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A Bayesian Approach in the Evaluation of Unit Weight of Mineral and Organic Soils Based on Dilatometer Tests (DMT)

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Abstract: Recently, geotechnical problems that are characterized by a high degree of complexity and uncertainty with respect to input data have been solved using Bayesian analysis. One example is the problem of cautious estimation of geotechnical parameters according to Eurocode 7 requirements. The research included various types of soil such as peat, gyttja, organic mud, and clays. These were studied in order to develop an empirical correlation for determining the unit weight of mineral and organic soils. The compiled database of documented field research sites for different types of soil was used to investigate and develop direct relationships between measured results and dilatometer (DMT) readings, i.e., p_o and p_1 together with pore water pressure (u_o) and pressure (P_a). The soil unit weights were determined for both mineral and organic soils. The paper addresses the applicability of the Bayesian approach in geotechnics via a simple example related to the determination of characteristic values of geotechnical parameters for design structures. The results show that it is possible to conduct a more reliable forecast with improved statistical measures compared to other available methods for multilayer subsoils.

Keywords: Bayes' theorem; DMT; unit weight; mineral and organic soils

1. Summary

Knowledge of the unit weight of soil is needed to calculate the total and effective vertical geostatic stress and the lateral stress index (K_D) for the assessment of undrained shear strength and constrained modulus, which is used in classification nomogram charts. This analysis was based on a set of dilatometer (DMT) test results from the Antoniny, Koszyce, Nielisz, Stegna, and "Szkoły Głównej Gospodarstwa Wiejskiego in Warsaw"—SGGW campus sites, coupled with data obtained from the laboratory for 56 samples with an undisturbed structure. The paper addresses the applicability of the Bayesian approach in geotechnics via a simple example related to the determination of characteristic values of geotechnical parameters for design structures. The results show that it is possible to conduct a more reliable forecast with improved statistical measures compared to other available methods for multilayer subsoils.

2. Introduction

According to the compulsory construction standards in Poland, each project that applies for a building permit should, depending on the needs, contain geological and engineering data. This documentation should consist of developed field and laboratory test results [1]. The advantage of field tests is that they take place in natural conditions, whose reproduction in the laboratory is often difficult. An example of such research is a borehole drilled to determine the soil state. Among many available methods, cone penetration tests (CPT) and dilatometer (DMT) tests are used by major research centers. In order to reduce the need of using various types of equipment, field research methods that allow for the interpretation of results obtained in a wide range based on single research techniques are being sought [2,3]. One of the field studies that meets this requirement is the Marchetti dilatometer [4], which is commonly used in Poland.

The largest advantage of studying dilatometer data are its fast and low-complexity measurements, based on which the profiles of many soil parameters may be determined. The interpretation of soil parameters is based on the use of empirical relationships correlating the measurement results with values of soil parameters. This paper is based on such a type of field measurements. The data was made available from the Department of Geoengineering of the Warsaw University of Life Sciences. Based on the DMT test results carried out at the Antoniny, Kosice, Nielisz, Stegny, and SGGW campus sites, new dependencies have been determined to calculate parameters describing soil properties. Finally, a new formula for the soil unit weight calculation of mineral and organic soils from DMT tests is offered.

3. Background

Deduction in classical mathematical statistics is based on random samples drawn from a given population. In an alternative approach, derived from the Bayes' theorem [5–9], deduction can be based not only on a random sample, but also on a priori information. The *a posteriori* information is generated from these two sets of data. A priori knowledge may refer to expert knowledge, deriving from earlier investigations whose results are known, but not for the entire dataset. The character of knowledge does not allow us to take advantage of it in the classical approach. Therefore, the Bayesian approach should be used. If the same data can be used in both cases, classical and Bayesian analysis would give similar conclusions. In its most basic form, the Bayes theorem presents the dependence of the conditional probability of event A on condition B and the conditional probabilities of event B on condition A, provided that event A is completed and the probability of event A and its complement, which can be expressed in the following form: $\mathrm{P}(A \mid B) = \frac{\mathrm{P}(B|A)\cdot\mathrm{P}(A)}{\mathrm{P}(B|A)\mathrm{P}(A)+\mathrm{P}(B|\overline{A})\mathrm{P}(\overline{A})}$. The formula above is generalized to a situation in which the occurrence of many mutually exclusive events is considered, and not as in the given relationship between only event A and its complement: $\mathrm{P}(\theta \mid x) = \frac{\mathrm{P}(x|\theta)\cdot\mathrm{P}(\theta)}{\sum_i \mathrm{P}(x|\theta_i)\mathrm{P}(\theta_i)}$. Here, $f(\theta)$ means a function of the *a priori* probability density of parameter θ, while $f(x|\theta)$ is a function of reliability, i.e., a function of the density of the conditional observation result at a given value of θ. The symbol Ω used under the integral indicates a set of possible values of the estimated parameter θ. To the left of the formula is the density function of the *a posteriori* probability of the parameter θ, after observing the result of the sample x. Thus, based on Bayes' theorem, the function of the a priori probability density of parameter θ is updated, using information from the sample. The population parameters to be estimated, e.g., parameter θ (for example mean, standard deviation, specified type element fraction) are treated differently in the two cases. In the classical approach, parameter values are specific but remain unknown. In the Bayesian analysis, they are treated as random variables.

For random variables with a continuous probability distribution, Bayes' theorem may be presented as follows:

$$\mathrm{f}(\theta \mid x) = \frac{f(\theta \mid x)\cdot f(\theta)}{\int_{\Omega} f(\theta \mid x)\cdot f(\theta) d\theta} \tag{1}$$

$\mathrm{f}(\theta|x)$—*a posteriori* density function of parameter θ, after sample result x has been observed
$\mathrm{f}(\theta)$—*a priori* distribution density function of parameter θ;
Ω—set of possible values of parameter θ.

Therefore, based on Bayes' theorem, *the a priori* density function of parameter θ is updated with the use of information from the sample.

In the present paper, Bayesian analysis has been adopted for a case, in which the unknown distribution of parameter θ is to be estimated, and θ is the mean in a normal population. The population standard deviation σ_0 is known. It is derived from a priori knowledge that the mean θ is a normal random variable with parameters m_1 and σ_1. If, in turn, the average of an n-element sample drawn from the population is equal to m_2, the *a posteriori* distribution of the random variable θ is also normal, with the mean m and standard deviation σ, calculated as follows:

$$m = \frac{\left(1/\sigma_1^2\right) \times m_1 + \left(n/\sigma_0^2\right) \times m_2}{\left(1/\sigma_1^2\right) + \left(n/\sigma_0^2\right)} \tag{2}$$

$$\sigma = \frac{1}{\left(1/\sigma_1^2\right) + \left(n/\sigma_0^2\right)} \tag{3}$$

The presented Bayes' theorem gives a valuable practical possibility of the successive inclusion of new information coming from consecutively drawn random samples. During a consecutive step, knowledge about the posterior distribution of parameter θ is treated as a priori knowledge of this parameter. Including new portions of information does not affect the final result.

When the probability distribution of the examined parameter is estimated, a credible set of the parameter can be constructed. The credible set (the highest posterior density set) is an interval, to which the value of the parameter drops with a defined probability. This is the analog of a confidence interval concept that is used in classical statistical analysis. The confidence interval predicts the unknown value of the parameter with an assumed probability.

The selection of parameters requires careful and cautious use of statistical analysis, including the use of the Bayesian approach. In Bayesian statistical analysis, results of research from the soil of the analyzed sites were subsequently included. In the calculations, it was stated that the final result is independent of the order of the objects. In addition, the mean values of the unit weight of organic soils, Pliocene clays, and boulder clays are the same in the case of classical analysis and the Bayesian approach.

Bayesian analysis (i.e., Equations (1)–(3)) is performed in order to provide an a posteriori probability distribution function for model parameters, the most likely thickness of layers of the soil, and its unit weight. Here, the asymptotic technique is used to estimate the posterior model parameters. The asymptotic technique includes an approximation of the *a posteriori* probability function as a Gaussian probability distribution function (PDF), and the *a posteriori* probability distribution function for model parameters is a combined Gaussian probability distribution function with a mean value equal to the most probable posterior probability distribution function [10]. The asymptotic technique was successfully used in previous studies to interpret the undrained shear strength of mineral and organic soils from DMT tests.

In the Bayesian approach, some preliminary knowledge about the distribution of parameter values is modified after confronting the data. Using the *a priori* distribution and knowledge about the sample taken, a new parameter distribution is determined, which takes into account both the original *a priori* beliefs and the empirical data obtained. An important characteristic of the Bayesian approach is that the sequential modification of knowledge about the distribution of the tested parameter gives the same result as in the case, when all doses of information are included in the conclusion at once—that is, if successively taken samples are treated as one larger sample. It also implies that the order of attaching new portions of information is arbitrary. Therefore, a question should be answered: When is the Bayesian approach worth applying in practice—that is, when will the classical approach not give better results? The classical approach will not give better results when the a priori information is only in the form of results of the analyses, but the tests on the basis of which these analyses were made are no longer available (therefore, it is impossible to extend the data sample based on which the inference is made in the classic way).

The application of statistical analysis of the test results in geotechnical design by the Bayesian approach allows for taking into account the influence of the distance of the test site from the designed object.

Previous statistical methods mostly using the least squares estimation, used data from field and laboratory studies as a closed number of random variables to determine the expected values, variances, and correlations for a given expression as a function of the random variable. The main disadvantage of the least squares method used to estimate the parameters is the difficulty in the representation of knowledge about the expected values of parameters, which should result from the estimation process. There has been significant progress in parameter estimation, as shown by increasingly numerous examples of applications in various fields, such as Bayesian analysis, in which a set of test results may be increased by new data—and on this basis, the probability of occurrence may be determined.

Bayesian approaches were used in the analysis presented in this paper. A total of 56 samples with an undisturbed structure were used for determining the unit weight test results, and the corresponding DMT measurements were taken into account. In laboratory tests, soil samples taken with thin "Shelby"-type samplers with dimensions of Ø 88.9 mm were used. In the laboratory, specimen samples for oedometer and triaxial test measurements with dimensions of 50 mm × 100 mm were cut out of soils with a non-disturbed structure taken with the SHELBY cylinder. Combining the measurement of soil unit weight with the structure of the sample—85 mm × 15 mm, h = 50 mm—the unit weight of the natural soil was calculated. Results of the DMT test, which allowed to determine the p_0 and p_1 values, were compared with results obtained from laboratory tests of the selected samples, obtaining a mean square relative deviation (MRSD) below 10%. Based on the collected results both from laboratory and dilatometer tests (DMT) examinations, a data set was developed (a regional database and a general database) in the form of a table.

4. Methodology and Interpretation of Dilatometer Test (DMT) Results

More than 30 years ago, Dr. Silvano Marchetti designed and constructed the first dilatometer at the L'Aquila University in Italy; this device and principles of soil investigation were presented in 1975 at the conference of the American Society of Civil Engineers (ASCE) in Raleigh [11]. Dilatometer investigations consist of measurements of gas pressure acting on the membrane of the dilatometer blade at selected depths. In soil tests, two pressure measurements (*A* and *B*) are usually carried out, which force the membrane center to move by 0.05 mm to the ground (*A* reading) and the diaphragm center to the ground by approximately 1.05 mm (*B* reading). In order to extend the dilatometer tests, pressure measurements are sometimes carried out when the membrane returns to the ground contact position (*C* reading). The values of readings *A*, *B*, and *C* are corrected due to the inertia of the diaphragm and marked as p_o, p_1, and p_2, respectively. The value of the vertical component of effective vertical stress σ'_{vo} are used to determine the following dilatometer indexes: material index I_D, lateral stress index K_D, and dilatometer modulus E_D [12–18]. The seismic dilatometer (SDMT) is a combination of a standard dilatometer presented by Marchetti (1980) and a module measuring the velocity of the seismic propagation V_S. Supplementing the equipment used to perform dilatometer tests with two geophones in the SDMT seismic dilatometer extended the possibilities of interpretation of dilatometer tests. It was first introduced by Hepton and then refined at Georgia Institute of Technology in Atlanta, USA. A new system with a seismic dilatometer has been recently developed in Italy [19–24].

Material index I_D:

$$I_D = \mathrm{f}(A,\ B,\ u_0) = \frac{p_1 - p_0}{p_0 - u_0} \tag{4}$$

Horizontal stress index K_D:

$$K_D = \mathrm{f}(A,\ u_0,\ \sigma'_{vo},\ B) = \frac{p_0 - u_0}{\sigma'_{vo}} \tag{5}$$

Dilatometer modulus E_D:

$$E_D = \mathrm{f}(A,\ B) = 34.7 \times (p_1 - p_0) \tag{6}$$

Pore pressure index U_D:

$$U_D = \mathrm{f}(A,\ C,\ u_0,\ B) = \frac{p_2 - u_0}{p_0 - u_0} \tag{7}$$

5. Previous Methods of Determining Soil Unit Weight

Soil total unit weight (γ_t) is classified as the basic physical characteristics of both mineral and organic soils. This feature is most often determined in laboratory tests. Recently, many formulas appeared in the literature to determine the soil total unit weight on the basis of in situ tests, e.g., cone penetration tests (CPTU) and DMT investigations. The article describes the dependencies between (γ_t) and parameters determined from dilatometer tests (DMT). Although so far no database for (γ_t) has been presented, Marchetti and Crapps [25] have suggested that it should be estimated according to the I_D and E_D values (Figure 1).

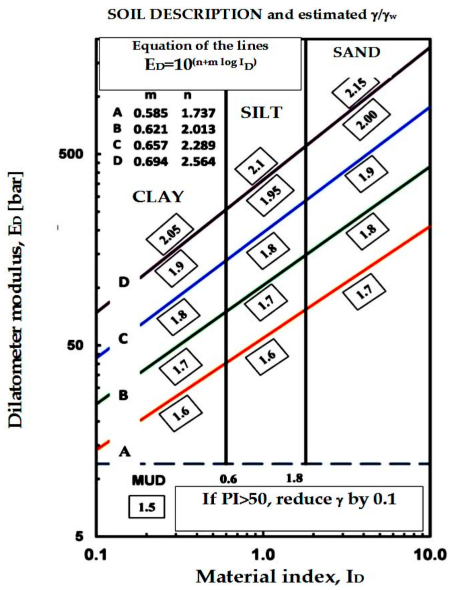

Figure 1. Unit weight estimate from dilatometer (DMT) indexes E_D and I_D [25]. Reproduced with permission from [25], Marchetti, S., 2019.

Similarly, a relationship was developed for the results of DMT tests to assess the soil total unit weight. These dependencies are presented in Equation (8) [26,27]:

$$\gamma_t \approx 1.12 \times \gamma_w \times \left(\frac{E_D}{\sigma_{atm}}\right)^{0.1} \times I_D^{-0.05} \tag{8}$$

where: $\rho = (\gamma_T/(\gamma_W)$, and σ_{atm} =100 kPa is atmospheric pressure.

So far, attempts have been made to estimate the soil unit weight based on DMT tests for selected types of mineral soils [28,29] or to develop new dependencies between the total unit weight and DMT indexes [30]. These studies focused mainly on natural clays with a soft or hard consistency. These soils are homogeneous, which is often associated with high groundwater level.

$$\frac{\gamma}{\gamma_w} = 1.31\left(\frac{p_1}{p_a}\right)^{0.164} \tag{9}$$

Ozer et al. [30,31] proposed a correlation to estimate the total unit weight in terms of p_1 pressure from DMT. This model provides a fairly accurate prediction of laboratory results for soft to medium compact clays from the "Lake Bonneville" valley. The formula is as follows:

$$\frac{\gamma}{\gamma_w} = 1.31\left(\frac{p_1}{p_a}\right)^{0.164} \tag{10}$$

Ouyang and Mayne [32] found that there is a relationship between soil total unit weight (γ_t), contact pressure (p_0), and depth (z) for clays in the range from normally consolidated (NC) to lightly preconsolidated (LOC), with soft to hard consistency. The newly defined slope parameter (m_{p0} = $\Delta p_0/\Delta z$) with a forced intersection equal to 0 has been set for homogeneous inorganic clays. It was found to be related with soil unit weight, as expressed by the following formulas in Equations (11) and (12):

$$\gamma_t\left(kN/m^3\right) = \gamma_w + 0.22 \times m_{p0} \tag{11}$$

$$m_{p0}(-) = \frac{\Delta_{p0}}{\Delta z} \tag{12}$$

where:

γ_w—water unit weight
Δ_{po} and Δz—slope parameters (p_o is the pressure and z is the depth)

The cited authors have stated that in the case of inorganic clays, these formulas (Equations (11) and (12)) do not give adequate results for OL (Organic Low liquid limit) or OH (Organic High liquid limit) soils, or soils with a significant organic content. The values are based on data obtained mainly from inorganic and non-sensitive clays.

6. Geotechnical Conditions of the Test Sites

This paper includes the test results of mineral and organic subsoils obtained from the Antoniny, Koszyce, and Mielimąka sites located in the Noteć river valley in the Wielkopolska province, the Nielisz site located in the Wieprz river valley in the Lublin province, and the SGGW Campus and Stegny sites located in Warsaw, where the Department of Geotechnical Engineering SGGW is located, and where a laboratory and field testing program has been carried out under and outside of the main dam embankment [33,34] (Figure 2). The grain-size distribution curve obtained from laboratory tests for mineral soils from these sites is presented in Table 1 (Figure 3). All the test sites are located in Poland.

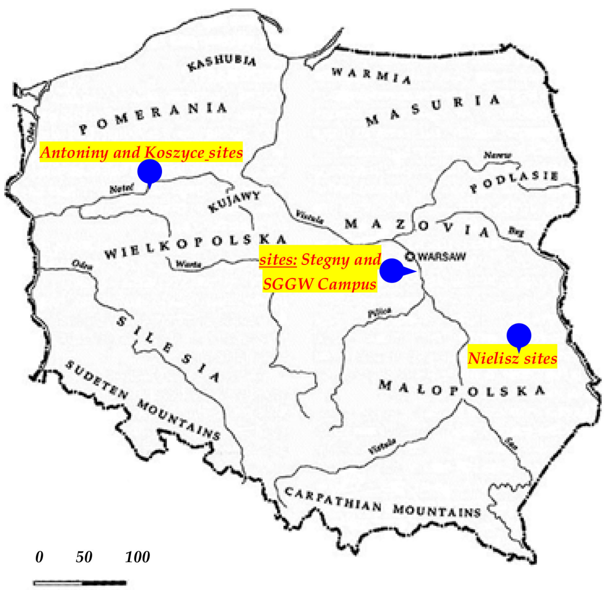

Figure 2. Location of test sites in Poland (all the test sites are located in Poland).

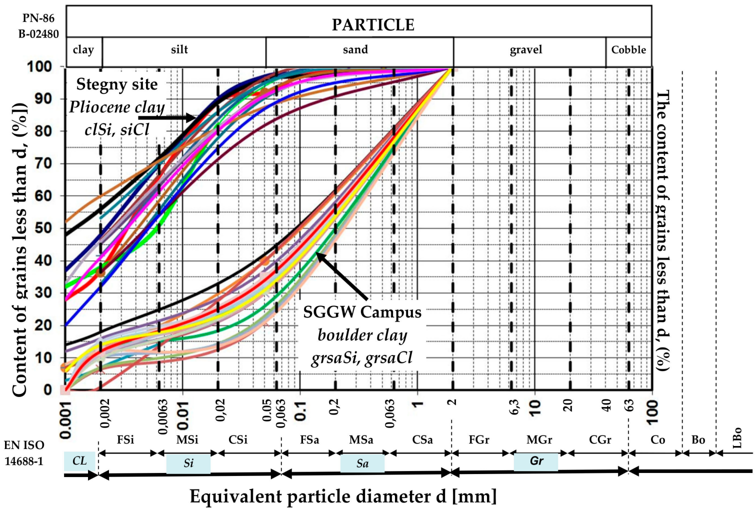

Figure 3. Grain size distribution of clay from the tested sites.

This paper presents the test results of mineral and organic subsoils obtained from the following sites: Antoniny, Koszyce, Nielisz, Stegny and Warsaw University of Life Sciences (WULS)-SGGW Campus. The Antoniny test embankment was designed and performed in the frame of cooperation between the Department of Geotechnical Engineering SGGW and the *Swedish Geotechnical Institute* (SGI). The physical properties of soil from the Antoniny site were determined during earlier WULS-SGGW tests [33–36]. The Koszyce test dam is located in the Ruda river valley. In the central part of the dam subsoil, a layer of soft organic soils was discovered. The organic soils are Quaternary deposits of an oxbow lake [33,34]. The Nielisz site is located in the Wieprz river valley in the Lublin province, the SGGW Campus site is located within the Department of Geotechnical Engineering SGGW, and the Stegny site is located in Warsaw, where a laboratory and field testing program was performed under and outside the main dam embankment [34,35]. The area of the SGGW campus is located in the southern part of Warsaw in the Ursynów commune. It is limited by: the Nowoursynowska street 166 from the north-east, Ciszewskiego street from the south-west, Rosoła street from the south-east, and the area of forts, behind which the Służewiecka valley runs, from the north-east. At a distance of about 700 m from the plot, toward the north-east, runs the edge of the Vistula escarpment (nature protection area). At a distance of about 700 m to the south-west runs Warsaw Metro I line [37]. The "Stegna" experimental field was founded by an academic center, following two research projects of the Scientific Research Committee implemented by the Department of Engineering Geology of the University of Warsaw and the Department of Geoengineering SGGW at the Warsaw University of Life Sciences. It is located in the southern part of the city of Warsaw, in the Mokotów district, at the Stegny housing estate. The soils that are subject to the presented research are Mio-Pliocene clays, belonging to the Poznań Formation. The location of all analyzed objects is shown in Figure 2. The index properties of mineral and organic soils and the grain size distribution curve obtained from laboratory tests for mineral soils from the described sites are presented in Table 1 and Figure 3.

Table 2 was compiled based on the results obtained from the analysis of well profiles and dilatometer tests. It contains data used to compare the unit weight of mineral and organic soils obtained from laboratory tests based on DMT results from the Antoniny, Koszyce, Nielisz, Stegny and WULS-SGGW campus sites. Data of dilatometer test readings (DMT), effective vertical stresses σ_{vo}, water pressure u_o, and soil unit weight (γ) from laboratory tests were collected for each study site. Individual soil fractions (f_{clay}, f_{silt} and f_{sand}) and corresponding pressures from the dilatometer, such as pressures (p_0, p_1) and dilatometer indexes (I_D, K_D and E_D), were also collected. The preconsolidation ratio (*OCR*) of these soils generally ranged from 1 to 3 across most of the subsoil profile. Groundwater levels are generally high in organic soils (organic muds, peat, and gyttja), usually at the depth of 0.2 to 2 or 3 m.

Table 1. Index properties of mineral and organic soils in the Antoniny, Koszyce, Nielisz, Stegny and Warsaw University of Life Sciences (WULS-SGGW) campus test sites.

Sites	Antoniny		Koszyce			Nielisz		Stegny	WULS-SGGW Campus
Type of Soil	Amorphous Peat	Calcareous Gyttja	Amorphous Peat	Calcareous Gyttja (G_y)	Calcareous Gyttja (G_y)	Organic Mud (M_{or})	Organic Mud (M_{or})	Pliocene Clays	Boulder Clay
Organic content I_{om} (%)	65–75	5–20	70–85	10–20	15–20	20–30	10–20	-	-
$CaCO_3$ content (%)	10–15	65–90	5–15	65–80	65–75	-	-	-	-
Water content w_n (%)	310–340	105–140	400–550	120–160	180–220	120–150	105–120	19.20–28.50	5.20–20.10
Liquid limit w_L (%)	305–450	80–110	450	80–110	100–110	130–150	110–130	67.6–88.0	21.9–26.6
Density unit soil weight ρ (t/m^3)	1.05–1.10	1.25–1.40	1.05–1.1	1.20–1.35	1.25–1.30	1.25–1.30	1.30–1.45	2.1–2.2	2.0–2.2
Density specific soil weight ρ_s (t/m^3)	1.45–1.50	2.2–2.30	1.45–1.50	2.1–2.25	2.2	2.25–2.3	2.30–2.40	2.68–2.73	2.68–2.73

Table 2. List of DMT and borehole number (OW) test results.

Warsaw Sites	Depth [m]	Profile Number	Borehole Number	Soil Type	Symbol	*From Laboratory* γ (kN/m^3)	Clay Particle (%)	Silt Particle (%)	Sand Particle (%)	p_0 (kPa)	p_1 (kPa)	I_D (-)	K_D (-)	E_D (MPa)	*From DMT* γ (kN/m^3) *(Marchetti 1980)*	σ_{v0} (MPa)	u_0 (MPa)	w_n	W_L	W_P	I_P	I_L	I_c	OCR
Stegny	6.0	DMT1	OW-1	silt	Si	21.1	48	49	3	509	887.5	0.787	4.70	13.13	14.5	0.1023	0.0280	26.03	78.38	25.89	5249	0.003	0.997	4
	9.0	DMT1	OW-1	silt	Si	21.1	37	55	8	846	1597.5	0.95	5.4	26.1	14.5	0.1305	0.000	28.53	88.11	31.16	56.95	−0.03	1.03	5
	12.0	DMT1	OW-1	silty clay	siCl	22.2	38	56	6	1101	1847.5	0.74	5.2	25.9	16.5	0.1980	0.000	19.84	67.60	25.51	42.09	−0.135	1.135	4
	15.0	DMT1	OW-1	silty clay	siCl	22.2	56	40	4	1223	2147.5	0.837	4.61	32.08	22	0.2398	0.118	21.37	87.41	24.97	62.44	−0.058	1.058	4
	9.0	DMT2	OW-1	silt	Si	21.1	37	55	8	846	1597.5	0.95	5.4	26.1	14.5	0.1305	0.058	28.53	88.11	31.16	56.95	−0.03	1.03	5
	12.0	DMT2	OW-1	silty clay	siCl	22.2	38	56	6	1101	1847.5	0.74	5.2	25.9	16.5	0.1980	0.088	19.84	67.60	25.51	42.09	−0.135	1.135	4
	9.0	DMT4	OW-1	silty clay	siCl	21.1	37	55	8	672	1210	0.80	4.6	18.7	14.5	0.1305	0.058	28.53	88.11	31.16	56.95	−0.03	1.03	4
	9.0	DMT5	OW-1	silty clay	siCl	21.1	37	55	8	658	1024	0.56	4.5	12.7	14.5	0.1305	0.058	28.53	88.11	31.16	56.95	−0.03	1.03	4
	9.0	DMT6	OW-1	silty clay	siCl	21.1	37	55	8	586	976	0.66	4.0	13.5	14.5	0.1305	0.058	28.53	88.11	31.16	56.95	−0.03	1.03	3
	9.0	DMT7	OW-1	silty clay	siCl	21.1	37	55	8	578	1019.5	0.76	4.0	15.3	14.5	0.1305	0.058	28.53	88.11	31.16	56.95	−0.03	1.03	3
	9.0	DMT8	OW-1	silty clay	siCl	21.1	37	55	8	859	1158	0.37	5.6	10.4	14.5	0.1305	0.046	28.53	88.11	31.16	56.95	−0.03	1.03	5
	12.0	DMT8	OW-1	silty clay	siCl	22.2	38	56	6	951	1408	0.52	4.5	15.9	16.5	0.1980	0.078	19.84	67.60	25.51	42.09	−0.135	1.135	4
	9.0	DMT9	OW-1	clay	Cl	21.1	37	55	8	666	1058	0.63	4.2	13.6	14.5	0.1305	0.048	28.53	88.11	31.16	56.95	−0.03	1.03	3
	12.0	DMT9	OW-1	silty sand	siSa	22.2	38	56	6	435	1268	2.33	1.9	28.9	16.5	0.1980	0.078	19.84	67.60	25.51	42.09	−0.135	1.135	1
	9.0	DMT10	OW-1	silty clay	siCl	21.1	37	55	8	691	1085	0.61	4.4	13.7	14.5	0.1305	0.048	28.53	88.11	31.16	56.95	−0.03	1.03	3
	12.0	DMT10	OW-1	silty clay	siCl	22.2	38	56	6	889	1325	0.54	4.2	15.1	16.5	0.1980	0.078	19.84	67.60	25.51	42.09	−0.135	1.135	3
Aula/WULS-SGGW	2.0	DMT2	OW-2	medium sand	MSa	17.5	1	4	95	390	1970	4.055	9.94	54.83	19.1	0.0392	0.0	No data available	No data available	No data available	No data available	No data available	No data available	12
	2.5	DMT2	OW-2	clayey sand	clSa	20	14	17	69	351	445	0.270	8.07	3.28	21.1	0.0434	0.0	13.3	24.32	11.78	12.54	0.120	0.88	9
	6.0	DMT2	OW-2	clayey sand	clSa	20	14	19	67	1279	2145	0.677	11.84	30.06	20.6	0.1079	0.0	13.3	24.32	11.78	12.54	0.120	0.88	6
	2.0	DMT3	OW-3	medium sand	MSa	17.5	0	1	99	390	1970	4.055	9.94	54.83	19.6	0.0392	0.0	No data available	No data available	No data available	No data available	No data available	No data available	8
	6.0	DMT3	OW-3	clayey sand	clSa	20	13	19	68	1301	2445	0.880	11.80	39.71	20.6	0.1102	0.0	8.9	22.90	11.76	11.14	−0.258	1.258	6
	3.0	DMT4	OW-11	medium sand/ slightly clayey sand	MSa/clSa	20	2	5	93	507	1840	2.63	9.6	46.3	16.7	0.0501	0.0	No data available	No data available	No data available	No data available	No data available	No data available	6

Table 2. *Cont.*

Warsaw Sites	Depth [m]	Profile Number	Borehole Number	Soil Type	Symbol	*From Laboratory* γ (kN/m^3)	Clay Particle (%)	Silt Particle (%)	Sand Particle (%)	p_0 (kPa)	p_1 (kPa)	I_D (-)	K_D (-)	E_D (MPa)	*From DMT* γ (kN/m^3) *(Marchetti 1980)*	σ_{v0} (MPa)	u_0 (MPa)	w_n	W_L	W_P	I_P	I_L	I_c	OCR
Aula/WULS-SGGW	4.5	DMT4	OW-11	medium sand/ clayey sand	MSa/clSa	20	10	21	69	901	2255	1.504	10.9	47.00	20.6	0.0819	0.012	17.5	23.45	12.22	11.23	0.47	0.53	8
	2.0	DMT5	OW-8	fine sand	FSa	17.5	0	0	100	390	1970	4.055	9.94	54.83	19.6	0.0392	0.0	No data available	No data available	No data available	No data available	No data available	No data available	6
	3.6	DMT5	OW-8	medium sand	MSa	17.5	0	0	100	227	920	3.05	3.4	24.0	18.6	0.0670	0.0	No data available	No data available	No data available	No data available	No data available	No data available	5
	4.5	DMT5	OW-8	clayey sand	clSa	20	13	20	67	702	2140	2.05	8.3	49.9	18.6	0.0837	0.0	9.1	23.7	10.67	13.03	–0.120	1.12	9
	9.0	DMT5	OW-8	clayey sand	clSa	20	14	25	61	1390	2058	0.480	8.06	23.17	20.6	0.1726	0.0	10.0	26.55	11.78	14.77	–0.118	1.118	9
Parking/ WULS-SGGW	1.5	DMT1	OW-9	fine sand	FSa	17.5	0	1	99	390	1970	4.055	9.94	54.83	20.0	0.0280	0.0	No data available	No data available	No data available	No data available	No data available	No data available	6
	3.0	DMT1	OW-9	medium sand	MSa	17.5	0	1	99	458	2080	3.54	7.6	56.3	20.0	0.0600	0.0	No data available	No data available	No data available	No data available	No data available	No data available	8
	4.5	DMT1	OW-9	slightly clayey sand / clayey sand	clSa	20	10	21	69	264	9200	1.649	3.0	22.8	20.0	0.0880	0.0	15.2	18.3	10.7	7.6	0.59	0.41	2
	7.0	DMT1	OW-9	clayey sand	clSa	20	11	22	67	1274	2245	0.763	9.92	33.70	20.0	0.1285	0.0	10.4	20.0	10.6	9.4	–0.02	1.02	6
	1.5	DMT2	OW-20	clayey sand	clSa	20	10	17	73	131	645	1.694	4.1	17.9	20.0	0.0320	0.0	No data available	No data available	No data available	No data available	No data available	No data available	3
	1.8	DMT2	OW-20	slightly clayey sand	clSa	20	5	11	84	148	1000	1.534	4.1	27.3	20.0	0.0360	0.0	No data available	No data available	No data available	No data available	No data available	No data available	3
	5.0	DMT2	OW-20	slightly clayey sand	clSa	20	5	26	69	417	1005	1.41	4.2	20.4	20.0	0.1000	0.0	No data available	No data available	No data available	No data available	No data available	No data available	3
	7.0	DMT2	OW-20	clayey sand	clSa	20	15	18	67	1415	2465	0.742	10.8	36.44	20.0	0.1308	0.0	10.3	20.6	10.2	10.4	0.01	0.99	8
underground passage/ WULS-SGGW	5.0	DMT1	OW-4	clayey sand	clSa	20	11	26	63	1166	2338	1.005	12.8	40.66	20.6	0.091	0.0	11.8	19.9	10.3	9.6	0.160	0.84	8
	1.5	DMT2	OW-5	slightly clayey sand	clSa	20	4	18	78	611	905	0.481	17.3	10.20	18.6	0.0354	0.0	No data available	No data available	No data available	No data available	No data available	No data available	9
Sport stadium/ WULS-SGGW	1.8	DMT1	OW-12	sandy silt	saSi	20	7	35	58	343	600	0.751	11.0	8.93	17.7	0.0311	0.0	No data available	No data available	No data available	No data available	No data available	No data available	6
	1.5	DMT2	OW-5	fine sand	FSa	17.5	1	4	95	292	770	1.63	11.8	16.6	17.7	0.0248	0.0	No data available	No data available	No data available	No data available	No data available	No data available	6
	3.5	DMT2	OW-5	clayey sand	clSa	20	13	19	68	351	445	0.720	8.07	3.28	21.1	0.0717	0.0	No data available	No data available	No data available	No data available	No data available	No data available	9

The scope of soil laboratory tests (physical properties of samples) in the analyzed sites included: grain-size analysis (sieve and areometric methods) and determination of soil unit weight (soil density for undisturbed samples, unit weight of soil skeleton, and soil skeleton specific density). The results of the laboratory tests of soil physical properties are presented in Table 2. The laboratory tests were carried out in accordance with the PN-88/B-04481 standard for building soils. The soil samples, types, and geotechnical conditions were determined in accordance with the PN-86/B-02480 [38] standard for building soils. The terms, symbols, subdivision, and description of the soils were made in accordance with ASTM D 2487-93 [39].

7. New Ways of Determining the Unit Weight of Mineral and Organic Soils Using the Marchetti Dilatometer (DMT)

The next task was to search for a direct correlation between the results of laboratory tests of soil unit weight and DMT test results for mineral (sand, silt, and clay) and organic soils (peat, organic mud, and gyttja) in the range of stresses from normally consolidated (NC) to heavy preconsolidated (HOC). The basic impulse of searching for a new form of dependence to determine soil unit weight based on DMT readings was to extend the range of soil types, including organic soils. Therefore, a comprehensive series of multiple regression analyses was performed, using both arithmetic and logarithmic scaling. A full set of regression attempts was not included here, because they were too large for the discussion. Analysis of dilatometer pressure readings showed that parameters p_0 (kPa), p_1 (kPa), and u_0 (kPa), and atmospheric pressure σ_{atm} = 100 kPa are sufficient to obtain a reasonable estimate of γ; thus, they do not need to be based on DMT indexes such as I_D, K_D, or E_D, without loss of statistical significance. The obtained statistical parameters are (n = 1021, R^2 = 0.69, S.E.Y. (Standard Error of the dependent variable) = 0.1011).

Analysis of the data presented in Figure 3, Table 2 showed that for soils such as peat, gyttja, silty clay, and Pliocene clay, some soil unit weight characteristics of the tested soils may differ from linear equations. On the other hand, with virgin subsoil, soil unit weight values did not differ between sites, and can be predicted by a generalized non-linear equation. Based on the analysis of the soil unit weight characteristics presented in Figure 3 and Table 2, to describe non-linear changes in coefficients (k_i), i = 1, 2, and 3 (Table 3) for a suitable soil as a function of dilatometer pressure p_0, p_1, water unit weight (γ_w), pore water pressure (u_0), and atmospheric pressure σ_{atm} = 100 kPa, a non-linear model was adopted, i.e., a power equation (Equation (13)).

Linking dilatometer pressures p_0 (kPa) and p_1 (kPa), and the calculated values of pore water pressure u_0 (kPa) and atmospheric pressure σ_{atm} = 100 (kPa), as well as multiple regression analysis combined with the results of unit weight γ correlate well with data obtained from laboratory tests based on the following formula (Table 2):

$$\frac{\gamma}{\gamma_w} = k_1 \times log\left[64 \times \left[\frac{p_0 - u_0}{p_1}\right]\right] + k_2 \times log\left(\frac{p_1}{\sigma_{atm}}\right) + k_3 \quad (13)$$

where: p_0, p_1, and γ_w are expressed in kN/m^3, u_0 is expressed in kPa, and σ_{atm} = 100 kPa.

Table 3. Values of empirical coefficients k_1, k_2, and k_3 for Equation (13).

Soil Types	Material Index Interval Values I_D (-)	Coefficients (k_i), I = 1, 2, and 3			
		k_1 (-)	k_2 (-)	K_3 (-)	σ_{atm} (kPa)
Peat	I_D < 0.3	0.231	0.25	0.75	100
Gyttja (Gy)	0.3< I_D < 0.6	0.231	0.25	0.75	100
Organic mud, mud	0.3 < I_D < 0.6	0.231	0.35	0.96	100
Clayey sand (Sicl)	0.6 < I_D <1.8	0.576	−0.23	1.45	100
Boulder clay (Cl)	0.6 < I_D <1.8	0.576	−0.23	1.45	100
Sand (Sa, CSa, MSa, FSa)	I_D > 1.8	0.576	−0.23	1.40	100

8. Bayes' Approach in the Interpretation of Test Results from the Proposed Formula

Statistical analysis has been applied on the measurement results obtained from the DMT field tests [40,41]. Additionally, p_0 and p_1 pressures were measured. For each of the indicators, there were 338 measurement results (in 30 screenings) in the peat, gyttja, and organic mud layers, and 683 results (in 65 tests) in the sand, clayey silt, and Pliocene clay layers. The measurement results can be treated as observations of continuous random variables with specific probability distributions. For each profile, there were from 8 to 22 DMT test results in the peat, gyttja, and organic mud layers, and from 45 to 65 DMT test results in the sand, clayey silt, and Pliocene clay layers.

The investigator is obliged to check if the profiles of every layer can be examined together—in other words, if the layers have been distinguished correctly. If new measurement results are included to the calculations according to Bayes' law and full data about previously examined samples are not available (thus, the standard statistical approach is not applicable), the only thing that can be done is to test each new sample independently. The type of probability distribution has been checked by the Shapiro–Wilk tests, which are applicable to small samples. For the majority of the tests, the null hypothesis that the samples come from a normally distributed population has not been rejected [5]. No other type of probability distribution has been found. On the other hand, the assumption of the normal distribution of all random variables under investigation is reasonable in accordance with the central limit theorem. Therefore, the formulas (Equations (1)–(3)) for every six (p_0, p_1 and γ) group of tests have been applied. Since the population standard deviation σ_0 is unknown, it has been decided to use its estimator from the samples in the consecutive steps of formula application (Equations (1)–(3)).

The results of calculations are presented in Table 4, Table 5, Table 6 and Figure 4. Credible sets for the mean indicator p_0, calculated for the assumed probability 0.95 in the peat, gyttja, organic mud, clayey silt, Pliocene clay, and sand layers are as follows: (176.04; 179.16 MPa), (561.94; 568.46 MPa), and (299.54; 313.06 MPa), respectively. Additionally, for comparison, estimators of confidence intervals are presented in Tables 4–6. In turn, the mean values of indicator p_1, calculated for the assumed probability 0.95 in the peat, gyttja, organic mud, clayey silt, Pliocene clay, and sand layers are as follows: (212.58; 217.02 MPa), (1479.58; 1498.42 MPa), and (1654.85; 1715.15 MPa), respectively. Additionally, for comparison, estimators of confidence intervals are presented in Tables 4–6. The mean values of indicator γ, calculated for the assumed probability 0.95 in the peat, gyttja, organic mud, clayey silt, Pliocene clay, and sand layers, are as follows: (13.43; 13.47 kN/m^3), (20.43; 20.45 kN/m^3), and (17.91; 17.99 kN/m^3), respectively.

Additionally, for comparison, estimators of confidence intervals are presented in Tables 4–6. Normally, they cannot be calculated because of the lack of full information about previously tested samples. In our case, the calculations are feasible, because we only present one of the possible applications of Bayes' law, and thus full data are available. Visible discrepancies between credible sets and confidence intervals probably derive from the incomplete fulfillment of the assumption of normality and also from the lack of full knowledge of standard deviation for the populations (it has been estimated on the basis of the samples).

Not all the parameters presented in Table 2 have a normal distribution. For this purpose, a series of analyses of the normality test were carried out for p_0 (kPa), p_1 (kPa), and u_0 (kPa). The analysis consists of a two-step calculation [42–47]. The first stage is to check the normality test together for data from the Stegny and SGGW campus sites. The second stage is to make a separate statistical analysis for each site. The results of these analyses are presented in Table 6 and in the correlation matrix drawing (Figures 5 and 6). After the transformation of (stage 1: $p_0 \rightarrow \log(p_0)$, $p_1 \rightarrow \log(p_1)$, $u_0 \rightarrow \log(u_0)$; stage 2: $p_0 \rightarrow p_0^2$, $p_1 \rightarrow p_1^2$, $u_0 \rightarrow u_0^2$; stage 3: $p_0 \rightarrow \sqrt{p_0}$, $p_1 \rightarrow \sqrt{p_1}$) parameters, a normal distribution was obtained only in the case of $\log(p_0)$ and $\sqrt{p_0}$ (Table 6, Figure 5). The correlation for selected variables is presented in Table 7 and Figure 5, and Figure 6 contains the matrix of extended charts for selected variables. The correlation coefficient determined is significant with $p < 0.05$, N = 15.

Table 4. Final results for indicators p_0, p_1, and γ in the peat, gyttja, and organic mud layers.

Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_0 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_1 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator γ (kN/m^3) Based on Bayes' Approach		
Number of Steps (Samples)	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**
30	177.6	4.364	30	214.8	6.192	30	13.45	0.056
credible set for p_0 indicator mean (prob. = 0.95) (176.04; 179.16)			credible set for p_1 indicator mean (prob. = 0.95) (212.58; 217.02)			credible set for γ indicator mean (prob. = 0.95) (13.43; 13.47)		
For comparison: confidence intervals for indicator means with confidence coefficient = 0.95								
(177.28; 177.92)			(214.21; 215.38)			(12.87; 14.03)		

Table 5. Final results for indicators p_0, p_1, and γ in the clayey silt and Pliocene clay layers.

Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_0 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_1 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator γ (kN/m^3) Based on Bayes' Approach		
Number of Steps (Samples)	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**
45	565.2	11.15	45	1489	32.24	45	20.44	0.034
credible set for p_0 indicator mean (prob. = 0.95) (561.94; 568.46)			credible set for p_1 indicator mean (prob. = 0.95) (1479.58; 1498.42)			credible set for γ indicator mean (prob. = 0.95) (20.43; 20.45)		
For comparison: confidence intervals for the indicator means with confidence coefficient = 0.95								
(560.99; 569.41)			(1477.90; 1500.09)			(20.29; 20.59)		

Table 6. Final results for indicators p_0, p_1, and γ in the sand layers.

Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_0 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator p_1 (kPa) Based on Bayes' Approach			Final Estimators of Statistical Parameters Calculated for the Mean of Indicator γ (kN/m^3) Based on Bayes' Approach		
Number of Steps (Samples)	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**	**Number of Steps (Samples)**	**Average**	**Std Deviation**
20	306.3	14.44	20	1685	64.43	20	17.95	0.095
credible set for p_0 indicator mean (prob. = 0.95)			credible set for p_1 indicator mean (prob. = 0.95)			credible set for γ indicator mean (prob. = 0.95)		
(299.54; 313.06)			(1654.85; 1715.15)			(17.91; 17.99)		
For comparison: confidence intervals for indicator means with confidence coefficient = 0.95								
(302.87; 309.72)			(1666.16; 1703.84)			(17.75; 18.15)		

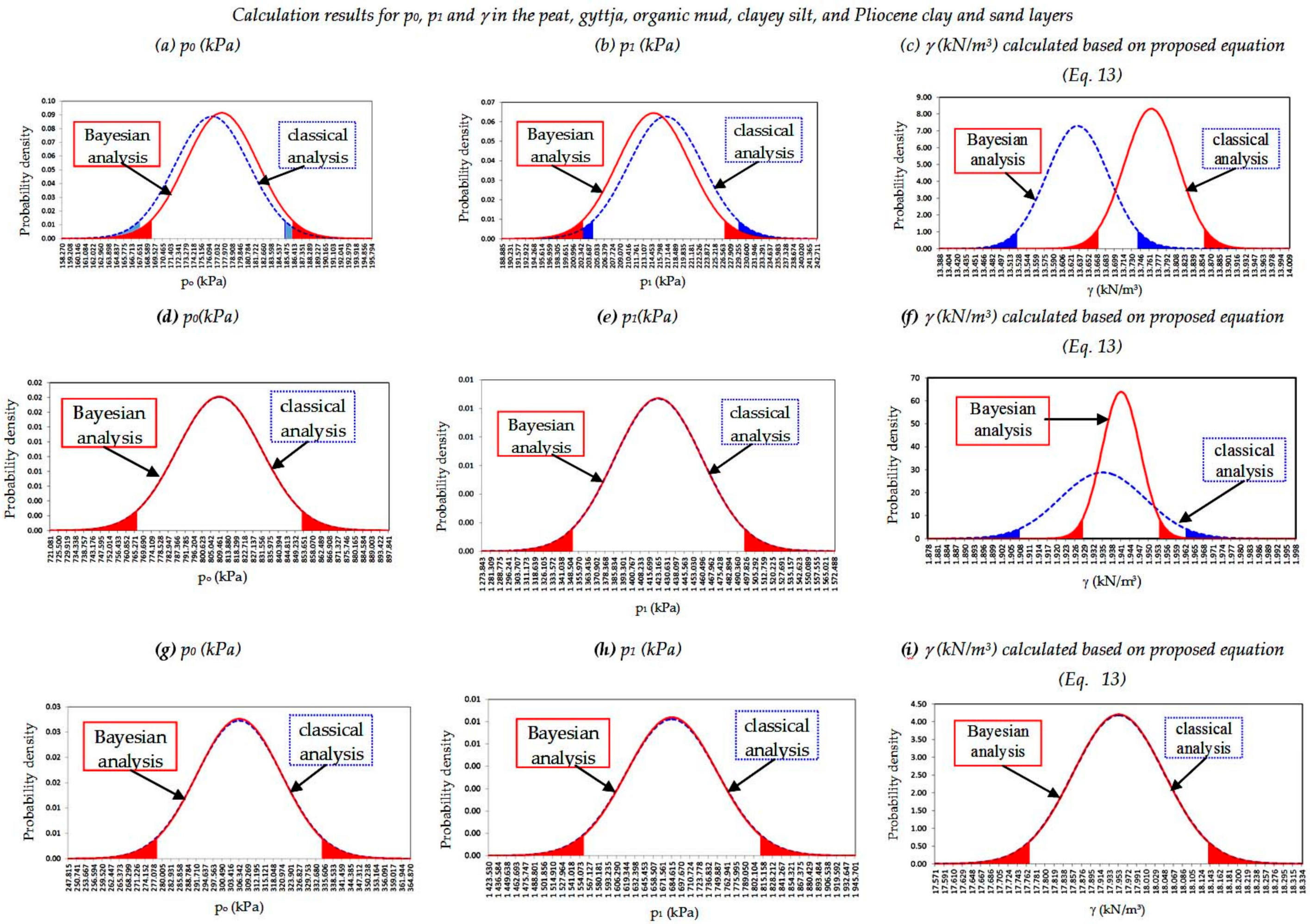

Figure 4. Dependence of calculated results for p_0, p_1, and γ for peat, gyttja (**a**–**c**), organic mud (**d**–**f**), clayey silt, Pliocene clay, and sand (**g**–**i**); layers of probability density performed in the Antoniny, Koszyce, Nielisz Stegny, and SGGW Campus sites.

Table 7. Correlation for selected variables from Table 2.

	p_0 (kPa)	log (p_0) (kPa)	p_0^2 (kPa)	$\sqrt{p_0}$ (kPa)	$p1$ (kPa)	log (p_1) (kPa)	p_1^2 (kPa)	$\sqrt{p_1}$ (kPa)	u_0 (kPa)	log (u_0) (kPa)	u_0^2 (kPa)
p_0 (kPa)	1	**0.99**	**0.99**	**1.00**	**0.78**	**0.80**	**0.76**	**0.79**	0.50	0.27	**0.62**
log (p_0) (kPa)	**0.99**	1	**0.96**	**1.00**	**0.74**	**0.76**	**0.71**	**0.75**	0.41	0.21	**0.52**
p_0^2 (kPa)	**0.99**	**0.96**	1	**0.98**	**0.80**	**0.81**	**0.78**	**0.81**	**0.57**	0.32	**0.70**
$\sqrt{p_0}$ (kPa)	**1.00**	**1.00**	**0.98**	1	**0.77**	**0.78**	**0.74**	**0.78**	0.46	0.24	**0.57**
p1 (kPa)	**0.78**	**0.74**	**0.80**	**0.77**	1	**0.99**	**0.99**	**1.00**	0.28	−0.06	0.47
log (p_1) (kPa)	**0.80**	**0.76**	**0.81**	**0.78**	**0.99**	1	**0.97**	**1.00**	0.33	0.02	0.50
p_1^2 (kPa)	**0.76**	**0.71**	**0.78**	**0.74**	**0.99**	**0.97**	1	**0.98**	0.22	−0.12	0.44
$\sqrt{p_1}$ (kPa)	**0.79**	**0.75**	**0.81**	**0.78**	**1.00**	**1.00**	**0.98**	1	0.30	−0.02	0.49
u_0 (kPa)	0.50	0.41	0.57	0.46	0.28	0.33	0.22	0.30	1	**0.93**	**0.96**
log (u_0) (kPa)	0.27	0.21	0.32	0.24	−0.06	0.02	−0.12	−0.02	**0.93**	1	**0.79**
u_0^2 (kPa)	**0.62**	**0.52**	**0.70**	**0.57**	0.47	0.50	0.44	0.49	**0.96**	**0.79**	1

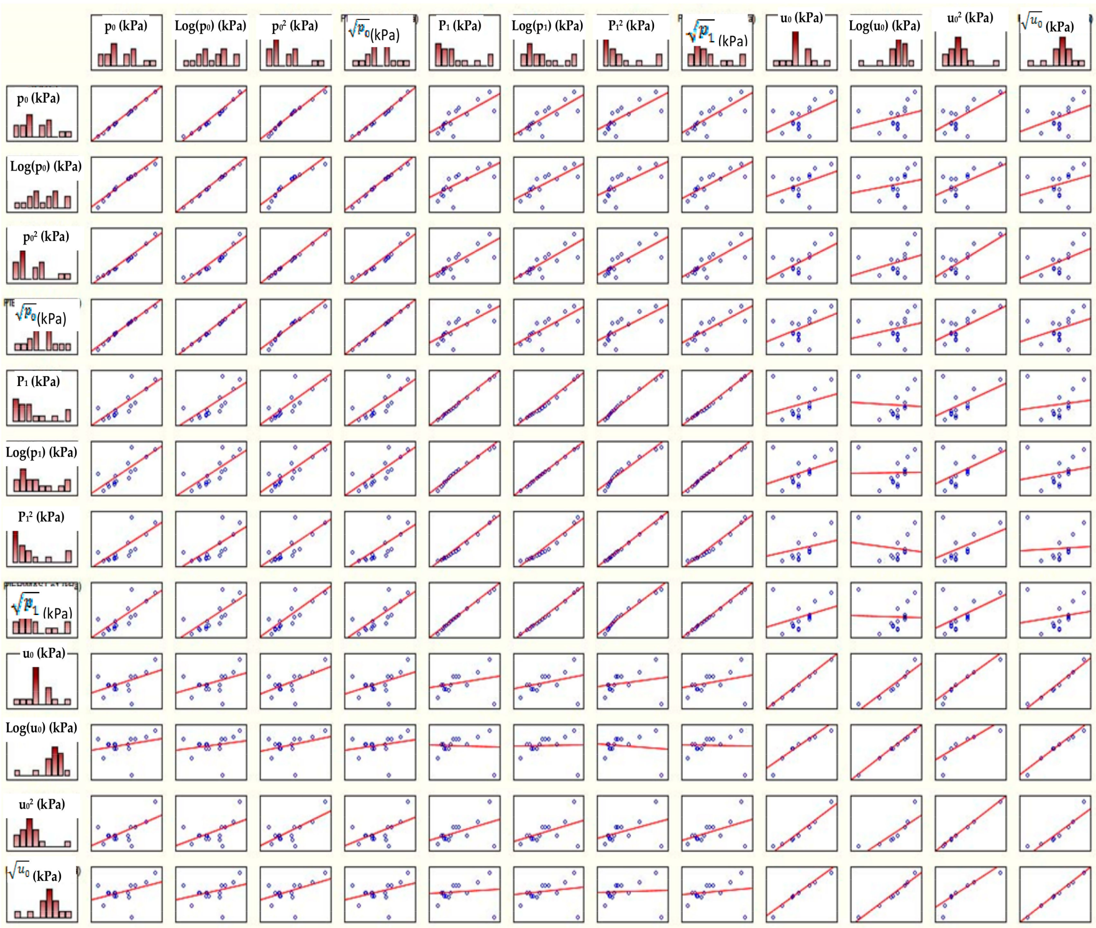

Figure 5. Matrix of extended charts for selected variables (dilatometer parameters).

Unit weight, as a basic physical feature of soil, is an elementary quantity, and knowledge of this parameter is necessary in each geotechnical and geoengineering task. Estimation of this quantity can be made both with laboratory and field techniques. Particular care should be taken when determining the characteristics, in which the value of unit weight is of particular importance for the quantity to be determined. This is especially applicable to e.g., the dynamic shear modulus G_{max} (or G_o, M_o). In such cases, unit weight should be determined directly or from local correlations.

Before obtaining Equation (13), a number of statistical analyses were performed with individual factors. Finally, a series of calculations were carried out in the form:

$$\varepsilon = \frac{\gamma}{\gamma_w} - k_1 \times log\left[64 \times \left[\frac{p_0 - u_0}{p_1}\right]\right] - k_2 \times log\left(\frac{p_1}{\sigma_{atm}}\right) - k_3 \qquad (14)$$

A separate analysis ($\frac{\gamma}{\gamma_w}$, $k_1 \times log\left[64 \times \left[\frac{p_0-u_0}{p_1}\right]\right] - k_2 \times log\left(\frac{p_1}{\sigma_{atm}}\right) - k_3$ *and* ε) was carried out for each soil peat, gyttja (Gy), organic mud, mud, clayey sand (Sicl), boulder clay (Cl), and sand (Sa, CSa, MSa, FSa). Then, the normality of the Shapiro–Wilk method was checked for each type of soil. The results of this analysis are presented in Figure 6. The results of the final stage are presented below:

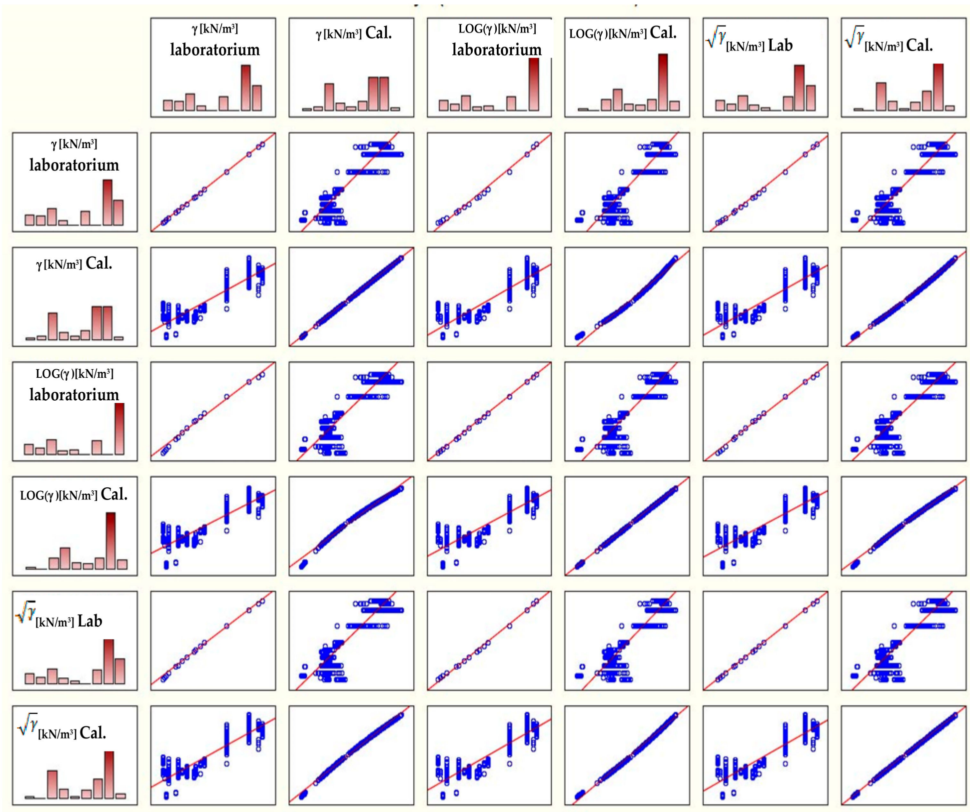

Figure 6. Matrix of extended charts for selected variables (soil unit weight).

This article does not assume that all variables are normal in the multiple regression analysis. The least squares method was used in Equation (13), which does not require the use of a normal distribution, but normality of the rest of the model is worth checking to verify the stability of the equation's parameters. The purpose of this paper is not to give up, substitute, or criticize the best practice used in geotechnics; rather, it is only to supplement these tests where possible using the dilatometer of Marchetti (DMT).

Next, we continue the transformation process for formula ($\frac{\gamma}{\gamma_w}, k_1 \times log\left[64 \times \left[\frac{p_0-u_0}{p_1}\right]\right] - k_2 \times log\left(\frac{p_1}{\sigma_{atm}}\right) - k_3$ *and* ε). The following modification was introduced for: $p_0 \rightarrow \log(p_0)$, $p_1 \rightarrow \log(p_1)$, $u_0 \rightarrow \log(u_0)$, $\sigma_{atm} \rightarrow \log(\sigma_{atm})$ parameters. A separate analysis was carried out for each soil peat, gyttja (Gy), organic mud, mud, clayey sand (Sicl), for boulder clay (Cl), u_0 = 0 kPa, sand (Sa, CSa, MSa, FSa)). Then, the normality of the Shapiro–Wilk method was checked for each type of soil. A normal distribution was obtained after the transformation. The results of the analysis are presented in Figure 7 and Table 8 below.

Table 8. Comparison of MRD, MRSD, and the rest of the obtained values before and after transformation for Equation (13).

Soil Type	$\frac{\gamma}{\gamma_w}=k_1\times log\left[64\times\left[\frac{p_0-u_0}{p_1}\right]\right]+k_2\times log\left(\frac{p_1}{\sigma_{atm}}\right)+k_3+\varepsilon$			for: $log\frac{\gamma}{\gamma_w}$, $p_0 \rightarrow$ log (p_0), $p_1 \rightarrow$ log (p_1), $u_0 \rightarrow$ log(u_0), $\sigma_{atm} \rightarrow$ log(σ_{atm})		
	MRD (%)	**MRSD (%)**	**Rest ε**	**MRD (%)**	**MRSD (%)**	**Rest ε**
peat and gyttja (Gy)	25.0	7.4	−5.53 ÷ 1.29	15.0	3.0	−0.05 ÷ 0.17
organic mud, mud	8.8	2.4	−0.93 ÷ 1.28	6.3	4.4	−0.01 ÷ 0.07
clayey sand (Sicl)	18.0	8.5	−1.03 ÷ 3.78	5.0	2.7	−0.03 ÷ 0.06
boulder clay (Cl)	16.5	4.0	−3.3 ÷ 1.68	*	*	*
sand (Sa, CSa, MSa, FSa)	22.0	6.3	−3.9 ÷ 2.72	*	*	*
For all analyzed data	**22.2**	**10.2**	**−5.53÷ 3.78**	**0.50**	**0.34**	**−0.19÷ 0.09**

* calculation was not possible because u_0 = 0 kPa for this subsoil; MRD—maximum relative deviation, MRSD—mean square relative deviation.

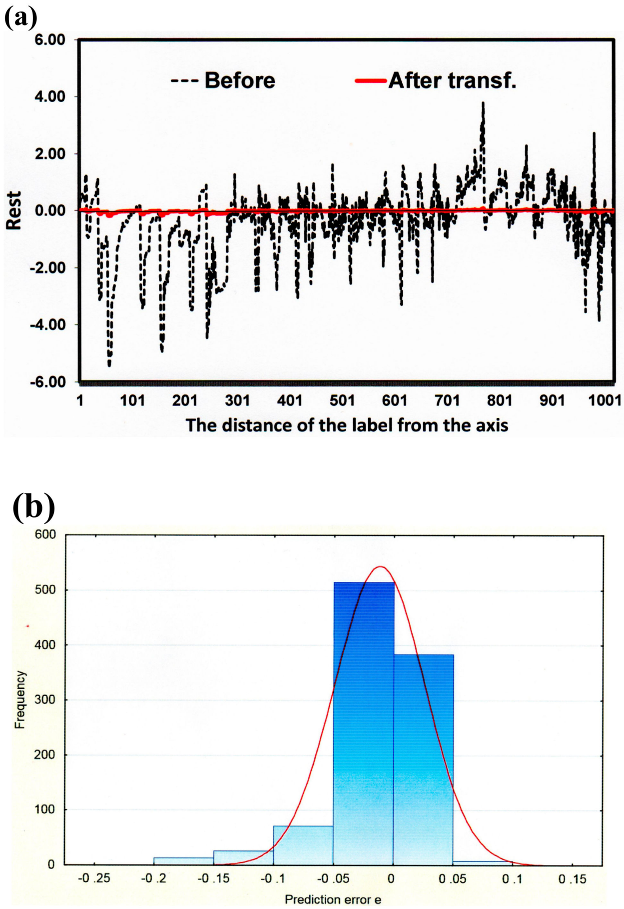

Figure 7. Profile of values: (**a**) before and after transformation; (**b**) histogram of the prediction error = $\varepsilon = \frac{\gamma}{\gamma_w} - k_1 \times log\left[64 \times \left[\frac{p_0-u_0}{p_1}\right]\right] - k_2 \times log\left(\frac{p_1}{\sigma_{atm}}\right) - k_3$.

Previous statistical methods using mostly the least squares estimation took data from field and laboratory studies as a closed number of random variables to determine the expected values, variances, and correlations for a given expression as a function of a random variable. The main disadvantage of the least squares method used to estimate the parameters is the difficulty in the representation of knowledge about the expected values of parameters, which should result from the estimation process. A significant progress in parameter estimation, as shown by increasingly numerous examples of applications in various fields, is the Bayesian analysis, in which a set of test results may be increased by new data, and on this basis, the probability of occurrence is determined. In order to determine the values of characteristic geotechnical parameters (X_k), Schneider (1997) [48] proposed a formula based on comparative calculations, proposing to use the following formula to select the characteristic value of geotechnical parameters (Table 9): $\gamma_k = \gamma_m - 0.5 \times S_d$, where: X_m—mean value; S_d—standard deviation.

Table 9. Summary of mean characteristic values of soil unit weight obtained from seismic dilatometer (SDMT) tests for the analyzed sites (peat, gyttja (Gy)), organic mud, mud, clayey sand (Sicl), boulder clay (Cl), and sand (Sa, CSa, MSa, FSa).

Soil Type	Mean Values of Soil Unit Weight γ_m (kN/m^3)	Standard Deviation S_d (kN/m^3)	Characteristic Values of Soil Unit Weight $\gamma_k = \gamma_m - 0.5 \times S_d$ (kN/m^3)
Peat gyttja (Gy) organic mud, mud	13.45	0.056	13.42
clayey sand (Sicl) boulder clay (Cl)	20.44	0.034	20.42
sand (Sa, CSa, MSa, FSa)	17.95	0.095	17.90

Unit weight parameters were calculated for the soil found in each site. The calculation was based on the use of Marchetti's patterns existing in the literature and on Marchetti's nomogram, which has not changed since 1980. Based on the nomogram chart for soils with E_D < 1.2 MPa, soil unit weight less than 15.0 kN/m^3 was not determined. However, other formulas gave the possibility to determine this parameter (γ) with a different result. The results obtained were compared with those from laboratory tests that may be considered as reference. Since the obtained results do not show adequate results consistent with the results of laboratory tests, both on the basis of Marchetti's nomogram and literature models, there is a need for a new design. Very promising results have been obtained using the unit weight values proposed in this article (Equation (13)) for all soils (mineral and organic soils). The obtained results are presented in Table 10.

Table 10. Comparison of results obtained based on empirical correlations in the literature with results calculated by the proposed formula (Equation (13)) and the results from laboratory tests.

Sites	Soil Type	Marchetti (1980) [4] (Nomogram Chart)	Schmertmann, 1986 [26] and Mayne et al., (2002)	Ozer et al. 2013 [30]	Ouyang and Mayne (2016) [32]	Rabarijoely (Equation (13) in This Paper)	Laboratory Test
		Soil unit weight γ (kN/m^3)					
Antoniny	Peat (oc)	15.0	12.8 ÷ 14.3	11.4 ÷ 13.8	m_{po} = 17 kN/m^3 γ = 14 kN/m^3	10.4 ÷ 14.2	10.5 ÷ 14.5
	Gyttja (oc)	15.0	12.8 ÷ 14.3	11.4 ÷ 13.8	m_{po} = 17 kN/m^3 γ =14	10.4 ÷ 14.2	10.5 ÷ 14.5
	Peat (nc)	17.0 ÷ 18.0	15.8 ÷ 17.5	15.6 ÷ 17.4	m_{po} = 27.2 kN/m^3 γ = 16 kN/m^3	11.8 ÷ 13.5	10.5 ÷ 14.5
	Gyttja(nc)	17.0 ÷ 18.0	15.8 ÷ 17.5	15.6 ÷ 17.4	m_{po} = 27.2 kN/m^3 γ = 16 kN/m^3	11.8 ÷ 13.5	10.5 ÷ 14.5
Koszyce	Peat (oc)	15.0	12.0 ÷ 15.0	11.7 ÷ 15.4	m_{po} = 18 kN/m^3 γ = 14	10.3÷13.8	10.3 ÷ 14.5
	Gyttja(oc)	15.0	12.0 ÷ 15.0	11.7 ÷ 15.4	m_{po} = 18 kN/m^3 γ = 14	10.3 ÷ 13.8	10.3 ÷ 14.5
	Peat (nc)	17.0 ÷ 18.0	15 ÷ 16.6	15.0 ÷ 16.3	m_{po} = 28,3 kN/m^3 γ = 16,2	10.6 ÷ 11.7	10.3 ÷ 14.5
	Gyttja(nc)	17.0 ÷ 18.0	15 ÷ 16.6	15.0 ÷ 16.3	m_{po} = 28,3 kN/m^3 γ = 16.2	10.6 ÷ 11.7	10.3 ÷ 14.5
Nielisz	Mud, organic mud (oc)	15.0	13.6 ÷ 16.0	13.6 ÷ 16.0	m_{po} = 33.8 kN/m^3 γ = 17.4	12.2 ÷ 14.8	12.0 ÷ 15.0
	Mud, organic mud (nc)	~18	15.1 ÷ 16.8	14.6 ÷ 16.3	m_{po} = 40.8 kN/m^3 γ = 19	13.2 ÷ 15.0	12.0 ÷ 15.0
SGGW Campus	Boulder clay	16.0 ÷ 21.0	17.0 ÷ 21.0	16.0 ÷ 24.0	m_{po} ≥ 60 kN/m^3 γ ≥ 23 kN/m^3	19.5 ÷ 22.1	20.0 ÷ 22.0
Stegny	Pliocene clay	15.0 ÷ 21.0	18.0 ÷ 21.0	18.0 ÷ 23.0	m_{po} ≥ 60 kN/m^3 γ ≥ 23kN/m^3	20.5 ÷ 22.0	21.0 ÷ 22.0

The collected research material allowed the analysis of laboratory characteristics of soil unit weight and the development of non-linear empirical models, enabling forecasting its changes in the function of water unit weight (γ_w), pore water pressure (u_0), and atmospheric pressure σ_{atm} = 100 kPa. In the non-linear model, an important parameter determining the shape of this relationship were dilatometer pressures p_0, p_1.

The non-linear model was developed in the paper. The basic premise that determined the construction of this model for soil unit weight (γ) was not a very precise determination of the value of the free expression by linear multiple regression models. The second reason for the development of non-linear changes in soil unit weight (γ) as a function of dilatometer pressure p_0, p_1, water unit weight (γ_w), pore water pressure (u_0), and atmospheric pressure σ_{atm} = 100 kPa is the analysis of soil unit weight correlation for peat, gyttja, mud, organic mud, silty clay, and Pliocene clay, originating from the same soil profile, but taken from different sites.

The process of the presented measurement characteristics are illustrated against the background of the general linear trend equation resulting from the alignment of all measurement data of soil unit weight characteristics presented in Figure 8 and the average value of soil unit weight determined by means of the dilatometer test. Linear alignment for both presented measurement relationships does not lead to the determination of the soil unit weight value with an appropriate level of accuracy.

The results obtained herein are satisfactory because when using the proposed equation (Equation (13)) to determine the soil unit weight of mineral and organic soils based on Marchetti's dilatometer (DMT) from the analyzed sites, the performed analyses prove that 69% of results ((a): Antoniny; (b): Koszyce; (c): Nielisz; (d): Stegny; (e): SGGW Campus) of the soil unit weight are within the limit of the accepted error (mean square relative deviation: MRSD ≤ 6.0%). In addition, the percentage difference between the soil unit weight obtained in the laboratory and calculated from the proposed equation (Equation (13) was analyzed. In the case of the soil unit weight of peat, gyttja, and organic mud, the difference obtained in all the results was averagely only MRSD = 4.9%. In the case of the unit weight of Pliocene clay and clay, it was at MRSD = 5.6%, and of the sand, it was at MRSD = 6.0% (Figure 8).

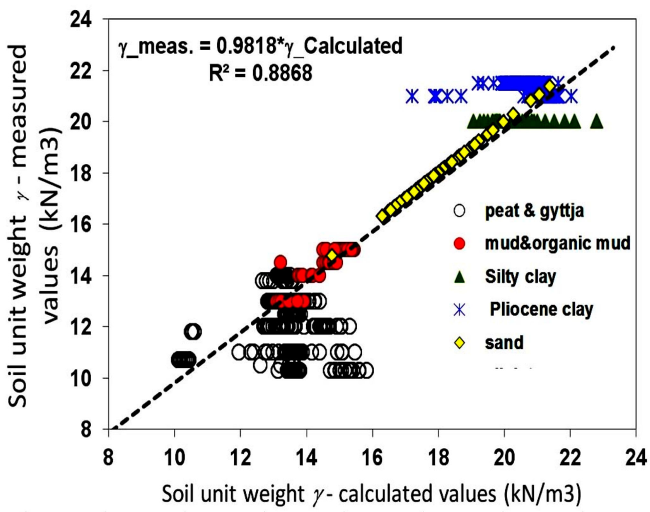

Figure 8. Comparison of soil unit weight obtained from laboratory tests and calculated from the proposed equation (Equation (13)) based on dilatometer test (DMT) results.

9. Conclusions

The paper proposes a new formula for determining the unit weight of mineral and organic soils based on a multifactorial relationship. For this purpose, the diagnosis of dilatometer parameters, such as p_0 and p_1 pressures, the calculated value of pore water pressure u_0, and atmospheric pressure σ_{atm} were measured in the analyzed sites.

The maximum and mean square relative deviation values obtained for the proposed dependence indicate that the dependencies—taking into account the three factors—give the smallest mean square relative deviation.

Comparison of the dependencies used to determine the unit weight for mineral and organic soils confirms that the multifactor relationship proposed herein (Equation (13), Figure 8) provides a wide range of its applicability both for mineral and organic soils in terms of stress (HOC) and (NC).

The calculated value of the mean square relative deviation of unit weight for mineral and organic soils based on the proposed multifactorial relation was obtained using values of empirical coefficients k_1, k_2, and k_3. The values of empirical coefficients k_1, k_2, and k_3 are presented in Table 3. The mean square relative deviation values of unit weight for mineral and organic soils for the studied peats, gyttja, and organic muds are in the range of 4.9 ÷ 6.0%, while for the studied Pliocene clays, clays, and sands, they are in the range of 4.9 ÷ 6.0%. The sand results work better than the other types of soil (Figure 8), because various factors influence the value of soil unit weight for organic and cohesive soil. These factors include the artesian pressure occurring in the organic layer, apart from the hydrostatic pressure, which causes the vertical component of the effective stress and the preconsolidation stress σ'_p.

The new computational dependency proposed herein may be recommended not only in Poland, but also in countries with similar ground properties.

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