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# Finite Memory Structure Filtering and Smoothing for Target Tracking in Wireless Network Environments

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**Abstract:** In this paper, a state estimation problem is considered for a target tracking scheme in wireless network environments. Firstly, a unified algorithm of finite memory structure (FMS) filtering and smoothing is proposed for a discrete-time state-space model. As shown in the terminology *unified*, both FMS filter and smoother are derived by solving one optimization problem directly with incorporation of the unbiasedness constraint. Hence, the unified algorithm provides simultaneously the current state estimate as well as the lagged state estimate using only finite measurements and inputs on the most recent window. The proposed unified algorithm of FMS filtering and smoothing shows that there are some unique properties such as unbiasedness, deadbeat, time-invariance and intrinsic robustness, which cannot be obtained by the recursive infinite memory structure (IMS) filtering such as Kalman filter. The on-line computational complexity of the proposed unified algorithm is discussed. Secondly, as an application of the proposed unified algorithm, a target tracking scheme in wireless network environments is considered via computer simulations for moving target's accelerations of various shapes. The proposed unified algorithm-based target tracking scheme provides estimates for position as well as acceleration of moving target in real time, while eliminating unwanted noise effects and maintaining desired moving positions. Due to intrinsic robustness and deadbeat properties, the proposed unified algorithm-based scheme can outperform the existing IMS filtering-based scheme when acceleration suddenly changes.

**Keywords:** filter; finite memory structure; infinite memory structure; smoother; target tracking

## 1. Introduction

The Kalman filter and smoother [1–5] have been the most commonly fundamental tools for filtering and smoothing in statistical time series analysis. Thus, the Kalman filter and smoother have been a standard choice and a beautiful reference for the state estimation and applied successfully for diverse engineering problems. However, due to their recursive formulations and infinite memory structure (IMS), the Kalman filter and smoother may exhibit performance degradation and even divergence in severe cases for mismodeling and temporary uncertainties.

Therefore, as an alternative to the Kalman filter, the finite memory structure (FMS) filter has been designed for state estimation and shown inherently to have BIBO stability and be more robust against temporary uncertainties [6–9]. This FMS filter has been applied successfully in various engineering fields [10–12]. Likewise the recursive Kalman filter in [1–5], the FMS filter in [6–9] is a causal filter that provides state estimates at given times based only on the relative past. Hence, as a noncausal filter, the FMS smoother has been also designed for estimation problems when there is a fixed delay  $d$  between the original state and the estimated state [13–16]. The FMS smoother has been shown to be much less

computationally complex and more robust against temporary uncertainties than the recursive Kalman smoother. Of course, noncausal filters such as the recursive Kalman smoother and the FMS smoother naturally yield more accurate estimates when there is a fixed delay  $d$  between the original state and the estimated state. However, the FMS smoother could not be better than the FMS filter when there is no fixed delay between the original state and the estimated state. That is, the FMS filter should be applied when there is no fixed delay between signal generation and signal estimation.

Meanwhile, a target tracking problem in indoor positioning systems for wireless network environments has been an interesting research problem and thus applied successfully for locating and tracking people within a building and products stored within a warehouse using wireless sensors and mobile devices [17–19]. Due to a variety of indoor wireless channel characteristics, an accurate tracking estimation of a moving target is required for indoor positioning systems. However, a couple of noises such as system noise and measurement noise can contaminate the estimated position and thus can cause deterioration of target tracking performance. Hence, these noises must be filtered for tracking a true path of a moving target. To provide the most accurate position and velocity of a moving target on the real-time indoor positioning in wireless network environments, some research adopted the Kalman filter to reduce the effect of noises [20–22]. Actually, past measurement data might have a little information about the target's current motion since the moving target changes its motion dynamics. In other words, the valid duration of the moving target's dynamics might be limited to the recently finite time interval. In addition, estimation filters are typically designed with the state-space model called the constant velocity motion model. The constant velocity motion model assumes that targets move with constant velocity, that is, zero-acceleration within a short sampling time. However, in real situations, moving targets maneuver and change velocity and thus move temporarily with nonzero acceleration. Although this can be a temporary uncertainty and thus effects typically occur over a short time interval, the state estimation filter should be robust to diminish the effects of the temporary uncertainty. Therefore, the FMS filter and smoother might be very appropriate for target tracking approaches. The FMS filter was applied successfully for the target tracking in wireless network environments [23,24] while the FMS smoother has not been addressed.

In this paper, a state estimation problem is considered for a target tracking scheme in wireless communication environments. Firstly, an alternative FMS filtering and smoothing algorithm is derived for both cases with delay and without delay between the original state and the estimated state. This algorithm is thus called the unified algorithm of FMS filter and smoother. As shown in the terminology *unified* which can also mean *united*, *combined*, *integrated*, both FMS filter and smoother are derived by solving one optimization problem directly with incorporation of the unbiasedness constraint using only finite measurements and inputs on the most recent window  $[i - M, i]$ . Thus, the proposed unified algorithm provides simultaneously state estimates at the current time  $i$  as well as at the lagged time  $i - d$ , given finite measurements and inputs on the most recent window  $[i - M, i]$  where  $i - M$  is the window initial time and  $i$  is the current time. The proposed unified algorithm of FMS filtering and smoothing shows that there are some unique properties such as unbiasedness, deadbeat, time-invariance and intrinsic robustness while the recursive IMS filter such as Kalman filter does not have these properties. The on-line computational complexity of the proposed unified algorithm is discussed and compared with the IMS filter. Secondly, a target tracking in wireless network environments is considered as an application of the proposed unified algorithm of FMS filtering and smoothing. Through extensive computer simulations for moving target's accelerations of various shapes, the proposed unified algorithm-based target tracking scheme is shown to provide estimates for position as well as acceleration of moving target in real time, while eliminating unwanted noise effects and maintaining desired moving positions. In particular, due to intrinsic robustness and deadbeat properties, the performance of the proposed unified algorithm-based scheme can be shown to be better than the existing IMS filtering-based scheme for suddenly changing acceleration.

This paper has the following structure. In Section 2, a unified algorithm of FMS filtering and smoothing is proposed. In Section 3, a target tracking scheme using the proposed unified algorithm is considered via extensive computer simulations. Then, concluding remarks are given in Section 4.

### 2. Unified Algorithm of Finite Memory Structure Filter and Smoother

Consider the following linear discrete-time state-space model:

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i + Gw_i, \\ z_i &= Cx_i + v_i, \end{aligned} \tag{1}$$

where  $x_i \in \mathbb{R}^n$  is the system state variable,  $u_i \in \mathbb{R}^p$  is the control input,  $z_i \in \mathbb{R}^q$  is the sensor measurement output. A couple of noises, the system noise  $w_i \in \mathbb{R}^p$  and the measurement noise  $v_i \in \mathbb{R}^q$ , are zero-mean white Gaussian. These noises are mutually uncorrelated and their covariances are denoted by positive definite matrices  $Q$  and  $R$ , respectively.

At the current time  $i$ , finite measurements  $Z_i$  and inputs  $U_i$  on the most recent window  $[i - M, i]$  can be expressed by the regression form with the current state  $x_i$  as follows:

$$Z_i - \Xi U_i = \Gamma x_i + \Lambda W_i + V_i, \tag{2}$$

where

$$Z_i \triangleq \begin{bmatrix} z_{i-M} \\ z_{i-M+1} \\ \vdots \\ z_{i-2} \\ z_{i-1} \end{bmatrix}, U_i \triangleq \begin{bmatrix} u_{i-M} \\ u_{i-M+1} \\ \vdots \\ u_{i-2} \\ u_{i-1} \end{bmatrix}, W_i \triangleq \begin{bmatrix} w_{i-M} \\ w_{i-M+1} \\ \vdots \\ w_{i-2} \\ w_{i-1} \end{bmatrix}, V_i \triangleq \begin{bmatrix} v_{i-M} \\ v_{i-M+1} \\ \vdots \\ v_{i-2} \\ v_{i-1} \end{bmatrix}, \tag{3}$$

and matrices  $\Gamma, \Xi, \Lambda$  are defined by

$$\begin{aligned} \Gamma &\triangleq \begin{bmatrix} CA^{-M} \\ CA^{-M+1} \\ \vdots \\ CA^{-2} \\ CA^{-1} \end{bmatrix}, \Xi \triangleq - \begin{bmatrix} CA^{-1}B & CA^{-2}B & \dots & CA^{-M}B \\ 0 & CA^{-1}B & \dots & CA^{-M+1}B \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & CA^{-2}B \\ 0 & 0 & \dots & CA^{-1}B \end{bmatrix}, \\ \Lambda &\triangleq - \begin{bmatrix} CA^{-1}G & CA^{-2}G & \dots & CA^{-M}G \\ 0 & CA^{-1}G & \dots & CA^{-M+1}G \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & CA^{-2}G \\ 0 & 0 & \dots & CA^{-1}G \end{bmatrix}. \end{aligned} \tag{4}$$

Meanwhile, at the lagged time  $i - d$ , the lagged state  $x_{i-d}$  can be represented by

$$x_{i-d} = A^{-d}x_i - \tilde{\Xi}U_i - \tilde{\Lambda}W_i, \tag{5}$$

where matrices  $\tilde{\Xi}$  and  $\tilde{\Lambda}$  is defined by

$$\begin{aligned} \tilde{\Xi} &\triangleq - \left[ \overbrace{0 \ 0 \ \dots \ 0}^{M-d} \ A^{-1}B \ A^{-2}B \ \dots \ A^{-d}B \right], \\ \tilde{\Lambda} &\triangleq - \left[ \overbrace{0 \ 0 \ \dots \ 0}^{M-d} \ A^{-1}G \ A^{-2}G \ \dots \ A^{-d}G \right]. \end{aligned}$$

From (2) and (5), the current state  $x_i$  and the lagged state  $x_{i-d}$  can be represented by the regression form with the augmented state  $X_i$  as follows:

$$\tilde{Z}_i - \tilde{\Xi}U_i = \tilde{\Gamma}X_i + \tilde{\Lambda}W_i + \tilde{V}_i, \tag{6}$$

where

$$\begin{aligned} X_i &\triangleq \begin{bmatrix} x_i \\ x_{i-d} \end{bmatrix}, \quad \tilde{Z}_i \triangleq \begin{bmatrix} Z_i \\ 0 \end{bmatrix}, \quad \tilde{V}_i \triangleq \begin{bmatrix} V_i \\ 0 \end{bmatrix}, \\ \tilde{\Gamma} &\triangleq \begin{bmatrix} \Gamma & 0 \\ A^{-d} & -1 \end{bmatrix}, \quad \tilde{\Xi} \triangleq \begin{bmatrix} \Xi \\ \tilde{\Xi} \end{bmatrix}, \quad \tilde{\Lambda} \triangleq \begin{bmatrix} \Lambda \\ \tilde{\Lambda} \end{bmatrix}. \end{aligned}$$

The noise term  $\tilde{\Lambda}W_i + \tilde{V}_i$  in (6) is zero-mean white Gaussian and its covariance given by the positive definite matrix  $\Pi$  as follows:

$$\Pi \triangleq \tilde{\Lambda}\tilde{Q}\tilde{\Lambda}^T + \begin{bmatrix} \tilde{R} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Lambda\tilde{Q}\Lambda^T + \tilde{R} & \Lambda\tilde{Q}\tilde{\Lambda}^T \\ \tilde{\Lambda}\tilde{Q}\Lambda^T & \tilde{\Lambda}\tilde{Q}\tilde{\Lambda}^T \end{bmatrix}, \tag{7}$$

where

$$\tilde{Q} \triangleq \left[ \text{diag}(\overbrace{Q \ Q \ \dots \ Q}^M) \right], \quad \tilde{R} \triangleq \left[ \text{diag}(\overbrace{R \ R \ \dots \ R}^M) \right]. \tag{8}$$

Based on the approach of *best linear unbiased estimation* in [25], the unified algorithm of FMS filter and smoother is derived using the following series of equations. The unified algorithm of FMS filter and smoother  $\hat{X}_i$  is assumed to be obtained from a matrix form using only finite measurements  $\tilde{Z}_i$  and inputs  $U_i$  on the most recent window  $[i - M, i]$  as follows:

$$\hat{X}_i = \begin{bmatrix} \hat{x}_i \\ \hat{x}_{i-d} \end{bmatrix} \triangleq \mathcal{H}(\tilde{Z}_i - \tilde{\Xi}U_i), \tag{9}$$

where  $\mathcal{H}$  is the unified gain matrix. When taking the expectation both sides of (9) as follows:

$$\mathbf{E}[\hat{X}_i] = \mathbf{E} \begin{bmatrix} \hat{x}_i \\ \hat{x}_{i-d} \end{bmatrix} = \mathbf{E}[\mathcal{H}(\tilde{Z}_i - \tilde{\Xi}U_i)] = \mathcal{H}\tilde{\Gamma}\mathbf{E}[X_i], \tag{10}$$

$\hat{X}_i$  is unbiased, i.e.,  $\mathbf{E}[\hat{X}_i] = \mathbf{E}[X_i]$ , with the following constraint:

$$\mathcal{H}\tilde{\Gamma} = I. \tag{11}$$

Thus, the constraint (11) can be called the *unbiasedness constraint* for the unified algorithm of FMS filter and smoother  $\hat{X}_i$ .

Subject to the unbiasedness constraint (11), the objective is now to obtain the gain matrix  $\mathcal{H}_*$  in order that the error of  $\hat{X}_i$  has a minimum variance as follows:

$$\mathcal{H}_* = \arg \min_{\mathcal{H}} \mathbf{E} \left[ (X_i - \hat{X}_i)^T (X_i - \hat{X}_i) \right]. \tag{12}$$

Then, by solving the optimization problem directly with incorporation of the unbiasedness constraint, the unified algorithm of FMS filter and smoother  $\hat{X}_i$  is obtained by the solution of (12) as follows:

$$\hat{X}_i = \begin{bmatrix} \hat{x}_i \\ \hat{x}_{i-d} \end{bmatrix} = \mathcal{H}_* (\bar{Z}_i - \bar{\mathbf{E}}U_i), \tag{13}$$

where

$$\mathcal{H}_* = (\bar{\Gamma}^T \Pi^{-1} \bar{\Gamma})^{-1} \bar{\Gamma}^T \Pi^{-1}. \tag{14}$$

The proposed unified algorithm (13) of FMS filtering and smoothing shows that there are some unique properties such as unbiasedness, deadbeat, time-invariance and intrinsic robustness. The gain matrix  $(\bar{\Gamma}^T \Pi^{-1} \bar{\Gamma})^{-1} \bar{\Gamma}^T \Pi^{-1}$  in (14) of the unified algorithm requires only once computation on the interval  $[0, M]$  and then is used for all windows, which means the time-invariance property. As shown in (10) and (11), the unified algorithm has the unbiasedness property *by design*. In addition, the proposed unified algorithm  $\hat{X}_i$  in (13) for both the current state estimate  $\hat{x}_i$  and the lagged state estimate  $\hat{x}_{i-d}$  has the deadbeat property as following theorem.

**Theorem 1.** *When  $M \geq n$ , the proposed unified algorithm of FMS filter and smoother  $\hat{X}_i$  on the most recent window  $[i - M, i]$  is exact for noise-free systems.*

**Proof of Theorem 1.** When there are no noises on the most recent window  $[i - M, i]$  for the discrete-time state-space model (1),  $\bar{Z}_i - \bar{\mathbf{E}}U_i$  (6) is represented by

$$\bar{Z}_i - \bar{\mathbf{E}}U_i = \bar{\Gamma} X_i = \bar{\Gamma} \begin{bmatrix} x_i \\ x_{i-d} \end{bmatrix}.$$

Hence, the following is satisfied:

$$\begin{aligned} \hat{X}_i = \begin{bmatrix} \hat{x}_i \\ \hat{x}_{i-d} \end{bmatrix} &= (\bar{\Gamma}^T \Pi^{-1} \bar{\Gamma})^{-1} \bar{\Gamma}^T \Pi^{-1} (\bar{Z}_i - \bar{\mathbf{E}}U_i) \\ &= (\bar{\Gamma}^T \Pi^{-1} \bar{\Gamma})^{-1} \bar{\Gamma}^T \Pi^{-1} \bar{\Gamma} \begin{bmatrix} x_i \\ x_{i-d} \end{bmatrix} \\ &= \begin{bmatrix} x_i \\ x_{i-d} \end{bmatrix}. \end{aligned}$$

This completes the proof of the deadbeat property.  $\square$

The deadbeat property means that the unified algorithm of FMS filter and smoother  $\hat{X}_i$  in (13) tracks exactly the current state  $x_i$  and the lagged state  $x_{i-d}$  at every time for noise-free systems although

the proposed unified algorithm has been designed assuming a couple noises such as  $w_i$  and  $v_i$  in the discrete-time state-space model (1). Since the deadbeat property indicates finite convergence time and fast tracking ability, of the proposed unified algorithm of FMS filter and smoother. Thus, it can be expected that the proposed unified algorithm of FMS filter and smoother might be appropriate for a variety of applications requiring fast tracking capability such as fault detection and diagnosis for dynamic process systems as well as maneuver detection and target tracking for indoor positioning systems. Moreover, in contrast to the recursive IMS filter such as the Kalman filter, the proposed unified algorithm can have intrinsic robustness due to its finite memory structure. This means that the proposed unified algorithm can be more robust against round-off errors, mismodeling and temporary uncertainties.

The on-line computational complexity of the proposed unified algorithm is discussed. To simply compare the computational complexity of the proposed unified algorithm and the IMS filter, the measurement  $z_i$  is assumed as a scalar-valued (i.e.,  $q = 1$ ). In the case of the IMS filter such as the fixed-lag Kalman smoothing filter, the coefficient values for fixed-lag Kalman filtering and fixed-lag smoothing algorithms must be always computed for  $d$  iterations by on-line computing before inputting any data to the fixed-lag Kalman smoothing filter algorithm stage. Thus, the IMS filter such as the fixed-lag Kalman smoothing filter with  $n$ th dimension can be done with  $4n^3 + 8n^2 + 6n + 1$  flops as shown in [26]. On the other hand, as mentioned before, the gain matrix (14) of the unified algorithm requires only once computation on the interval  $[0, M]$  and then is used for all windows. Thus, FMS filter and smoother with  $n$ th dimension in the proposed unified algorithm can be done respectively with  $n \times M$  flops as shown in Table 1. Thus, as the dimension grows, the on-line computational complexity of the proposed unified algorithm is shown to be much less than that of the fixed-lag Kalman smoothing filter. However, the proposed unified algorithm requires the memory with size of  $M$  to store finite measurements on the most recent window  $[i - M, i]$  and the memory shift with  $M - 1$  operations. On the other hand, the fixed-lag Kalman smoothing filter requires to store only intermediate estimates with size of  $d$ . Since  $M > d$ , the fixed-lag Kalman smoothing filter can be better than the proposed unified algorithm in terms of the memory management.

**Table 1.** Comparison of on-line computation and memory size.

	<b>FMS Filter and Smoother</b>	<b>Fixed-Lag Kalman Smoothing Filter</b>
On-line computation (flops)	$n \times M$ (respectively)	$4n^3 + 8n^2 + 6n + 1$
Memory shift	$M - 1$	None
Memory size	$M$	$d$

### 3. Application for Target Tracking Problem

As an application of the proposed unified algorithm of FMS filtering and smoothing, a target tracking problem is considered through extensive computer simulations. In these days, a target tracking problem in indoor positioning systems for wireless network environments has been an interesting research problem and thus been applied successfully for locating and tracking people within a building and products stored within a warehouse using wireless sensors and mobile devices. Due to a variety of indoor wireless channel characteristics, an accurate tracking estimation of a moving target is required for indoor positioning systems. However, a couple of noises such as system noise and measurement noise can contaminate the estimated position and thus can cause deterioration of target tracking performance. These noises can be caused by various factors such as environmental interference, turbulence affecting the target’s movement, inaccuracies of sensor measurement data, and human’s navigation error in a perfect straight

line. Hence, these noises must be filtered for tracking a true path of a moving target. To provide the most accurate position and velocity of a moving target on the real-time indoor positioning in wireless network environments, some research adopted both IMS and FMS filters to reduce the effect of noises [20].

This paper considers only the one dimensional moving target for the  $X$  direction for the simplicity. The state estimation filter for the target tracking is typically designed with the state-space model called the constant velocity motion model. The constant velocity motion model assumes that targets move with constant velocity, that is, zero-acceleration within a short sampling time. However, in real situations, moving targets maneuver and change velocity and thus move temporarily with nonzero acceleration. Thus, the moving target's successive locations for the  $X$  direction can be represented by the 3rd-order discrete-time state-space model with sampling time  $T$  and acceleration term:

$$\begin{aligned}x_{i+1} &= Ax_i + Gw_i, \\z_i &= Cx_i + v_i\end{aligned}\quad (15)$$

where

$$x_i \triangleq \begin{bmatrix} x_p : \text{position} \\ x_v : \text{velocity} \\ x_a : \text{acceleration} \end{bmatrix}, A = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, G = \begin{bmatrix} T/2 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, C = [1 \ 0 \ 0]. \quad (16)$$

The state vector  $x_i$  at time  $i$  consists of three state variables,  $x_p$  to represent the moving target's random *position*,  $x_v$  to represent the corresponding *velocity*,  $x_a$  to represent the corresponding *accelerations*. Typically, since the acceleration occurs over a short time interval, this can be considered as a temporary uncertainty, that is, an unknown input and thus considered as an unknown state term. Hence, since the target tracking problem in this paper does not need the control input term, the proposed unified algorithm (13) is applied with assuming that the control input matrix is zero, that is,  $B = 0$ . Because of random disturbances by fading and shadowing, the state vector  $x_i$  cannot be measured directly. Thus, the measurement vector  $z_i$  is modeled by only the position to take these effects into account filtering or smoothing estimates. As shown in [23], target tracking applications using the constant velocity motion model often adopt the state-space model where the state equation is linear and only the measurement equation is nonlinear. Since little research is going on in the case of the FMS smoother for nonlinear systems, the proposed unified algorithm is currently difficult to use this type of state space model. Thus, this paper consider only the linear measurement equation. The application of the nonlinear measurement equation can be a future work of the current paper.

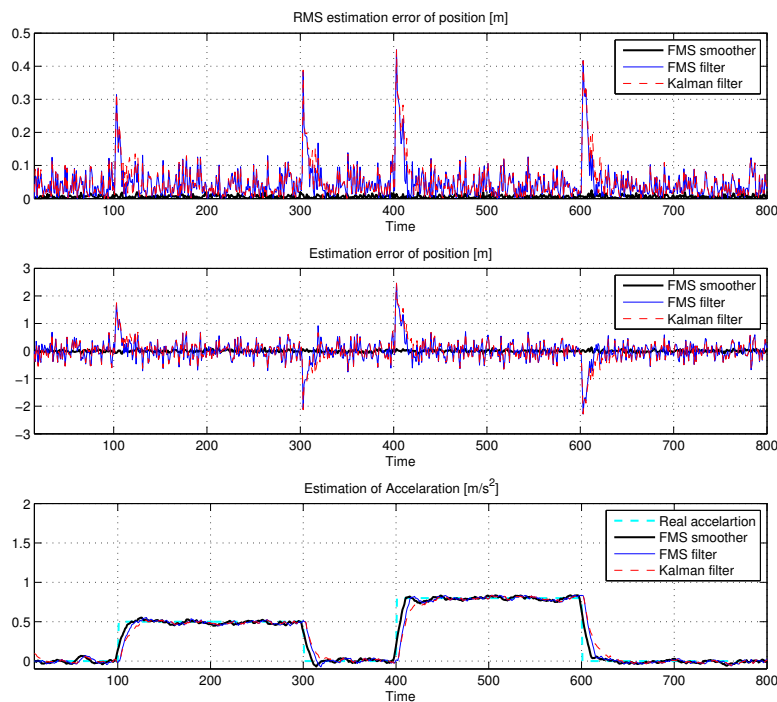
Using (13) with (15) and (16), the proposed unified algorithm of a FMS filtering and smoothing-based target tracking scheme provides estimates for position as well as acceleration of moving target in real time, while eliminating unwanted noise effects and maintaining desired moving positions. Via extensive computer simulations, the performance of the proposed unified algorithm-based target tracking scheme is evaluated and compared with the existing IMS filtering-based scheme of [20–22]. Computer simulations are performed for moving target's accelerations of four kinds of shapes. The first and second scenarios consider that the target moves with suddenly changing acceleration such as step-type and ramp-type. The third case considers that the target moves with slowly changing acceleration such as triangle-type. The last case considers that the target moves with randomly changing acceleration. The sampling period is taken by  $T = 2$ . The covariance of the system noise is  $Q = \text{diag}(0.173^2 \ 0.01^2)$  and the covariance of measurement noise is  $R = 0.05^2$ . The window length is taken by  $M = 15$ . The lagged length is taken by  $d = 5$ . Simulations of 30 runs are performed using different system and measurement noises to make the comparison clearer. Each single simulation run lasts 800.



Figures 1–4 show simulation results for moving target’s accelerations of four kinds of shapes. The first plot of figures shows the root-mean-square(RMS) estimation error of the moving target’s random position for 30 simulations. The second plot of all figures shows the estimation error of the moving target’s random position for one of 30 simulations. The last plot of figures shows the estimate of unknown acceleration. In addition, time averaged values of RMS estimation errors are presented by Table 2. As shown in simulation results, the proposed unified algorithm-based target tracking scheme can outperform the IMS filtering-based target tracking scheme when the target moves with suddenly changing acceleration. On the interval where the acceleration varies suddenly, the estimation error of the proposed unified algorithm-based scheme is remarkably smaller than that of the IMS filtering-based scheme. In addition, when the acceleration varies constantly, the convergence of the estimation error for the proposed unified algorithm-based scheme is much faster than that of the IMS filtering-based scheme. These observations for simulation results might come from the fast convergent time and the fast tracking ability due to intrinsic robustness and deadbeat properties of the proposed unified algorithm-based scheme.

**Table 2.** Comparison of mean of RMS estimation errors.

	Step-Type	Ramp-Type	Triangle-Type	Randomly Changing
FMS Smoother	0.0062	0.0060	0.0059	0.0059
FMS Filter	0.0477	0.0441	0.0408	0.0445
Kalman Filter	0.0505	0.0456	0.0415	0.0481



**Figure 1.** Simulation result for suddenly changing acceleration(step-type).



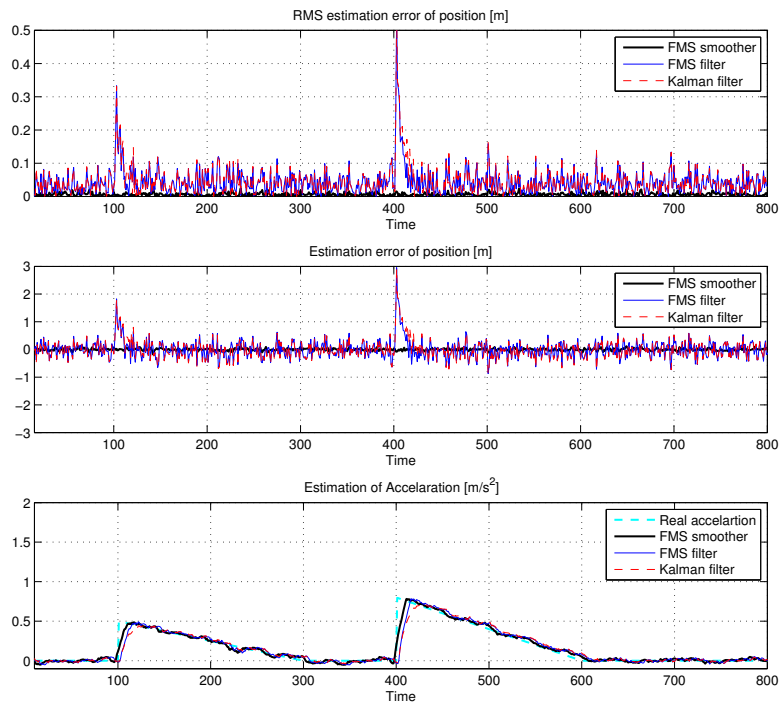


Figure 2. Simulation result for suddenly changing acceleration(ramp-type).

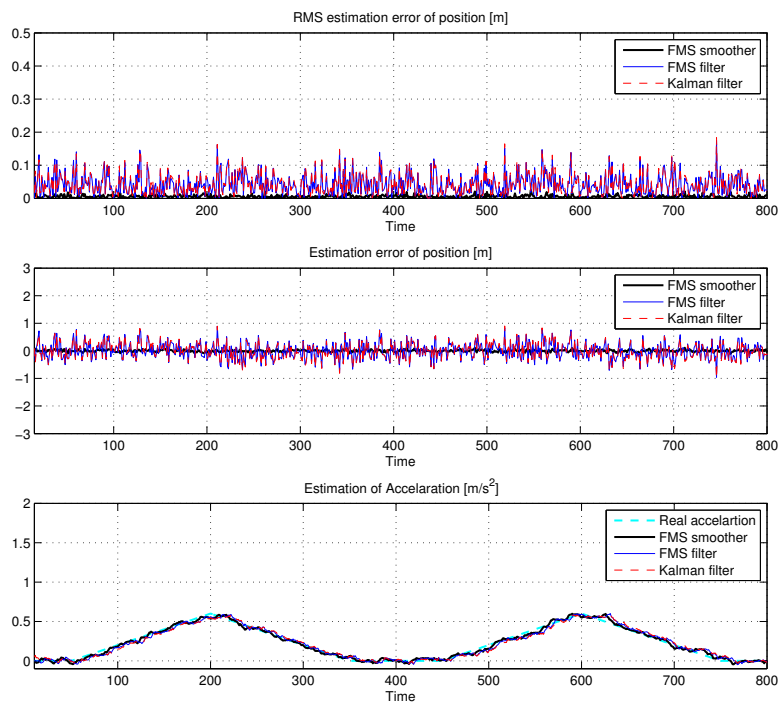


Figure 3. Simulation result for slowly changing acceleration(triangle-type).

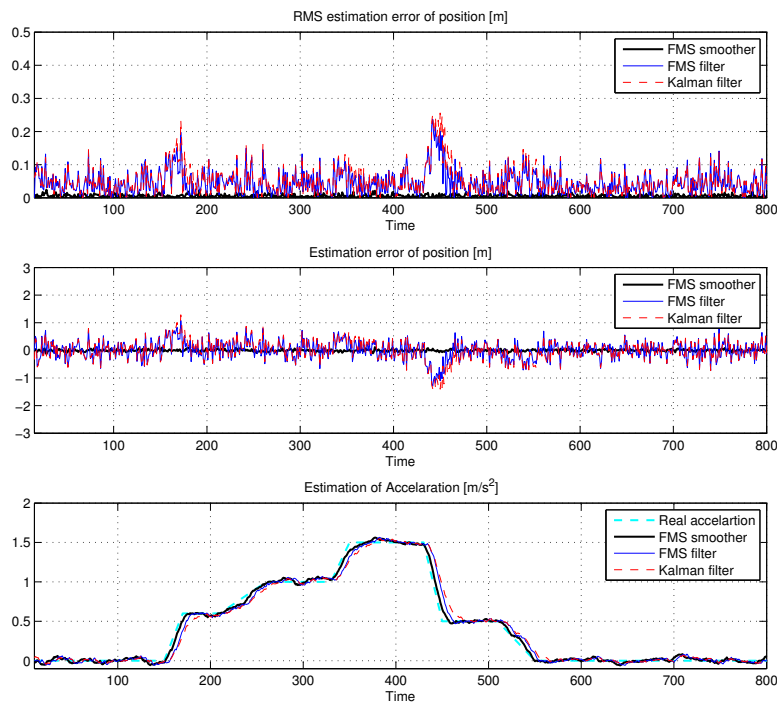


Figure 4. Simulation result for randomly changing acceleration.

#### 4. Conclusions

This paper has dealt with the state estimation problem for the target tracking scheme in wireless communication environments. Firstly, the unified algorithm of FMS filtering and smoothing has been proposed. The unified algorithm has been derived by solving one optimization problem directly with incorporation of the unbiasedness constraint using only finite measurements and inputs on the most recent window. The proposed unified algorithm provides simultaneously the current state estimate as well as the lagged state estimate. The proposed unified algorithm has shown that there are some unique properties such as unbiasedness, deadbeat, time-invariance and intrinsic robustness while the recursive IMS filter such as Kalman filter does not have these properties. The on-line computational complexity of the proposed unified algorithm has been compared with the IMS filter such as the fixed-lag Kalman smoothing filter. Secondly, the target tracking in wireless network environment has been considered via extensive computer simulations as an application of the proposed unified algorithm of FMS filtering and smoothing. Through simulation results for moving targets' accelerations of various shapes, the proposed unified algorithm-based target tracking scheme has been shown to provide estimates for position as well as acceleration of moving target in real time, while eliminating unwanted noise effects and maintaining desired moving positions. It has been shown that the performance of the proposed unified algorithm-based target tracking scheme can be better than the IMS filtering-based scheme for suddenly changing acceleration.

Actually, research on the FMS filter and smoother in nonlinear systems is relatively inactive compared to research in linear systems. There is some research, such as an adoption of nonlinear measurement model, on the FMS filter for nonlinear systems, but little research is going on in the case of the FMS smoother for nonlinear systems. Thus, for nonlinear systems, research on the FMS smoother should be preceded and

then a unified algorithm of FMS filter and smoother can be researched as a future work. In addition, the estimation of target's acceleration can be extended to identification and classification of moving targets, which could be another future work.

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## References

1. Grewal, M. Applications of Kalman filtering in aerospace 1960 to the present. *IEEE Control Syst.* **2010**, *30*, 69–78. [[CrossRef](#)]
2. Auger, F.; Hilaiet, M.; Guerrero, J.M.; Monmasson, E.; Orłowska-Kowalska, T.; Katsura, S. Industrial applications of the Kalman filter: A review. *IEEE Trans. Ind. Electron.* **2013**, *60*, 5458–5471. [[CrossRef](#)]
3. Rhudy, M.; Salguero, R.; Holappa, K. A Kalman filtering tutorial for undergraduate students. *Int. J. Comput. Sci. Eng. Surv.* **2017**, *8*, 1–18. [[CrossRef](#)]
4. Barrau, A.; Bonnabel, S. Invariant Kalman filtering. *Ann. Rev. Control Robot. Auton. Syst.* **2018**, *1*, 237–257. [[CrossRef](#)]
5. Aravkin, A.; Burke, J.V.; Ljung, L.; Lozano, A.; Pilonetto, G. Generalized Kalman smoothing: Modeling and algorithms. *Automatica* **2017**, *86*, 63–86. [[CrossRef](#)]
6. Kim, P.S. An alternative FIR filter for state estimation in discrete-time systems. *Digit. Signal Process.* **2010**, *20*, 935–943. [[CrossRef](#)]
7. Shmaliy, Y.S.; Simon, D. Iterative unbiased FIR state estimation: A review of algorithms. *EURASIP J. Adv. Signal Process.* **2013**, *2013*, 113. [[CrossRef](#)]
8. Shmaliy, Y.S.; Zhao, S.; Ahn, C.K. Unbiased finite impulse response filtering: An iterative alternative to Kalman filtering ignoring noise and initial conditions. *IEEE Control Syst.* **2017**, *37*, 70–89.
9. Shmaliy, Y.S.; Neuvo, Y.; Khan, S. Review of unbiased FIR filters, smoothers, and predictors for polynomial signals. *Front. Signal Process.* **2018**, *2*, 1–29.
10. Zhao, S.; Shmaliy, Y.S.; Liu, F. Fast Kalman-Like optimal unbiased FIR filtering with applications. *IEEE Trans. Signal Process.* **2016**, *64*, 2284–2297. [[CrossRef](#)]
11. Shmaliy, Y.; Khan, S.H.; Zhao, S.; Ibarra-Manzano, O. General unbiased FIR filter with applications to GPS-based steering of oscillator frequency. *IEEE Trans. Control Syst. Technol.* **2017**, *25*, 1141–1148. [[CrossRef](#)]
12. Kim, P.S. A design of finite memory residual generation filter for sensor fault detection. *Meas. Sci. Rev.* **2017**, *17*, 75–81. [[CrossRef](#)]
13. Shmaliy, Y.S.; Morales-Mendoza, L.J. FIR smoothing of discrete-time polynomial signals in state space. *IEEE Trans. Signal Process.* **2010**, *58*, 2544–2555. [[CrossRef](#)]
14. Kim, P.S. A computationally efficient fixed-lag smoother using recent finite measurements. *Measurement* **2013**, *46*, 846–850. [[CrossRef](#)]
15. Kwon, B.K.; Quan, Z.; Han, S.H. A robust fixed-lag receding horizon smoother for uncertain state space models. *Adapt. Control Signal Process.* **2015**, *29*, 1354–1366. [[CrossRef](#)]
16. Kim, P.S. A finite memory structure smoother with recursive form using forgetting factor. *Math. Probl. Eng.* **2017**, *2017*, 1–6. [[CrossRef](#)]
17. Ciuonzo, D.; Buonanno, A.; D'Urso, M.; Palmieri, F.A.N. Distributed classification of multiple moving targets with binary wireless sensor networks. In Proceedings of the 14th International Conference on Information Fusion, Chicago, IL, USA, 5–8 July, 2011; pp. 1–8.
18. Ciuonzo, D.; Willett, P.K.; Bar-Shalom, Y. Tracking the tracker from its passive sonar ML-PDA estimates. *IEEE Trans. Aerosp. Electron. Syst.* **2014**, *50*, 573–590. [[CrossRef](#)]
19. Ez-Zaidi, A.; Rakrak, S. A comparative study of target tracking approaches in wireless sensor networks. *J. Sens.* **2016**, *2016*, 1–11. [[CrossRef](#)]

20. Lee, S.; Cho, B.; Koo, B.; Ryu, S.; Choi, J.; Kim, S. Kalman filter-based indoor position tracking with self-calibration for RSS variation mitigation. *Int. J. Distrib. Sens. Netw.* **2015**, *2015*, 1–10. [[CrossRef](#)]
21. Lee, S.H.; Lim, I.K.; Lee, J.K. Method for improving indoor positioning accuracy using extended Kalman filter. *Mob. Inf. Syst.* **2016**, *2016*, 1–15. [[CrossRef](#)]
22. Fariz, N.; Jamil, N.; Din, M.M.; Rusli, M.E.; Sharudin, Z.; Mohamed, M.A. An improved indoor location technique using Kalman filter. *Int. J. Eng. Technol.* **2018**, *7*, 1–4. [[CrossRef](#)]
23. Pak, J.M.; Kim, P.S.; You, S.H.; Lee, S.S.; Song, M.K. Extended least square unbiased FIR filter for target tracking using the constant velocity motion model. *Int. J. Control Autom. Syst.* **2017**, *15*, 947–951. [[CrossRef](#)]
24. Kim, P.S.; Lee, E.H.; Jang, M.S.; Kang, S.Y. A finite memory structure filtering for indoor positioning in wireless sensor networks with measurement delay. *Int. J. Distrib. Sens. Netw.* **2017**, *13*, 1–8. [[CrossRef](#)]
25. Mendel, J. *Lessons in Estimation Theory for Signal Processing, Communications, and Control*; Prentice-Hall: Englewood Cliffs, NJ, USA, 1995.
26. Grewal, M.S.; Andrews, A.P. *Kalman Filtering—Theory and Practice*; Prentice-Hall: Englewood Cliffs, NJ, USA, 1993.



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