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Determination of the Electromagnetic Field on the Surface of the Beam Splitter of Laser Interferometer by Measuring the Displacements of Control Object Surfaces

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Abstract: The paper describes in detail the governing relationships for the electromagnetic field on the surface of the laser beam splitter of a laser interferometer. These expressions are intended for numerical simulation of measuring the displacements of control object surfaces by using a laser interferometer. We consider the cases of perpendicular and parallel polarizations of the radiation source. Moreover, the most relevant particular cases of constructing optical circuits of the interferometer in question are treated. The presented theoretical results have been used in numerical studies of the functional characteristics of promising contactless optical means of measuring displacements. The use of the described results has improved the quality and informativeness of the numerical simulation results of the measurement process of displacements by a laser interferometer.

Keywords: laser interferometer; displacement measurement; electromagnetic field; perpendicular polarization; parallel polarization

1. Introduction, Purpose and Statement of the Problem

At present, the development and use of contactless optical measuring instruments based on laser interferometers is one of the most relevant and promising scientific and technical areas. This is due to the need to improve the information content and accuracy of measurements of displacements of control surfaces. Such studies allow us to establish the features of the formation of defects in advanced materials and to perform diagnostics of the state of structural materials and goods. The measuring instruments [1–4], created on the base of a two-way laser interferometer with combined branches, are the most applicable and preferred for the solution of the pointed problems. The scheme of the laser interferometer is shown in Figure 1, where the beam splitter 1 with known amplitude transmission is located in a plane S , related to a associated Cartesian coordinate system $Oxyz$. The reflector 2 is in a plane S_r that is rotated relative to the plane S by an angle α . The reflector 2 is rigidly fixed on the surface of the control object and can perform linear (Δh) and angular ($\Delta\alpha$) displacements simultaneously with it. The beam splitter 1 receives coherent radiation from a point source of radiation in the form of an electric dipole located at the point O' (x_0, y_0, z_0). The photodetectors for analyzing the interference field are located in the direction of reflection from the beam splitter of the source radiation, at a certain distance from the beam splitter corresponding to the Fresnel zone. This device allows us to determine

the coordinates of the source of radiation from the interference pattern created, or if fixed, to find the movement of the interferometer reflector in space.

To determine the spatial displacement of the radiation source or reflector, it is necessary to define the change in the distribution of the electromagnetic field scattered by the device under consideration in the zone of observation of the interference pattern [5], i.e., in half-space $y > 0$.

Using the tensor Green's function [6] with mixed Neumann-Dirichlet boundary conditions [7], and also taking into account the fact that the electromagnetic field has a transverse character and corresponds to the Maxwell equations, for its determination at the observation points it is sufficient to know only the tangential components of the electric field on the surface where the sources are located, i.e., on the beam splitter.

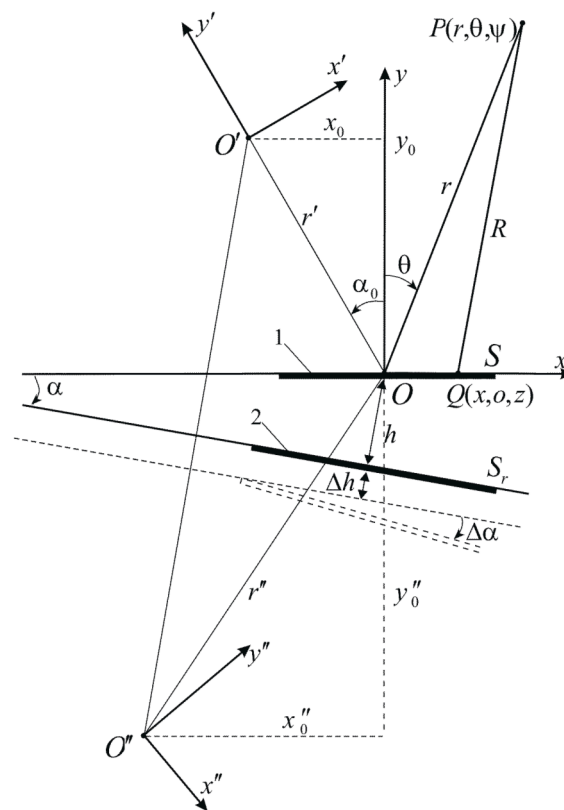


Figure 1. Circuits of interferometer under consideration.

To determine the tangential components of the electric field intensity vector E , the image method [8] is used, in accordance with which the total field is a superposition of the fields from the source at the point O' and the imaginary mirror source located at the point O'' taking into account the transmission of the beam splitter. Thus, the problem of determining the distribution of the intensity of the optical field in the area of the photodetector devices is solved successively in two stages, and initially, the field is determined on the beam splitter surface, and further, the field is determined in the area of the photodetector devices. At a given distribution of the intensity field in the area of photodetectors, it is possible to obtain displacements of the surface of the test object, uniquely associated with this area.

The aim of the work is to obtain the constitutive relations for determining the electromagnetic field on the surface of the beam splitter of the laser interferometer under consideration for the cases of perpendicular and parallel polarizations of the source radiation. These relations can be used for numerical simulation of the process of measuring the displacements of control object surfaces.

2. Determination of the Electromagnetic Field on the Beam Splitter Surface

The relationships for the spherical components of the field of the source of electromagnetic radiation are sufficiently well-known [9–11]. If the beam splitter is flat, has a rectangular shape and is completely illuminated, then the field on its surface is convenient to express through the Cartesian components of the vector E , and in the case where the source field is a light spot on the beam splitter surface, it is more convenient to consider the cylindrical components of the vector E .

Let us consider the first case when the beam splitter is fully illuminated. For this, it is necessary, by transforming from spherical to Cartesian coordinates, to express the tangential components of this field on the beam splitter surface through the Cartesian components.

By solving the problem, it is convenient to additionally introduce auxiliary local Cartesian and associated spherical coordinate systems with origins at the points O' and O'' . If the coordinates are written without strokes, then they correspond to the main system with the origin on the beam splitter surface at the point O , and the coordinates with one and two strokes refer, respectively, to the coordinate systems with the origin at the points O' and O'' (see Figure 1). Note that the coordinate system $O'x'y'z'$ is obtained from the system $Oxyz$ by turning around the z -axis by an angle α_0 in the positive direction and then parallel translation along the x -axis on the length x_0 and along the y -axis on the length y_0 . The coordinate system $O''x''y''z''$ is obtained from the system $O'x'y'z'$ by mirroring it and then changing the direction of the y -axis to the opposite.

The Cartesian coordinate systems introduced are related to each other by the following equations:

$$\begin{aligned} x' &= (x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0; \\ y' &= -(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0; \\ z' &= z; \end{aligned} \tag{1}$$

$$\begin{aligned} x'' &= (x + x''_0) \cos(\alpha_0 + 2\alpha) - (y + y''_0) \sin(\alpha_0 + 2\alpha); \\ y'' &= (x + x''_0) \sin(\alpha_0 + 2\alpha) + (y + y''_0) \cos(\alpha_0 + 2\alpha); \\ z'' &= z, \end{aligned} \tag{2}$$

where

$$\begin{aligned} x''_0 &= x_0 \cos 2\alpha + 2h \sin \alpha + y_0 \sin 2\alpha; \\ y''_0 &= -x_0 \sin 2\alpha + 2h \cos \alpha + y_0 \cos 2\alpha. \end{aligned} \tag{3}$$

In the expressions (1)–(3), the values of the parameters x_0 , y_0 , α_0 and α , h determine the position of the radiation source and the reflector in space (see Figure 1).

3. Case of Perpendicular Polarization of Source Radiation

If the dipole is parallel to the z -axis (see Figure 1), i.e., the electric field intensity vector lies in the plane perpendicular to the plane of incidence, then in the notations for the tangential components of the electric field intensity vector, we use the notation \perp .

Expressions that determine the relationships between Cartesian and spherical coordinates are known and have the following forms:

(i) for the basic coordinate system

$$x = r \sin \theta \sin \psi; \quad y = r \cos \theta; \quad z = r \sin \theta \cos \psi; \tag{4}$$

Lamé coefficients are equal, respectively:

$$h_r = 1; \quad h_\theta = r; \quad h_\psi = r \sin \theta; \tag{5}$$

(ii) for an auxiliary coordinate system with origin at the point O' :

$$x' = r' \sin \theta' \cos \psi'; \quad y' = r' \sin \theta' \sin \psi'; \quad z' = r' \cos \theta'; \quad (6)$$

(iii) for an auxiliary coordinate system with origin at the point O'' :

$$x'' = r'' \sin \theta'' \cos \psi''; \quad y'' = r'' \sin \theta'' \sin \psi''; \quad z'' = r'' \cos \theta''. \quad (7)$$

By substituting Expression (1) into Expression (6), and Expression (2) into Expression (7), we find the relationships of the basic Cartesian coordinate system x, y, z and the auxiliary spherical ones:

(i) for the auxiliary coordinate system $O'r'\theta'\psi'$:

$$\begin{aligned} r' \sin \theta' \cos \psi' &= (x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0; \\ r' \sin \theta' \sin \psi' &= -(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0; \\ r' \cos \theta' &= z; \end{aligned} \quad (8)$$

(ii) for the auxiliary coordinate system $O''r''\theta''\psi''$:

$$\begin{aligned} r'' \sin \theta'' \cos \psi'' &= (x + x''_0) \cos(\alpha_0 + 2\alpha) - (y + y''_0) \sin(\alpha_0 + 2\alpha); \\ r'' \sin \theta'' \sin \psi'' &= (x + x''_0) \sin(\alpha_0 + 2\alpha) + (y + y''_0) \cos(\alpha_0 + 2\alpha); \\ r'' \cos \theta'' &= z. \end{aligned} \quad (9)$$

From Equation (1), we obtain:

$$y = y_0 + x' \sin \alpha_0 + y' \cos \alpha_0; \quad x = -x_0 + x' \cos \alpha_0 - y' \sin \alpha_0; \quad z = z'. \quad (10)$$

By substituting Expression (6) into Relationships (10), we obtain:

$$\begin{aligned} y &= y_0 + r' \sin \theta' \cos \psi' \sin \alpha_0 + r' \sin \theta' \sin \psi' \cos \alpha_0; \\ x &= -x_0 + r' \sin \theta' \cos \psi' \cos \alpha_0 - r' \sin \theta' \sin \psi' \sin \alpha_0; \\ z &= r' \cos \theta'. \end{aligned} \quad (11)$$

Similarly to Expression (11), we have

$$\begin{aligned} y &= y''_0 - r'' \sin \theta'' \cos \psi'' \sin(\alpha_0 + 2\alpha) + r'' \sin \theta'' \sin \psi'' \cos(\alpha_0 + 2\alpha); \\ x &= -x''_0 + r'' \sin \theta'' \cos \psi'' \cos(\alpha_0 + 2\alpha) + r'' \sin \theta'' \sin \psi'' \sin(\alpha_0 + 2\alpha); \\ z &= r'' \cos \theta''. \end{aligned} \quad (12)$$

To obtain the relationships connecting the components of the vector E in the selected coordinate systems, we apply the well-known formula relating the components of vectors in different orthogonal coordinate systems [12]:

$$\tilde{E}_k \tilde{h}_k = \frac{\partial x^m}{\partial \tilde{x}^k} E_m h_m, \quad (13)$$

where \tilde{E}_k and E_m are the physical components of vector E in the first and second coordinate systems, respectively, \tilde{h}_k and h_m are the corresponding metric Lamé coefficients.

By assuming the “first” coordinate system is Cartesian, the “second” coordinate system is spherical $O'r'\theta'\psi'$ or $O''r''\theta''\psi''$ and using Expression (13), we can obtain relationships for the components of the vector E' of source field O' or O'' in the Cartesian coordinate system through the spherical components of this vector.

For the incident field, we obtain

$$\begin{aligned} E'_{x\perp} h_x &= \frac{\partial r'}{\partial x} E'_{r'} h_{r'} + \frac{\partial \theta'}{\partial x} E'_{\theta'} h_{\theta'} + \frac{\partial \psi'}{\partial x} E'_{\psi'} h_{\psi'}; \\ E'_{z\perp} h_z &= \frac{\partial r'}{\partial z} E'_{r'} h_{r'} + \frac{\partial \theta'}{\partial z} E'_{\theta'} h_{\theta'} + \frac{\partial \psi'}{\partial z} E'_{\psi'} h_{\psi'}; \\ E'_{y\perp} h_y &= \frac{\partial r'}{\partial y} E'_{r'} h_{r'} + \frac{\partial \theta'}{\partial y} E'_{\theta'} h_{\theta'} + \frac{\partial \psi'}{\partial y} E'_{\psi'} h_{\psi'}. \end{aligned} \tag{14}$$

In accordance with Expression (13), the tangential components of the electric field intensity vector on the surface $y = 0$ can be written in the form:

$$E'_{x\perp} = \frac{\partial x}{\partial \theta'} \cdot \frac{1}{h_{\theta'}} E_{\theta'}; \quad E'_{z\perp} = \frac{\partial z}{\partial \theta'} \cdot \frac{1}{h_{\theta'}} E_{\theta'}. \tag{15}$$

Here $E'_{x\perp}$ and $E'_{z\perp}$ are present via $E'_{\theta'}$, since the other components of the dipole radiation in the Fresnel zone $E'_{\psi'} = E'_{r'} = 0$.

From Equations (11), we obtain

$$\begin{aligned} \frac{\partial x}{\partial \theta'} &= (r' \cos \psi' \cos \alpha_0 - r' \sin \psi' \sin \alpha_0) \cos \theta' = r' \cos(\psi' + \alpha_0) \cos \theta'; \\ \frac{\partial z}{\partial \theta'} &= -r' \sin \theta'. \end{aligned} \tag{16}$$

By substituting Expression (16) and corresponding Lamé coefficients into Expression (15), we obtain

$$E'_{x\perp} = \cos(\psi' + \alpha_0) \cos \theta' \cdot E_{\theta'}; \quad E'_{z\perp} = -\sin \theta' \cdot E_{\theta'}. \tag{17}$$

If the radiation source is removed from the beam splitter by a distance corresponding to the Fresnel zone and further, then in the spherical coordinate system $O'r'\theta'\psi'$, the field on the surface $y = 0$ can be represented only by the transverse components of the vectors E' and H' [9–11] as

$$H'_{\psi'} = -\frac{ikl'l}{4\pi r'} \sin \theta' \cdot e^{ikr'} = -A_0 \frac{ik}{r'} \sin \theta' \cdot e^{ikr'}; \quad E'_{\theta'} = A_0 \frac{ik^2}{\omega \epsilon r'} \sin \theta' \cdot e^{ikr'}. \tag{18}$$

Other components of the vectors E' and H' can be neglected, then, assuming $A_0 = 1$, we have:

$$H'_{\psi'} = -\frac{ik}{r'} \sin \theta' \cdot e^{ikr'}; \quad E'_{\theta'} = -\frac{ik^2}{\omega \epsilon r'} \sin \theta' \cdot e^{ikr'}, \tag{19}$$

where $r' = \sqrt{(x + x_0)^2 + y_0^2 + z^2}$, because $y = 0$.

Then Expression (17) transform to

$$E'_{x\perp} = \frac{ik^2}{\omega \epsilon} \cdot \frac{\cos \theta' \sin \theta' \cos(\psi' + \alpha_0)}{r'} e^{ikr'}; \quad E'_{z\perp} = -\sin \theta' \cdot \frac{ik^2}{\omega \epsilon} \cdot \frac{\sin \theta'}{r'} e^{ikr'}. \tag{20}$$

From Relationship (8), we obtain:

$$\cos \theta' = \frac{z}{r'} = \frac{z}{\sqrt{(x + x_0)^2 + (y - y_0)^2 + z^2}}; \tag{21}$$

$$\tan \psi' = \frac{-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0}{(x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0}. \tag{22}$$

From Expression (21), we obtain:

$$\sin \theta' = \sqrt{1 - \frac{z^2}{(r')^2}} = \frac{\sqrt{(x + x_0)^2 + (y - y_0)^2}}{\sqrt{(x + x_0)^2 + (y - y_0)^2 + z^2}}. \tag{23}$$

From Expression (8), we obtain:

$$\cos \psi' = \frac{(x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0}{r' \sin \theta'}. \tag{24}$$

From Expression (23), we obtain:

$$(r')^2 \sin^2 \theta' = (x + x_0)^2 + (y - y_0)^2. \tag{25}$$

Then, instead of Relationship (24), we obtain:

$$\cos \psi' = \frac{(x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0}{\sqrt{(x + x_0)^2 + (y - y_0)^2}}. \tag{26}$$

From Expression (8), we obtain:

$$\sin \psi' = \frac{-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0}{r' \sin \theta'}. \tag{27}$$

By substituting Expression (25) into Relationship (27), we obtain:

$$\sin \psi' = \frac{-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0}{\sqrt{(x + x_0)^2 + (y - y_0)^2}}. \tag{28}$$

Let us write the Cartesian components of the vectors E' and H' in the basic coordinate system $Oxyz$; with this aim, we substitute Expressions (21), (23), (26), (28), into (20) and taking into account that $y = 0$, we obtain:

$$E'_{x\perp} = \frac{ik^2}{\omega\varepsilon} \cdot \frac{z(x + x_0)}{(r')^3} e^{ikr'}; E'_{z\perp} = -\frac{ik^2}{\omega\varepsilon} \cdot \frac{(x + x_0)^2 + y_0^2}{(r')^3} e^{ikr'}, \tag{29}$$

where $r' = \sqrt{(x + x_0)^2 + y_0^2 + z^2}$.

Similarly, one can obtain expressions for the tangent field components from the imaginary source:

$$E''_{x\perp} = \frac{ik^2}{\omega\varepsilon} \cdot \frac{z(x + x''_0)}{(r'')^3} e^{ikr''}; E''_{z\perp} = -\frac{ik^2}{\omega\varepsilon} \cdot \frac{(x + x''_0)^2 + y''_0^2}{(r'')^3} e^{ikr''}, \tag{30}$$

where $r'' = \sqrt{(x + x''_0)^2 + y''_0^2 + z^2}$.

4. Case of Parallel Polarization of Source Radiation

If the dipole is parallel to the x' -axis, i.e., the vector of the electric field intensity is parallel to the plane of incidence, then in the notations for the tangential components of the electric field intensity vector we use the notation \parallel .

Expressions that determine the relationships between Cartesian and spherical coordinates are known and have the following forms:

- (i) for the basic coordinate system, they are found by Formulae (4);

(ii) for an auxiliary coordinate system with origin at the point O' :

$$x' = r' \cos \theta'; y' = r' \sin \theta' \cos \psi'; z' = r' \sin \theta' \sin \psi'; \tag{31}$$

(iii) for an auxiliary coordinate system with origin at the point O'' :

$$x'' = r'' \cos \theta''; y'' = r'' \sin \theta'' \cos \psi''; z'' = r'' \sin \theta'' \sin \psi''. \tag{32}$$

By substituting Relationship (31) into Expression (10), we obtain:

$$\begin{aligned} y &= y_0 + r' \cos \theta' \sin \alpha_0 + r' \sin \theta' \cos \psi' \cos \alpha_0; \\ x &= -x_0 + r' \cos \theta' \cos \alpha_0 - r' \sin \theta' \cos \psi' \sin \alpha_0; \\ z &= r' \sin \theta' \sin \psi'. \end{aligned} \tag{33}$$

Similarly to Expression (33), we obtain:

$$\begin{aligned} y &= y''_0 - r'' \cos \theta'' \sin(\alpha_0 + 2\alpha) + r'' \sin \theta'' \cos \psi'' \cos(\alpha_0 + 2\alpha); \\ x &= -x''_0 + r'' \cos \theta'' \cos(\alpha_0 + 2\alpha) + r'' \sin \theta'' \cos \psi'' \sin(\alpha_0 + 2\alpha); \\ z &= r'' \sin \theta'' \sin \psi''. \end{aligned} \tag{34}$$

From Relationship (33), we obtain:

$$\frac{\partial x}{\partial \theta'} = -r' \sin \theta' \cos \alpha_0 - r' \cos \theta' \cos \psi' \sin \alpha_0; \frac{\partial z}{\partial \theta'} = r' \cos \theta' \sin \psi'. \tag{35}$$

By substituting Expression (35) and the corresponding Lamé coefficients into relationships (15), we obtain:

$$E_x = -(\sin \theta' \cos \alpha_0 + \cos \theta' \cos \psi' \sin \alpha_0) \cdot E_{\theta'}; E_z = \cos \theta' \sin \psi' \cdot E_{\theta'}. \tag{36}$$

By substituting Expression (18) into Relationships (36), we have

$$E_x = -(\sin \theta' \cos \alpha_0 + \cos \theta' \cos \psi' \sin \alpha_0) \cdot \frac{ik^2}{\omega \varepsilon} \cdot \frac{\sin \theta'}{r'} e^{ikr'}; E_z = \cos \theta' \sin \psi' \cdot \frac{ik^2}{\omega \varepsilon} \cdot \frac{\sin \theta'}{r'} e^{ikr'}. \tag{37}$$

From Relationships (31) we find:

$$\cos \psi' = \frac{y'}{r' \sin \theta'}; \sin \psi' = \frac{z'}{r' \sin \theta'}; \cos^2 \psi' + \sin^2 \psi' = 1 = \frac{(y')^2 + (z')^2}{(r' \sin \theta')^2}. \tag{38}$$

Then

$$\sin \theta' = \frac{\sqrt{(y')^2 + (z')^2}}{r'}. \tag{39}$$

By substituting expressions for y' and z' from Expression (1) into Relationship (39), we obtain:

$$\sin \theta' = \frac{\sqrt{[-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0]^2 + z^2}}{\sqrt{(x + x_0)^2 + (y - y_0)^2 + z^2}}. \tag{40}$$

Then we substitute Expressions (1) and (40) into Relationship (38):

$$\cos \psi' = \frac{-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0}{\sqrt{[-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0]^2 + z^2}}; \tag{41}$$

$$\sin \psi' = \frac{z'}{\sqrt{[-(x + x_0) \sin \alpha_0 + (y - y_0) \cos \alpha_0]^2 + z^2}}. \tag{42}$$

Now, by using Expression (31), we obtain:

$$\cos \theta' = \frac{x'}{r'} = \frac{(x + x_0) \cos \alpha_0 + (y - y_0) \sin \alpha_0}{\sqrt{(x + x_0)^2 + (y - y_0)^2 + z^2}}. \tag{43}$$

Let us write the Cartesian components of the vectors E' and H' in the basic coordinate system $Oxyz$; with this aim, we substitute Expressions (40), (41), (42), (43), into (37) and taking into account that $y = 0$, we obtain:

$$\begin{aligned} E'_{x\parallel} &= -i \frac{k^2}{\omega \epsilon} \cdot \frac{[(x + x_0) \sin \alpha_0 + y_0 \cos \alpha_0]y_0 + z^2 \cos \alpha_0}{(r')^3} e^{ikr'}; \\ E'_{z\parallel} &= i \frac{k^2}{\omega \epsilon} \cdot \frac{[(x + x_0) \cos \alpha_0 - y_0 \sin \alpha_0]z}{(r')^3} e^{ikr'}, \end{aligned} \tag{44}$$

where $r' = \sqrt{(x + x_0)^2 + y_0^2 + z^2}$.

Similarly, one can obtain expressions for the tangent field components from the imaginary source:

$$\begin{aligned} E''_{x//} &= -i \frac{k^2 e^{ikr''}}{\omega \epsilon} \left[\frac{-(x + x''_0) \sin(\alpha_0 + 2\alpha) y''_0}{(r'')^3} + \frac{y''_0{}^2 \cos(\alpha_0 + 2\alpha) + z^2 \cos(\alpha_0 + 2\alpha)}{(r'')^3} \right]; \\ E''_{z//} &= i \frac{k^2 e^{ikr''}}{\omega \epsilon} \left[\frac{(x + x''_0) \cos(\alpha_0 + 2\alpha) z}{(r'')^3} + \frac{y''_0 \sin(\alpha_0 + 2\alpha) z}{(r'')^3} \right], \end{aligned} \tag{45}$$

where $r'' = \sqrt{(x + x''_0)^2 + (y''_0)^2 + z^2}$.

By using the Expressions (29), (30), (44), and (45), it is possible to determine the components of the electric field intensity vector of the incident or reflected wave tangential to the beam splitter surface for a given polarization of the source radiation.

The total field on the beam splitter surface for each polarization can be written, using the expressions:

$$E_x = E'_x + T_x(x, z)E''_x; \quad E_z = E'_z + T_z(x, z)E''_z. \tag{46}$$

Here $T(x, z)$ is the amplitude transmission of the beam splitter, it can be different for different polarizations of the irradiating light [13].

Thus, it can be noted that the relationships are obtained for determining the field distribution on the beam splitter surface of the laser interferometer under consideration, which is represented by the tangential components of the electric field intensity vector E_x and E_y expressed in the Cartesian coordinates of the basic system $Oxyz$, taking into account the polarization of the radiation source and the transmittance of the beam splitter.

5. Calculated Relationships for Particular Cases of Optical Circuits

If a circular diaphragm with diameter $2a$ is the source of radiation, whose center is located at the point $O' (x_0, y_0, 0)$ (see Figure 2), and the normal to the diaphragm aperture makes an angle α_0 with the y -axis, then in Relationships (17) and (36), the quantities $E'_{\theta'}$ and $E'_{\psi'}$ for the case of polarization in question will be determined by the expressions:

$$\begin{aligned} E'_{\theta'\parallel} &= -i \frac{\exp(ikr')}{r'} \cdot \frac{a \sin \psi'}{\sin \theta'} J_1(ka \sin \theta'); \quad E'_{\psi'\parallel} = -i \frac{\exp(ikr')}{r'} \cdot \frac{a \cos \theta' \cos \psi'}{\sin \theta'} J_1(ka \sin \theta'); \\ E'_{\theta'\perp} &= -i \frac{\exp(ikr')}{r'} \cdot \frac{a \cos \psi'}{\sin \theta'} J_1(ka \sin \theta'); \quad E'_{\psi'\perp} = -i \frac{\exp(ikr')}{r'} \cdot \frac{a \cos \theta' \sin \psi'}{\sin \theta'} J_1(ka \sin \theta'), \end{aligned} \tag{47}$$

where a is the diaphragm radius, J_1 is the Bessel function of the first order.

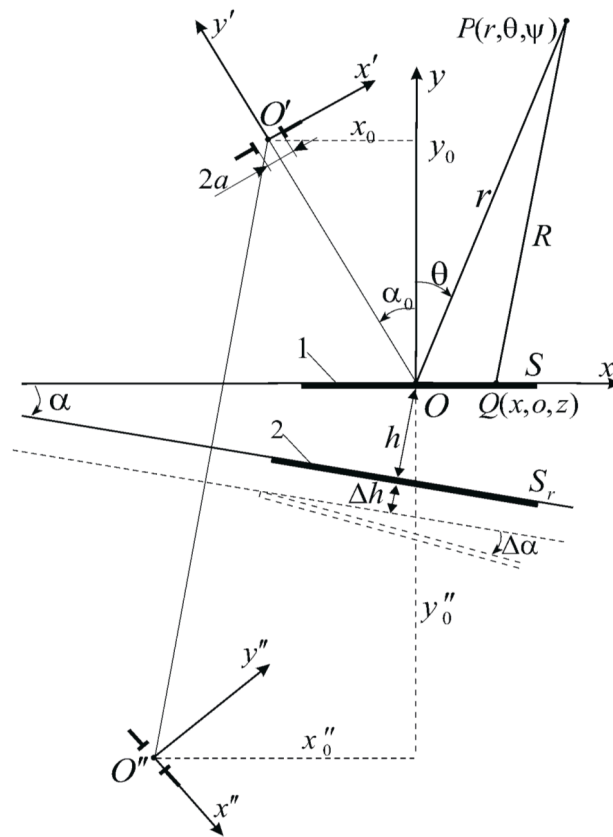


Figure 2. Special case of optical circuits (the source of radiation is the diaphragm).

Similar relationships can be obtained for a mirror diaphragm, where instead of the coordinates of the system denoted by a single stroke, it is necessary to write coordinates with two strokes. By substituting Relationship (47) in Expressions (17) and (36), and carrying out transformations similar to those given above, one can obtain expressions for calculating the field on the beam splitter surface.

In the case when the beam splitter is not completely illuminated (represents itself a round “spot”, described on the surface of the beam splitter by cylindrical coordinates ρ and φ), it is convenient to represent the source field by the cylindrical components of the vector E_ρ and E_φ . On the surface $y = 0$, the expressions for these components have the form:

$$\begin{aligned}
 E_\rho &= \frac{\bar{r}'}{\bar{r}''} \left[\sin \varphi \sin \alpha_0 + \frac{\bar{y}'(\rho + x_0 \sin \varphi)}{(\bar{r}')^2} \right] E'_{\theta'} + \frac{\bar{r}''(-x_0 \cos \alpha_0 + y_0 \sin \alpha_0) \cos \varphi}{\rho^2 \cos^2 \varphi + (\bar{x})^2} E'_{\psi'}; \\
 E_\varphi &= \frac{\bar{r}' \cos \varphi}{\bar{r}''} \left[\sin \alpha_0 + \frac{\bar{y}' x_0}{(\bar{r}')^2} \right] E'_{\theta'} + \frac{\bar{r}''(\rho \cos \alpha_0 + x_0 \sin \varphi \cos \alpha_0 - y_0 \sin \varphi \sin \alpha_0)}{\rho^2 \cos^2 \varphi + (\bar{x})^2} E'_{\psi'},
 \end{aligned} \tag{48}$$

where $\bar{r}' = \sqrt{\rho^2 + 2\rho x_0 \sin \varphi + x_0^2 + y_0^2}$; $\bar{x}' = (\rho \sin \varphi + x_0) \cos \alpha_0 - y_0 \sin \alpha_0$;

$$\bar{y}' = -(\rho \sin \varphi + x_0) \sin \alpha_0 - y_0 \cos \alpha_0; \quad \bar{r}'' = \sqrt{(\bar{r}')^2 - (\bar{y}')^2}. \tag{49}$$

The same relationships can be obtained by expanding the vector E written in a spherical coordinate system associated with a mirror diaphragm. To do this, we need to replace the notations with one stroke by two strokes in Expressions (48) and (49), and replace x_0, y_0, α_0 by values of $x''_0, y''_0, -(\alpha_0 + 2\alpha)$ from Expression (3), respectively.

The presented relationships have been used in the original software [14–18] for numerical studies of the functional characteristics of promising contactless means designed to measurements of optical displacements [19,20]. The use of the relations described in the paper has made it possible to improve the quality and informativeness of the numerical simulation of the process of measuring displacements by a laser interferometer [19].

6. Conclusions

In the paper, the constitutive relations are obtained for the determination of the electromagnetic field on the surface of the beam splitter of the laser interferometer for the cases of perpendicular and parallel polarization of the source radiation. The relationships are also obtained for particular cases of optical schemes for the interferometer under consideration. These relationships allow us to perform numerical simulation of the process of measuring the displacements of control object surfaces using a laser interferometer.

The described results have been used in numerical studies of the functional characteristics of promising contactless optical means for measuring displacements. Their use has improved the quality and informativeness of the numerical simulation of the measurement process of displacements by a laser interferometer.

The obtained relationships can also be used in theoretical and numerical modeling and for justification of new contactless optical interference measurement methods and advanced optical means for measuring displacements of control object surfaces.

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