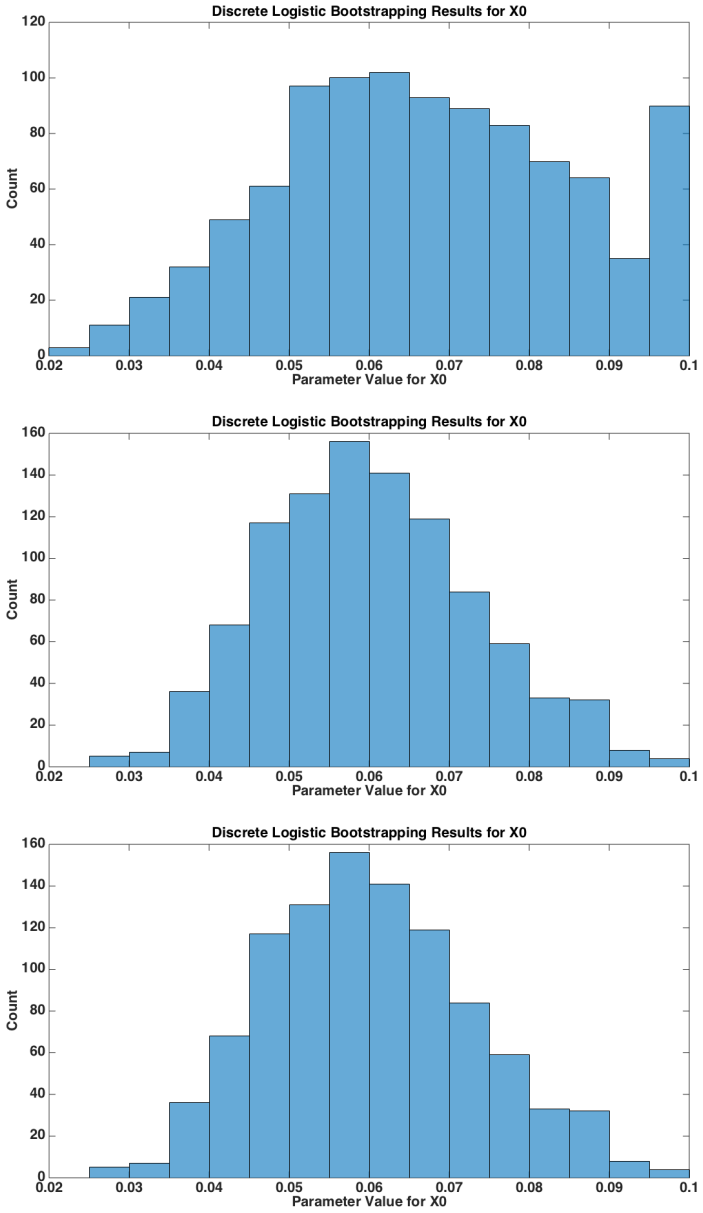

Figure S7. Plots of the DEL curve X_0 parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

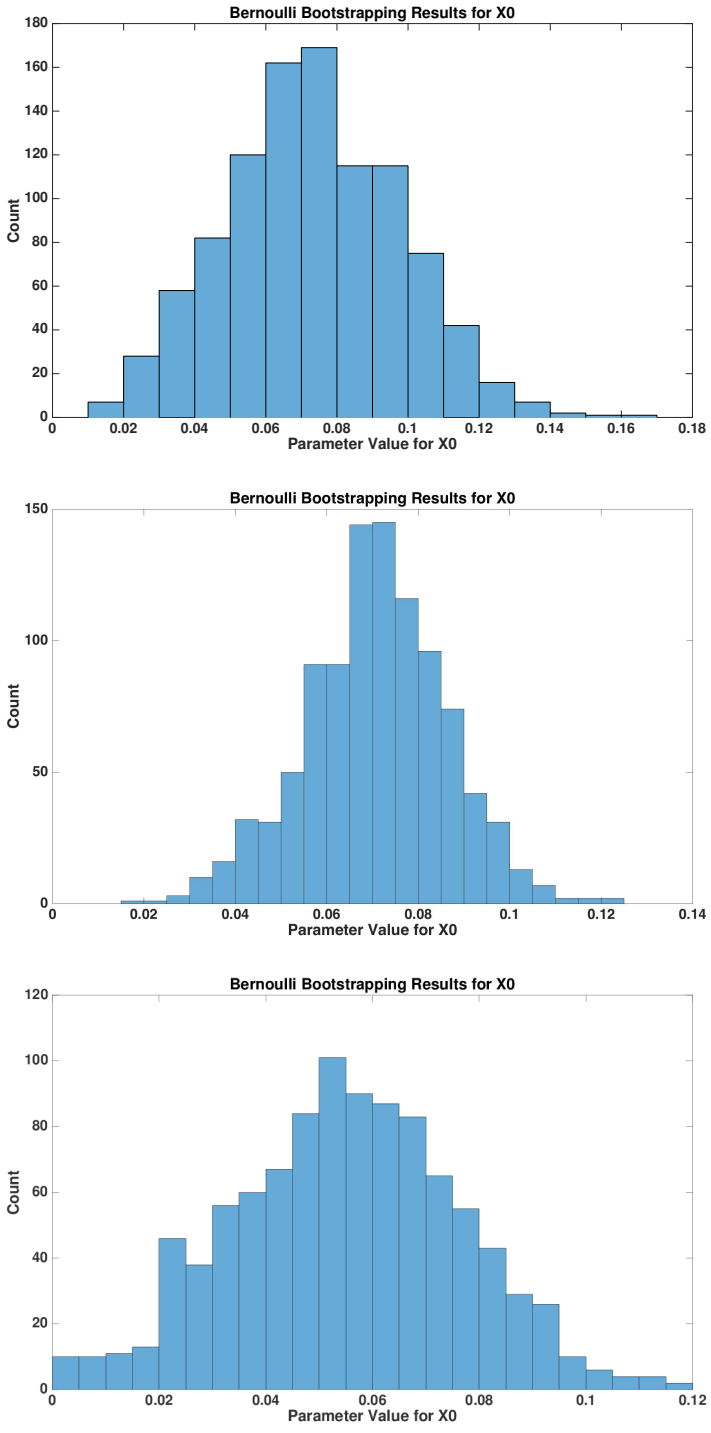

Figure S8. Plots of the Bernoulli curve X_0 parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

3. Bootstrapping Results: Growth Rate

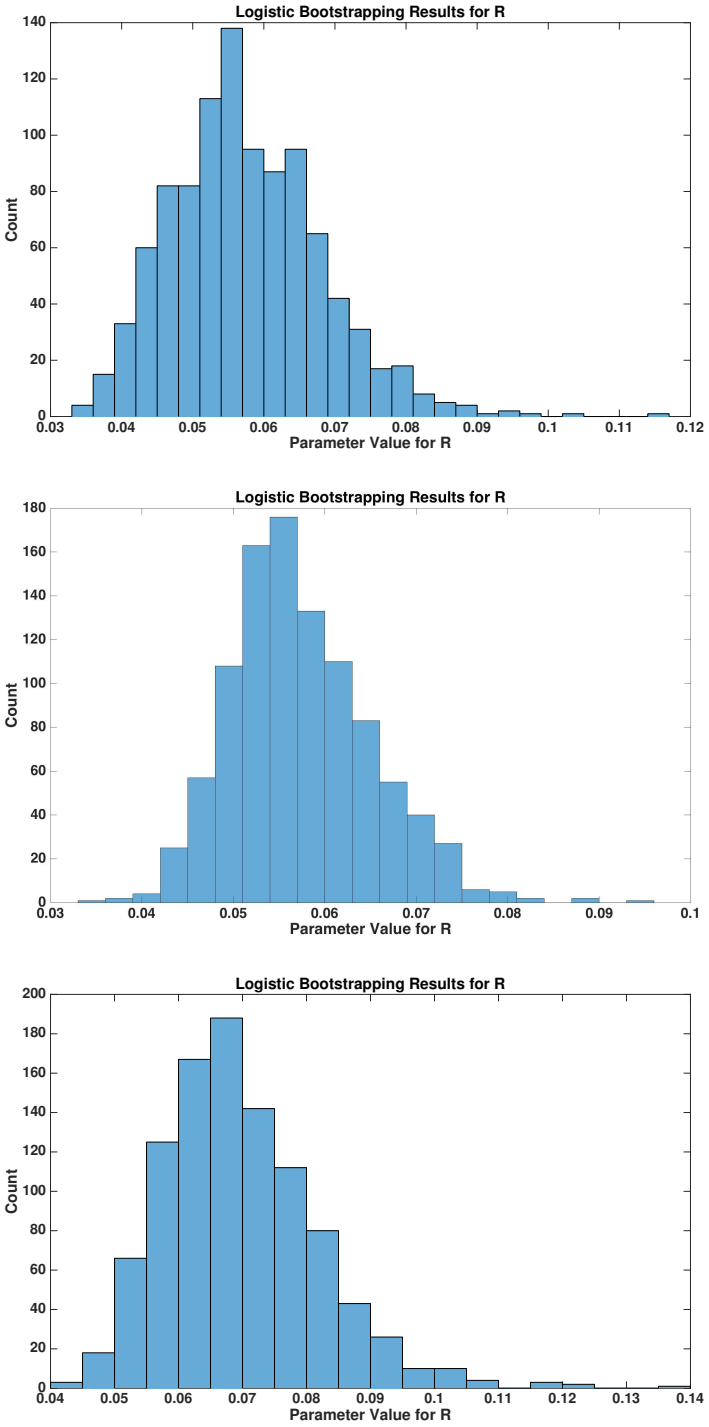

Figure S9. Plots of the Logistic curve *R* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

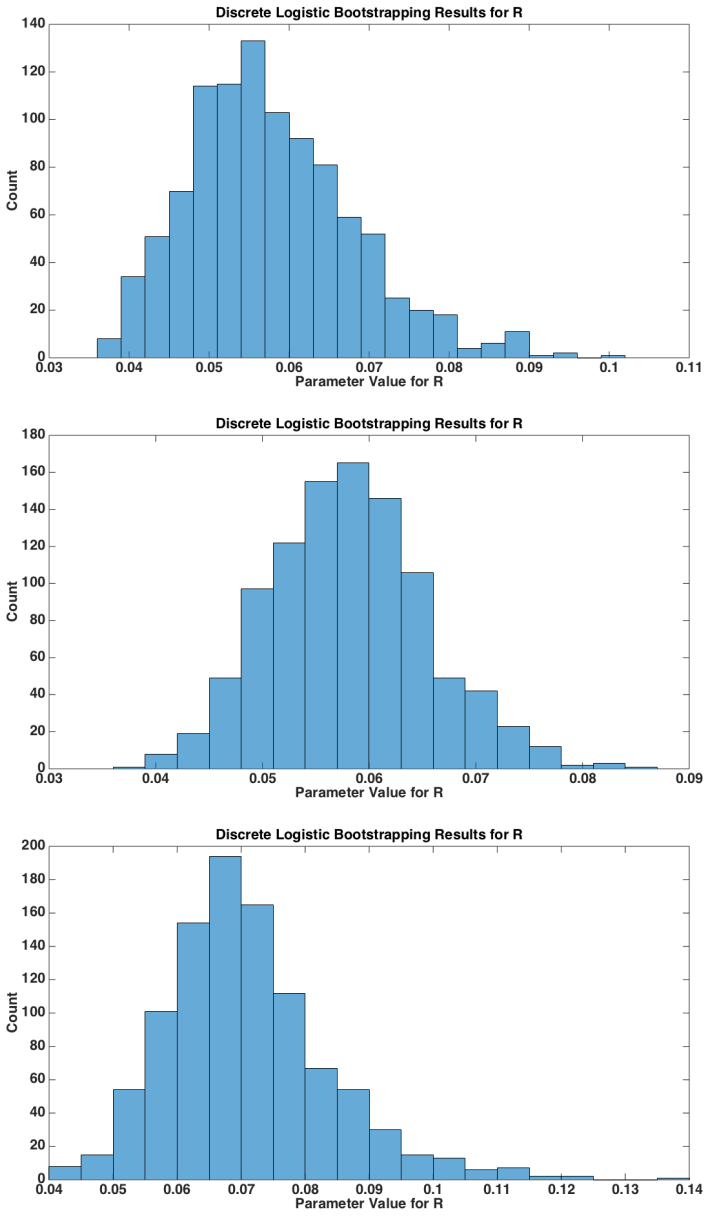

Figure S10. Plots of the DEL curve *R* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

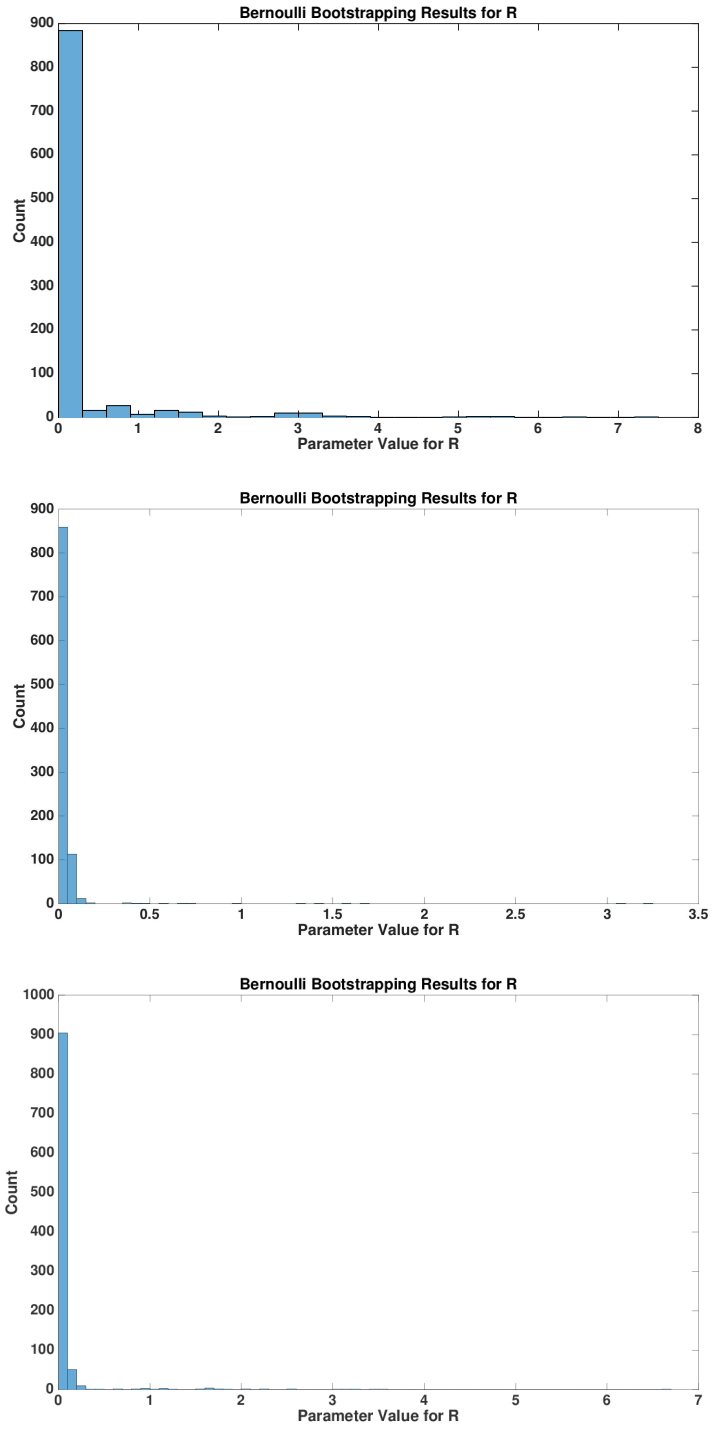

Figure S11. Plots of the Bernoulli curve *R* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

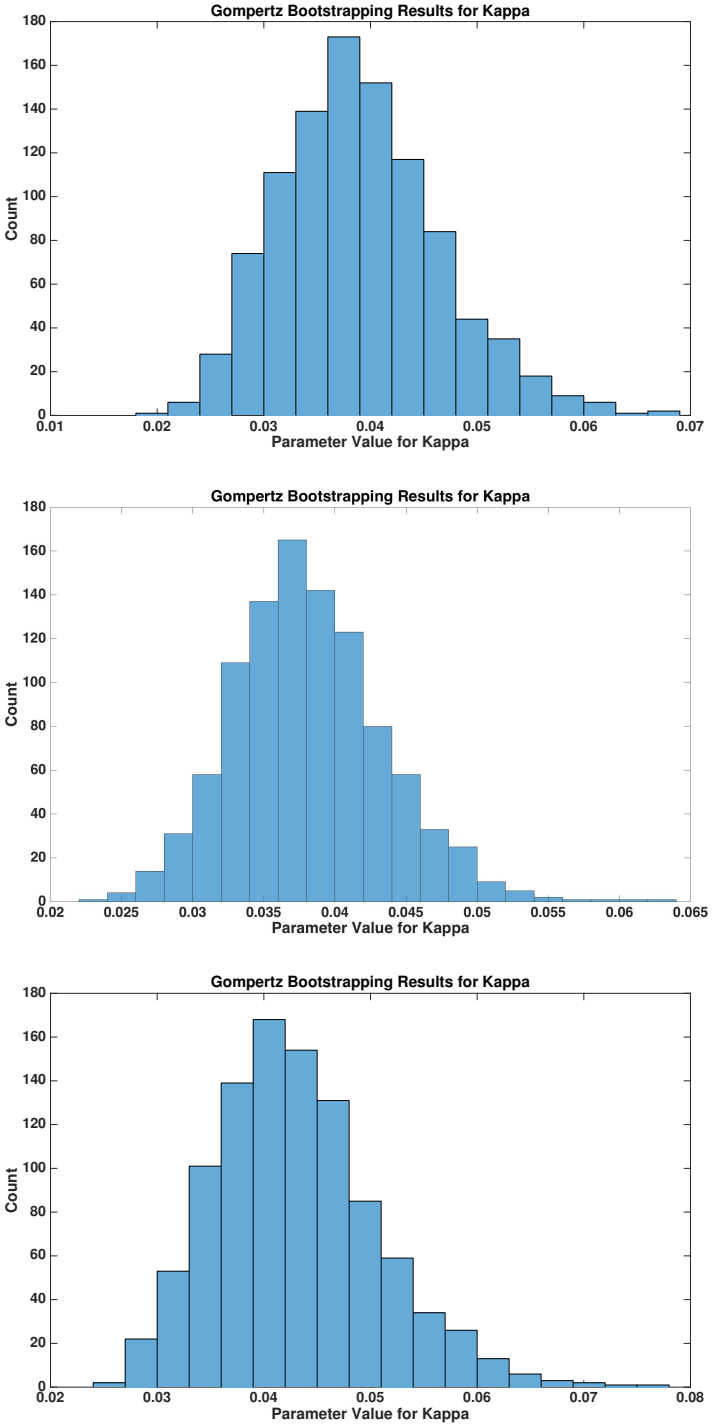

Figure S12. Plots of the Gompertz curve κ parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

4. Bootstrapping Results: Saturation Parameter

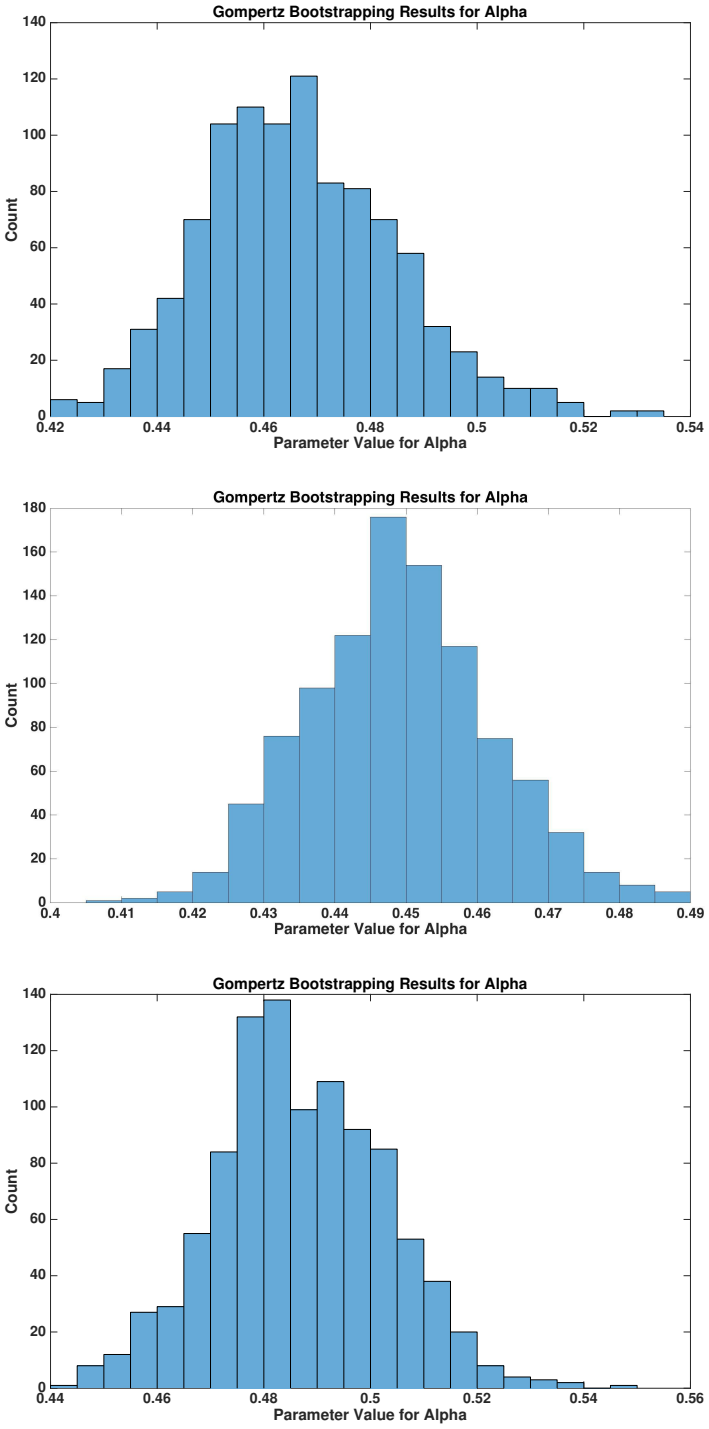

Figure S13. Plots of the Gompertz curve *K* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

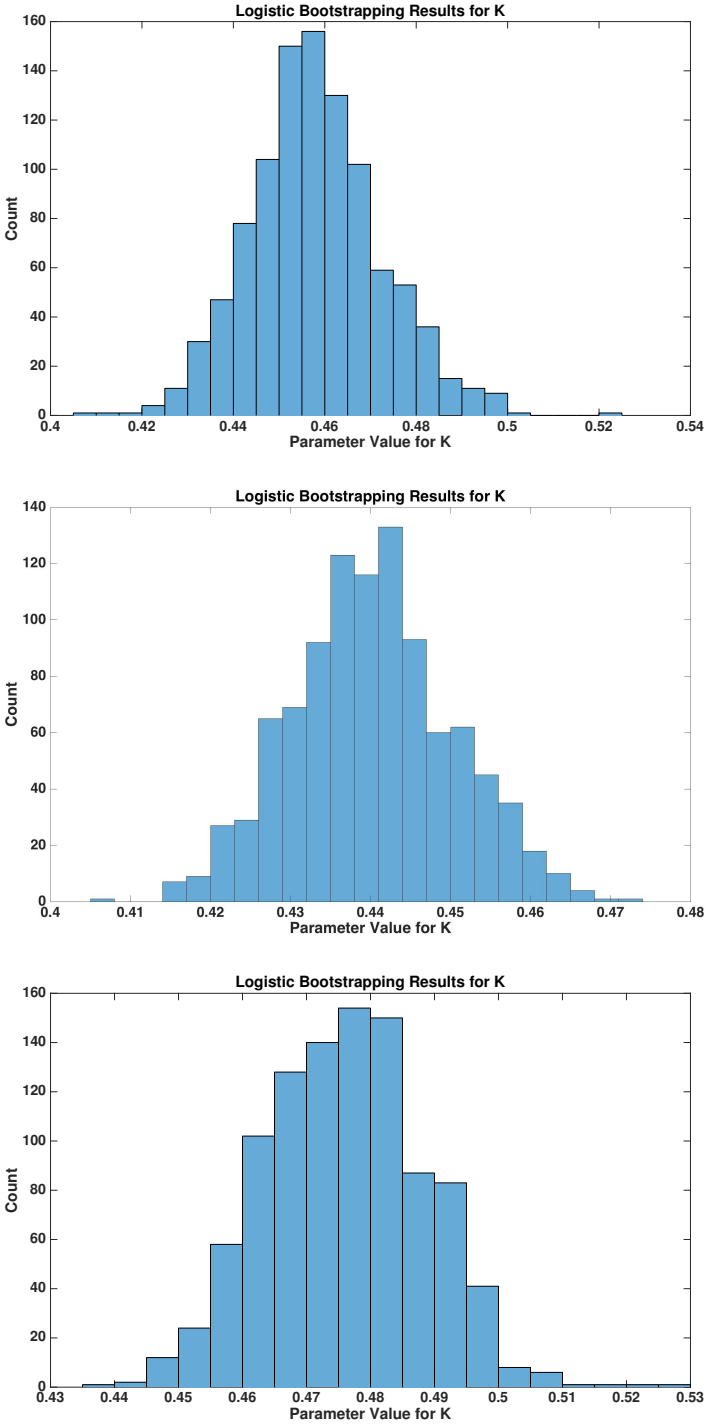

Figure S14. Plots of the Logistic curve K parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

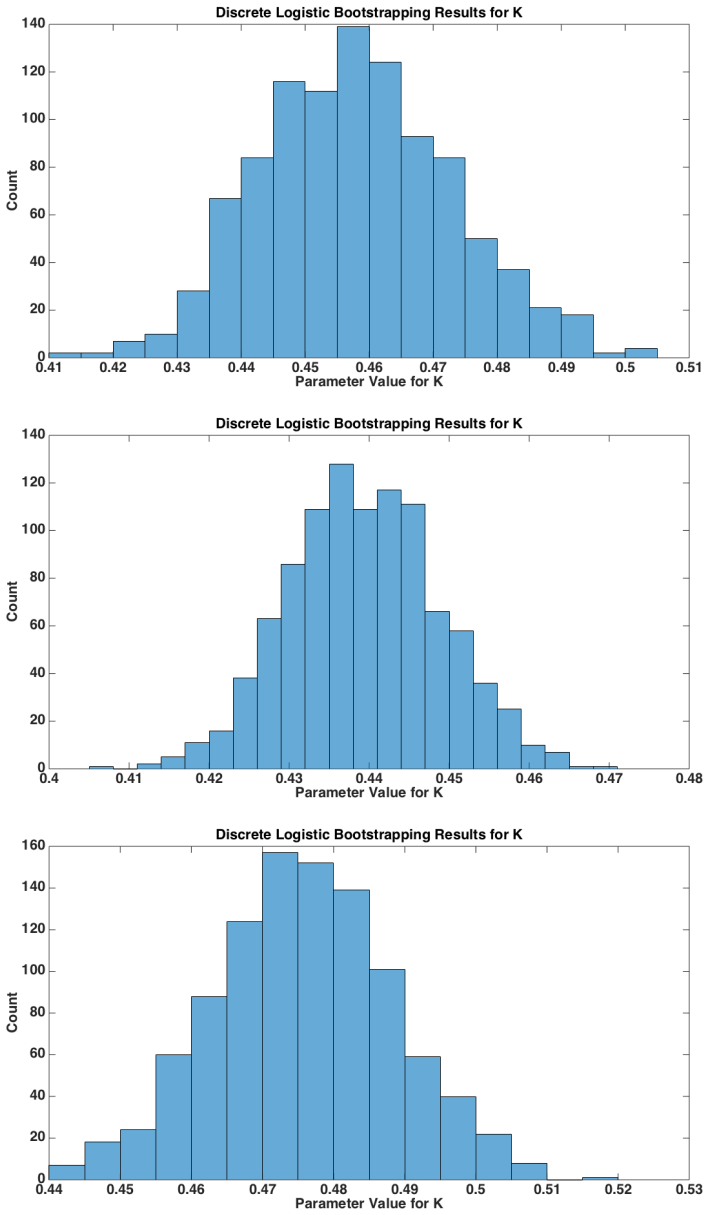

Figure S15. Plots of the DEL curve *K* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

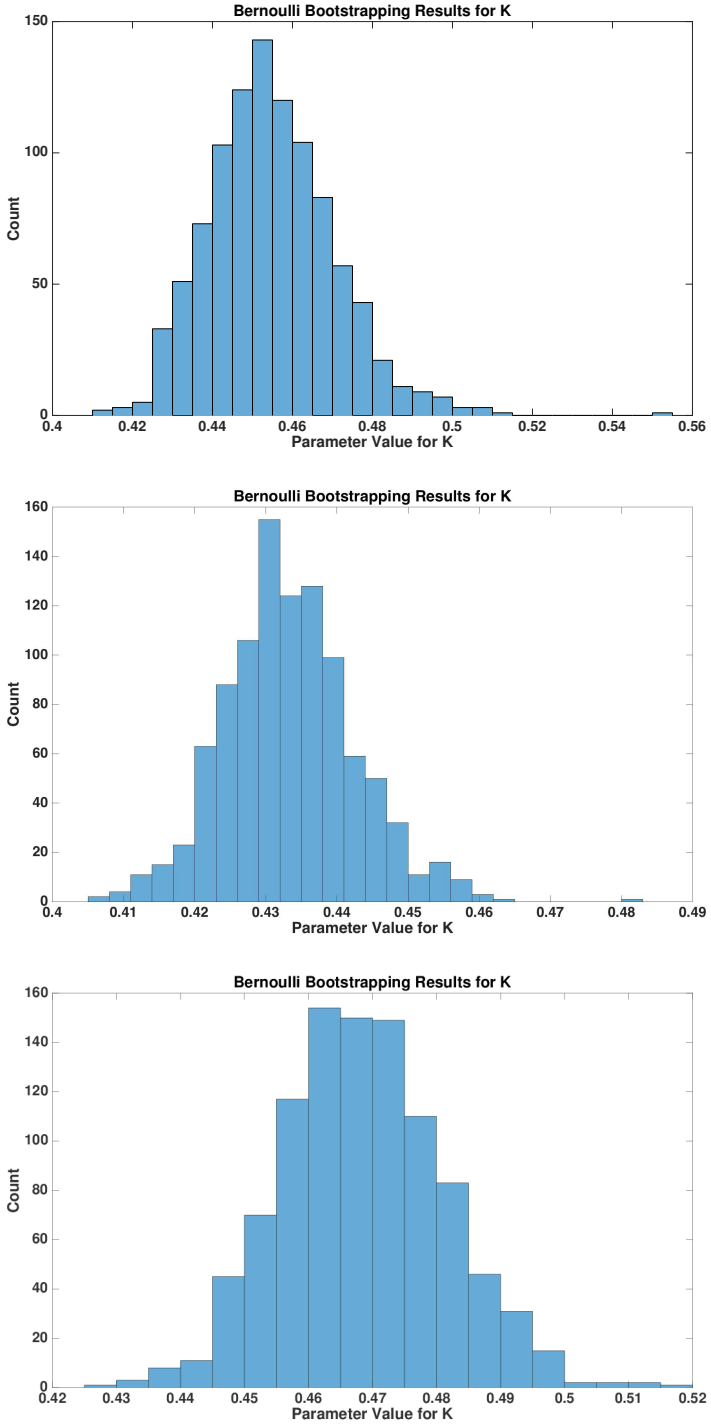

Figure S16. Plots of the Bernoulli curve *K* parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

5. Bootstrapping: Bernoulli Model Parameter β

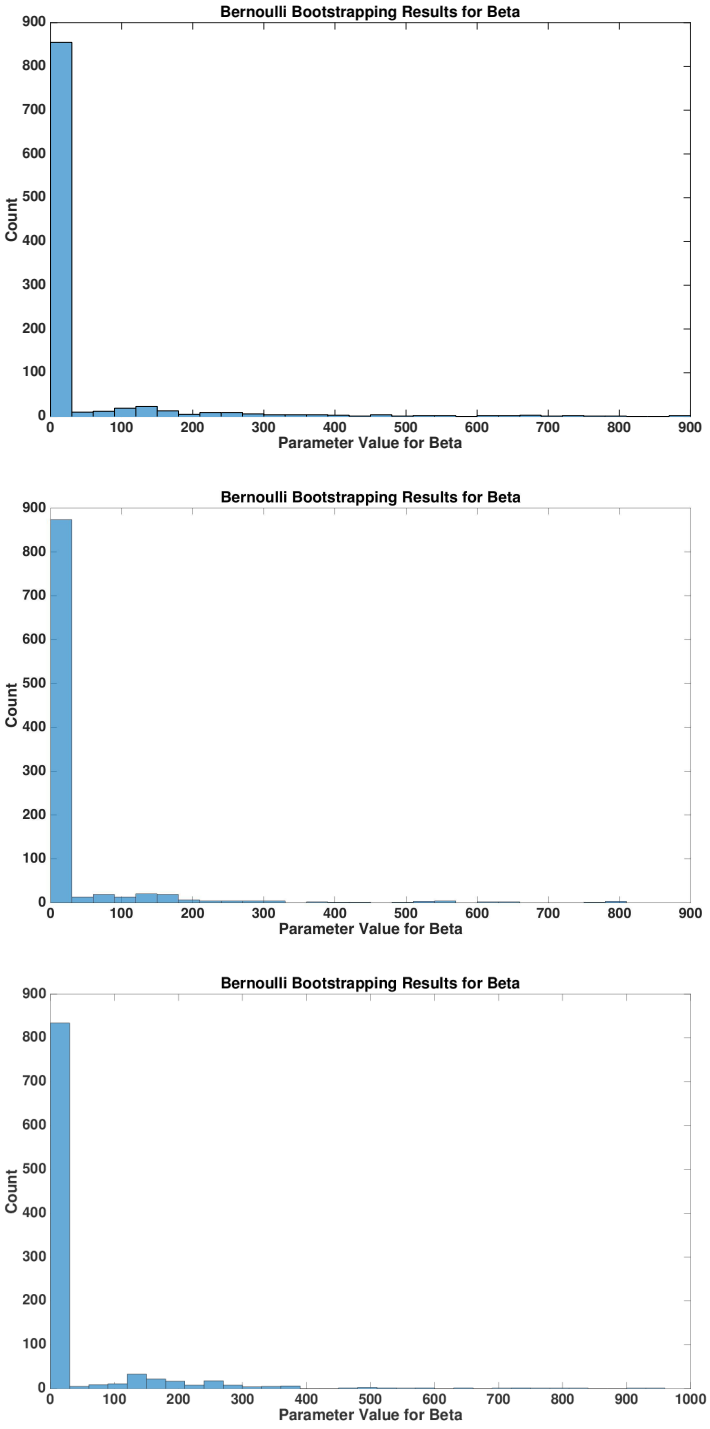

Figure S17. Plots of the Bernoulli curve β parameter estimate distributions for the three replicates of the data. Replicate one is on top and three is on the bottom.

6. Uncertainty Analysis: Initial Condition

Table S1. X_0 estimate and standard error for the Gompertz model.

Asymptotic Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0488	0.0066
	2	0.0421	0.0047
	3	0.0308	0.0055
Bootstrapping Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0495	0.0234
	2	0.0450	0.0159
	3	0.0333	0.0179

Table S2. X_0 estimate and standard error for the Logistic model.

Asymptotic Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0633	0.0193
	2	0.0569	0.0128
	3	0.0434	0.0156
Bootstrapping Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0633	0.0192
	2	0.0580	0.0132
	3	0.0448	0.0154

Table S3. X_0 estimate and standard error for the discrete Euler-method logistic (DEL) model.

Asymptotic Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0564	0.0096
	2	0.0572	0.0063
	3	0.0688	0.0078
Bootstrapping Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0651	0.0182
	2	0.0624	0.0128
	3	0.0607	0.0173

Table S4. X_0 estimate and standard error for the Bernoulli model.

Asymptotic Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0749	0.0272
	2	0.0722	0.0164
	3	0.0434	0.0229
Bootstrapping Results: X_0	**Replicate**	**Estimate**	**SE**
	1	0.0737	0.0244
	2	0.0707	0.0156
	3	0.0551	0.0214

7. Uncertainty Analysis: Growth Rate

Table S5. R estimate and standard error for the Logistic model.

Asymptotic Results : R	**Replicate**	**Estimate**	**SE**
	1	0.0560	0.0143
	2	0.0567	0.0098
	3	0.0680	0.0126
Bootstrapping Results: R	**Replicate**	**Estimate**	**SE**
	1	0.0578	0.0149
	2	0.0571	0.0097
	3	0.0695	0.0123

Table S6. R estimate and standard error for the DEL model.

Asymptotic Results: R	**Replicate**	**Estimate**	**SE**
	1	0.0626	0.0077
	2	0.0560	0.0056
	3	0.0676	0.0084
Bootstrapping Results: R	**Replicate**	**Estimate**	**SE**
	1	0.4525	0.0166
	2	0.4350	0.0115
	3	0.4816	0.0180

Table S7. R estimate and standard error for the Bernoulli model.

Asymptotic Results: R	**Replicate**	**Estimate**	**SE**
	1	0.0361	0.0209
	2	0.0319	0.0089
	3	0.0422	0.0171
Bootstrapping Results: R	**Replicate**	**Estimate**	**SE**
	1	0.2476	0.7313
	2	0.0535	0.1752
	3	0.1148	0.3390

Table S8. κ estimate and standard error for the Gompertz model.

Asymptotic Results: κ	**Replicate**	**Estimate**	**SE**
	1	0.0379	0.0094
	2	0.0380	0.0069
	3	0.0421	0.0080
Bootstrapping Results: κ	**Replicate**	**Estimate**	**SE**
	1	0.0391	0.0078
	2	0.0382	0.0055
	3	0.0431	0.0078

8. Uncertainty Analysis: Saturation Parameter

Table S9. *K* estimate and standard error for the Gompertz model.

Asymptotic Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4642	0.0881
	2	0.4465	0.0588
	3	0.4844	0.0544
Bootstrapping Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4658	0.0185
	2	0.4484	0.0135
	3	0.4869	0.0159

Table S10. *K* estimate and standard error for the Logistic model.

Asymptotic Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4565	0.0099
	2	0.4390	0.0071
	3	0.4739	0.0111
Bootstrapping Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4581	0.0106
	2	0.4409	0.0074
	3	0.4758	0.0118

Table S11. *K* estimate and standard error for the DEL model.

Asymptotic Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4559	0.0052
	2	0.4384	0.0036
	3	0.4731	0.0096
Bootstrapping Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4525	0.0107
	2	0.4350	0.0071
	3	0.4816	0.0092

Table S12. *K* estimate and standard error for the Bernoulli model.

Asymptotic Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4522	0.1138
	2	0.4317	0.0341
	3	0.4674	0.0654
Bootstrapping Results: *K*	**Replicate**	**Estimate**	**SE**
	1	0.4548	0.0158
	2	0.4334	0.0092
	3	0.4685	0.0128