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Finite Element Steady-State Vibration Analysis Considering Frequency-Dependent Soil-Pile Interaction

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Abstract: The vibration response of equipment foundation structures is not only affected by the structural stiffness and mass, but also greatly influenced by the degree of a soil-foundation structural interaction. Furthermore, the vibratory performance of equipment foundation structures supported by pile systems largely depends on the soil-pile dynamic stiffness and damping, which are variable in nature within the speed range that machines operate at. This paper reviews a method for evaluating effective soil-pile stiffness and damping that can be computed by Novak's method or by commercial software (DYNA6, University of Western Ontario). A series of Finite Element (FE) time history and steady-state analyses using SAP2000 have been performed to examine the effects of dynamic soil-pile-foundation interaction on the vibration performance of equipment foundations, such as large compressor foundations and steam/gas turbine foundations. Frequency-dependent stiffness is estimated to be higher than frequency-independent stiffness, in general, and, thus, affects the vibration calculation. This paper provides a full-spectrum steady-state vibration solution, which increases the reliability of the foundation's structural design.

Keywords: FEM (Finite Element Method); DYNA6; soil-structure interaction; soil-pile dynamic stiffness

1. Introduction

The importance of foundational dynamic stiffness and damping in the vibration's assessment has been addressed in many literatures such as Novak's method [1], Kausel's approach [2], and Roesset's research [3].

The conventional method for evaluating soil stiffness and damping is based on the classical theory of vibrations of a disk supported on top of an elastic half-space [4]. This theoretical solution is limited to few simple foundation configurations and soil profiles. The applications of the theory often overestimate the degree of soil damping, and, thus, underestimates the vibration amplitude. Field experiences show that the beneficial effects of radiation damping in mitigating foundational vibrations, as predicted by the elastic half-space vibration theory, may not always be effective [5]. Such cases include large compressor foundations, combustion turbine generator/steam turbine generator (CTG/STG) table-top foundations, and other large foundational structures that support high-speed (30 Hz or higher) vibratory equipment. In special soil media, such as rock stratum found very close to the bottom of the foundation, the radiation damping from the theory may be impractical to vibration analyses. This minimal radiation damping is due to the elastic rebounding of waves at the soft soil-rock interface, which is different from the theoretical solution. Currently, a few modelling guidelines [5] exist for assessing such a soil-structure interaction damping on foundation vibrations using the Finite Element

Method. This paper proposes several state-of-the-art methods for evaluating effective soil damping that could be modelled in FE (Finite Element) software i.e., SAP2000 and GTSTRUDL. For the formulation of modal damping ratios in the dynamic equation of motion, two different approaches, i.e., direct use of effective damping values and proportional damping (either mass-proportional or stiffness-proportional damping, or both) are employed in this study.

The frequency-dependent interaction effect [6,7] is accomplished by numerical simulations utilizing DYNA6 [8] and is incorporated in the frequency domain steady-state vibration analysis in this paper.

2. Dynamic Soil Profile

The idealized soil profile located beneath the sample CTG and STG foundations is depicted in Figure 1. The dynamic characteristics of soil properties, i.e., shear wave velocity, shear modulus, unit weight, and Poisson ratio for each layer, are summarized. The frequency-independent soil stiffness and damping are formulated by the Elastic Half-Space Method [4] and is used initially in the analysis. The consideration of soil uncertainty is not accounted for in the sample analyses to minimize the computational effort.

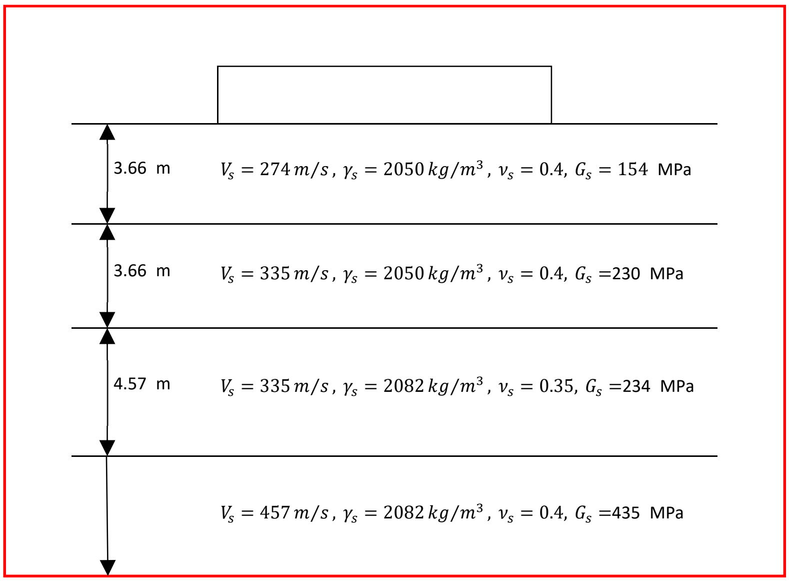


Figure 1. Dynamic soil profiles under CTG/STG Foundations V_S: Shear wave velocity of soil, γ_S: Unit weight of soil, ν_S: Poisson ratio of soil, G_S: Shear modulus of soil.

3. Analytical CTG/STG FE Models

The FE models of the sample CTG and STG foundations were developed with two commercial programs, SAP2000 [9] and GTSTRUDL [10]. The two programs provide eight-node solid element equipped with different shape functions, i.e., bending improved and non-bending improved. The CTG foundation (5.49 m wide, 18.29 m long, and 1.52 m thick) supports a set of turbine and generator equipment, which weighs a total of 210.01 t. Both the SAP2000 and GTSTRUDL CTG models consist of 624 eight-node solid elements and 972 joints (2916 degrees of freedom). The STG foundation (18.90 m wide, 36.58 m long, and 13.11 m tall) also supports a set of turbine and generator equipment together with a heavy condenser. The weight of the turbine and generator is estimated to be 857.29 t and the condenser is assumed to weigh 578.33 t.

The STG foundation models consist of 3828 eight-node solid elements and 6335 joints (19,005 degrees of freedom). The SAP2000 models additionally include nine incompatible bending shape functions in the eight-node solid elements so that the bending behaviour of the element can be significantly improved, which is not featured in GTSTRUDL models. The eight-node solid elements used in GTSTRUDL models adopt the 'IPSL (Isoparametric Solid Linear Displacement)' elements. The dynamic FE models are illustrated in Figures 2 and 3. The material property of both CTG and STG foundations is assumed to be 27.6 MPa cylindrical concrete strength with 24,856 MPa Young's modulus. The unit weight of concrete is of typical 2403 kg/m^3.

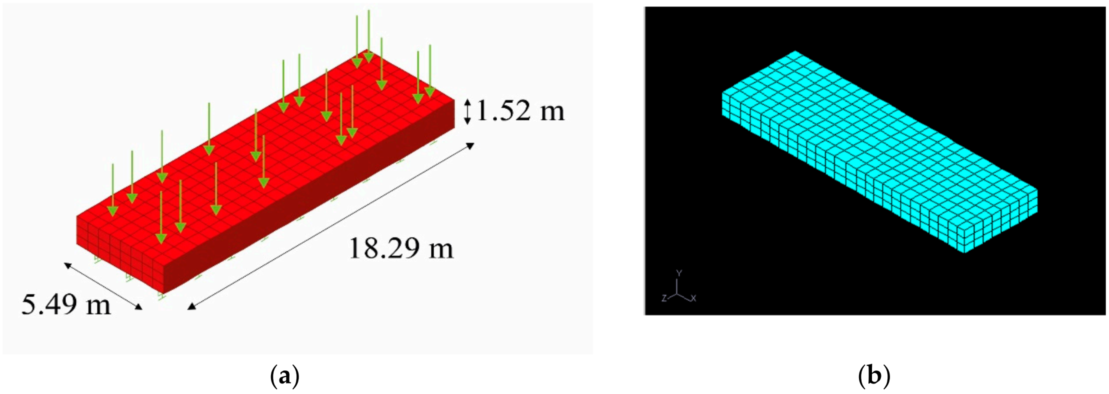


Figure 2. CTG FE Model: (**a**) SAP2000, (**b**) GTSTRUDL.

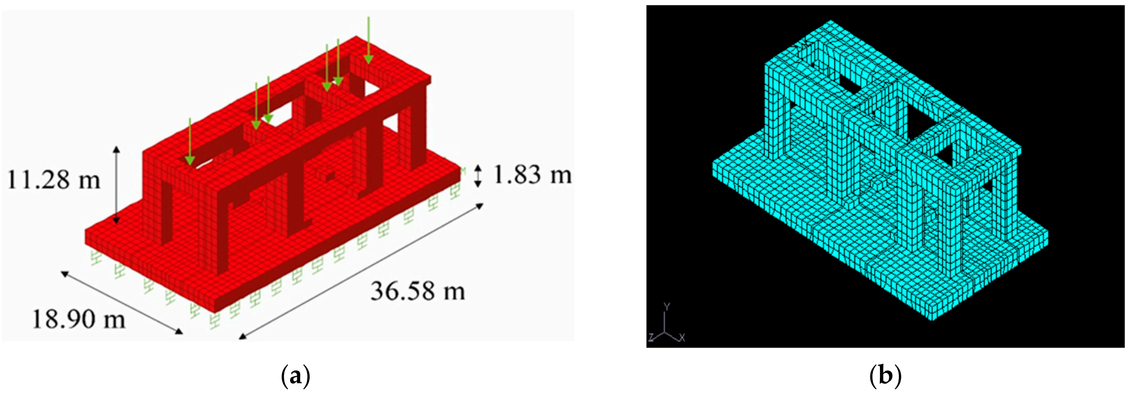
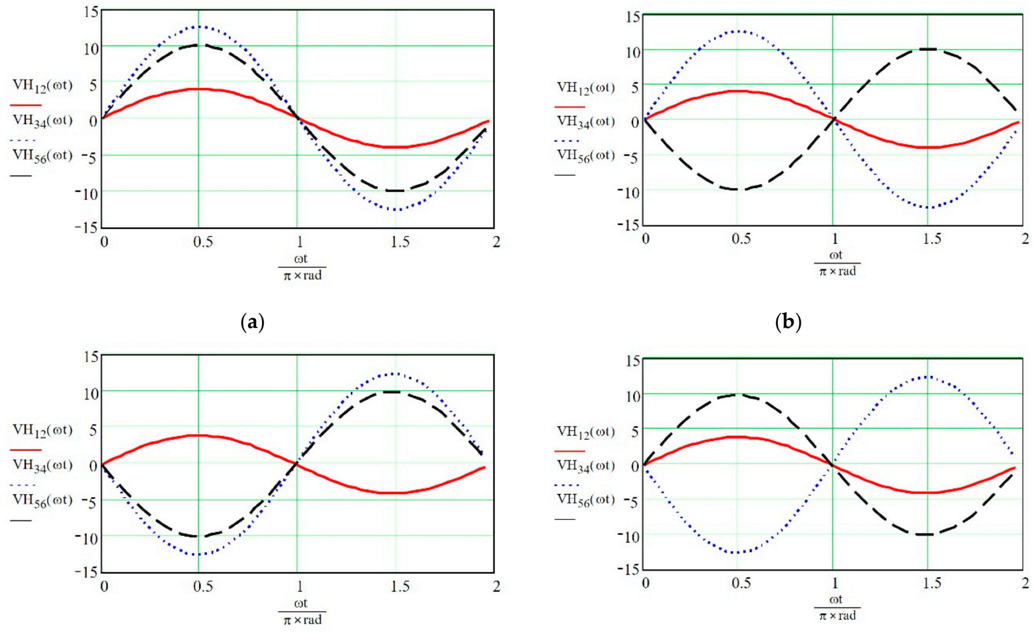


Figure 3. STG FE Model: (**a**) SAP2000, (**b**) GTSTRUDL.

4. Dynamic Unbalanced Forces

The dynamic unbalanced forces at 60 Hz excitation frequency are applied to the CTG and STG FE models. The CTG foundation force amplitudes and locations are all designated by the CTG vendor. Two scenario-based phase angle conditions are considered in the analysis. One is VTX60A, which is the case when all X, Y, and Z directional loads on the turbine and generator are in-phase. The other is VTX60B, which is the case when all the loads on the turbine are 180° out-of-phase with the loads on the generator.

The STG foundation force amplitudes and phase angle cases are plotted in Figure 4. A total of eight load cases were analysed, to simulate meaningful phase angle cases, i.e., V60A, H60A, V60B, H60B, V60C, H60C, V60D, and H60D. V60A and H60A are the vertical and transverse load cases, respectively, where all the loads are in-phase. V60B and H60B are also vertical and transverse load cases where unbalanced forces at bearings 5 and 6 are 180° out-of-phase with those at bearings 1, 2, 3, and 4. V60C and H60C are also vertical and transverse load cases where unbalance forces at bearings 1 and 2 are 180° out-of-phase with those at bearings 3, 4, 5, and 6. V60D and H60D are also vertical and transverse load cases where unbalanced forces at bearings 3 and 4 are 180° out-of-phase with those at bearings 1, 2, 5, and 6. The unbalanced force amplitudes at 60 Hz are assumed to be 1.81 t at bearings 1 and 2, 5.67 t at bearings 3 and 4, and 4.54 t at bearings 5 and 6, as per the vendor recommendation. The location of bearing, i.e., point of unbalanced force, is specified by the equipment vendor. However, in general, it is two ends of each combustion/steam turbine generator segment at the foundation center in the axial direction.



Phase Angle Case	Force Direction at Bearings 1 and 2	Force Direction at Bearing s3 and 4	Force Direction at Bearings 5 and 6
A	↑	↑	↑
B	↑	↑	↓
C	↑	↓	↓
D	↑	↓	↑

Figure 4. STG Foundation Dynamic Unbalance Force at 60 Hz: (**a**) V60A, H60A, (**b**) V60B, H60B, (**c**) V60C, H60C, (**d**) V60D, H60D.

5. Analysis Methods for CTG and STG Foundational Vibration Assessment

A number of analysis methods can be proposed for the vibration assessment of the CTG and STG foundations. The critical issue is how to model soil radiation damping, and, therefore, how much the contribution of soil damping would affect the foundation's vibration response at 60 Hz. Two dynamic analysis methods are considered: modal superposition time history and direct integration time history. In the modal superposition time history analysis, five different soil damping cases are considered: (1) a direct soil damping coefficient from the elastic half-space solution plus 2% concrete material damping, (2) a direct soil damping coefficient from the elastic half-space solution with a 20% cut-off from the EPRI (Electric Power Research Institute) plus 2% concrete material damping, (3) all modal damping at 2%, (4) all modal damping at 4%, and (5) all modal damping at 10%. The modal damping of the (1) and (2) models is, therefore, computed by SAP2000 systematically depending on the modal deformation shapes, i.e., the energy is proportional at each mode and at the interface of the foundation and the soil.

For the direct integration time history analysis, the conventional Newmark β method (β = 0.25 assumed) [11] is used throughout the analyses and three sets of proportional damping parameters (α: mass proportional damping and β: stiffness-proportional damping) [12] are considered: (1) ξ = 4% at 1st modal frequency and ξ = 4% at 60 Hz, (2) ξ = 10% at 1st modal frequency and ξ = 4% at 60 Hz, and (3) ξ = 20% at the first modal frequency and ξ = 4% at 60 Hz. The general form of α and β based on two frequency points is given by the following equations.

$$\alpha := 2 \cdot \frac{\xi_1 \cdot \omega_2 - \xi_2 \cdot \omega_1}{\left(\frac{\omega_2}{\omega_1} - \frac{\omega_1}{\omega_2}\right)} \cdot sec \tag{1}$$

$$\beta := 2 \cdot \frac{\xi_1 \cdot \omega_1 - \xi_2 \cdot \omega_2}{{\omega_1}^2 - {\omega_2}^2} \cdot \frac{1}{sec} \tag{2}$$

in which ξ_1 and ξ_2 are damping ratios that correspond to the two frequencies, ω_1 and ω_2, respectively.

It should be noted that both dynamic analysis methods are approximate in terms of their solution techniques. For instance, the modal superposition solution (steady-state part only) depends on only the diagonal terms of the damping matrix but does not account for non-diagonal damping terms in the dynamic equation of motion. On the other hand, the direct integration method utilizes every term in the damping matrix. From this point of view, for the particular case where coupled modes contribute significantly to the total response, the modal superposition solution may not result in an accurate response representation. On the contrary, the direct integration method depends on the time increment, and the two frequency point proportional dampings may lead to an approximate response as well. Therefore, foundation engineers must pay close attention to the selection of damping parameters and analysis methods. A summary table of the applied analysis methods for the vibration assessment of the CTG and STG foundations is given in Table 1.

Table 1. Analysis methods for the CTG and STG foundation vibration assessment.

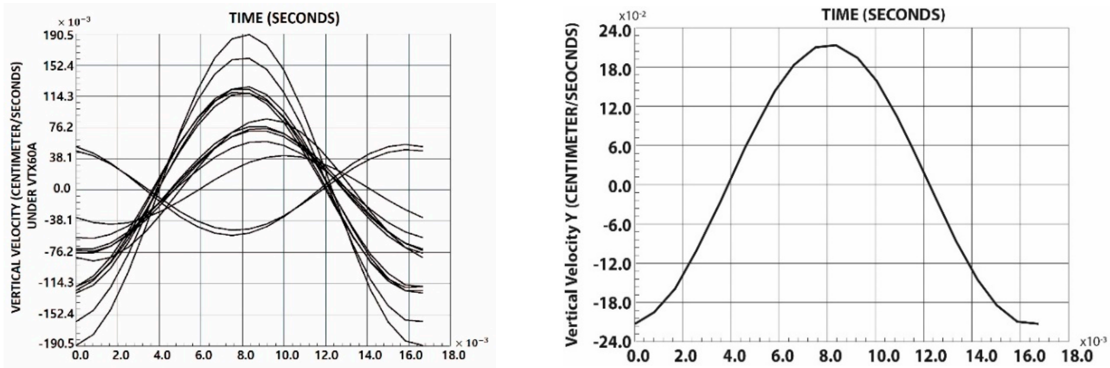
<table>
<tr><th colspan="3">Analysis Method (Damping)</th><th>CTG Foundation</th><th>STG Foundation</th></tr>
<tr><td rowspan="5">Modal Superposition Time History Analysis</td><td colspan="2">Direct Soil Damping Coefficient from Elastic Half Space Solution Plus 2% Concrete Material Damping</td><td rowspan="2">SAP2000 (20 Modes)</td><td rowspan="2">SAP2000 (120 Modes)</td></tr>
<tr><td colspan="2">Direct Soil Damping Coefficient from Elastic Half Space Solution with 20% Cutoff by EpRI Plus 2% Concrete Material Damping</td></tr>
<tr><td rowspan="3">Modal Damping for All Modes</td><td>2%</td><td rowspan="3">SAP2000, GTSTRUDL (20 Modes)</td><td rowspan="3">SAP2000, GTSTRUDL (120 Modes)</td></tr>
<tr><td>4%</td></tr>
<tr><td>10%</td></tr>
<tr><td rowspan="3">Direct Integration Time History Analysis (Newmark β=0.25)</td><td rowspan="3">Rayleigh Damping</td><td>ξ=4% at ω_{1st}
ξ =4% at 60Hz</td><td rowspan="3">GTSTRUDL</td><td rowspan="3">N/A *</td></tr>
<tr><td>ξ =10% at ω_{1st}
ξ =4% at 60Hz</td></tr>
<tr><td>ξ =20% at ω_{1st}
ξ =4% at 60Hz</td></tr>
</table>

* It requires significant computation memory and time.

6. Steady State Time History and Direct Integration Time History Response

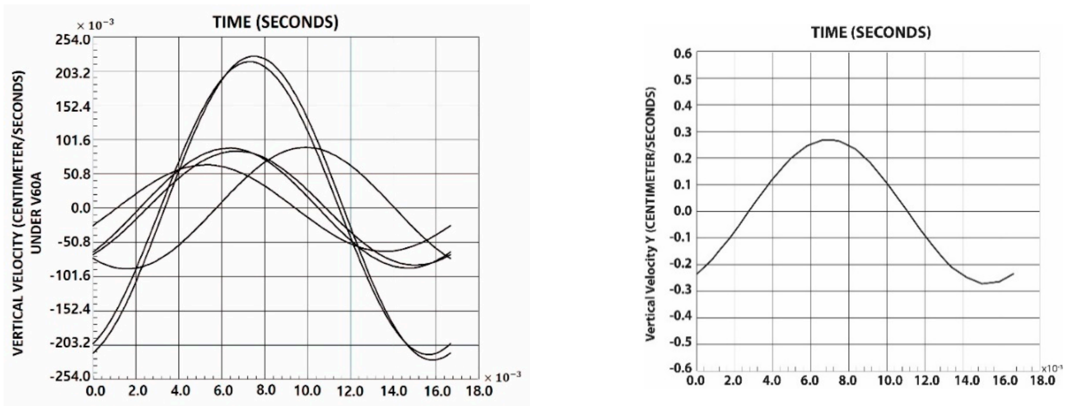
The steady-state time history solutions by modal superposition analysis were executed by both SAP2000 and GTSTRUDL. To compute the steady-state modal solution, eigenvalue/eigenvector analysis must be performed before conducting forced vibrational analysis. A total of 20 and 120 modes were considered in the CTG and STG foundation models, respectively, and the higher modes, i.e., more than 120% of excitation frequency, should be included in the modal solution computation. Then, modal combination using either the SRSS (square root of the sum of squares) or CQC (complete quadratic combination) method is performed to compute the total solution. To simplify, only one cycle response was considered using a periodic time history motion.

The direct integration analysis of CTG foundations was also performed with a full damping matrix. Twenty integration points per cycle for a total of 50 cycles were used for the convergence of the steady-state solution. Results of the time history response are illustrated in Figures 5–7.



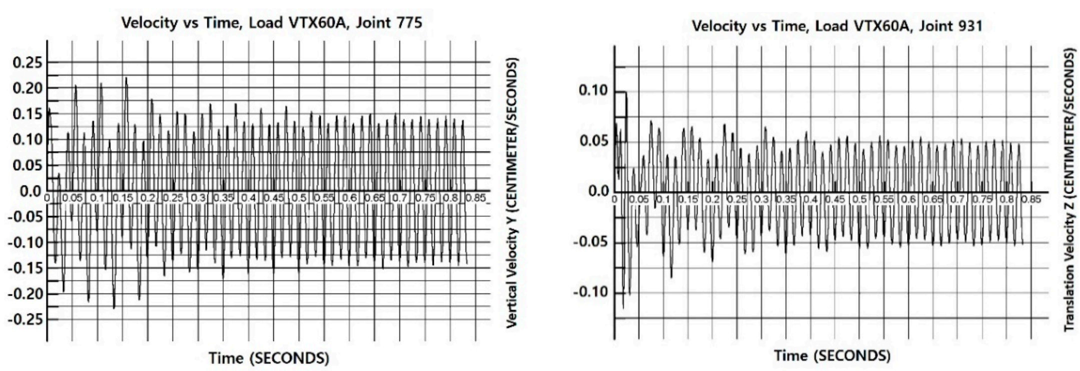
(**a**) (**b**)

Figure 5. Steady-state time history velocity response (CTG Foundation under VTX60A Load): (**a**) SAP2000, (**b**) GTSTRUDL.



(**a**) (**b**)

Figure 6. Steady-State Time History Velocity Response (STG Foundation under V60A Load): (**a**) SAP2000 and (**b**) GTSTRUDL.



(**a**) (**b**)

Figure 7. Direct Integration Time History Velocity Response. (CTG Foundation under VTX60A Load, 50 Cycles): (**a**) Max.Velocity a Joint 775, (**b**) Max.Velocity at Joint 931.

A complete maximum steady-state (or permanent) velocity response at the bearings of CTG and STG foundations are tabulated in the following four tables (Tables 2–5). A separate table is provided for each directional velocity, i.e., vertical and transverse.

Table 2. Maximum vertical velocity steady-state eesponse at bearings of the CTG foundation (cm/s).

Damping Method		VTX60A	VTX60B
Direct soil damping value with 2% material damping SAP 2000		0.132	0.155
Direct soil damping value with 20% cutoff by EPRI 2% material damping SAP 2000		0.178	0.302
Modal damping 2%	SAP 2000	0.191	0.351
	GTSTRUDL	0.213	0.442
Modal damping 4%	SAP 2000	0.185	0.333
	GTSTRUDL	0.201	0.391
Modal damping 10%	SAP 2000	0.170	0.274
	GTSTRUDL	0.170	0.290
Rayleigh damping (α = 5.3174, β = 0.00017) GTSTRUDL		0.152	0.127
Rayleigh damping (α = 15.4648, β = 0.0001) GTSTRUDL		0.147	0.122
Rayleigh damping (α = 32.3771, β = 0.0) GTSTRUDL		0.142	0.117

Table 3. Maximum transverse velocity steady-state response at bearings of CTG foundation (cm/s).

Damping Method		VTX60A	VTX60B
Direct soil damping value with 2% material damping SAP 2000		0.127	0.112
Direct soil damping value with 20% cutoff by EPRI 2% material damping SAP 2000		0.132	0.145
Modal damping 2 %	SAP 2000	0.135	0.163
	GTSTRUDL	0.127	0.165
Modal damping 4%	SAP 2000	0.132	0.152
	GTSTRUDL	0.124	0.150
Modal damping 10%	SAP 2000	0.124	0.122
	GTSTRUDL	0.117	0.114
Rayleigh damping (α = 5.3174, β = 0.00017) GTSTRUDL		0.056	0.033
Rayleigh damping (α = 15.4648, β = 0.0001) GTSTRUDL		0.053	0.030
Rayleigh damping (α = 32.3771, β = 0.0) GTSTRUDL		0.051	0.030

Table 4. Maximum vertical velocity steady-state response at bearings of STG foundation (cm/s).

Damping Method		V60A	H60A	V60B	H60B	V60C	H60C	V60D	H60D
Direct soil damping value with 2% material damping SAP 2000		0.198	0.008	0.163	0.008	0.208	0.008	0.178	0.008
Direct soil damping value with 20% cutoff by EPRI plus 2% material damping SAP 2000		0.224	0.053	0.185	0.028	0.229	0.043	0.188	0.020
Modal damping 2%	SAP 2000	0.226	0.124	0.175	0.061	0.221	0.086	0.160	0.061
	GTSTRUDL	0.272	0.147	0.241	0.069	0.310	0.127	0.262	0.081
Modal damping 4%	SAP 2000	0.226	0.058	0.191	0.028	0.234	0.051	0.196	0.025
	GTSTRUDL	0.267	0.074	0.213	0.033	0.300	0.066	0.236	0.033
Modal damping 10%	SAP 2000	0.226	0.013	0.175	0.008	0.239	0.013	0.196	0.008
	GTSTRUDL	0.244	0.015	0.173	0.008	0.262	0.015	0.203	0.010

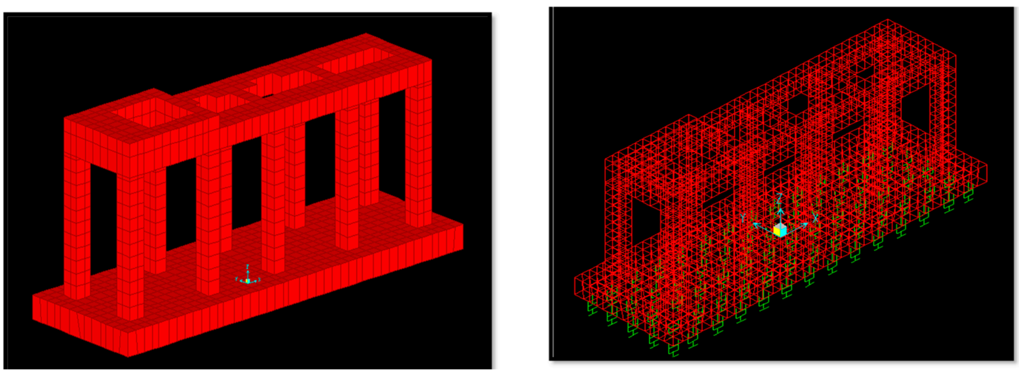
Table 5. Maximum transverse velocity steady-state response at bearings of STG foundation (cm/s).

Damping Method		V60A	H60A	V60B	H60B	V60C	H60C	V60D	H60D
Direct soil damping value with 2% material damping SAP 2000		0.008	0.094	0.010	0.069	0.005	0.089	0.008	0.074
Direct soil damping value with 20% cutoff by EPRI 2% material damping SAP 2000		0.028	0.140	0.030	0.079	0.038	0.135	0.018	0.114
Modal damping 2%	SAP 2000	0.061	0.193	0.071	0.104	0.089	0.168	0.046	0.221
	GTSTRUDL	0.056	0.168	0.053	0.056	0.069	0.168	0.064	0.178
Modal damping 4%	SAP 2000	0.030	0.150	0.028	0.081	0.038	0.137	0.018	0.135
	GTSTRUDL	0.030	0.145	0.025	0.061	0.030	0.132	0.023	0.132
Modal damping 10%	SAP 2000	0.010	0.114	0.008	0.074	0.010	0.104	0.008	0.084
	GTSTRUDL	0.010	0.112	0.008	0.071	0.010	0.107	0.005	0.076

7. Soil-Pile Interaction Study and Frequency-Dependent Stiffness and Damping

Another sample model has been used to investigate the effect of soil-pile-foundation interaction on the vibration performance of the equipment foundation. The material property of the foundation is assumed to be 27.6 MPa cylindrical concrete strength with 24.856 MPa Young's modulus. The unit weight of concrete is of typical 2403 kg/m^3.

A total of 2706 joints, 70 links, and 1170 solid elements were used in the example model (Figure 8, 3D solid element model with link element). The 3-D solid elements are expected to predict displacement/velocity response with higher accuracy, and the behaviour of the pile foundation with higher efficiency. This model used only three translational degrees of freedom at each joint. A direct soil-pile interaction study has been conducted using the DYNA6 program. A total of 70 CIP (600 mm diameter, 18 m long Cast-In-Place with 34.5 MPa cylindrical concrete strength) piles are installed beneath the large compressor foundation and are modelled by the FE link elements. Two layers of soil media, with shear wave velocities equal to Vs = 300 m/s for the upper 3 m of soil and V_b = 538 m/s below the soil, are expected to interact with the CIP piles, including a weak zone (Figure 9). The weak zone parameters are assumed to have a Poisson ratio of 0.3 and material damping of 5%. The compressor vendor has set the single machine speed at 66.5 Hz. Thus, all the foundational vibration responses up to 120% (80 Hz) of the operating speed were calculated and considered for resonance checking at any frequency. For a reference purpose, an eigenvalue analysis using frequency independent stiffness was performed. The fundamental modal period returned was 0.395 s (horizontal mode coupled with rocking) and 0.099 s (vertical mode), as shown in Figure 10.



(**a**) (**b**)

Figure 8. 3-D solid element model and FE link element: (**a**) 3-D Solid element model (31 m × 10 m × 12.5 m), (**b**) FE link element (Green).

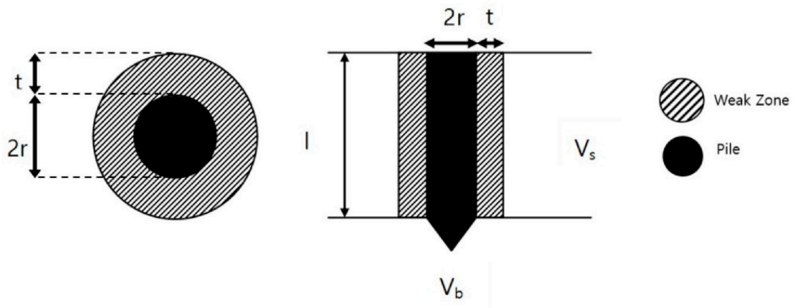
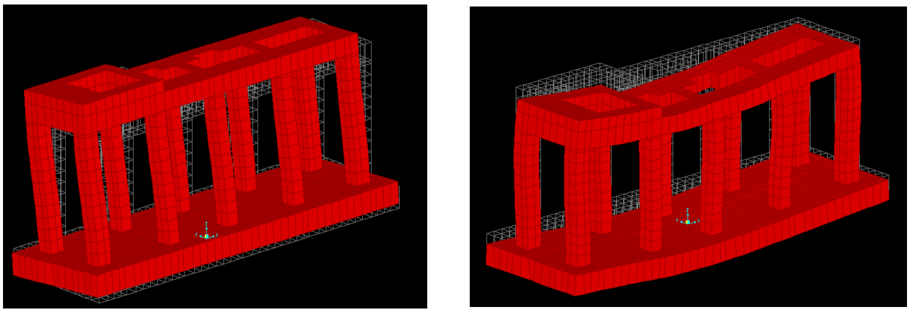


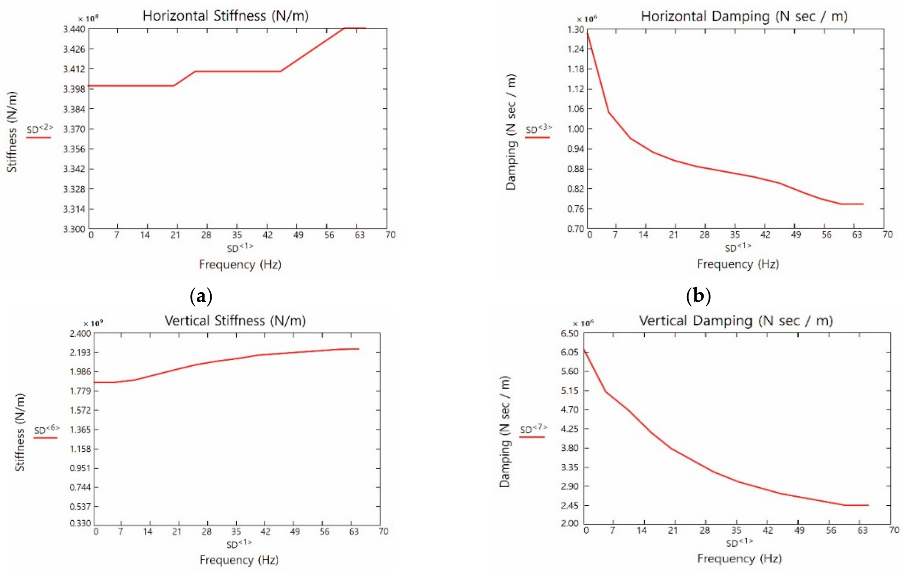
Figure 9. Soil-pile section and weak zone (l: Pile length, r: Radius of the pile, l/r: Slenderness ratio of the pile, t: Thickness of the weak zone, vs.: Shear wave velocity of soil, V_b: Shear wave velocity below the pile tip).



(**a**) (**b**)

Figure 10. Foundation's mode shapes. (**a**) 1st mode 0.395 s (Horizontal mode with rocking), (**b**) 4th mode 0.099 s (vertical mode).

The individual soil-pile interactions were numerically simulated in the DYNA6 platform [8]. The dynamic pile stiffness and damping along the frequency range have been generated and plotted in Figure 11. The stiffness and damping values were incorporated into the FE model (Figure 8, Right) via frequency-dependent link elements in SAP2000.



(**a**) (**b**) (**c**) (**d**)

Figure 11. Soil-pile stiffness and damping vs. frequency: (**a**) Horizontal stiffness vs. frequency, (**b**) Horizontal damping vs. frequency, (**c**) Vertical stiffness vs. frequency, (**d**) Vertical damping vs. frequency.

In general, the horizontal stiffness has little variation with the frequency range (less than 2%) while the vertical pile stiffness gradually increases up to 30%. Horizontal and vertical damping is drastically reduced as frequency increases. In particular, the frequency-dependent values are very different from frequency-independent values, as shown in Table 6. The frequency-dependent vertical stiffness is 10–13 times higher than the frequency-independent value. The frequency-dependent horizontal stiffness is 3.5 times higher.

Table 6. Individual pile stiffness and damping comparison.

Range 0–70 Hz	Vertical Stiffness (N/m)	Vertical Damping (N sec/m)	Horizontal Stiffness (N/m)	Horizontal Damping (N sec/m)
Frequency Independent	1.734×10^8	2.203×10^6	9.89×10^7	1.392×10^6
Frequency Dependent	1.82×10^9~ 2.2×10^9	6.05×10^6~ 2.45×10^6	3.398×10^8~ 3.44×10^8	1.3×10^6~ 7.6×10^5
Delta (0–70 Hz)	10~13 Times Increase	3~1 Times Increase	3.4~3.5 Times Increase	0.9~0.5 Times Reduction

8. FE (Finite Element) Steady-State Vibration Analysis

Frequency domain steady-state analysis has been proposed to calculate only the permanent vibration amplitude from the basic equation of motion as follows.

$$[M]\{u''(t)\} + [C]\{u'(t)\} + [K]\{u(t)\} = \{p(t)\} \tag{3}$$

Converting to complex form, then the following is true.

$$[K + iwC - w^2M]\, a = p \tag{4}$$

in which $K = [K + iwC - w^2M]$ is a complex impedance matrix, $K - w^2M$ represents stiffness and the inertia effect, and iwC is the imaginary part considering the damping effect. Thus, the equation of motion can be expressed with a function of frequency as follows.

$$K(w)\, a(w) = p(w) \tag{5}$$

The solution from Equation (5) for multi-degrees-of-freedom (MDOF) can be easily calculated through the finite element analysis solver. For the given vendor's dynamic unbalanced forces (F) at operating speed, one can generate: em factor = $F/(2\pi f)^2$. The steady-state function value can be given by $\omega^2 = (2\pi f)^2$. Taking advantage of the efficient steady-state solver in SAP2000, any nodal amplitude of interest can be generated, along with the frequency range. This gives a full-spectrum view of dynamic amplitudes over the frequency range and elevates the reliability of a vibrational assessment. For instance, several nodal amplitudes of the compressor foundation are plotted in Figure 12 for a hysteretic damping of 1% and 2%. The frequency-domain analysis results show that the dynamic amplitudes reflect a significant and meaningful engineering mode and modal responses in the sense of resonance checking. The hysteretic damping effect on the amplitude calculation over the range is clearly shown. Therefore, the importance of the selection of a reasonable damping value should be emphasized. From these observations, the steady-state vibration analysis technique enables foundation engineers to utilize the exact soil-pile stiffness and damping at certain frequencies for the corresponding amplitude calculations over such ranges.

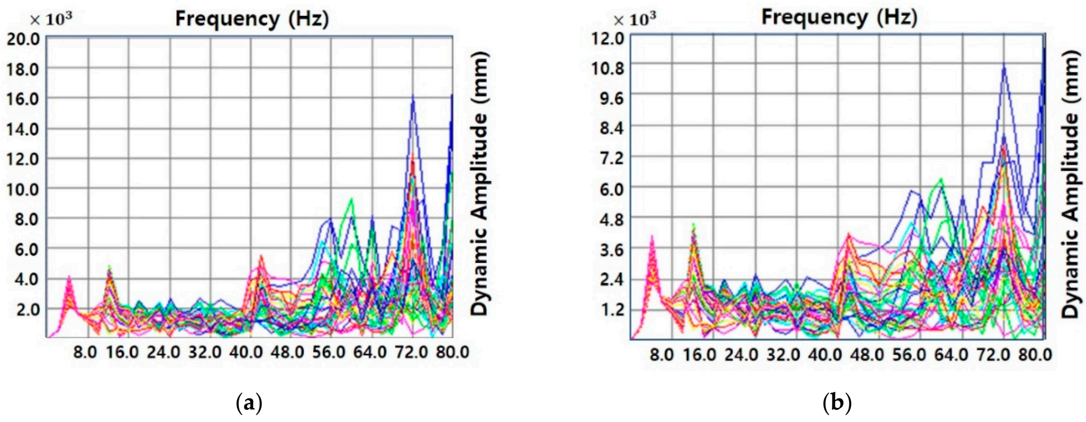


Figure 12. Steady-state vibration analysis considering frequency-dependent interaction: (**a**) hysteretic damping = 1% and (**b**) hysteretic damping = 2%

9. Discussion and Conclusions

The steady-state solutions by modal superposition analyses and direct integration time history analyses of the CTG and STG foundations were computed using several combinations of damping options, which would be generally available in commercial FE software. Frequency-domain steady-state analysis using a frequency-dependent soil-pile interaction was developed, and the effect on hysteretic damping was addressed. Based on the vibration response of the subject foundations, the following conclusions and recommendations can be made from this study.

1. The steady-state vibration velocity responses computed by SAP2000 and GTSTRUDL using modal superposition analysis shows good agreement in both CTG and STG foundations. The cases where there were minor differences are due to the consideration of the incompatible bending mode in the SAP2000 model, which significantly improves local bending behaviour at higher modal frequencies.
2. Modal damping analyses using 4% damping produce consistent vibration performances compared to those with soil radiation damping of 20% (cut-off by EPRI) plus 2% material damping. In other words, the total modal damping by the latter method, which largely depends on the deformation of the soil-foundation interface at the significant modal frequency, would be close to 4%. Modal superposition analyses using 2% and 10% modal damping to some extent overestimate and underestimate the vibration level, respectively. The direct use of damping ratios by the Elastic Half-Space Solution, without cut-off limits, results in an underestimation of the vibration response.
3. The two frequency-point based Rayleigh damping (proportional to mass and stiffness) in the direct integration analysis approximates the replacement of the participating modal damping. The CTG foundation's vibrational responses were almost the same, regardless of the variation of the first modal damping from 4% to 20%, together with 4% damping at 60 Hz. This is primarily due to the fast drop of damping after the first mode and the fast convergence of damping toward 4% until 60 Hz. Furthermore, the 60 Hz vibration response is generally less affected by the first modal damping. After the 60 Hz threshold, the Rayleigh damping increases gradually, but the increase would be negligible for the 60 Hz response.
4. Frequency-dependent vertical and horizontal stiffness gradually increased as the frequency grew. However, frequency-dependent vertical and horizontal damping decreased rapidly as the frequency increased.
5. A full-spectrum steady-state vibration solution compensates for the disadvantage of the modal time history solution, and, thus, increases the reliability of the foundational design. Foundation engineers will be able to estimate the prospective vibration level in a broad frequency range.

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