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An NHPP Software Reliability Model with S-Shaped Growth Curve Subject to Random Operating Environments and Optimal Release Time

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Abstract: The failure of a computer system because of a software failure can lead to tremendous losses to society; therefore, software reliability is a critical issue in software development. As software has become more prevalent, software reliability has also become a major concern in software development. We need to predict the fluctuations in software reliability and reduce the cost of software testing: therefore, a software development process that considers the release time, cost, reliability, and risk is indispensable. We thus need to develop a model to accurately predict the defects in new software reliability model, with S-shaped growth curve for use during the software development process, and relate it to a fault detection rate function when considering random operating environments. An explicit mean value function solution for the proposed model is presented. Examples are provided to illustrate the goodness-of-fit of the proposed model, along with several existing NHPP models that are based on two sets of failure data collected from software applications. The results show that the proposed model fits the data more closely than other existing NHPP models to a significant extent. Finally, we propose a model to determine optimal release policies, in which the total software system cost is minimized depending on the given environment.

Keywords: software reliability; non-homogeneous Poisson process; optimal release time; mean squared error

1. Introduction

'Software' is a generic term for a computer program and its associated documents. Software is divided into operating systems and application software. As new hardware is developed, the price decreases; thus, hardware is frequently upgraded at low cost, and software becomes the primary cost driver. The failure of a computer system because of a software failure can cause significant losses to society. Therefore, software reliability is a critical issue in software development. This problem requires finding a balance between meeting user requirements and minimizing the testing costs. It is necessary to know in the planning cycle the fluctuation of software reliability and the cost of testing, in order to reduce costs during the software testing stage, thus a software development process that considers the release time, cost, reliability, and risk is indispensable. In addition, it is necessary to develop a model to predict the defects in software products. To estimate reliability metrics, such as the number of residual faults, the failure rate, and the overall reliability of the software, various non-homogeneous Poisson process (NHPP) software reliability models have been developed using a fault intensity rate function and mean value function within a controlled testing environment. The purpose of many

NHPP software reliability models is to obtain an explicit formula for the mean value function, m(t), which is applied to the software testing data to make predictions on software failures and reliability in field environments [1]. A few researchers have evaluated a generalized software reliability model that captures the uncertainty of an environment and its effects on the software failure rate, and have developed a NHPP software reliability model when considering the uncertainty of the system fault detection rate per unit of time subject to the operating environment [2–4]. Inoue et al. [5] developed a bivariate software reliability growth model that considers the uncertainty of the change in the software failure-occurrence phenomenon at the change-point for improved accuracy. Okamura and Dohi [6] introduced a phase-type software reliability model and developed parameter estimation algorithms using grouped data. Song et al. [7,8] recently developed an NHPP software reliability model to consider a three-parameter fault detection rate, and applied a Weibull fault detection rate function during the software development process. They related the model to the error detection rate function by considering the uncertainty of the operating environment. In addition, Li and Pham [9] proposed a model accounting for the uncertainty of the operating environment under the condition that the fault content function is a linear function of the testing time, and that the fault detection rate is based on the testing coverage.

In this paper, we discuss a new NHPP software reliability model with S-shaped growth curve applicable to the software development process and relate it to the fault detection rate function when considering random operating environments. We examine the goodness-of-fit of the proposed model and other existing NHPP models that are based on several sets of software failure data, and then determine the optimal release times that minimize the expected total software cost under given conditions. The explicit solution of the mean value function for the new NHPP software reliability model is derived in Section 2. Criteria for the model comparisons and the selection of the best model are discussed in Section 3. The optimal release policy is discussed in Section 4, and the results of a model analysis and the optimal release times are discussed in Section 5. Finally, Section 6 provides some concluding remarks.

2. A New NHPP Software Reliability Model

2.1. Non-Homogeneous Poisson Process

The software fault detection process has been formulated using a popular counting process. The counting process {N(t), $t \ge 0$ } is a non-homogeneous Poisson process (NHPP) with an intensity function $\lambda(t)$, if it satisfies the following condition.

- (I) N(0) = 0
- (II) Independent increments
- (III) $\int_{t_1}^{t_2} \lambda(t) dt$, $(t_2 \ge t_1)$: the average of the number of failures in the interval $[t_1, t_2]$

Assuming that the software failure/defect conforms to the NHPP condition, $N(t)(t \ge 0)$ represents the cumulative number of failures up to the point of execution, and m(t) is the mean value function. The mean value function m(t) and the intensity function $\lambda(t)$ satisfy the following relationship.

$$\mathbf{m}(\mathbf{t}) = \int_0^t \lambda(\mathbf{s}) d\mathbf{s}_{,\prime} \frac{d\mathbf{m}(\mathbf{t})}{d\mathbf{t}} = \lambda(\mathbf{t}). \tag{1}$$

N(t) is a Poisson distribution involving the mean value function, m(t), and can be expressed as:

$$\Pr\{N(t) = n\} = \frac{\{m(t)\}^n}{n!} \exp\{-m(t)\}, \ n = 0, 1, 2, 3....$$
(2)

2.2. General NHPP Software Reliability Model

Pham et al. [10] formalized the general framework for NHPP-based software reliability and provided analytical expressions for the mean value function m(t) using differential equations.

The mean value function m(t) of the general NHPP software reliability model with different values for a(t) and b(t), which reflects various assumptions of the software testing process, can be obtained with the initial condition N(0) = 0.

$$\frac{\mathrm{d}\,\mathbf{m}(t)}{\mathrm{d}t} = \mathbf{b}(t)[\mathbf{a}(t) - \mathbf{m}(t)]. \tag{3}$$

The general solution of (1) is

$$m(t) = e^{-B(t)} \left[m_0 + \int_{t_0}^t a(s)b(s)e^{B(s)}bs \right]$$
(4)

where $B(t) = \int_{t_0}^t b(s) ds$, and $m(t_0) = m_0$ is the marginal condition of (2).

2.3. New NHPP Software Reliability Model

Pham [3] formulated a generalized NHPP software reliability model that incorporated uncertainty in the operating environment as follows:

$$\frac{\mathrm{d}\,\mathbf{m}(t)}{\mathrm{d}t} = \eta[\mathbf{b}(t)][\mathbf{N} - \mathbf{m}(t)], \tag{5}$$

where η is a random variable that represents the uncertainty of the system fault detection rate in the operating environment with a probability density function g; b(t) is the fault detection rate function, which also represents the average failure rate caused by faults; N is the expected number of faults that exists in the software before testing; and, m(t) is the expected number of errors detected by time t (the mean value function).

Thus, a generalized mean value function, m(t), where the initial condition m(0) = 0, is given by

$$\mathbf{m}(t) = \int_{\eta} \mathbf{N} \left(1 - e^{-\eta \int_0^t b(\mathbf{x}) d\mathbf{x}} \right) dg(\eta).$$
 (6)

The mean value function [11] from (4) using the random variable η has a generalized probability density function g with two parameters $\alpha \ge 0$ and $\beta \ge 0$ and is given by

$$\mathbf{m}(t) = \mathbf{N} \left(1 - \frac{\beta}{\beta + \int_0^t \mathbf{b}(s) ds} \right)^{\alpha},\tag{7}$$

where b(t) is the fault detection rate per fault per unit of time.

We propose an NHPP software reliability model including the random operating environment using Equations (3)–(5) and the following assumptions [7,8]:

- (a) The occurrence of a software failure follows a non-homogeneous Poisson process.
- (b) Faults during execution can cause software failure.
- (c) The software failure detection rate at any time depends on both the fault detection rate and the number of remaining faults in the software at that time.
- (d) Debugging is performed to remove faults immediately when a software failure occurs.
- (e) New faults may be introduced into the software system, regardless of whether other faults are removed or not.
- (f) The fault detection rate b(t) can be expressed by (6).
- The random operating environment is captured if unit failure detection rate b(t) is multiplied by a factor (g) what remember to the uncertainty of the system fault detection rate in the field
- η that represents the uncertainty of the system fault detection rate in the field

In this paper, we consider the fault detection rate function b(t) to be as follows:

$$b(t) = \frac{a^2 t}{1 + at}, a > 0, a, b > 0,$$
(8)

We obtain a new NHPP software reliability model with S-shaped growth curve subject to random operating environments, m(t), that can be used to determine the expected number of software failures detected by time t by substituting function b(t) above into (5) so that:

$$\mathbf{m}(\mathbf{t}) = \mathbf{N} \left(1 - \frac{\beta}{\beta + \mathbf{at} - \ln(1 + \mathbf{at})} \right)^{\alpha}.$$
(9)

3. Criteria for Model Comparisons

Theoretically, once the analytical expression for mean value function m(t) is derived, then the parameters in m(t) can be estimated using parameter estimation methods (MLE: the maximum likelihood estimation method, LSE: the least square estimation method); however, in practice, accurate estimates may not be obtained by the MLE, particularly under certain conditions where the mean value function m(t) is too complex. The model parameters to be estimated in the mean value function m(t) can then be obtained using a MATLAB program that is based on the LSE method. Six common criteria; the mean squared error (MSE), Akaike's information criterion (AIC), the predictive ratio risk (PRR), the predictive power (PP), the sum of absolute errors (SAE), and R-square (R^2) will be used for the goodness-of-fit estimation of the proposed model, and to compare the proposed model with other existing models, as listed in Table 1. These criteria are described as follows.

The MSE is

$$MSE = \frac{\sum_{i=0}^{n} (\hat{m}(t_i) - y_i)^2}{n - m}.$$
 (10)

AIC [12] is

$$AIC = -2\log L + 2m. \tag{11}$$

The PRR [13] is

$$PRR = \sum_{i=0}^{n} \left(\frac{\hat{m}(t_i) - y_i}{\hat{m}(t_i)} \right)^2.$$
 (12)

The PP [13] is

$$PP = \sum_{i=0}^{n} \left(\frac{\hat{m}(t_i) - y_i}{y_i} \right)^2.$$
(13)

The SAE [8] is

$$SAE = \sum_{i=0}^{n} |\hat{m}(t_i) - y_i|.$$
(14)

The correlation index of the regression curve equation (R^2) [9] is

$$R^{2} = 1 - \frac{\sum_{i=0}^{n} (\hat{m}(t_{i}) - y_{i})^{2}}{\sum_{i=0}^{n} (y_{i} - \overline{y_{i}})^{2}}.$$
(15)

Here, $\hat{m}(t_i)$ is the estimated cumulative number of failures at t_i for $i = 1, 2, \dots, n$; y_i is the total number of failures observed at time t_i ; n is the actual data which includes the total number of observations; and, m is the number of unknown parameters in the model.

The MSE measures the distance of a model estimate from the actual data that includes the total number of observations and the number of unknown parameters in the model. AIC is measured to compare the capability of each model in terms of maximizing the likelihood function (L), while considering the degrees of freedom. The PRR measures the distance of the model estimates from the actual data against the model estimate. The PP measures the distance of the model estimates from the actual data. The SAE measures the absolute distance of the model. For five of these criteria, i.e., MSE, AIC, PRR, PP, and SAE, the smaller the value is, the closer the model fits relative to other models run on the same dataset. On the other hand, R² should be close to 1.

We use (8) below to obtain the confidence interval [13] of the proposed NHPP software reliability model. The confidence interval is described as follows;

$$\hat{\mathbf{m}}(\mathbf{t}) \pm \mathbf{Z}_{\alpha/2} \sqrt{\hat{\mathbf{m}}(\mathbf{t})},\tag{16}$$

where, $Z_{\alpha/2}$ is $100(1 - \alpha)$, the percentile of the standard normal distribution.

Table 1 summarizes the different mean value functions of the proposed new model and several existing NHPP models. Note that models 9 and 10 consider environmental uncertainty.

No.	Model	m(t)
1	GO Model [14]	$m(t) = a \Big(1 - e^{-bt} \Big)$
2	Delayed S-shaped Model [15]	$\mathbf{m}(t) = \mathbf{a} \left(1 - (1 + \mathbf{b}t)\mathbf{e}^{-\mathbf{b}t} \right)$
3	Inflection S-shaped Model [16]	$\mathbf{m}(\mathbf{t}) = \frac{\mathbf{a}(1 - \mathbf{e}^{-\mathbf{b}\mathbf{t}})}{1 + 6\mathbf{e}^{-\mathbf{b}\mathbf{t}}}$
4	Yamada ImperfectDebugging Model [17]	$\mathbf{m}(t) = \mathbf{a} \left[1 - \mathbf{e}^{-bt} \right] \left[1 - \frac{\alpha}{b} \right] + \alpha \mathbf{a} t$
5	PNZ Model [10]	$m(t) = \frac{a[1-e^{-bt}][1-\frac{\alpha}{b}] + \alpha at}{1+\beta e^{-bt}}$
6	PZ Model [18]	$m(t) = \frac{\left((c+a)\left[1 - e^{-bt}\right] - \left[\frac{ab}{b-\alpha}\left(e^{-\alpha t} - e^{-bt}\right)\right]\right)}{1 + 6e^{-bt}}$
7	Dependent Parameter Model [19]	$m(t) = m_0 \Big(\frac{\gamma t + 1}{\gamma t_0 + 1} \Big) e^{-\gamma (t - t_0)} + \alpha (\gamma t + 1) (\gamma t - 1 + (1 - \gamma t_0) e^{-\gamma (t - t_0)}$
8	Testing Coverage Model [4]	$m(t) = N \left[1 - \left(\frac{\beta}{\beta + (at)^b} \right)^{lpha} ight]$
9	Three parameter Model [7]	$\mathrm{m}(\mathrm{t}) = \mathrm{N} \left[1 - \left(rac{eta}{eta - rac{\mathrm{a}}{\mathrm{b}} \ln \left(rac{(1+\mathrm{c})\mathrm{e}^{-\mathrm{b} \mathrm{t}}}{1+\mathrm{c}\mathrm{e}^{-\mathrm{b} \mathrm{t}}} ight)} ight) ight]$
10	Proposed New Model	$\mathbf{m}(\mathbf{t}) = \mathbf{N} \left(1 - \frac{\beta}{\beta + at - \ln(1 + at)} \right)^{\alpha' + 2}$

Table 1. NHPP software reliability models.

4. Optimal Software Release Policy

In this section, we next discuss the use of the software reliability model under varying situations to determine the optimal software release time, and to determine the optimal software release time, T*, which minimizes the expected total software cost. Many studies have been conducted on the optimal software release time and its related problems [20–24]. The quality of the system will normally depend on the testing efforts, such as the testing environment, times, tools, and methodologies. If testing is short, the cost of the system testing is lower, but the consumers may face a higher risk e.g., buying an unreliable system. This also involves the higher costs of the operating environment because it is much more expensive to detect and correct a failure during the operational phase than during the testing phase. In contrast, the longer the testing costs for the system will also increase. Therefore, it is very important to determine when to release the system based on test cost and reliability. Figure 1 shows the system development lifecycle considered in the following cost model: the testing phase before release time T, the testing environment period, the warranty period, and the operational life in the actual field environment, which is usually quite different from the testing environment [24].



Figure 1. System cost model infrastructure.

The expected total software $\cot C(T)$ [24] can be expressed as

$$C(T) = C_0 + C_1 T + C_2 m(T) \mu_y + C_3 (1 - R(x|T)) + C_4 [m(T + T_w) - m(T)] \mu_w$$
(17)

where, C_0 is the set-up cost of testing, C_1T is the cost of testing, $C_2m(T)\mu_y$ is the expected cost to remove all errors detected by time T during the testing phase, $C_3(1 - R(x|T))$ is the penalty cost owing to failures that occurs after the system release time T, and $C_4[m(T + T_w) - m(T)]\mu_w$ is the expected cost to remove all of the errors that are detected during the warranty period $[T, T + T_w]$. The cost that is required to remove faults during the operating period is higher than during the testing period, and the time that is needed is much longer.

Finally, we aim to find the optimal software release time, T*, with the expected minimum in the environment as follows:

$$Minimize C(T). (18)$$

5. Numerical Examples

5.1. Data Information

Dataset #1 (DS1), presented in Table 2, was reported by Musa [25] based on software failure data from a real time command and control system (RTC&CS), and represents the failures that were observed during system testing (25 hours of CPU time). The number of test object instructions delivered for this system, which was developed by Bell Laboratories, was 21,700.

Hour Index	Failures	Cumulative Failures	Hour Index	Failures	Cumulative Failures
1	27	27	14	5	111
2	16	43	15	5	116
3	11	54	16	6	122
4	10	64	17	0	122
5	11	75	18	5	127
6	7	83	19	1	128
7	2	84	20	1	129
8	5	89	21	2	131
9	3	92	22	1	132
10	1	93	23	2	134
11	4	97	24	1	135
12	7	104	25	1	136
13	2	106	-	-	-

Table 2. Dataset #1 (DS1) : real time command and control system (RTC&CS) data set.

Dataset #2 (DS2), as shown in Table 3, is the second of three releases of software failure data collected from three different releases of a large medical record system (MRS) [26], consisting of 188 software components. Each component contains several files. Initially, the software consisted of 173 software components. All three releases added new functionality to the product. Between three and seven new components were added in each of the three releases, for a total of 15 new components. Many other components were modified during each of the three releases as a side effect of the added functionality. Detailed information of the dataset can be obtained in the report by Stringfellow and Andrews [26].

Dataset #3 (DS3), as shown in Table 4, is from one of four major releases of software products at Tandom Computers (TDC) [27]. There are 100 failures that are observed within testing CPU hours. Detailed information of the dataset can be obtained tin the report by Wood [27].

Week Index	Failures	Cumulative Failures	Week Index	Failures	Cumulative Failures
1	90	90	10	0	190
2	17	107	11	2	192
3	19	126	12	0	192
4	19	145	13	0	192
5	26	171	14	0	192
6	17	188	15	11	203
7	1	189	16	0	203
8	1	190	17	1	204
9	0	190	-	-	-

Table 3. DS2: medical record system (MRS) data set.

Table 4. DS3: Tandom Computers (TDC) data set.

Time Index (CPU hours)	Cumulative Failures	Time Index (CPU hours)	Cumulative Failures	Time Index (CPU hours)	Cumulative Failures
519	16	4422	58	8205	96
968	24	5218	69	8564	98
1430	27	5823	75	8923	99
1893	33	6539	81	9282	100
2490	41	7083	86	9641	100
3058	49	7487	90	10,000	100
3625	54	7846	93	-	-

5.2. Model Analysis

Tables 5–7 summarize the results of the estimated parameters of all 10 models in Table 1 using the LSE technique and the values of the six common criteria: MSE, AIC, PRR, PP, SAE, and R². We obtained the six common criteria at $t = 1, 2, \dots, 25$ from DS1 (Table 2), at $t = 1, 2, \dots, 17$ from DS2 (Table 3), and at cumulative testing CPU hours from DS3 (Table 4). As can be seen in Table 5, when comparing all of the models, the MSE and AIC values are the lowest for the newly proposed model, and the PRR, PP, SAE, and R^2 values are the second best. The MSE and AIC values of the newly proposed model are 7.361, 114.982, respectively, which are significantly less than the values of the other models. In Table 6, when comparing all of the models, all criteria values for the newly proposed model are best. The MSE value of the newly proposed model is 60.623, which is significantly lower than the value of the other models. The AIC, PRR, PP, and SAE values of the newly proposed model are 151.156, 0.043, 0.041, and 98.705, respectively, which are also significantly lower than the other models. The value of R^2 is 0.960 and is the closest to 1 for all of the models. In Table 7, when comparing all of the models, all the criteria values for the newly proposed model are best. The MSE value of the newly proposed model is 6.336, which is significantly lower than the value of the other models. The PRR, PP, and SAE values of the newly proposed model are 0.086, 0.066, and 36.250, respectively, which are also significantly lower than the other models. The value of \mathbb{R}^2 is 0.9940 and is the closest to 1 for all of the models.

Table 5. Model parameter estimation and comparison criteria from RTC&CS data set (DS1). Least-squares estimate (LSE); mean squared error; Akaike's information criterion (AIC); predictive ratio risk (PRR); predictive power (PP), sum absolute error (SAE), correlation index of the regression curve equation (\mathbb{R}^2).

Model	LSE's	MSE	AIC	PRR	PP	SAE	R ²
GOM	$\hat{a} = 136.050, \hat{b} = 0.138$	33.822	121.878	0.479	0.262	118.530	0.972
DSM	$\hat{a} = 124.665, \hat{b} = 0.356$	134.582	210.287	12.787	1.181	239.335	0.889
ISM	$\hat{a} = 136.050, \hat{b} = 0.138$ $\hat{\beta} = 0.0001$	35.363	123.878	0.479	0.262	118.532	0.972
YIDM	$\hat{a} = 81.252, \hat{b} = 0.340$ $\hat{\alpha} = 0.0333$	9.435	116.403	0.035	0.031	60.842	0.993
PNZM	$\hat{a} = 81.562, \hat{b} = 0.337$ $\hat{\alpha} = 0.033, \hat{\beta} = 0.00$	9.888	118.388	0.037	0.032	60.877	0.993
PZM	$\hat{a} = 0.01, \hat{b} = 0.138$ $\hat{\alpha} = 800.0, \hat{\beta} = 0.00, \hat{c} = 136.04$	38.895	127.878	0.479	0.262	118.530	0.972
DPM	$\hat{\alpha} = 28650, \ \hat{\beta} = 0.003$ $t_0 = 0.00, \ m_0 = 71.8$	274.911	382.143	0.857	3.568	304.212	0.792
TCM	$\hat{a} = 0.000035, \hat{b} = 0.734,$ $\hat{\alpha} = 0.29, \hat{\beta} = 0.002, \hat{N} = 427$	7.640	116.932	0.019	0.019	47.304	0.995
3PFDM	$\hat{a} = 1.696, \hat{b} = 0.001$ $\hat{c} = 6.808, \hat{\beta} = 1.574$ $\hat{N} = 173.030$	17.827	119.523	0.137	0.100	81.313	0.987
New Model	$\hat{a} = 0.277, \hat{\alpha} = 0.328$ $\hat{\beta} = 17.839, \hat{N} = 228.909$	7.361	114.982	0.022	0.022	47.869	0.994

Table 6. Model parameter estimation and comparison criteria from MRS data set (DS2).

Model	LSE's	MSE	AIC	PRR	PP	SAE	R ²
GOM	$\hat{a} = 197.387, \hat{b} = 0.399$	80.678	184.331	0.170	0.101	104.403	0.939
DSM	$\hat{a} = 192.528, \hat{b} = 0.882$	232.628	331.857	1.291	0.333	142.544	0.823
ISM	$\hat{a} = 197.354, \hat{b} = 0.399$ $\hat{\beta} = 0.000001$	86.440	186.334	0.171	0.101	104.370	0.939
YIDM	$\hat{a} = 182.934, \hat{b} = 0.464$ $\hat{\alpha} = 0.0071$	78.837	157.825	0.128	0.087	100.617	0.944
PNZM	$\hat{a} = 183.124, \hat{b} = 0.463$ $\hat{\alpha} = 0.007, \hat{\beta} = 0.00$	84.902	159.873	0.128	0.087	100.608	0.944
PZM	$\hat{a} = 195.990, \hat{b} = 0.3987$ $\hat{\alpha} = 1000.00, \hat{\beta} = 0.00, \hat{c} = 1.390$	100.989	190.332	0.172	0.102	104.354	0.939
DPM	$\hat{\alpha} = 26124.0, \hat{\gamma} = 0.0044$ $t_0 = 0.00, m_0 = 147.00$	769.282	480.341	0.415	0.712	334.128	0.494
TCM	$\hat{a} = 0.053, \hat{b} = 0.774,$ $\hat{\alpha} = 181.0, \hat{\beta} = 38.6, \hat{N} = 204.1$	72.283	158.933	0.052	0.048	103.196	0.956
3PFDM	$ \hat{a} = 0.028, \hat{b} = 0.210 \hat{c} = 9.924, \hat{\beta} = 0.005 \hat{N} = 206.387 $	81.090	163.797	0.073	0.061	106.341	0.951
New Model	$\hat{a} = 0.008, \hat{\alpha} = 0.275,$ $\hat{\beta} = 0.001, \hat{N} = 207.873$	60.623	151.156	0.043	0.041	98.705	0.960

Figures 2–4 show the graphs of the mean value functions for all 10 models for DS1, DS2, and DS3, respectively. Figures 5–7 show the graphs of the 95% confidence limits of the newly proposed model for DS1, DS2, and DS3. Tables A1–A3 in Appendix A list the 95% confidence intervals of all 10 NHPP software reliability models for DS1, DS2, and DS3. In addition, the relative error value of the proposed software reliability model confirms its ability to provide more accurate predictions as it remains closer to zero when compared to the other models (Figures 8–10).

Model	LSE's	MSE	AIC	PRR	РР	SAE	R ²
GOM	$\hat{a} = 133.835, \hat{b} = 0.000146$	8.620	86.136	0.556	0.242	42.166	0.991
DSM	$\hat{a} = 101.918, \hat{b} = 0.000507$	45.783	117.316	22.692	1.318	101.659	0.951
ISM	$\hat{a} = 133.835, \hat{b} = 0.000146$ $\hat{\beta} = 0.000001$	9.127	88.136	0.556	0.242	42.166	0.991
YIDM	$\hat{a} = 130.091, \hat{b} = 0.00015$ $\hat{\alpha} = 0.000003$	9.084	88.267	0.561	0.243	42.052	0.991
PNZM	$\hat{a} = 121.178, \hat{b} = 0.000163$ $\hat{\alpha} = 0.000009, \hat{\beta} = 0.00$	9.532	90.326	0.530	0.234	41.538	0.991
PZM	$\hat{a} = 122.259, = 0.0002$ $\hat{\alpha} = 9955.597, \hat{\beta} = 0.305$ $\hat{c} = 0.569$	11.491	92.020	0.643	0.268	44.848	0.990
DPM	$\hat{\alpha} = 123.193, \hat{\gamma} = 0.0001$ t ₀ = 0.0001, m ₀ = 38.459	156.480	212.867	0.917	2.879	196.360	0.851
ТСМ	$ \hat{a} = 0.000013, \hat{b} = 0.78, \hat{\alpha} = 141.399, \hat{\beta} = 54.71, \hat{N} = 254.707 $	7.090	90.758	0.091	0.068	37.880	0.9937
3PFDM	$ \hat{a} = 0.016, \hat{b} = 0.07 \hat{c} = 0.00001, \hat{\beta} = 157.458 \hat{N} = 205.025 $	9.410	92.360	0.420	0.200	39.909	0.992
New Model	$\hat{a} = 0.064, \hat{\alpha} = 0.731,$ $\hat{\beta} = 2509.898, \hat{N} = 337.765$	6.336	88.885	0.086	0.066	36.250	0.9940

 Table 7. Model parameter estimation and comparison criteria from MRS data set (DS3).



Figure 2. Mean value function of the ten models for DS1.



Figure 3. Mean value function of the ten models for DS2.



Figure 4. Mean value function of the ten models for DS3.



Figure 5. 95% confidence limits of the newly proposed model for DS1.



Figure 6. 95% confidence limits of the newly proposed model for DS2.



Figure 7. 95% confidence limits of the newly proposed model for DS3.



Figure 8. Relative error of the ten models for DS1.



Figure 9. Relative error of the ten models for DS2.



Figure 10. Relative error of the ten models for DS3.

5.3. Optimal Software Release Time

Factor η captures the effects of the field environmental factors based on the system failure rate as described in Section 2. System testing is commonly carried out in a controlled environment, where we can use a constant factor η equal to 1. The newly proposed model becomes a delayed S-shaped model when $\eta = 1$ in (7). Thus, we apply different mean value functions m(t) to the cost model C(T) of (8) when considering the three conditions described below. We apply the cost model to these three conditions using DS1 (Table 2). Using the LSE method, the parameters of the delayed S-shaped model and the newly proposed model are obtained, as described in Section 5.2.

(1) The expected total software cost with controlled environmental factor ($\eta = 1$) is

$$C_1(T) = C_0 + C_1 T + C_2 m(T) \mu_y + C_3 (1 - R(x|T)) + C_4 [m(T + T_w) - m(T)] \mu_w$$
(19)

where

$$m(T) = a(1 - (1 + bT)e^{-bT}), m(T + T_w) = a\left(1 - (1 + b(T + T_w))e^{-b(T + T_w)}\right).$$
(20)

(2) The expected total software cost with a random operating environmental factor ($\eta = f(x)$) is

$$C_2(T) = C_0 + C_1 T + C_2 m(T) \mu_y + C_3 (1 - R(x|T)) + C_4 [m(T + T_w) - m(T)] \mu_w$$
(21)

where

$$m(T) = N\left(1 - \frac{\beta}{\beta + aT - ln(1 + aT)}\right)^{\alpha}, \ m(T + T_w) = N\left(1 - \frac{\beta}{\beta + a(T + T_w) - ln(1 + a(T + T_w))}\right)^{\alpha}.$$
(22)

(3) The expected total software cost between the testing environment ($\eta = 1$) and field environment ($\eta = f(x)$) is

$$C_3(T) = C_0 + C_1 T + C_2 m_1(T) \mu_y + C_3 (1 - R(x|T)) + C_4 [m_2(T + T_w) - m_1(T)] \mu_w$$
(23)

where

$$m_1(T) = a \left(1 - (1 + bT)e^{-bT} \right), \ m_2(T + T_w) = N \left(1 - \frac{\beta}{\beta + a(T + T_w) - \ln(1 + a(T + T_w))} \right)^{\alpha}.$$
(24)

We consider the following coefficients in the cost model for the baseline case:

$$C_0 = 100, C_1 = 20, C_2 = 50, C_3 = 2000, C_4 = 400, T_w = 10, x = 20, \mu_y = 0.1, \mu_w = 0.2$$
 (25)

The results of the baseline case are listed in Table 8, and the expected total cost for the three conditions above is 1338.70, 2398.24, and 2263.33, respectively. For the second condition, the expected total cost and the optimal release time are high. The expected total cost is the lowest for the first condition, and the optimal release time is shortest for the third condition.

Table 8. Optimal release time T* subject to the warranty period.

Warrnaty Period	C ₁ (T)	T *	C ₂ (T)	T *	C ₃ (T)	T *
$T_w = 2$	1173.41	14.2	1403.78	11.6	599.88	10.5
$T_w = 5$	1286.95	14.9	1928.63	22.8	1334.72	11.3
$T_w = 10(basic)$	1338.70	15.1	2398.24	34.7	2263.33	12.3
$T_{w} = 15$	1348.88	15.2	2702.33	42.7	2969.88	13.0

To study the impact of different coefficients on the expected total cost and the optimal release time, we vary some of the coefficients and then compare them with the baseline case. First, we evaluate the impact of the warranty period on the expected total cost by changing the value of the corresponding warranty time and comparing the optimal release times for each condition. Here, we change the values

of T_w from 10 h to 2, 5, and 15 h, and the values of the other parameters remain unchanged. Regardless of the warranty period, the optimal release time for the third condition is the shortest, and the expected total cost for the first condition is the lowest overall. Figure 11 shows the graph of the expected total cost for the baseline case. Figures 12–14 show the graphs of the expected total cost subject to the warranty period for the three conditions.



Figure 11. Expected total cost for the baseline case.



Figure 12. Expected total cost subject to the warranty period for the 1st condition.



Figure 13. Expected total cost subject to the warranty period for the 2nd condition.

Next, we examine the impact of the cost coefficients, C_1 , C_2 , C_3 , and C_4 on the expected total cost by changing their values and comparing the optimal release times. Without loss of generality, we change only the values of C_2 , C_3 , and C_4 , and keep the values of the other parameters C_0 and C_1

unchanged, because different values of C₀ and C₁ will certainly increase the expected total cost. When we change the values of C_2 from 50 to 25 and 100, the optimal release time is only changed significantly for the second condition. As can be seen from Table 9, the optimal release time T* is 37.5 when the value of C₂ is 25, and 29.1 when the value of C₂ is 100. When we change the value of C₃ from 2000 to 500 and 4000, the optimal release time is only changed significantly for the first condition. As Table 10 shows, the optimal release time T^{*} is 16.5 when the value of C_3 is 500, and 14.6 when the value of C_3 is 4000. When we change the value of C_4 from 400 to 200 and 1000, the optimal release time is changed for all of the conditions. As can be seen from Table 11, the optimal release time T* is 14.3 for the first condition when the value of C_4 is 200, and 16.3 when the value of C_4 is 1000. In addition, the optimal release time T* is 20.0 for the second condition when the value of C_4 is 200, and 61.0 when the value of C_4 is 1000. The optimal release time T* is 11.6 for the third condition when the value of C_4 is 200, and 12.8 when the value of C_4 is 1000. Thus, the second condition has a much greater variation in optimal release time than the other conditions. As a result, we can confirm that the cost model of the first condition does not reflect the influence of the operating environment, and that the cost model of the second condition does not reflect the influence of the test environment. Figure 15 shows the graph of the expected total cost according to the cost coefficient C_2 in the 2nd condition. Figures 16–18 show the graphs of the expected total cost according to cost coefficient C_4 in the three conditions.



Figure 14. Expected total cost subject to the warranty period for the 3rd condition.

Table 9. Optimal release time T* according to cost coefficient C2.

Cost Coefficient C ₂	C ₁ (T)	T *	C ₂ (T)	T *	C ₃ (T)	T*
C ₂ = 25	1036.02	15.2	2013.25	37.5	1972.06	12.5
$C_2 = 50$ (basic)	1338.70	15.1	2398.24	34.7	2263.33	12.3
$C_2 = 100$	1943.64	15.1	3141.20	29.1	2843.35	12.1

Table 10. Optimal release time T* according to cost coefficient C₃.

Cost Coefficient C ₃	C ₁ (T)	T*	C ₂ (T)	T *	C ₃ (T)	T*
$C_3 = 500$	1270.65	16.5	2398.24	34.7	2262.96	12.4
$C_3 = 2000$ (basic)	1338.70	15.1	2398.24	34.7	2263.33	12.3
$C_3 = 4000$	1376.26	14.6	2398.24	34.7	2263.77	12.3

Table 11. Optimal release time T* according to cost coefficient C4.

Cost Coefficient C ₄	C ₁ (T)	T *	C ₂ (T)	T *	C ₃ (T)	T *	
$C_4 = 200$	1183.14	14.3	1859.29	20	1590.02	11.6	
$C_4 = 400$ (basic)	1338.70	15.1	2398.24	34.7	2263.33	12.3	
$C_4 = 1000$	1680.99	16.3	3272.23	61	4253.45	12.8	



Figure 15. Expected total cost according to cost coefficient C_2 for the 2nd condition.



Figure 16. Expected total cost according to cost coefficient C₄ for the 1st condition.



Figure 17. Expected total cost according to cost coefficient C_4 for the 2nd condition.



Figure 18. Expected total cost according to cost coefficient C₄ for the 3rd condition.

6. Conclusions

Existing well-known NHPP software reliability models have been developed in a test environment. However, a testing environment differs from an actual operating environment, so we considered random operating environments. In this paper, we discussed a new NHPP software reliability model, with S-shaped growth curve that accounts for the randomness of an actual operating environment. Tables 5–7 summarize the results of the estimated parameters of all ten models that are applied using the LSE technique and six common criteria (MSE, AIC, PRR, PP, SAE, and R²) for the DS1, DS2, and DS3 datasets. As can be seen from Tables 5–7, the newly proposed model displays a better overall fit than all of the other models when compared, particularly in the case of DS2. In addition, we provided optimal release policies for various environments to determine when the total software system cost is minimized. Using a cost model for a given environment is beneficial as it provides a means for determining when to stop the software testing process. In this paper, faults are assumed to be removed immediately when a software failure has been detected, and the correction process is assumed to not introduce new faults. Obviously, further work in revisiting these assumptions is worth the effort as our future study. We hope to present some new results on this aspect in the near future.

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Appendix A

Model	Time Index	1	2	3	4	5	6	7	8	9
	LCL	9.329	21.586	32.810	42.823	51.671	59.452	66.277	72.253	77.479
GOM	$\hat{\mathbf{m}}(t)$	17.537	32.814	46.121	57.713	67.811	76.607	84.269	90.944	96.758
	UCL	25.745	44.041	59.431	72.602	83.950	93.761	102.261	109.635	116.037
	LCL	1.352	11.192	24.289	37.792	50.271	61.120	70.191	77.571	83.458
DSM	$\hat{\mathbf{m}}(t)$	6.253	19.945	36.058	51.914	66.220	78.484	88.644	96.861	103.387
	UCL	11.154	28.698	47.827	66.035	82.170	95.847	107.097	116.150	123.316
	LCL	9.328	21.584	32.808	42.820	51.668	59.449	66.274	72.250	77.476
ISM	$\hat{\mathbf{m}}(t)$	17.535	32.811	46.118	57.709	67.807	76.603	84.266	90.941	96.755
	UCL	25.743	44.038	59.428	72.599	83.947	93.758	102.258	109.631	116.034
	LCL	14.263	28.936	40.449	49.466	56.637	62.468	67.333	71.506	75.185
YIDM	$\hat{\mathbf{m}}(t)$	23.831	41.573	54.982	65.305	73.433	79.998	85.451	90.111	94.209
	UCL	33.399	54.211	69.515	81.144	90.228	97.528	103.568	108.717	113.232
	LCL	14.191	28.840	40.360	49.400	56.598	62.455	67.343	71.534	75.227
PNZM	$\hat{\mathbf{m}}(t)$	23.741	41.460	54.879	65.230	73.389	79.984	85.462	90.143	94.255
	UCL	33.291	54.080	69.399	81.059	90.179	97.513	103.581	108.752	113.283
	LCL	9.329	21.586	32.810	42.823	51.671	59.452	66.277	72.253	77.479
PZM	$\hat{\mathbf{m}}(t)$	17.537	32.813	46.121	57.713	67.811	76.607	84.269	90.944	96.758
	UCL	25.745	44.041	59.431	72.602	83.950	93.761	102.261	109.635	116.037
	LCL	55.306	55.649	56.223	57.028	58.068	59.344	60.859	62.614	64.611
DPM	$\hat{\mathbf{m}}(t)$	71.929	72.316	72.964	73.874	75.047	76.485	78.189	80.162	82.403
	UCL	88.551	88.984	89.706	90.720	92.026	93.626	95.520	97.710	100.195
	LCL	17.974	30.408	39.981	47.851	54.561	60.419	65.621	70.302	74.555
TCM	$\hat{\mathbf{m}}(t)$	28.423	43.306	54.443	63.465	71.086	77.695	83.535	88.768	93.508
	UCL	38.872	56.204	68.905	79.080	87.611	94.971	101.449	107.234	112.461
	LCL	12.011	25.458	36.751	46.227	54.252	61.123	67.065	72.251	76.816
3PFDM	$\hat{\mathbf{m}}(t)$	20.991	37.452	50.708	61.611	70.737	78.487	85.151	90.942	96.021
	UCL	29.970	49.447	64.665	76.995	87.221	95.851	103.237	109.633	115.227
	LCL	18.358	30.526	39.950	47.745	54.426	60.283	65.501	70.208	74.492
New	$\hat{\mathbf{m}}(t)$	28.893	43.444	54.407	63.344	70.933	77.542	83.401	88.663	93.438
	UCL	39.428	56.363	68.864	78.944	87.440	94.801	101.300	107.118	112.384

Table A1. 95% Confidence interval of all 10 models (DS1).

Table A1. Cont.

Model	Time Index	10	11	12	13	14	15	16	17
	LCL	82.045	86.033	89.514	92.552	95.201	97.512	99.527	101.284
GOM	$\hat{\mathbf{m}}(t)$	101.823	106.235	110.078	113.426	116.342	118.882	121.095	123.023
	UCL	121.600	126.436	130.641	134.300	137.483	140.253	142.663	144.762
	LCL	88.083	91.672	94.431	96.534	98.127	99.326	100.225	100.895
DSM	$\hat{\mathbf{m}}(t)$	108.498	112.457	115.494	117.807	119.557	120.874	121.861	122.596
	UCL	128.914	133.241	136.557	139.080	140.988	142.423	143.497	144.298
	LCL	82.043	86.031	89.512	92.550	95.200	97.511	99.526	101.283
ISM	$\hat{\mathbf{m}}(t)$	101.820	106.232	110.076	113.424	116.340	118.881	121.094	123.022
	UCL	121.597	126.434	130.639	134.298	137.481	140.251	142.662	144.761
	LCL	78.512	81.588	84.485	87.257	89.939	92.558	95.133	97.678
YIDM	$\hat{\mathbf{m}}(t)$	97.905	101.316	104.523	107.586	110.546	113.433	116.267	119.064
	UCL	117.298	121.044	124.561	127.916	131.153	134.307	137.401	140.451
	LCL	78.562	81.642	84.540	87.310	89.987	92.600	95.168	97.703
PNZM	$\hat{\mathbf{m}}(t)$	97.960	101.376	104.584	107.645	110.600	113.479	116.305	119.092
	UCL	117.339	121.110	124.028	127.980	131.212	134.338	137.442	140.481
PZM	LCL	82.045	86.033	89.514	92.552	95.201	97.512	99.527	101.284
	m(t)	101.823	106.235	110.078	113.426	116.342	118.882	121.095	123.023
	UCL .	121.000	120.450	150.041	134.300	157.405	140.235	142.005	144.702
DPM	$\hat{\mathbf{LCL}}$	66.855 84.016	69.346 87.701	72.087	75.081	78.331	81.838	85.606	89.636
	UCL	102 977	106.055	109 432	94.095 113.105	97.704 117.078	101.393	105.762	130 788
	LCL	TO 150	00.000	107.102	00.500	01.416	04.155	06 744	00.101
TCM	$\hat{\mathbf{m}}(t)$	78.453 97.840	82.050 101.828	85.387	88.500	91.416 112 174	94.157 115 193	96.744 118.038	99.191 120 726
ICIVI	UCL	117.227	121.606	125.654	129.418	132.933	136.229	139.332	142.261
	ICI	20.862	94 47E	07 710	00.647	02 202	05 724	07.040	00.074
3PEDM	$\hat{\mathbf{m}}(t)$	100 512	04.475 104 512	108.096	90.047 111.326	93.303 114 253	95.724 116.917	97.940 119.352	99.974 121 586
511 DW	UCL	120.162	124.549	128.473	132.006	135.203	138.110	140.764	143.198
	LCL	78 423	82 051	85 417	88 555	91 491	94 248	96 844	99 296
New	$\hat{\mathbf{m}}(t)$	97.806	101.829	105.554	109.019	112.257	115.293	118.148	120.841
	UCL	117.189	121.607	125.690	129.484	133.024	136.338	139.452	142.387
Model	Time	18	19	20	21	22	23	24	25
Model	Time Index	18	19	20	21	22	23	24	25
Model	Time Index LCL	18 102.8153	19 104.15	20 105.3133	21 106.3271	22 107.2106	23 107.9804	24 108.6512 121.0010	25 109.2357 121.721
Model GOM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146 5892	19 104.15 126.165 148 1799	20 105.3133 127.4392 149.565	21 106.3271 128.5491 150 7711	22 107.2106 129.516 151.8214	23 107.9804 130.3582 152.736	24 108.6512 131.0919 153.5326	25 109.2357 131.731 154 2263
Model GOM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892	19 104.15 126.165 148.1799	20 105.3133 127.4392 149.565	21 106.3271 128.5491 150.7711	22 107.2106 129.516 151.8214	23 107.9804 130.3582 152.736	24 108.6512 131.0919 153.5326	25 109.2357 131.731 154.2263
Model GOM	$ Time Index LCL \hat{m}(t) UCL LCL \hat{m}(t)$	18 102.8153 124.7022 146.5892 101.3931 123.1427	19 104.15 126.165 148.1799 101.7621 123.5475	20 105.3133 127.4392 149.565 102.0346 123.8463	21 106.3271 128.5491 150.7711 102.2353 124.0664	22 107.2106 129.516 151.8214 102.3828 124.2281	23 107.9804 130.3582 152.736 102.4909 124.3466	24 108.6512 131.0919 153.5326 102.57 124.4333	25 109.2357 131.731 154.2263 102.6278 124.4967
Model GOM DSM	$\begin{array}{c} \textbf{Time} \\ \textbf{Index} \\ \\ \textbf{LCL} \\ \hat{\textbf{m}}(t) \\ \textbf{UCL} \\ \\ \\ \textbf{LCL} \\ \hat{\textbf{m}}(t) \\ \textbf{UCL} \end{array}$	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924	19 104.15 126.165 148.1799 101.7621 123.5475 145 3328	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734	23 107.9804 130.3582 152.736 102.4909 124.3466 146 2023	24 108.6512 131.0919 153.5326 102.57 124.4333 146 2967	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656
Model GOM DSM	Time IndexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656
Model GOM DSM	$\begin{array}{c} \text{Time} \\ \text{Index} \\ \\ \text{LCL} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131 7306
Model GOM DSM ISM	Time Index LCL m̂(t) UCL LCL m̂(t) UCL LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259
Model GOM DSM ISM	Time Index LCL m(t) UCL LCL m(t) UCL LCL m(t) UCL LCL m(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547
Model GOM DSM ISM	Time Index LCL m(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138 2097	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215
Model GOM DSM ISM YIDM	Time Index LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883
Model GOM DSM ISM YIDM	Time Index LCL m(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587
Model GOM DSM ISM YIDM PNZM	Time Index LCL m(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477
Model GOM DSM ISM YIDM PNZM	Time Index LCL m(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.423 105.2037 127.3191 149.4345	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084
Model GOM DSM ISM YIDM PNZM	Time Index LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357
Model GOM DSM ISM YIDM PNZM	Time Index LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731
Model GOM DSM ISM YIDM PNZM PZM	Time Index LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263
Model GOM DSM ISM YIDM PNZM PZM	Time Index LCL m̂(t) UCL LCL m̂(t) UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157
Model GOM DSM ISM YIDM PNZM PZM DPM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038
Model GOM DSM ISM YIDM PNZM PZM DPM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919
Model GOM DSM ISM YIDM PNZM PZM DPM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944 147.6682	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996 150.174	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057 161.1389
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349 101.8497	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944 147.6682 103.5836	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996 150.174 105.1914	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635 106.6864	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467 108.0801	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271 110.6019	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057 161.1389 111.7464
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349 101.8497 123.6436	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944 103.5836 125.5443	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3126 127.3191 149.4345 103.3268 125.2629 147.199 105.8252 127.9996 150.174 105.1914 127.3057	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635 106.6864 128.9424 151.1022	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467 108.0801 130.4673 153.2514 113.4157 155.3226 108.0801 130.4673 153.2514 113.4157 155.3226 154.8467 108.0801 130.4673 155.3514 155.3514 155.3526 154.8467 108.0801 130.4673 155.3514 155.3514 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 155.3526 107.2106 155.3266 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 107.2106 155.3526 109.7585 132.3026 155.3527 155.3527 155.3526 155.3526 155.3527 155.3526 155.3526 155.3527 155.35	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4951 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032 109.3824 131.8914	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271 110.6019 133.2244 155.5407	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057 161.1389 111.7464 134.4748
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349 101.8497 123.6436 145.4374	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944 103.5836 125.5443 147.505	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3126 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996 150.174 105.1914 127.3057 149.4199	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635 106.6864 128.9424 151.1983	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467 108.0801 130.4673 152.8544	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4951 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032 109.3824 131.8914 154.4004	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.1866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271 110.6019 133.2244 155.8469	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057 161.1389 111.7464 134.4748 157.2032
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349 101.8497 123.6436 145.4374 101.6161 123.6235	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6442 103.7205 125.5443 147.505 103.8169 125.243 147.505	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996 150.174 105.1914 127.3057 149.4199 105.9085	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635 106.6864 128.9424 151.1983 107.8999	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467 108.0801 130.4673 152.8544 109.799	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032 109.3824 131.8914 154.4004 111.6129	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.866 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271 110.6019 133.2244 155.8469 113.3476	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 156.1038 180.5919 115.0726 138.1057 161.1389 111.7464 134.4748 157.2032
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM New	Time Index LCL $\hat{m}(t)$ UCL	18 102.8153 124.7022 146.5892 101.3931 123.1427 144.8924 102.8143 124.7012 146.588 100.2015 121.8354 143.4693 100.2169 121.8523 143.4693 100.2169 121.8523 143.4877 102.8153 124.7022 146.5892 93.93128 114.9445 135.9577 101.5124 123.2736 145.0349 101.8497 123.6436 145.4374 101.6161 123.3873 145.1584	19 104.15 126.165 148.1799 101.7621 123.5475 145.3328 104.1492 126.164 148.1789 102.7107 124.5875 146.4644 102.7154 124.5927 146.47 104.15 126.165 148.1799 98.49414 119.961 141.4278 103.7205 125.6944 147.6682 103.8366 125.5443 147.505 103.8169 125.8 147.782	20 105.3133 127.4392 149.565 102.0346 123.8463 145.658 105.3126 127.4383 149.5641 105.2103 127.3263 149.4423 105.2037 127.3191 149.4345 105.3133 127.4392 149.565 103.3268 125.2629 147.199 105.8252 127.9996 150.174 105.1914 127.3057 149.4199 105.9085 128.0908 129.722	21 106.3271 128.5491 150.7711 102.2353 124.0664 145.8975 106.3265 128.5484 150.7703 107.7037 130.0555 152.4073 107.6855 130.0356 152.3856 106.3271 128.5491 150.7711 108.4316 130.8518 153.2719 107.8352 130.1994 152.5635 106.6864 128.9424 151.1983 107.8999 130.2702 152.4044	22 107.2106 129.516 151.8214 102.3828 124.2281 146.0734 107.21 129.5154 151.8207 110.1934 132.7779 155.3625 110.1632 132.7449 155.3266 107.2106 129.516 151.8214 113.8108 136.7288 159.6469 109.7585 132.3026 154.8467 108.0801 130.4673 152.8544 109.799 132.3469	23 107.9804 130.3582 152.736 102.4909 124.3466 146.2023 107.9799 130.3577 152.7354 112.6811 135.4956 158.31 112.6385 135.4491 158.2597 107.9804 130.3582 152.736 119.4664 142.8956 166.3248 111.6019 134.3169 157.032 109.3824 131.8914 154.4004 111.6129 134.3289 157.045	24 108.6512 131.0919 153.5326 102.57 124.4333 146.2967 108.6508 131.0914 153.5321 115.1679 138.2097 161.2516 115.1129 138.1497 161.2516 108.6512 131.0919 153.5326 125.4007 149.3535 173.3063 113.3714 136.2493 159.1271 110.6019 133.2244 155.8469 113.3476 136.2233 159.090	25 109.2357 131.731 154.2263 102.6278 124.4967 146.3656 109.2353 131.7306 154.2259 117.6547 140.9215 164.1883 117.587 140.8477 164.1084 109.2357 131.731 154.2263 131.6157 164.1084 138.05919 115.0726 138.1057 161.1389 111.7464 134.4748 157.2032 115.009 138.0364 115.009 138.0364

Model	Time index	1	2	3	4	5	6	7	8	9
	LCL	49.147	88.100	114.753	132.788	144.944	153.123	158.620	162.312	164.791
GOM	$\hat{\mathbf{m}}(t)$	64.942	108.518	137.757	157.376	170.540	179.373	185.300	189.276	191.945
	UCL	80.737	128.935	160.761	181.963	196.135	205.623	211.980	216.241	219.099
DSM	$\hat{\mathbf{m}}(t)$	29.754 42 537	81.610 101 340	119.319 142 735	141.607 166 930	153.581 179.867	159.671 186.433	162.660 189.651	164.090 191 191	164.762 191 914
DSM	UCL	55.320	121.071	166.151	192.253	206.153	213.194	216.643	218.291	219.066
	LCL	49.138	88.084	114.731	132.764	144.918	153.095	158.591	162.282	164.761
ISM	$\hat{\mathbf{m}}(t)$	64.931	108.500	137.733	157.349	170.511	179.343	185.269	189.245	191.913
	UCL	80.725	128.915	160.736	181.935	196.104	205.590	211.946	216.207	219.065
	LCL	51.989	90.820	116.125	132.604	143.452	150.736	155.770	159.386	162.108
YIDM	m(t)	68.171 84 354	111.517	139.254	157.176	168.926 194 400	202 858	182.228	186.125 212.864	189.057 216.007
		51 946	90 780	116 107	132 609	143 476	150 772	155 811	159 427	162 145
PNZM	$\hat{\mathbf{m}}(t)$	68.123	111.474	139.234	157.181	168.952	176.835	182.272	186.169	189.097
	UCL	84.300	132.168	162.361	181.753	194.428	202.899	208.733	212.912	216.049
PZM	LCL	49.064	88.017	114.677	132.724	144.892	153.080	158.585	162.284	164.769
	$\hat{\mathbf{m}}(t)$	64.848	108.425	137.674	157.306	170.483	179.327	185.263	189.247	191.921
	UCL	80.631	128.834	160.672	181.888	196.074	205.573	211.940	216.210	219.074
DPM	$\hat{\mathbf{m}}(t)$	123.469 147 252	124.167 148.012	125.336	126.981 151.071	129.105 153.379	131.715	134.816 159 574	138.411 163.470	142.507 167 904
DIM	UCL	171.036	171.857	173.230	175.161	177.652	180.709	184.333	188.529	193.301
	LCL	60.749	93,551	114.901	129.743	140.446	148.355	154.305	158.844	162.346
TCM	$\hat{\mathbf{m}}(t)$	78.066	114.526	137.919	154.071	165.673	174.225	180.648	185.542	189.313
	UCL	95.384	135.501	160.936	178.399	190.901	200.096	206.991	212.239	216.281
	LCL	57.549	93.474	115.850	130.807	141.307	148.941	154.636	158.970	162.319
3PFDM	$\tilde{m}(t)$	74.461 91.374	114.441 135.408	138.954	155.226	166.605	174.858	181.005	185.677	189.284
		62.246	02.042	114 265	179.043	1/0.426	148 466	154 422	158 020	162 275
New	$\hat{\mathbf{m}}(t)$	62.346 79.861	93.042 113.965	137.225	129.445	165.652	140.400	180.786	185.634	189.345
	UCĹ	97.377	134.889	160.184	178 048	190.878	200.224	207.138	212,338	216.315
				100.101	170.010				212.000	210.010
Model	Time index	10	11	100.101	12	13.	14	15	16	17
Model	Time index LCL	10 166.455	11 167.	572 16	12 58.321 16	13. 168.824 16	14 59.162 16	15 59.389 16	16 59.541	17 169.643
Model GOM	Time index LCL $\hat{\mathbf{m}}(t)$	10 166.455 193.735 221.016	11 167 194.	572 16 937 19	12 12 13 15 15 16 17 17 17 18 18 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19 19	13. 68.824 16 66.284 19 19.742 22	14 59.162 16 96.647 19	15 59.389 16 96.890 19	16 59.541 97.054	17 169.643 197.163
Model GOM	Time index LCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016	11 167. 194. 222.	572 16 937 19 302 22	12 12 58.321 16 55.743 19 13.164 22 57.200 10	13. 58.824 16 66.284 19 23.743 22	14 59.162 16 56.647 19 24.132 22	15 59.389 16 96.890 19 24.392 22	16 59.541 07.054 24.567	17 169.643 197.163 224.684
GOM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$	10 166.455 193.735 221.016 165.073 192.249	1: 167. 194. 222. 165. 192	572 16 937 19 302 22 216 16 402 19	12 (8.321 16) (5.743 19) (3.164 22) (5.280 16) (2.472 19)	13. 58.824 16 66.284 19 23.743 22 55.309 16 52.503 19	14 59.162 16 66.647 19 24.132 22 55.322 16 22 517	15 59.389 16 56.890 19 24.392 22 55.328 16 52.328 16 52.523 19	16 59.541 57.054 24.567 55.331 52.526	17 169.643 197.163 224.684 165.332 192.527
Model GOM DSM	$\begin{tabular}{c} \hline Time \\ index \\ LCL \\ \hat{m}(t) \\ UCL \\ LCL \\ \hat{m}(t) \\ UCL \\ \hline \end{array} \end{tabular}$	10 166.455 193.735 221.016 165.073 192.249 219.424	1: 167. 194. 222. 165. 192. 219.	1 572 16 937 19 302 22 216 16 402 19 588 21	12 58.321 16 55.743 19 23.164 22 55.280 16 52.472 19 9.663 21	13. 68.824 16 66.284 19 13.743 22 55.309 16 92.503 19 9.696 21	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 19.711 21	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21	16 59.541 07.054 24.567 55.331 02.526 19.721	17 169.643 197.163 224.684 165.332 192.527 219.722
Model GOM DSM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL LCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425	1: 167 194. 222. 165 192. 219. 167.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16	12 88.321 16 165.743 19 13.164 22 55.280 16 12,472 19 9.663 21 58.291 16	13. 88.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.8794 16	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 99.711 23 59.131 16	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612
Model GOM DSM ISM	$\begin{tabular}{ c c c c } \hline Time \\ index \\ \hline LCL \\ \hat{m}(t) \\ UCL \\ \hline LCL \\ \hat{m}(t) \\ UCL \\ \hline LCL \\ \hat{m}(t) \\ \hline \end{tabular}$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703	11 167. 194. 222. 165. 192. 219. 167. 194.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19	12 8.321 16 55.743 19 13.164 22 52.280 16 12.472 19 9.663 21 88.291 16 55.710 19	13. 58.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.794 16 66.251 19	14 59.162 16 66.647 19 94.132 22 55.322 16 92.517 19 99.711 22 59.131 16 96.614 19	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 69.6857 19	16 59.541 07.054 24.567 55.331 92.526 19.721 59.510 97.021	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130
Model GOM DSM ISM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981	1: 167. 194. 222. 165. 192. 219. 167. 194. 222.	1001101 1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22	12 88.321 16 12 19 13.164 22 55.280 16 12.472 19 9.663 21 18.291 16 15.710 19 13.129 22	13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 9.696 21 58.794 16 66.251 19 13.708 22	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 19.711 22 59.131 16 66.614 19 24.096 22	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 66.857 19 24.357 22	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649
Model GOM DSM ISM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 101.282	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 196. 166.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16	12 12 18.321 16 19.5743 19 13.164 22 55.280 16 12.772 19 9.663 21 55.710 19 13.129 22 77.661 16 56.222 16	13. 58.824 16 66.284 19 93.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 19 93.708 22 99.107 17 99.107 17	14 59.162 10 66.647 19 94.132 22 55.322 10 92.517 19 99.711 21 59.131 10 96.614 19 24.096 22 70.464 17 80.047 11	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 90.44 21	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 90.900	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147
Model GOM DSM ISM YIDM	$\begin{tabular}{lllllllllllllllllllllllllllllllllll$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220.	1 572 16 937 19 302 22 216 16 402 19 5588 21 542 16 904 19 267 22 076 16 328 19 328 19	12 88.321 16 15.743 19 13.164 22 55.280 16 12.472 19 9.663 21 88.321 16 12.472 19 9.663 21 18.291 16 19.5.710 19 13.129 22 77.661 16 16.5.033 19 12.405 22	13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 12 33.708 22 99.107 17 766.587 19 20.107 17 44.068 22	14 59.162 16 66.647 19 74.132 22 55.322 16 92.517 19 99.711 21 59.131 16 96.614 19 24.096 22 70.464 17 88.047 19 25.629 22	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 12 24.357 22 71.767 17 99.446 20 27.126 22	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 88.583	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014
Model GOM DSM ISM YIDM	Time index LCL m̂(t) UCL LCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16	12 12 8.321 16 15.743 19 13.164 22 55.280 16 12.472 19 9.663 21 58.291 16 15.710 19 13.129 22 57.661 16 16.5.033 19 12.2.405 22 17.669 16	13. 88.824 16 66.284 19 93.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 19 93.708 22 59.107 17 96.587 19 94.068 22 99.101 17	14 99.162 16 96.647 19 94.132 22 55.322 16 92.517 19 99.711 27 59.131 16 96.644 12 70.464 12 98.047 19 95.529 22 70.445 12	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.763 12	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 90.809 28.583 72.986	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217
Model GOM DSM ISM YIDM PNZM	$\begin{tabular}{ c c c c } \hline Time \\ index \\ \hline ICL \\ \hline m(t) \\ UCL \\ \hline LCL \\ \hline m(t) \\ \hline n(t) \\ $	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16 349 19	12 12 8.321 16 15.743 19 13.164 22 52.280 16 12.472 19 9.663 21 88.291 16 15.710 19 13.129 22 37.661 16 15.033 19 12.4405 22 37.669 16 15.041 19	13. 18.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.794 16 66.251 19 13.708 22 59.107 17 66.587 19 94.068 22 59.101 17 76.581 19	14 59.162 16 66.647 19 24.132 22 55.322 16 12.517 19 19.711 22 59.131 16 66.614 19 24.096 22 70.464 12 15.529 22 70.464 15 15.629 22 70.445 17 98.025 19	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 66.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.733 17 99.410 20	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 90.809 28.583 72.986 90.756	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078
Model GOM DSM ISM YIDM PNZM	$\begin{tabular}{ c c c c } \hline Time \\ index \\ \hline ICL \\ \hline m(t) \\ UCL \\ \hline LCL \\ \hline m(t) \\ UCL \\ \hline ext{tabular}$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16 349 19 602 22	12 12 88.321 16 15.743 19 13.164 22 52.80 16 12.472 19 9.663 21 16.5.710 19 13.129 22 7.661 16 5.033 19 12.405 22 7.669 16 5.041 19 12.413 22	13. 18.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.824 16 16.251 19 17.708 22 99.107 17 17.65.87 19 99.101 17 17.65.81 19 94.068 22 99.101 17 96.581 19 94.061 22	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 19.711 22 59.131 16 66.614 19 24.096 22 70.464 15 15.5629 22 70.445 15 15.5606 22	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 66.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.733 17 99.410 20 27.087 22	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940
Model GOM DSM ISM YIDM PNZM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437	11 167. 194. 222. 165. 192. 219. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16 349 19 602 22 557 16	12 12 18.321 16 15.743 19 13.164 22 5.280 16 12.472 19 9.663 21 18.291 16 15.710 19 13.129 22 7.661 16 15.2033 19 12.405 22 7.669 16 15.041 19 12.413 22 8.309 16	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 9.696 21 58.794 16 66.251 19 13.708 22 99.107 17 76.587 19 94.068 22 99.101 17 76.581 19 94.061 22 88.814 16	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 99.711 21 99.731 16 96.644 12 94.096 22 70.464 12 88.047 19 82.5629 22 70.445 12 88.025 19 82.5606 22 99.152 16	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.733 17 92.410 20 27.087 22 59.380 16	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 08.809 28.583 72.986 00.756 28.526 59.532	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635
Model GOM DSM ISM YIDM PNZM PZM	$\begin{tabular}{ c c c c } \hline Time index \\ index \\ LCL \\ \hline m(t) \\ UCL \\ \\ LCL \\ \hline m(t) \\ UCL \\ \\ LCL \\ \hline m(t) \\ UCL \\ \\ \\ LCL \\ \hline m(t) \\ UCL \\ \\ \\ LCL \\ \hline m(t) \\ \\ UCL \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 (20.995	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 165. 194. 222. 165. 192. 219. 167. 194. 222. 165. 192. 219. 167. 194. 222. 165. 192. 219. 167. 194. 222. 165. 192. 219. 167. 194. 222. 165. 192. 219. 167. 194. 222. 165. 192. 219. 167. 194. 222. 165. 194. 222. 165. 194. 222. 165. 194. 222. 165. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 220. 166. 193. 220. 167. 194. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 193. 220. 167. 194. 220. 167. 194. 220. 167. 194. 193. 220. 167. 194. 194. 220. 167. 194. 19	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16 349 19 602 22 557 16 921 19 925 20	12 12 18.321 16 19.5743 19 13.164 22 55.280 16 12.772 19 19.663 21 18.291 16 19.5710 19 13.129 22 77.661 16 19.50.33 19 12.405 22 77.669 16 19.50.41 19 12.413 22 18.309 16 15.729 12 12.3150 27	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 99.696 21 58.794 16 96.251 19 13.708 22 99.107 17 96.587 19 94.068 22 99.101 17 96.581 19 94.061 22 98.814 16 96.2721 23	14 59.162 16 66.647 19 94.132 22 55.322 16 92.517 19 99.711 21 59.131 16 96.614 19 94.096 22 70.464 17 98.047 19 95.529 22 70.445 17 98.025 19 99.152 16 96.636 14 99.152 16 96.636 14	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.733 17 99.410 20 27.087 22 99.380 16 96.881 16 96.482 27	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 90.8.893 72.986 90.756 28.526 59.532 97.045 20.526	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.65
Model GOM DSM ISM YIDM PNZM PZM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.100	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 222. 165. 194. 194. 219. 195. 19	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 5580 22 0557 16 921 19 285 22	12 12 38.321 16 55.743 19 33.164 22 55.280 16 92.472 19 9.663 21 55.280 16 55.710 19 33.129 22 57.661 16 55.033 19 22.405 22 57.669 16 55.041 19 22.413 22 38.309 16 55.729 15 33.150 22	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 19 13.708 22 59.107 17 765.587 19 94.068 22 99.101 17 765.581 19 44.068 22 38.814 16 66.272 19 33.731 22	14 59.162 10 66.647 19 74.132 22 55.322 10 92.517 19 99.711 22 59.131 10 96.614 19 70.464 12 70.464 12 70.464 12 70.445 12 70.445 12 99.152 10 96.636 19 99.152 10 96.636 19 94.120 22	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 27.126 22 59.380 16 96.4851 15 24.382 22 27.020 14	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 97.045 24.558 52.526	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675
Model GOM DSM ISM YIDM PNZM PZM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 191.415 218.531 166.437 193.716 220.995 147.109 172.880	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 167. 194. 222. 167. 194. 222. 167. 194. 178. 167. 194. 193. 167. 193. 167. 193. 167. 193. 167. 193. 167. 179. 179. 179. 167. 179. 179. 167. 179. 179. 167. 179. 179. 167. 179. 167. 179. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 194. 220. 166. 179. 220. 166. 179. 220. 166. 179. 220. 166. 179. 220. 166. 179. 220. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 167. 179. 179. 179. 179. 179. 177. 179. 177. 179. 177. 179. 177. 179. 177. 17	1 572 16 937 19 302 22 216 16 402 19 558 21 542 16 904 19 267 22 076 16 328 19 550 22 557 16 921 19 285 22 223 15 401 18	12 12 8.321 16 15.743 19 13.164 22 52.280 16 12.472 19 9.663 21 8.291 16 15.710 19 13.129 22 37.661 16 15.033 19 12.4405 22 37.669 16 15.729 19 13.150 22 37.833 16 14.474 19	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.794 16 66.251 19 93.708 22 99.107 17 76.587 19 99.101 17 76.581 19 44.068 22 88.814 16 66.272 19 123.731 22 44.006 17 92.3731 22 44.006 17 92.101 17	14 59.162 16 66.647 19 74.132 22 55.322 16 92.517 19 99.711 21 59.131 16 66.614 15 70.464 17 70.465 17 88.027 19 99.152 16 66.636 19 25.606 22 59.152 16 66.636 19 24.120 22 70.687 17 78.8286 20	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 71.766 22 71.767 17 99.446 20 27.126 22 59.380 16 66.881 19 24.382 22 59.380 16 66.881 19 24.382 22 77.901 18 6.034 21	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 14.349	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235
Model GOM DSM ISM YIDM PNZM PZM DPM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650	11 167. 194. 222. 165. 192. 167. 194. 219. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 222. 152. 178. 204.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 557 16 921 19 285 22 223 15 401 18 580 21	12 12 88.321 16 15.743 19 13.164 22 52.80 16 12.5.280 16 12.5.280 16 12.472 19 9.663 21 16.5.710 15 13.129 22 37.661 16 15.033 19 12.405 22 37.669 16 15.729 19 13.150 22 37.853 16 34.474 19 1.094 21	13. 18.824 16 66.284 19 13.743 22 55.309 16 12.503 19 9.696 21 58.824 16 16.2503 19 9.696 21 18.794 16 16.251 19 19.3708 22 19.107 17 16.587 19 14.068 22 19.101 17 16.581 19 14.061 22 18.814 16 16.272 19 13.731 22 34.006 17 94.006 17 91.101 19 8.195 22	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 19.711 22 59.131 16 66.614 19 24.096 22 70.464 17 85.629 22 70.445 17 85.660 22 99.152 16 96.636 19 24.120 22 70.687 17 88.286 20 25.885 23	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 66.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.733 17 99.410 20 27.087 22 59.380 16 96.881 19 24.382 22 77.901 18 96.034 21 34.167 24	16 99.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 97.045 24.558 35.654 14.349 13.045	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519
Model GOM DSM ISM YIDM PNZM PZM DPM	$\begin{tabular}{ c c c c } \hline Time \\ index \\ \hline ICL \\ \hline m(t) \\ UCL \\ \hline LCL \\ \hline m(t) \\ UCL \\ \hline m(t) \\ \hline$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073	11 167 194. 222 165. 192. 167 194. 222 166. 193. 220 166 193 220 166 193 220 166 193 220 166 193 220 167 194 222 178 204 167	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 557 16 921 19 285 22 15 401 580 21 214 16	12 12 8.321 16 15.743 19 13.164 22 5.280 16 12.5.280 16 12.5.280 16 12.5.280 16 12.472 19 9.663 21 18.291 16 15.710 19 13.129 22 7.661 16 5.033 19 12.405 22 7.669 16 5.041 19 12.413 22 37.569 16 5.729 19 31.50 22 37.853 16 44.474 19 1.094 21 38.906 17	13. 13. 18.824 16 16.284 19 13.743 22 5.309 16 12.503 19 9.696 21 18.794 16 16.251 19 13.708 22 19.107 17 16.587 19 19.4068 22 19.101 17 16.581 19 14.068 22 19.101 17 16.581 19 14.061 22 13.731 22 34.006 17 19.101 19 18.195 22 70.251 17	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 99.711 22 59.131 16 66.614 19 24.096 22 70.464 17 82.5629 22 70.445 17 82.626 19 99.152 16 96.636 19 24.120 22 70.687 17 82.886 20 25.885 23 71.327 17	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 66.857 19 24.357 22 71.767 17 92.446 22 27.126 22 71.767 17 92.410 22 27.087 22 59.380 16 96.881 19 24.382 22 77.901 18 96.034 21 34.167 24	16 99.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 14.349 13.045 72.889	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073 192.249	11 167. 194. 222. 165. 192. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 202. 152. 178. 204. 167. 194. 204.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 557 16 921 19 285 22 223 15 401 18 580 21 214 16 552 19	12 12 18.321 16.5.743 19.5.743 19.164 22.5.280 16.92.472 19.663 19.663 19.663 19.663 19.663 19.663 19.663 19.2472 19.9.663 19.25710 19.3129 22.77.661 16.5033 19.2.405 22.405 22.405 22.405 22.405 22.413 22.413 22.413 22.413 22.413 22.729 19.3150 22.77.853 10.94 11.094 21.094 88.906 17 66.371 92.927	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 9.696 21 58.824 16 96.92 13 88.794 16 96.251 19 13.708 22 99.107 17 46.587 19 94.068 22 99.101 17 46.581 19 42.061 22 88.814 16 66.272 19 13.731 22 74.006 17 11.01 19 8.195 22 70.251 17 77.818 19	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 99.711 21 59.161 16 90.711 21 99.131 16 96.614 19 24.096 22 70.464 12 98.047 19 95.629 22 70.445 12 99.152 16 66.636 19 24.120 22 70.687 12 70.828 22 71.327 12 71.327 12 72.642 12	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.767 17 99.410 20 27.087 22 99.380 16 96.881 19 92.4382 22 77.901 18 16.034 21 34.167 24 72.192 17 799.903 20	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 92.82.583 72.986 90.756 28.526 59.532 77.045 24.558 55.654 14.349 13.045 72.889 00.653	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455 201.260 202.67
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073 192.249 219.424	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 222. 152. 178. 204. 167. 194. 221.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 095 16 349 19 285 22 257 16 921 19 285 22 223 15 401 18 580 21 214 16 552 19 890 22	12 12 18.321 16 15.743 19 13.164 22 5.280 16 12.77 19 9.663 21 18.291 16 19.5710 19 13.129 22 7.661 16 15.2033 19 12.2405 22 7.669 16 15.729 19 13.150 22 77.853 16 94.4.74 19 1.094 21 38.806 17 96.371 19 3.837 22	13. 13. 58.824 16 66.284 19 93.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 19 93.708 22 99.107 17 96.587 19 94.068 22 99.101 17 96.581 19 44.061 22 58.814 16 96.272 19 33.731 22 54.006 17 91.101 17 98.814 16 96.272 19 33.731 22 54.006 17 92.101 17 93.1731 22 92.0251 17 92.251 17 92.53.84 22 92.53.84 22	14 59.162 10 66.647 19 24.132 22 55.322 10 92.517 19 99.711 21 59.131 10 66.614 19 24.096 22 70.464 17 98.047 19 95.520 22 70.445 17 98.025 19 99.152 10 96.636 19 24.120 22 70.6487 17 98.286 20 25.885 23 71.327 17 98.974 19 26.621 27	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.767 17 99.410 20 27.087 22 59.380 16 96.881 19 24.382 22 277.901 18 96.034 21 34.167 24 27.192 17 99.903 20 27.614 22	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 97.021 24.532 73.035 90.8583 72.986 90.756 28.526 59.532 97.045 24.358 85.654 14.349 13.045 72.889 90.653 28.416	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455 201.260 229.065
Model GOM DSM ISM YIDM PNZM PZM DPM TCM	Time indexLCL $\hat{m}(t)$ UCLLCL $\hat{m}(t)$ UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 147.109 192.249 219.424 165.073 192.249 219.424 164.940 193.15 105.073 192.249 219.424 165.073 192.249 219.424 164.940 193.15 105.074 105.073 1	11 167. 194. 222. 165. 192. 219. 167. 194. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 167. 194. 222. 152. 178. 204. 167. 194. 221. 167. 194. 221.	1 1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 5580 22 076 16 328 19 5580 22 095 16 349 19 285 22 223 15 401 18 5580 21 214 16 552 19 890 22 013 16	12 12 18.321 16 15.743 19 13.164 22 55.280 16 12.77 19 9.663 21 55.280 16 15.710 19 13.129 22 57.661 16 15.033 19 12.405 22 57.669 16 15.041 19 12.413 22 57.853 16 44.474 19 1.094 21 33.837 22 88.606 17 96.371 19 33.837 22 88.6668 15	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 99.696 21 58.794 16 66.251 19 13.708 22 59.107 17 76.587 19 94.068 22 99.101 17 76.581 19 44.068 22 99.101 17 96.581 19 44.061 22 93.731 22 93.731 22 94.006 17 91.101 17 98.8195 22 70.251 17 77.818 19 95.384 22 70.001 17 92.5384 22 70.001 17 75.48 16	14 59.162 10 66.647 19 74.132 22 55.322 10 92.517 19 99.711 22 59.131 10 96.614 19 70.464 12 70.464 12 70.465 12 70.464 12 70.445 12 70.445 12 99.152 16 96.636 19 44.102 22 70.667 12 788.286 20 55.885 23 71.327 12 74.083 19 26.621 22 71.083 19 71.083 19	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.767 17 99.446 20 27.09 16 96.881 19 24.382 22 270.90 18 96.034 21 34.167 24 27 29 77.901 18 96.032 20 27.614 22 27.614 22 27.614 22 27.99.68 27	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 77.021 24.567 73.035 90.8.583 72.986 90.756 59.532 97.045 24.558 35.654 14.349 14.349 90.653 28.416 72.697 00.446	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.303 201.960 229.065
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073 192.249 219.424 164.940 192.105 219.270	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 204. 167. 194. 221. 167. 194. 221. 167. 194. 221.	1 572 16 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 267 22 076 16 328 19 580 22 2075 16 349 19 96 921 2657 16 921 19 285 222 15 401 18 580 21 223 15 401 18 552 19 935 19 921 19 214 16 552 19 99 22 013 16 335 19 658 22	12 12 18.321 16 19.3164 22 55.280 16 19.663 21 52.80 16 52.743 19 9.663 21 52.80 16 55.710 19 33.129 22 57.661 16 55.033 19 52.405 22 88.309 16 55.729 19 33.150 22 33.150 22 33.150 22 88.906 17 66.371 19 33.837 22 88.668 17 96.115 19 33.563 22	13. 13. 58.824 16 66.284 19 13.743 22 55.309 16 92.503 19 9.696 21 58.794 16 66.251 19 93.708 22 99.107 17 766.587 192 99.101 17 766.587 192 99.101 17 766.581 122 38.814 16 66.272 19 13.731 222 70.251 17 77.818 19 25.384 22 70.001 17 77.548 19 25.096 22	14 59.162 16 66.647 19 74.132 22 55.322 16 92.517 19 99.711 21 59.131 16 70.464 12 70.464 12 70.465 12 70.445 12 70.445 12 70.6667 12 88.025 16 99.152 16 66.636 12 70.687 17 88.286 22 71.327 17 88.974 19 26.621 22 71.083 17 78.8.711 19	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 22 27.087 22 70.90446 22 27.087 22 70.901 18 66.881 19 94.382 22 27.901 18 90.380 16 60.034 21 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.614 22 27.957 22	16 59.541 77.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 90.8583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 44.349 13.045 72.889 00.653 28.416 72.697 0.446 28.195	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 202.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455 201.260 229.065 173.303 201.097 228.891
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM	Time index LCL m̂(t) UCL	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073 192.249 219.424 164.940 192.105 219.270 165.057	11 167. 194. 222. 165. 192. 219. 167. 194. 222. 166. 193. 220. 166. 193. 220. 167. 194. 222. 152. 178. 204. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167.	1 572 16 937 19 302 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 075 16 921 19 285 22 23 15 401 18 5580 21 214 16 552 19 890 22 013 16 335 19 658 22 175 16	12 12 8.321 16 15.743 19 13.164 22 5.280 16 12.5.280 16 12.5.280 16 12.5.280 16 12.5.280 16 12.5.280 16 12.472 19 9.663 21 18.291 16 15.710 19 13.129 22 7.661 16 15.033 19 12.405 22 7.669 16 15.729 19 13.150 22 13.150 22 10.094 21 38.309 16 14.474 19 13.837 22 88.668 17 13.563 22 88.668 17 13.563 22 13.87 22	13. 13. 58.824 16 66.284 16 13.743 22 55.309 16 12.503 16 12.503 16 12.503 16 12.503 16 16.6251 19 13.708 22 59.107 17 16.6587 19 19.101 17 16.6587 19 19.101 17 16.6581 19 14.068 22 18.814 16 16.272 19 13.731 22 14.061 12 12.3731 22 10.101 19 18.195 22 10.001 17 17.7.818 19 15.096 22 10.001 17 17.548 19 15.096 22 10.01 17 <td>14 59.162 16 66.647 19 76.647 19 76.647 19 99.711 27 55.322 16 99.711 27 59.131 16 66.614 19 70.464 17 88.047 12 88.047 12 99.152 16 66.636 19 25.606 22 99.152 16 66.636 19 24.120 22 70.687 17 88.286 20 25.885 23 71.327 13 26.621 22 71.083 12 26.340 22 71.380 12</td> <td>15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 99.358 16 99.358 16 99.358 16 99.358 16 71.767 17 72.126 22 71.767 17 79.446 20 27.126 22 59.380 16 96.881 19 24.382 22 77.901 18 16.034 21 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.192 17 99.662 20 27.357 22 72.319 17</td> <td>16 99.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 14.349 13.045 72.889 00.653 28.416 72.697 00.446 28.195 73.107</td> <td>17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455 201.097 228.891 173.773</td>	14 59.162 16 66.647 19 76.647 19 76.647 19 99.711 27 55.322 16 99.711 27 59.131 16 66.614 19 70.464 17 88.047 12 88.047 12 99.152 16 66.636 19 25.606 22 99.152 16 66.636 19 24.120 22 70.687 17 88.286 20 25.885 23 71.327 13 26.621 22 71.083 12 26.340 22 71.380 12	15 59.389 16 96.890 19 24.392 22 55.328 16 92.523 19 99.358 16 99.358 16 99.358 16 99.358 16 71.767 17 72.126 22 71.767 17 79.446 20 27.126 22 59.380 16 96.881 19 24.382 22 77.901 18 16.034 21 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.192 17 99.903 20 27.192 17 99.662 20 27.357 22 72.319 17	16 99.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 00.809 28.583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 14.349 13.045 72.889 00.653 28.416 72.697 00.446 28.195 73.107	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 224.675 193.951 223.235 252.519 173.455 201.097 228.891 173.773
Model GOM DSM ISM YIDM PNZM PZM DPM TCM 3PFDM New	$\begin{tabular}{ c c c c } \hline Time \\ index \\ index \\ lcL \\ \hline m(t) \\ UCL \\ lcL \\ \hline m(t) \\ ucc \\ lcL \\ m(t) \\ ucc \\ ucc \\ ucc \\ m(t) \\ ucc \\ uc$	10 166.455 193.735 221.016 165.073 192.249 219.424 166.425 193.703 220.981 164.269 191.383 218.498 164.298 191.415 218.531 166.437 193.716 220.995 147.109 172.880 198.650 165.073 192.249 219.270 164.940 192.105 219.270 165.057 192.219	11 167. 194. 222. 165. 192. 167. 194. 222. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 166. 193. 220. 167. 194. 221. 167. 194. 221. 167. 194. 221. 167. 194. 21. 167. 194.	1001101 5772 16 9377 19 3002 22 216 16 402 19 588 21 542 16 904 19 267 22 076 16 328 19 580 22 076 16 328 19 580 22 557 16 921 19 285 22 223 15 401 18 580 21 214 16 552 19 890 22 013 16 335 19 658 22 013 16 5335 19 658 22 175 16 510 19	12 12 8.321 16 15.743 19 13.164 22 5.280 16 12.5.280 16 12.5.280 16 12.5.280 16 12.472 19 9.663 21 18.291 16 15.710 19 13.129 22 7.661 16 15.710 12 12.405 22 7.669 16 15.729 19 13.150 22 7.853 16 14.474 19 1.094 21 8.906 17 6.371 19 3.837 22 8.668 17 16.115 19 3.563 22 8.872 17 96.335 19	13. 13. 58.824 16 66.284 19 13.743 22 5.309 16 92.503 19 9.696 21 18.794 16 66.251 19 13.708 22 19.107 17 16.587 19 19.4068 22 19.101 17 16.581 19 12.3.731 22 18.814 16 16.2.72 19 19.3.731 22 70.251 17 17.818 19 12.3.734 22 70.251 17 77.548 19 15.096 22 70.249 17 77.815 19	14 59.162 16 66.647 19 24.132 22 55.322 16 92.517 19 99.711 21 99.731 16 96.644 12 99.131 16 96.614 19 24.096 22 70.464 12 88.047 19 88.025 19 88.025 19 99.152 16 66.636 19 24.120 22 70.887 12 70.887 12 71.827 17 78.8974 19 26.621 22 71.083 17 98.711 19 26.340 22 71.380 17 99.031 20	15 59.389 16 66.890 19 24.392 22 55.328 16 92.523 19 19.718 21 59.358 16 96.857 19 24.357 22 71.767 17 99.446 20 27.126 22 71.767 17 92.410 20 27.087 22 59.380 16 66.881 19 24.382 22 77.901 18 16.034 21 34.167 24 27.192 17 99.903 20 27.614 22 27.357 22 72.319 17 90.040 20	16 59.541 97.054 24.567 55.331 92.526 19.721 59.510 77.021 24.532 73.035 02.8583 72.986 00.756 28.583 72.986 00.756 28.526 59.532 77.045 24.558 35.654 14.349 13.045 72.889 00.653 28.416 72.097 0.446 28.195 73.107 00.886	17 169.643 197.163 224.684 165.332 192.527 219.722 169.612 197.130 224.649 174.281 202.147 230.014 174.217 202.078 229.940 169.635 197.155 223.235 252.519 173.455 201.097 228.891 173.773 201.602

Table A2. 95% Confidence interval of all 10 models (DS2).

Model	Time Index	519	968	1430	1893	2490	3058	3625	4422	5218	5823
	LCL	3.641	9.407	15.376	21.176	28.274	34.589	40.464	48.022	54.803	59.482
GOM	$\hat{\mathbf{m}}(t)$	9.767	17.639	25.218	32.318	40.791	48.196	55.000	63.660	71.359	76.641
	UCL	15.892	25.870	35.060	43.460	53.309	61.803	69.535	79.298	87.916	93.799
DEM	LCL $\hat{m}(t)$	-0.409	3.059	8.744 16.770	15.540 25.422	24.802 36.671	33.366 46.770	41.233	50.791 66.811	58.514 75.550	63.256 80.883
DSIVI	UCL	6.342	14.760	24.796	35.305	48.540	40.770 60.174	70.537	82.832	92.586	98.510
	LCL	3 641	9 407	15.376	21 176	28 273	34 589	40 464	48 022	54 802	59 482
ISM	$\hat{\mathbf{m}}(t)$	9.767	17.639	25.218	32.318	40.791	48.196	55.000	63.660	71.359	76.641
	UCL	15.892	25.870	35.060	43.460	53.309	61.803	69.535	79.298	87.916	93.799
	LCL	3.631	9.384	15.337	21.122	28.202	34.503	40.367	47.916	54.698	59.387
YIDM	$\hat{\mathbf{m}}(t)$	9.751 15.871	17.608	25.171	32.254 43.385	40.707 53 213	48.096 61.689	54.888 69.409	63.539 79.162	71.241 87.784	76.534 93.680
	LCL	15.671	25.052	15 402	45.505	00.070	01.009	40.402	79.102	54.504	53.000
PNZM	$\hat{\mathbf{m}}(t)$	3.701 9.853	9.506 17 767	15.493 25.363	21.294 32.460	28.373 40.909	34.657 48.275	40.493 55.033	47.993	54.724 71.271	59.378 76.523
	UCL	16.005	26.028	35.234	43.627	53.445	61.893	69.573	79.261	87.817	93.668
PZM	LCL	3.458	9.130	15.085	20.933	28.150	34.608	40.627	48.354	55.238	59.940
	$\hat{\mathbf{m}}(t)$	9.499	17.277	24.856	32.024	40.646	48.218	55.187	64.039	71.852	77.157
	UCL	15.539	25.424	34.628	43.116	53.141	61.828	69.747	79.723	88.466	94.373
	LCL	26.407	26.678	27.161	27.873	29.166	30.827	32.943	36.756	41.628	46.096
DPM	$\hat{\mathbf{m}}(t)$	38.581 50.754	38.903 51.128	39.475 51 789	40.318 52 763	41.845 54 523	43.799 56 770	46.275 59.608	50.713 64.671	56.340 71.051	61.461 76.827
		50.734	11.051	17.426	22.703	20.071	24.406	20.605	46.449	52.916	E7 296
TCM	$\hat{\mathbf{m}}(t)$	5.929 12.993	20.787	27.763	22.634 34.075	20.071 41.497	34.406 47.982	59.605 54.009	40.440 61.863	52.816 69.110	74.278
rem	UCL	20.058	29.723	38.090	45.516	54.122	61.559	68.413	77.279	85.404	91.170
	LCL	3.990	9.962	16.016	21.804	28.789	34.941	40.631	47.938	54.522	59.105
3PFDM	$\hat{\mathbf{m}}(t)$	10.271	18.361	26.012	33.076	41.400	48.605	55.191	63.564	71.041	76.216
	UCL	16.552	26.759	36.008	44.348	54.011	62.270	69.752	79.190	87.561	93.327
	LCL	5.981	12.096	17.800	23.076	29.375	34.946	40.168	47.027	53.403	57.974
New	m(t)	13.066	21.099	28.210 38.621	34.606 46.135	42.091 54.807	48.611 62.276	54.658 69.148	62.525 78.023	69.774 86.146	74.941 91 909
	Time	20.101	00.102	00.021	10.100	01.007	02.270	07.110	70.020	00.110	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,
Model	index	6539	7083	7487	7846	8205	8564	8923	9282	9641	10,000
	LCL	64.535	68.049	70.489	72.543	74.495	76.350	78.112	79.786	81.376	82.886
GOM	$\hat{\mathbf{m}}(t)$	82.318	86.251	88.977	91.267	93.441	95.504 114.658	97.461	99.319	101.081	102.754
	LCL	(7.772)	70.52(70.2403	70 57(74.72(75 749	76.627	77.201	70.052	70.(2)
DSM	$\hat{\mathbf{m}}(t)$	67.773 85.943	70.526	72.248 90.939	73.576 92.418	74.736 93.710	75.748 94 834	76.627 95.812	77.391 96.660	78.053 97.395	78.626 98.031
DOW	UCL	104.112	107.510	109.629	111.260	112.683	113.921	114.997	115.930	116.738	117.437
	LCL	64.535	68.049	70.489	72.543	74.495	76.350	78.112	79.786	81.376	82.886
ISM	$\hat{\mathbf{m}}(t)$	82.318	86.251	88.977	91.267	93.441	95.504	97.461	99.319	101.081	102.754
	UCL	100.100	104.454	107.465	109.992	112.387	114.658	116.810	118.851	120.786	122.621
	LCL	64.460	67.996	70.457	72.532	74.508	76.389	78.181	79.887	81.511	83.059
YIDM	$\hat{m}(t)$	82.234 100.007	86.192 104 388	88.941 107.425	91.255 109.978	93.455 112 403	95.547 114 706	97.537	99.430 118 974	101.231	102.945 122.831
		100.007	104.500	107.425	107.770	114.100		1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	110.771	120.901	122.001
PNZM		64 417	67.026	70.280	72.462	74.440	76 229	79 121	70.9E4	81 E00	82 074
PNZM	$\hat{\mathbf{m}}(t)$	64.417 82.186	67.936 86.125	70.389 88.865	72.462 91.177	74.440 93.380	76.328	78.131	79.854 99.394	81.500 101.218	83.074 102.962
	$\hat{\mathbf{m}}(t)$ UCL	64.417 82.186 99.954	67.936 86.125 104.314	70.389 88.865 107.342	72.462 91.177 109.892	74.440 93.380 112.320	76.328 95.480 114.631	78.131 97.483 116.834	79.854 99.394 118.934	81.500 101.218 120.937	83.074 102.962 122.850
	$ \frac{\hat{m}(t)}{UCL} $ LCL	64.417 82.186 99.954 64.953	67.936 86.125 104.314 68.387	70.389 88.865 107.342 70.742	72.462 91.177 109.892 72.704	74.440 93.380 112.320 74.547	76.328 95.480 114.631 76.279	78.131 97.483 116.834 77.905	79.854 99.394 118.934 79.430	81.500 101.218 120.937 80.860	83.074 102.962 122.850 82.201
PZM	$ \begin{array}{r} \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \hline \\ \text{LCL} \\ \hat{\mathbf{m}}(t) \end{array} $	64.417 82.186 99.954 64.953 82.786	67.936 86.125 104.314 68.387 86.629	70.389 88.865 107.342 70.742 89.260	72.462 91.177 109.892 72.704 91.447	74.440 93.380 112.320 74.547 93.499	76.328 95.480 114.631 76.279 95.425	78.131 97.483 116.834 77.905 97.231	79.854 99.394 118.934 79.430 98.924	81.500 101.218 120.937 80.860 100.510	83.074 102.962 122.850 82.201 101.995
PZM	$ \begin{array}{c} \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \hline \text{LCL} \\ \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \end{array} $	64.417 82.186 99.954 64.953 82.786 100.619	67.936 86.125 104.314 68.387 86.629 104.872	70.389 88.865 107.342 70.742 89.260 107.777	72.462 91.177 109.892 72.704 91.447 110.189	74.440 93.380 112.320 74.547 93.499 112.451	76.328 95.480 114.631 76.279 95.425 114.571	78.131 97.483 116.834 77.905 97.231 116.558	79.854 99.394 118.934 79.430 98.924 118.418	81.500 101.218 120.937 80.860 100.510 120.160	83.074 102.962 122.850 82.201 101.995 121.789
PZM	$\frac{\hat{m}(t)}{UCL}$ $\frac{LCL}{\hat{m}(t)}$ UCL LCL LCL	64.417 82.186 99.954 64.953 82.786 100.619 52.288	67.936 86.125 104.314 68.387 86.629 104.872 57.681	70.389 88.865 107.342 70.742 89.260 107.777 62.083	72.462 91.177 109.892 72.704 91.447 110.189 66.287	74.440 93.380 112.320 74.547 93.499 112.451 70.771	76.328 95.480 114.631 76.279 95.425 114.571 75.541	78.131 97.483 116.834 77.905 97.231 116.558 80.600	79.854 99.394 118.934 79.430 98.924 118.418 85.954	81.500 101.218 120.937 80.860 100.510 120.160 91.607	83.074 102.962 122.850 82.201 101.995 121.789 97.562
PZM DPM	$ \begin{array}{c} \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \text{LCL} \\ \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \begin{array}{c} \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \hat{\mathbf{m}}(t) \\ \text{UCL} \\ \end{array} $	64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91 540	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812	76.328 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312
PZM DPM	$\begin{array}{c} \hat{\mathbf{m}}(t) \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \\ \mathbf{L}\mathbf{C}\mathbf{L} \\ \hat{\mathbf{m}}(t) \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \\ \\ \mathbf{L}\mathbf{C}\mathbf{L} \\ \hat{\mathbf{m}}(t) \\ \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 72.510	76.328 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.901	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312
PZM DPM		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355	76.328 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086
PZM DPM TCM		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190	76.328 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815 113.900	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082
PZM DPM TCM		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603 64.115	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237 67.651	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550 70.138	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408 72.257	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190 74.295	76.256 76.279 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815 113.900 76.256	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541 78.144	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116 79.963	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629 81.717	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082 83.410
PZM DPM TCM 3PFDM	$\begin{array}{c} \hat{\mathbf{m}}(t) \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \mathbf{M}(t) \\ \mathbf{U}\mathbf{C}\mathbf{L} \\ \mathbf{M}(t) \\ \mathbf$	64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603 64.115 81.847	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237 67.651 85.806	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550 70.138 88.586	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408 72.257 90.949	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190 74.295 93.218	76.259 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815 113.900 76.256 95.399	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541 78.144 97.497	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116 79.963 99.515	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629 81.717 101.459	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082 83.410 103.333
PZM DPM TCM 3PFDM		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603 64.115 81.847 99.578	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237 67.651 85.806 103.961	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550 70.138 88.586 107.033	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408 72.257 90.949 109.641	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190 74.295 93.218 112.142	76.279 95.425 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815 113.900 76.256 95.399 114.543	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541 78.144 97.497 116.849	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116 79.963 99.515 119.067	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629 81.717 101.459 121.202	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082 83.410 103.333 123.257
PZM DPM TCM 3PFDM		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603 64.115 81.847 99.578 63.115	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237 67.651 85.806 103.961 66.844	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550 70.138 88.586 107.033 69.522	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408 72.257 90.949 109.641 71.840	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190 74.295 93.218 112.142 74.104	$\begin{array}{c} 76.328\\ 95.480\\ 114.631\\ 76.279\\ 95.425\\ 114.571\\ 75.541\\ 94.604\\ 113.668\\ 75.730\\ 94.815\\ 113.900\\ 76.256\\ 95.399\\ 114.543\\ 76.315\\ 76$	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541 78.144 97.497 116.849 78.476	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116 79.963 99.515 119.067 80.590	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629 81.717 101.459 121.202 82.657	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082 83.410 103.333 123.257 84.680
PZM DPM TCM 3PFDM New		64.417 82.186 99.954 64.953 82.786 100.619 52.288 68.511 84.734 62.528 80.065 97.603 64.115 81.847 99.578 63.115 80.725 98.334	67.936 86.125 104.314 68.387 86.629 104.872 57.681 74.610 91.540 66.257 84.247 102.237 67.651 85.806 103.961 66.844 84.903 102.963	70.389 88.865 107.342 70.742 89.260 107.777 62.083 79.566 97.049 68.936 87.243 105.550 70.138 88.586 107.033 69.522 87.897 106.272	72.462 91.177 109.892 72.704 91.447 110.189 66.287 84.280 102.274 71.255 89.832 108.408 72.257 90.949 109.641 71.840 90.484 109.128	74.440 93.380 112.320 74.547 93.499 112.451 70.771 89.292 107.812 73.519 92.355 111.190 74.295 93.218 112.142 74.104 93.006 111.907	76.256 95.480 114.631 76.279 95.425 114.571 75.541 94.604 113.668 75.730 94.815 113.900 76.256 95.399 114.543 76.315 95.465 114.615	78.131 97.483 116.834 77.905 97.231 116.558 80.600 100.222 119.843 77.891 97.216 116.541 78.144 97.497 116.849 78.476 97.866 97.866	79.854 99.394 118.934 79.430 98.924 118.418 85.954 106.148 126.341 80.003 99.560 119.116 79.963 99.515 119.067 80.590 100.210 119.830	81.500 101.218 120.937 80.860 100.510 120.160 91.607 112.385 133.163 82.069 101.849 121.629 81.717 101.459 121.202 82.657 102.500	83.074 102.962 122.850 82.201 101.995 121.789 97.562 118.937 140.312 84.090 104.086 124.082 83.410 103.333 123.257 84.680 104.739 124.797

Table A3. 95% Confidence interval of all 10 models (DS3).

References

- 1. Pham, T.; Pham, H. A generalized software reliability model with stochastic fault-detection rate. *Ann. Oper. Res.* **2017**, 1–11. [CrossRef]
- 2. Teng, X.; Pham, H. A new methodology for predicting software reliability in the random field environments. *IEEE Trans. Reliab.* **2006**, *55*, 458–468. [CrossRef]
- 3. Pham, H. A new software reliability model with Vtub-Shaped fault detection rate and the uncertainty of operating environments. *Optimization* **2014**, *63*, 1481–1490. [CrossRef]
- 4. Chang, I.H.; Pham, H.; Lee, S.W.; Song, K.Y. A testing-coverage software reliability model with the uncertainty of operation environments. *Int. J. Syst. Sci. Oper. Logist.* **2014**, *1*, 220–227.
- 5. Inoue, S.; Ikeda, J.; Yamada, S. Bivariate change-point modeling for software reliability assessment with uncertainty of testing-environment factor. *Ann. Oper. Res.* **2016**, *244*, 209–220. [CrossRef]
- 6. Okamura, H.; Dohi, T. Phase-type software reliability model: Parameter estimation algorithms with grouped data. *Ann. Oper. Res.* **2016**, *244*, 177–208. [CrossRef]
- 7. Song, K.Y.; Chang, I.H.; Pham, H. A Three-parameter fault-detection software reliability model with the uncertainty of operating environments. *J. Syst. Sci. Syst. Eng.* **2017**, *26*, 121–132. [CrossRef]
- 8. Song, K.Y.; Chang, I.H.; Pham, H. A software reliability model with a Weibull fault detection rate function subject to operating environments. *Appl. Sci.* **2017**, *7*, 983. [CrossRef]
- 9. Li, Q.; Pham, H. NHPP software reliability model considering the uncertainty of operating environments with imperfect debugging and testing coverage. *Appl. Math. Model.* **2017**, *51*, 68–85. [CrossRef]
- 10. Pham, H.; Nordmann, L.; Zhang, X. A general imperfect software debugging model with S-shaped fault detection rate. *IEEE Trans. Reliab.* **1999**, *48*, 169–175. [CrossRef]
- 11. Pham, H. A generalized fault-detection software reliability model subject to random operating environments. *Vietnam J. Comput. Sci.* **2016**, *3*, 145–150. [CrossRef]
- 12. Akaike, H. A new look at statistical model identification. *IEEE Trans. Autom. Control* **1974**, *19*, 716–719. [CrossRef]
- 13. Pham, H. System Software Reliability; Springer: London, UK, 2006.
- 14. Goel, A.L.; Okumoto, K. Time dependent error detection rate model for software reliability and other performance measures. *IEEE Trans. Reliab.* **1979**, *28*, 206–211. [CrossRef]
- Yamada, S.; Ohba, M.; Osaki, S. S-shaped reliability growth modeling for software fault detection. *IEEE Trans. Reliab.* 1983, 32, 475–484. [CrossRef]
- 16. Ohba, M. Inflexion S-shaped software reliability growth models. In *Stochastic Models in Reliability Theory;* Osaki, S., Hatoyama, Y., Eds.; Springer: Berlin, Germany, 1984; pp. 144–162.
- 17. Yamada, S.; Tokuno, K.; Osaki, S. Imperfect debugging models with fault introduction rate for software reliability assessment. *Int. J. Syst. Sci.* **1992**, *23*, 2241–2252. [CrossRef]
- Pham, H.; Zhang, X. An NHPP software reliability models and its comparison. *Int. J. Reliab. Qual. Saf. Eng.* 1997, 4, 269–282. [CrossRef]
- 19. Pham, H. Software Reliability Models with Time Dependent Hazard Function Based on Bayesian Approach. *Int. J. Autom. Comput.* **2007**, *4*, 325–328. [CrossRef]
- 20. Li, X.; Xie, M.; Ng, S.H. Sensitivity analysis of release time of software reliability models incorporating testing effort with multiple change-points. *Appl. Math. Model.* **2010**, *34*, 3560–3570. [CrossRef]
- Pham, H. Software reliability and cost models: Perspectives, comparison, and practice. *Eur. J. Oper. Res.* 2003, 149, 475–489. [CrossRef]
- 22. Pham, H.; Zhang, X. NHPP software reliability and cost models with testing coverage. *Eur. J. Oper. Res.* 2003, 145, 443–454. [CrossRef]
- 23. Kimura, M.; Toyota, T.; Yamada, S. Economic analysis of software release problems with warranty cost and reliability requirement. *Reliab. Eng. Syst. Saf.* **1999**, *66*, 49–55. [CrossRef]
- 24. Sgarbossa, F.; Pham, H. A cost analysis of systems subject to random field environments and reliability. *IEEE Trans. Syst. Man Cybern. Part C Appl. Rev.* **2010**, *40*, 429–437. [CrossRef]
- 25. Musa, J.D.; Iannino, A.; Okumoto, K. Software Reliability: Measurement, Prediction, and Application; McGraw-Hill: New York, NY, USA, 1987.

- 26. Stringfellow, C.; Andrews, A.A. An empirical method for selecting software reliability growth models. *Empir. Softw. Eng.* **2002**, *7*, 319–343. [CrossRef]
- 27. Wood, A. Predicting software reliability. IEEE Comput. Soc. 1996, 11, 69–77. [CrossRef]



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