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Review

Numerical Reconstruction in Maritime Archaeology

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Abstract: Numerical reconstruction is the process of modeling, analyzing, and evaluating the performance of structures or a sequence of events using the finite element method and other numerical engineering methods. Although numerical analysis is used extensively in contemporary engineering problems, it can be equally useful in the study of ancient structures and events of the past. The materials and design of structures are different, e.g., when comparing a modern containership with an old galleon, but the main problem to be solved is essentially the same—will the ship sail efficiently and safely for many years? This paper aims to provide an overview of recent achievements in numerical reconstruction in maritime archaeology. Since it is clearly an interdisciplinary activity, research is often carried out within a specific project by project team members or interested groups of researchers. While the paper aims to provide a comprehensive overview of such efforts, special attention is paid to activities related to the AdriaS (Archaeology of Adriatic Shipbuilding and Seafaring) and NEREAS (Numerical Reconstruction in the Archaeology of Seafaring) projects supported by the Croatian Science Foundation, to which the authors actively contribute.

Keywords: ancient structures analysis; computational fluid dynamics; finite element method; maritime archaeology; numerical reconstruction

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1. Introduction

Maritime archaeology is a scientific discipline of archaeology that focuses on all human activities related to the sea [1]. As such, it examines all available material evidence, iconography and historical data and ultimately presents a narrative of our maritime past. However, such an achievement is not always simple, as the passage of time often reduces available sources of knowledge (e.g., shipwrecks may be heavily damaged by natural forces, to name just one example). In order to face such challenges, maritime archaeologists often need to perform methodological and systematic interpretations of the available data before drawing conclusions, [2].

Contemporary engineering branches, such as mechanical engineering and naval architecture, can analyze the statics and dynamics of structures with significant precision and reliability. This is the result of centuries of scientific achievements and the availability of state-of-the-art software packages, as well as significant computational power. Commonly, these engineering tools are used in the project design phase, where the response of the structure to given loads may be successfully predicted. In other words, by accurately understanding and modeling physical laws, it becomes possible to know in advance the properties of structures or the sequence of physical events involved [3]. Since the objects of interest in maritime archaeology are old structures, a justifiable course of action is to examine the possibility of using engineering tools to help archaeologists in the reconstruction and interpretation of the collected historical data.

When it comes to maritime transport as one of the main research topics of maritime archaeology, the similarity between historical and modern ships is greater than it seems at first glance. In both cases, the ship's structure is made of available materials, the shape of the hull has been optimised for performance by experience, and in its mature state, the ship as such is capable of successful and safe navigation for many years [4]. Shipbuilding, both historically as in the present, is limited by the simple, applicable laws of physics. Moreover, this can be stated for many other aspects of the maritime past: amphorae were the first standardised containers for maritime transport [5]; structural details are critical in ships, whether wood or steel; waves and wind generate environmental loads for each structure [6], and so forth. Therefore, in essence, it allows engineers to study both old and modern structures with the available state-of-the-art tools and methods. This paper presents an overview of such efforts.

2. Methodology

Examples of numerical reconstructions and consequent analysis of old structures and events in the literature are rare. However, a review of the available literature was done by consulting common scientific databases, such as Science Direct and Scopus. Only articles related to application of the finite element method and finite volume method in maritime archaeology were selected. The articles are then divided into groups, and a subsection in Section 4 is dedicated to each group.

3. On Numerical Reconstruction

In this paper, numerical reconstruction is defined as the creation of numerical models and the consequent numerical analysis of the performance of structures under given loads. For example, the strength of amphorae or simulation of a ship capsize can be analyzed this way.

The most common engineering method for assessing the strength of structures is the finite element method (FEM) [7]. At the user level, the large structure (continuum) is discretized by a series of small finite elements. Since the behaviour of each finite element is well-known, an assembly of all finite elements representing the structure is generated, and a system of equations is obtained that can be solved by a computer. As a result, the behaviour of the entire structure becomes known. Numerical analysis is usually divided into static and dynamic analysis. Static analysis solves a system of equations representing the equilibrium of internal forces (i.e., structural deformation) and external forces (i.e., applied load). These problems are easy to solve, and the results are reliable, especially for isotropic and homogenous materials commonly used in engineering, such as steel and aluminium. These materials have the same properties throughout the thickness of the material regardless of orientation. Matters can be more complex with other materials, such as wood. Wood is the material that remains after the end of the life of a living being. As such, its properties change through thickness and in different orientations, which is known to every builder of wooden vessels. Analysis of structures made of complex materials is more complicated. It should be emphasised that state-of-the-art engineering software is developed with contemporary engineering problems and materials in mind. However, today it is possible to analyze structures made of almost any material and even made of many different materials at once.

Discretisation of the structure into a mesh of finite elements is a specialist problem, but not the most important one in numerical reconstruction [7]. The numerical analysis will be most affected by a suitable choice and application of material models. A material model is essentially a fundamental set of equations that describe the properties and behaviour of a particular material. For simple materials, such as the aforementioned steel and aluminium, it is only necessary to know the modulus of elasticity (and Poisson's ratio). This is because there is a linear relationship between the applied force and deformation in structures made of such materials, i.e., twice the force, twice the deformation, [8]. To be sure, such a relationship cannot endure indefinitely. When excessive load is applied, a non-linear relationship between force and deformation will occur, and the structure will break. The linear relationship between applied force and deformation is also a property of ceramics, glass, wood, glass-fiber composite materials, and other materials that exhibit brittle behavior. Even so, many parameters are often required to define certain material models. Wood, for example, as one of the most interesting and important materials in maritime archaeology, is particularly difficult to model.

Another very important numerical method is the finite volume method (FVM), which is most widely used in computational fluid dynamics (CFD) [9]. Here, the physical domain, such as air and water, is discretised by finite volume elements. In general, transport of material is possible in each finite volume element, and the equilibrium of flux between adjacent finite elements is determined at each calculation time step. As a result, the velocities and pressure are known either within the domain or at its boundaries. CFD is widely used for the analysis of ship resistance, seakeeping problems, ship propulsion, wind turbine analysis, etc. CFD results are usually validated by experiments in a towing tank, where a ship model is manufactured and tested under controlled laboratory conditions.

Finally, it is possible to combine the two methods and analyze the fluid–structure interaction (FSI), i.e., directly assess the effect of the fluid on the structure and vice versa. An example of such an analysis is the study of a ship in waves—the ship interferes with the free propagation of the waves, and the waves affect the movement of the ship [10]. Such analyses are complex and expensive because they require powerful computers and are time consuming—one calculation can take several days or more. FSI analysis is able to capture much of the physics of the problem: different materials, environmental loads, gravity, contact, non-linear structural response, etc. This makes FSI analysis the most powerful numerical reconstruction method, but the obtained results must be carefully evaluated due to its complexity.

4. Numerical Reconstruction in Maritime Archaeology

The application of numerical reconstruction methods in maritime archaeology begins with the selection of problems that are of interest to archaeologists. Whether we are talking about history or the present, we live in a material world and skilled people transform materials from raw to usable form—wood is cut and reshaped into a boat, or clay is extracted and baked into useful amphorae. In any case, craftsmen must respect the limitations of the material and technology used and, in the end, put it to the test of reality. Maritime archaeologists and engineers should collaborate in selecting a problem suitable for numerical reconstruction. Maritime archaeologists are familiar with the historical data and have a list of open questions, while engineers estimate the feasibility of numerical reconstruction as a tool to obtain appropriate answers.

Two consecutive scientific projects financed by the Croatian Science Foundation can serve as an example of such interdisciplinary cooperation. The AdriaS project initiated and the NEREAS project continued the collaboration of archaeologists and engineers in the numerical reconstruction of maritime structures and events from the past. Within these two projects, the following open questions suitable for analysis were identified:

- Strength of amphorae;
- Strength of wooden structures and structural details;
- Simulation of a ship sailing, capsizing and sinking;
- Shipwreck forensics;
- Variations in the shape of the ship's hull.

Therefore, each topic will be discussed in the following subsections. However, this is not a complete list of numerical reconstruction topics. Other researchers have also considered the performance of ship sails, sedimentation at shipwreck sites, and other topics of interest to maritime archaeology.

4.1. Strength of the Amphorae

Amphorae are ceramic containers that have served humanity for millennia. They were made locally from material that was available near the production site. Since the material, i.e., clay, had different properties from site to site, the chemical composition of amphorae differed [11]. But their size and shape also differed, depending on the historical era, production skills, their purpose and the contents being transported. In any case, amphorae had to withstand operational loads and be suitable for manipulation and transport. They had to be strong enough to withstand the greatest possible load resulting from contact between them and nearby objects. The strength of the amphorae is partly determined by the properties of their material and partly by their shape.

One of the pioneering works in failure prediction and function determination, Kilikoglou et al. [12] applied FEM to estimate the strength limits of archaeological ceramics. Young's modulus of elasticity [13], and fracture strain data [14] were first calculated from transverse rupture strength (TRS) tests, Figure 1. Different combinations of quartz tempering and average temper size were considered, resulting in a variation of modulus of the elastic modulus from 6.88 to 22.4 GPa.

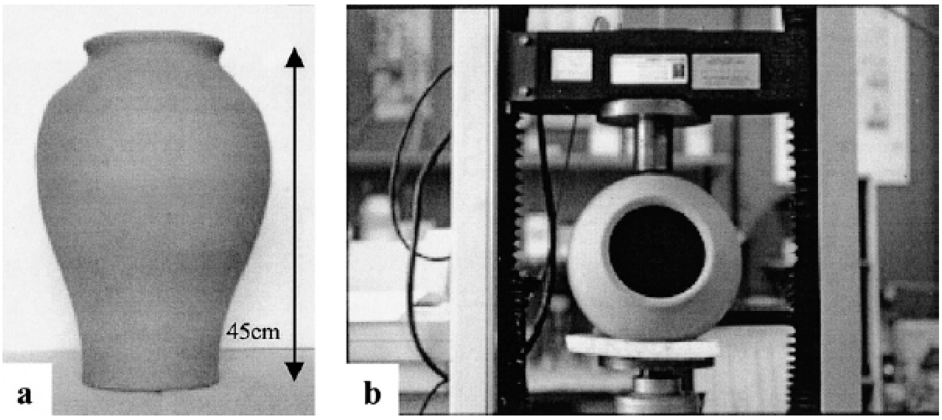

Figure 1. (**a**) Jar model manufactured for the development and testing of the FEM method. (**b**) Testing configuration of the jar model, reprinted from [12], copyright (2002), with permission from Elsevier.

Tsantini et al. [15] used a transverse rupture strength test (four-point bending test) to determine the strength of pre-Roman amphorae of various types and from various archaeological sites. Different equivalent fire temperatures (EFT) were also considered, showing a linear relation between TRS and EFT. Muller et al. [16] examined the impact resistance of archaeological ceramics based on variations in temper and firing temperatures. In this case, dynamic loads were considered using a pendulum impact tester. The relationship between TRS and firing temperature, and the amount of temper was determined.

In the FEM analysis by Kilikoglou et al. [12], the different wall thickness of the vessel ranged from 4 to 8 mm. As a result, the failure threshold line was determined as a function of Young's modulus of elasticity and strain [13].

Another pioneering work related to the strength of the amphorae is the consideration of the origin and function of the amphora's spike conducted by Radić Rossi et al. [17]. Many types of amphorae have a prominent spike at the bottom of the body. It is useful when the amphorae are placed in sand, soil, a bed of branches or another suitable substrate that makes their arrangement more stable. At the same time, the spike seems inconvenient in other cases, such as handling a single amphora, when it has to be left somewhere in an upright position. On the other hand, there are many amphorae with no prominent spike or no spike at all. Therefore, the question arises: what is the purpose of the amphora spike in the context of an amphora's strength? The chronological selection of amphora shapes is shown in Figure 2. The simplified evolution line shows an increase in size accompanied by an increase in the spike. The reduction in size in Late Antiquity and the Early Mediaeval periods again resulted in the elimination of the spike while retaining the rounded or conical bottom.

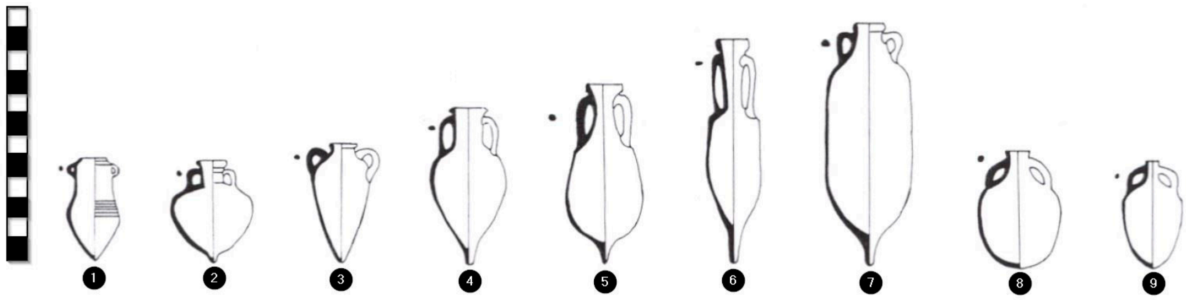

Figure 2. A chronological selection of amphora shapes: (1) Phoenician amphora, 7th century BC; (2) Corinthian amphora, 6th–2nd century BC; (3) Etruscan Py 3B amphora, 6th century BC; (4) Graeco-Italic amphora, 4th–3rd century BC; (5) Italic Lamboglia 2 amphora, 1st century BC; (6) Italic Dressel 1b amphora, 2nd–1st century BC; (7) North African Africana Grande (I) amphora, 2nd–5th century AD; (8) Aegean Late Roman 2 amphora, 4th–7th century AD; (9) East Mediterranean Late Roman 1 amphora, 5th–7th century AD; reprinted from [17], courtesy of the authors, 2023.

The analysis began with an estimation of the basic material properties, bending strength and modulus of elasticity on a ceramic sample that was partially exposed only to seawater, and partially to the joint action of sea sand and water. Finite element models of four representative amphora types were then generated: Py 3B, Lamboglia 2, Dressel 20 and the hypothetical Py 3B with a flat bottom. The models were exposed to different types of loads, and a linear static analysis was performed [17]. The results clearly showed that the conical shape of the amphora has a protective function. The prominent spike protects the amphora even more, especially in the case of vertical load (for example, when placing the amphora on a hard surface). Another conclusion is that a rounded body is desirable in case of horizontal loading (such as contact with other, similar containers).

Hein et al. [18] investigated the strength of the Koan amphorae from Halasarna; see Figure 3. The authors pointed out that amphorae had to fulfil requirements in terms of mechanical strength and toughness, but also in terms of standardisation of the vessel's shape and size. The detailed chemical composition of ceramics and clays was determined first. After that, FEM analysis of different amphorae was conducted. Hein et al. [18] concluded that the change in the shape of the Koan amphorae led to an improvement in mechanical performance, which is an example of the development of a functional design. Hein et al. [18] considered the modulus of elasticity value of 18 GPa, while Radić Rossi et al. [17] measured the modulus of elasticity value of 8 GPa according to the standard test procedure DIN 53427.

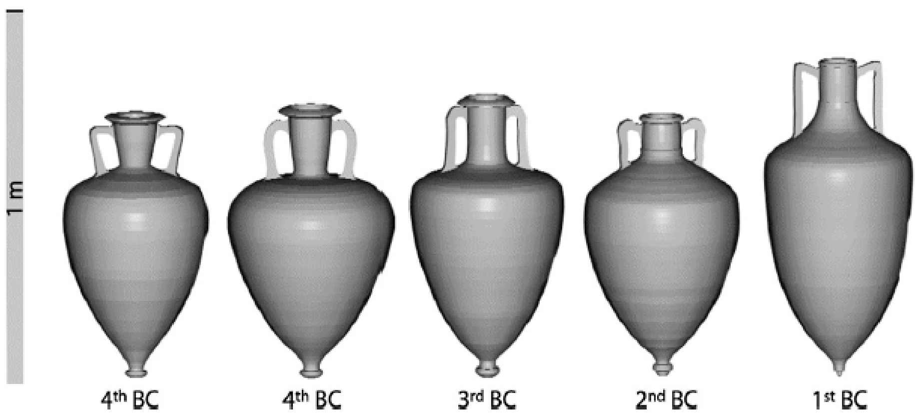

Figure 3. Digital models of amphorae, which were examined via FEM; reprinted from [18], copyright 2008, with permission from Elsevier.

De Francesco et al. [19] performed a similar analysis, which determined the archaeometric characterisation of Roman Imperial amphorae and bricks found in the villa near the town of Luzzi in Calabria in Italy.

Hein and Kilikoglou [20] presented a FEM analysis of the contact problem by simulating loads that occur in the transport of amphorae and that can lead to the breaking of ceramic vessels. A Hellenistic wine amphora was used as a case study. A very detailed FEM model consisting of volume finite elements of two bodies in contact was generated (Figure 4). The ANSYS software package was used to perform contact problem analysis. The authors found an increase in the contact area due to an increase in load and concluded, by comparing the obtained strain with a critical strain of 0.11%, that an amphora is subjected to failure when the load at the contact point exceeds 1000 N.

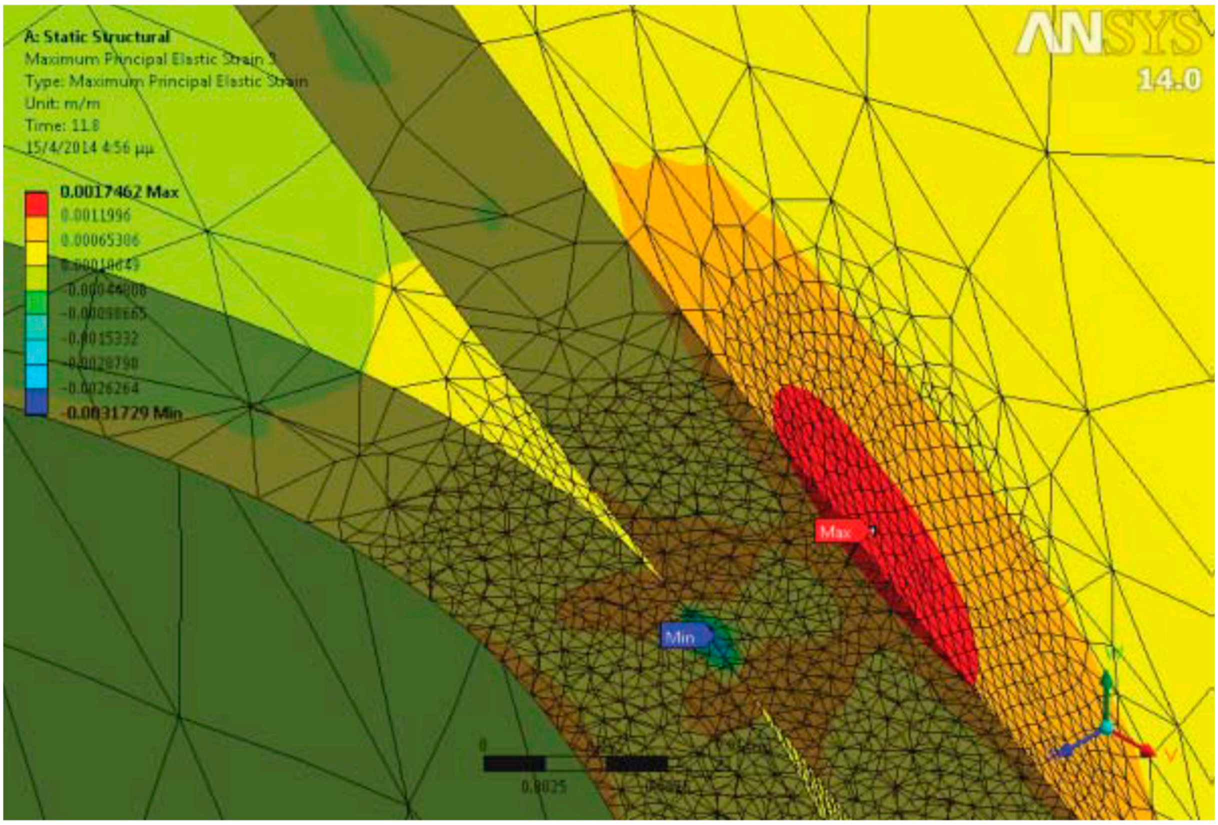

Figure 4. Simulated strain distribution in two Rhodian amphorae with a load of 1000 N. A section through the contact area is shown; reprinted from [20], courtesy of the authors, 2023.

Recently, Sviličić et al. [21] performed a sophisticated numerical non-linear FEM analysis to determine amphora resistance to ruptures and cracks. The LS-DYNA software package (version R11.1) was used for the calculation, and the Johnson–Holmquist material model was chosen for damage modeling. Since the parameters of the material models were not readily available, a sensitivity analysis was first performed to determine their most appropriate values (Figure 5).

Three different amphorae were taken for a free-fall drop test from different heights. Finite element models of the same amphorae were generated, and a simulation of the crush test was carried out with the correct physical constraints: gravity, drop height, ceramics material properties, etc. The results of experiments and numerical simulation were compared, as shown in Figure 6.

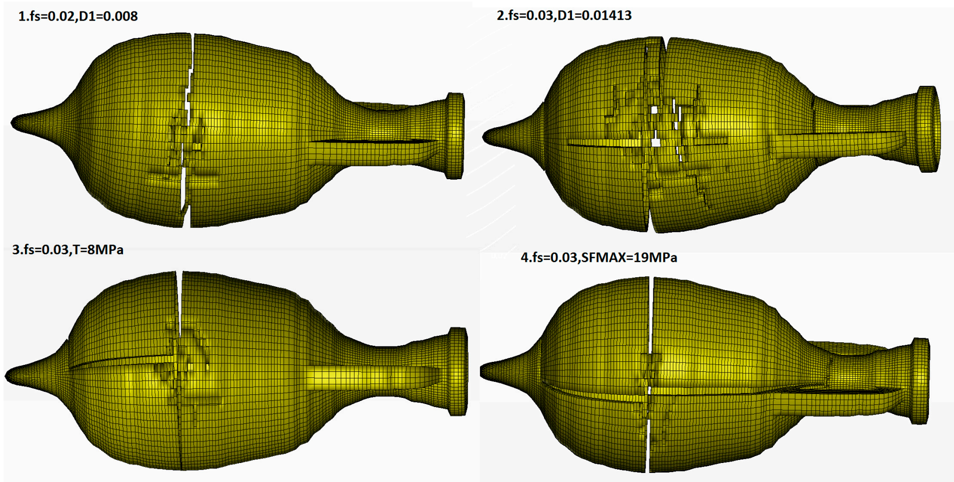

Figure 5. Sensitivity analysis of the Lamboglia 2 amphora type; reprinted from [21], courtesy of the authors, 2023.

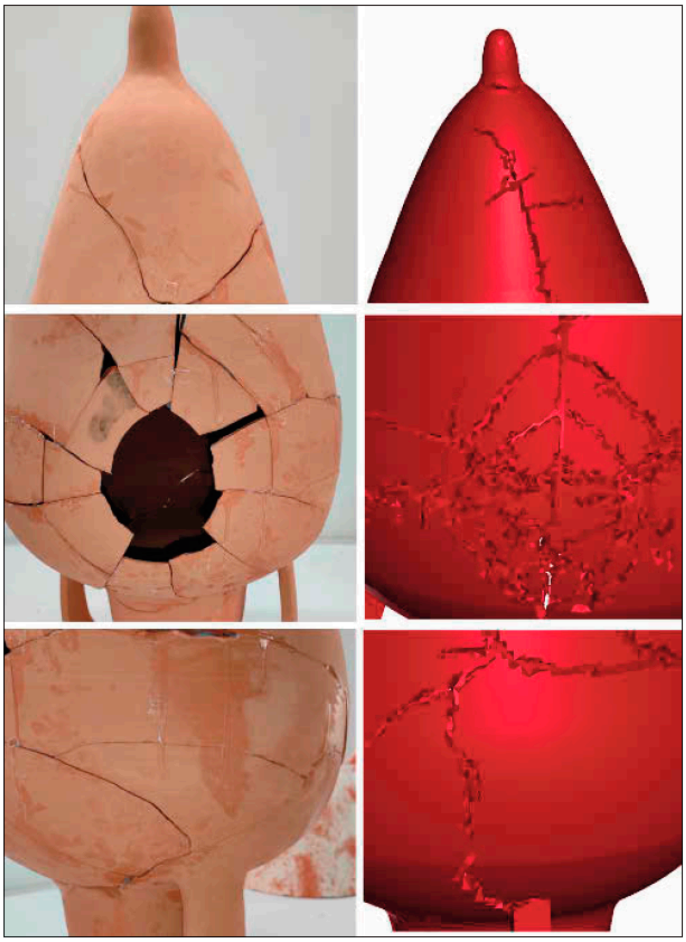

Figure 6. Comparison between experimental and numerical results for Model 2, reprinted from [21], courtesy of the authors, 2023.

FEM analysis again revealed the protection function of the amphora spike, but many other important findings were also obtained. The main contribution of the article, [21], is a set of material model parameters that can be used in future research, e.g., the evolution of the amphora shape. Furthermore, recommendations for the finite element modeling of ceramics are provided.

Valuable research on transport amphora fracture strength, deformation and distribution is presented in Hein et al. [22]. The authors attempted to propose a calibrated material model applicable to the numerical simulation of biaxial flexure tests, starting with a linear elastic model followed by a bilinear hardening model. Experimental and numerical tests were performed for different amphora-type ceramics: Koan (Hellenistic), Koan (Roman), Rhodian A and Rhodian B, as shown in Figure 7. The authors presented major element concentrations for each amphora type, clearly indicating possible differences in material strength. Indeed, the experimental test confirmed a variation of biaxial flexural strength ranging from 7.6 to 34.9 MPa. Numerical simulation of the biaxial flexural test indicated that the linear elastic material model over-predicts specimen response. The bilinear hardening model on the other hand enabled acceptable prediction of contact areas and load–displacement curves, but the authors indicated that further research is needed for material elastic and plastic moduli, that is, material model calibration.

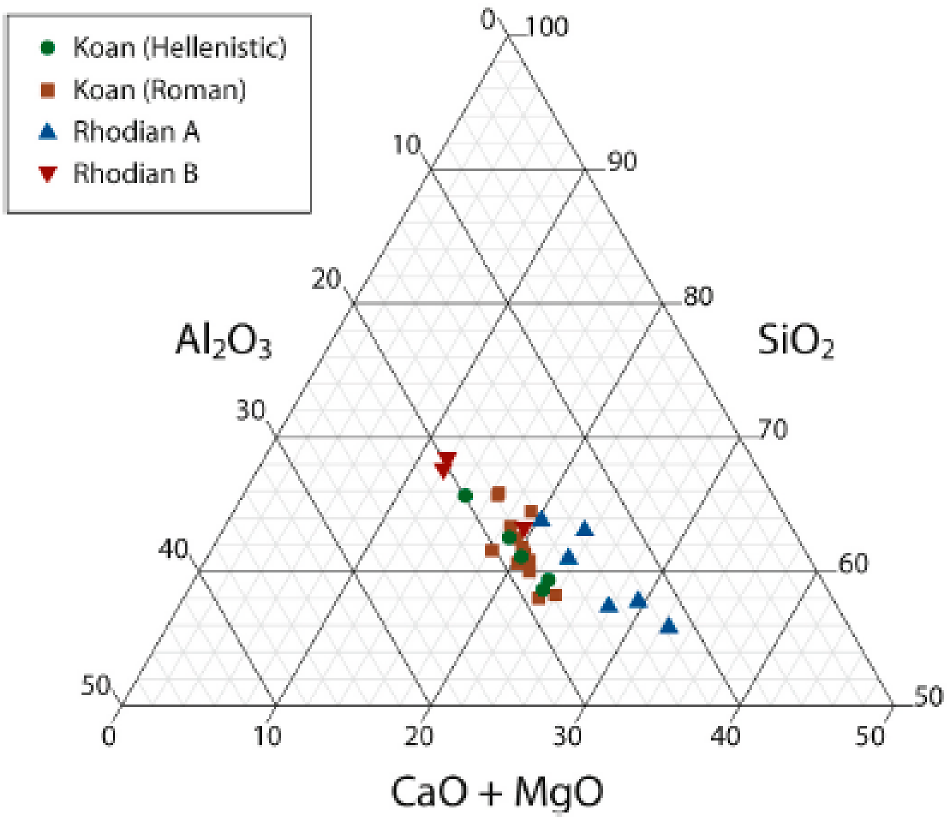

Figure 7. Major element concentrations in different amphora types, reprinted from [22], courtesy of the authors, 2023.

The role of amphorae strength in the context of their transportation was studied by Hein and Kilikoglou [23]. While the packing of the amphorae was not studied in detail, the authors proposed an assumed and idealised arrangement of amphorae in two layers. As expected, this resulted in a small contact area between amphorae, leading to the localization of the stress, as seen in Figure 8. Ten different amphorae finite element models were generated, using two different finite element types: solid shell and tetrahedrons. Finally, models were subjected to horizontal load and the resulting maximum tensile stress was estimated. Based on comparative analysis results, the authors concluded that mechanical performance, as a criterion in amphora development and design, is a valid assumption.

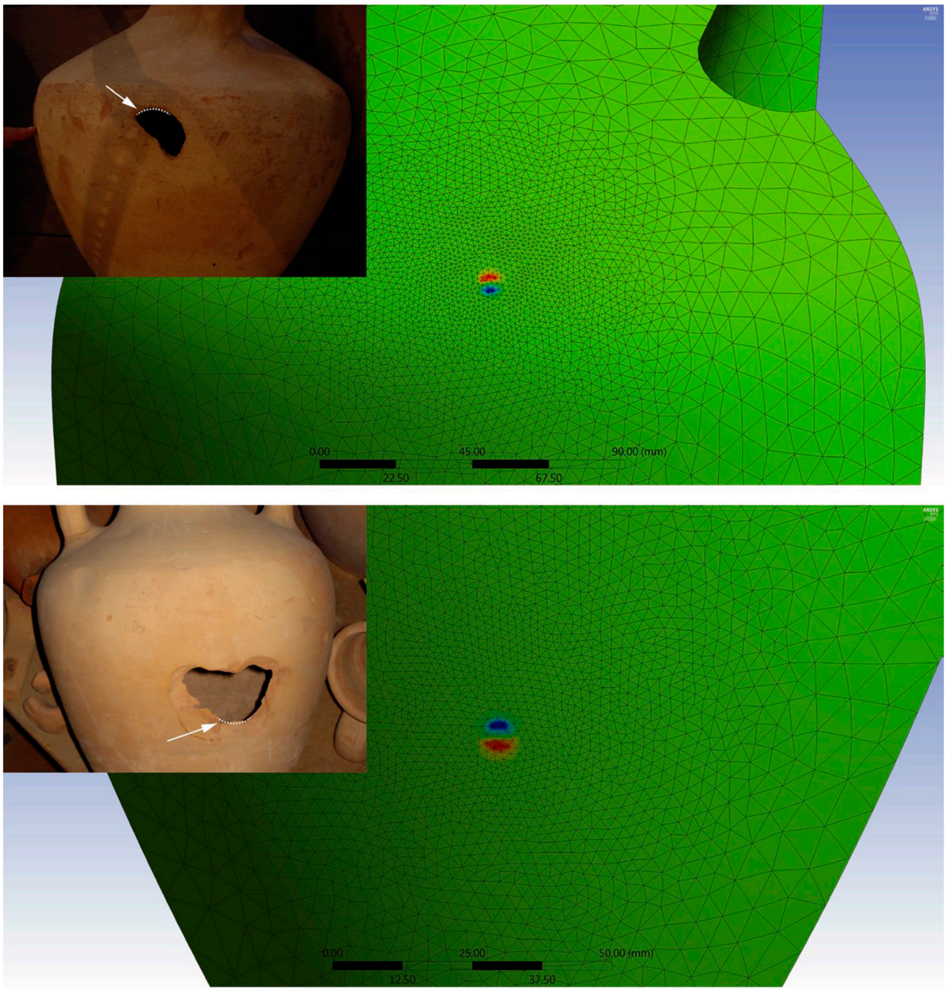

Figure 8. External surfaces of the two Rhodian amphorae: Crack initiation is expected in the areas of maximum tension (red) with the crack propagating perpendicular to the direction of the first principal stress, which in both cases is oriented toward the contact area (blue). The photographs present damages observed in amphorae of this type, which might be related to vertical loads during storage reprinted from. Arrows presenting observed area of crack initiation on real amphorae [23], courtesy of the authors, 2023.

4.2. Strength of Wooden Structures and Structural Details

The strength of wooden structures and their structural details are important in nautical archaeology, since wooden ships were the only ships that sailed around the world until recently [24], roughly a century and a half ago (some of the first ships made of steel are Redoutable, the French Navy ship launched in 1876, and HMS Iris, a British Royal Navy ship launched in 1877) [25]. Unlike steel, which is an isotropic and homogenous material with excellent manufacturing properties [24], wood is an organic, anisotropic material that requires the special design of its details and joints [24]. One should bear in mind the fact that a tree was once a living being with unique properties depending on its geographical location, growing conditions, etc.

The paper by Fujita et al. [26], "Shell strength tests and structural analysis of large wooden ships," is one of the pioneering works in this field. It was written in Japanese for the Society of Naval Architects of Japan, with a summary in English.

Another noteworthy pioneering work is Kenn Jensen's doctoral dissertation [27], which presents the documentation and analysis of ancient ships. The ship Helge Ask was used as a study case, and two numerical methods were applied for analysis: the beam model method and FEM. The dissertation provided valuable insights, such as the torsional moment distribution and distribution of the shear stresses along the hull length, through the application of FEM in the structural analysis of historical wooden ships. It also underscored the complexity of wood as a structural material in the context of numerical reconstruction and the need for careful determination of material model parameters, such as Young's modulus of elasticity.

Mackerle [28] published a review paper entitled "Finite element analyses in wood research: a bibliography". Of the approximately 300 listed references, only a few pertain to historical structures, such as bridges or household structures. Nevertheless, it showed the potential of FEM in the numerical reconstruction of wooden structures in general.

In 2009, Levy and Dawson [29] published an article using FEM to analyze an ancient whalebone house structure. In addition to the findings associated with whalebone, the authors concluded that they "believe FEM to be an effective tool for examining architectural practices in other archaeological context". In the same year, Guan and Zhu [30] presented a way to analyze anisotropic timber composite beams with openings using FEM.

Stoyanov et al. [31] presented a structural analysis of the Cutty Sark historic ship, the 140-year-old (at that time) last surviving clipper in the world. The authors defined the smeared shell modeling approach as a new technique suitable for the rapid analysis of complex heritage structures (Figure 9).

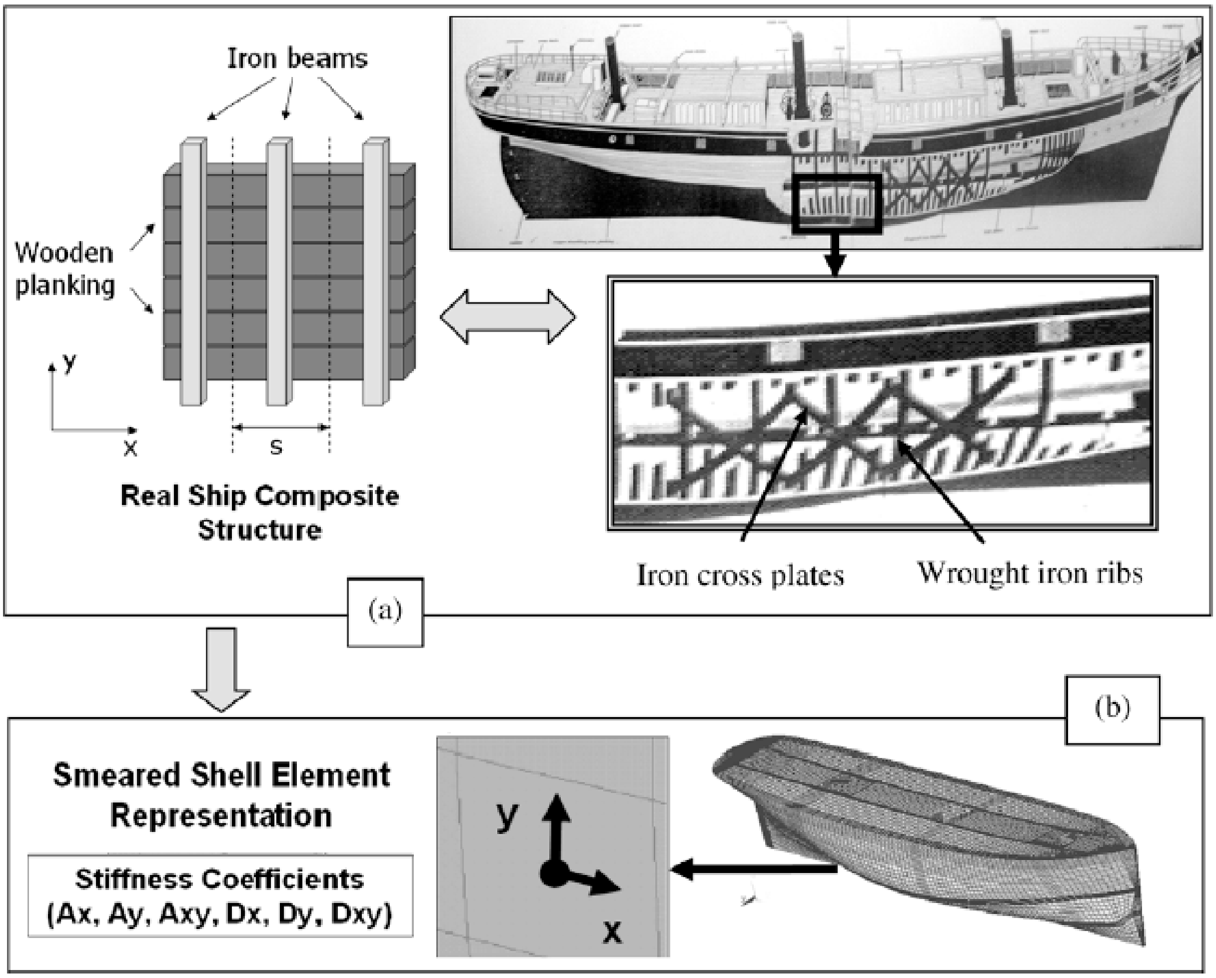

Figure 9. Composite iron–wood ship structure (**a**) and its modeling through shells with smeared properties (**b**); reprinted from [31], copyright (2010), with permission from Elsevier.

Dabbagh and Garza [32] conducted a FEM analysis of the bottom structure of the Vasa, a Swedish royal warship that sank on its maiden voyage shortly after launching. The Vasa was salvaged in 1961, but the current support system was not suitable for the ship's weight.

The authors analyzed the interaction between the new support system and the structure of the ship's bottom. The ABAQUS software package was applied for FEM analysis using a simple finite element model of the beam. However, structural joints and planking systems were taken into account and deformation and stresses were presented.

While not considering historical structures, del Coz Diaz et al. [33] presented a nonlinear numerical analysis of plywood board timber connections. FE analysis of the mechanical performance of timber joints was compared to experimental results, which can serve as a validation reference in the analysis of structural details of wood.

Invernizzi et al. [34] carried out numerical modeling and an assessment of the brig Ebe Schooner. Both two-dimensional and three-dimensional FEM models were generated to cross-check various simplifying hypotheses. The research was motivated by significant deformations on the deck and keel of the ship in the museum. Different wood properties were assigned to the two-dimensional FEM model, and stresses and deformations, including structural details, were ascertained in the cross-sectional model (Figure 10).

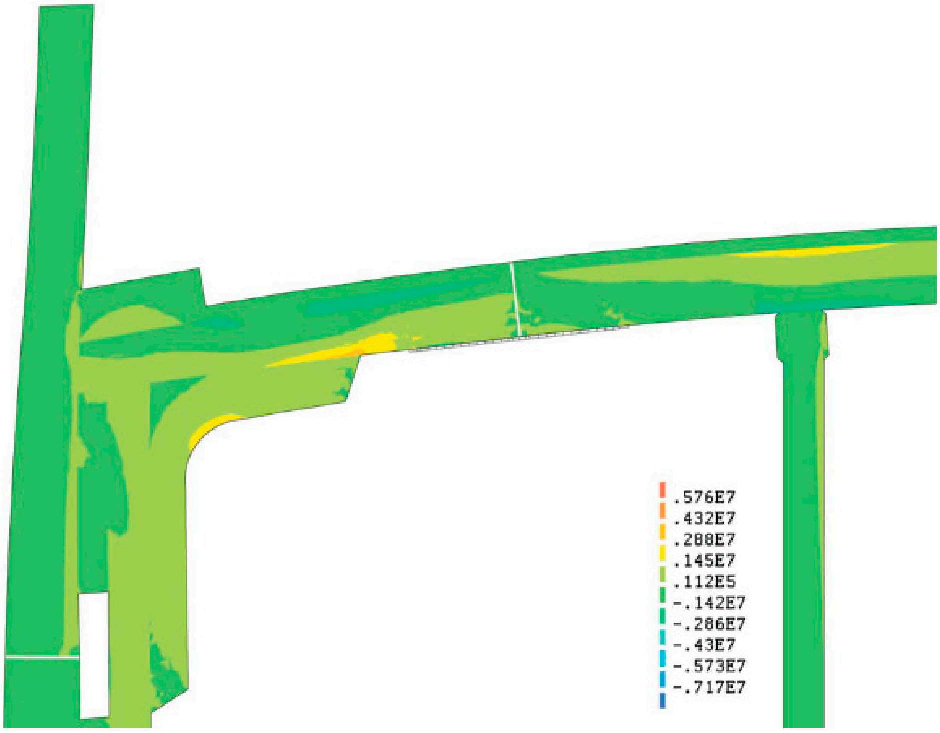

Figure 10. Contour plot of the stresses σ_0 parallel to the wood fibres, in the cut area of the upper deck beam (colour figure available online); reprinted from [34], copyright (2012), with permission from Taylor & Francis.

A three-dimensional FEM model of the entire schooner was generated using beam finite elements [34]. Such a model is inadequate for an analysis of structural details, but it takes into account the interaction of all primary structural elements. An important finding is that the time-dependent behaviour of wood should be considered.

Analysis of the strength of traditional wood ships in Indonesia was conducted by Malisan Johny et al. [35], but only theoretically. Koch et al. [36], on the other hand, experimentally and numerically conducted a multimode failure analysis of form-fitting timber connections. A tapered tenon joint was considered, as shown in Figure 11. The authors pointed out the importance of the eccentricity of the joints and the weakening of structural members by the mortise. Furthermore, three types of failure were identified: rolling shear failure and front side and back face failure. The ABAQUS software package was used for the analysis, with the use of volume finite elements and elastic material parameters.

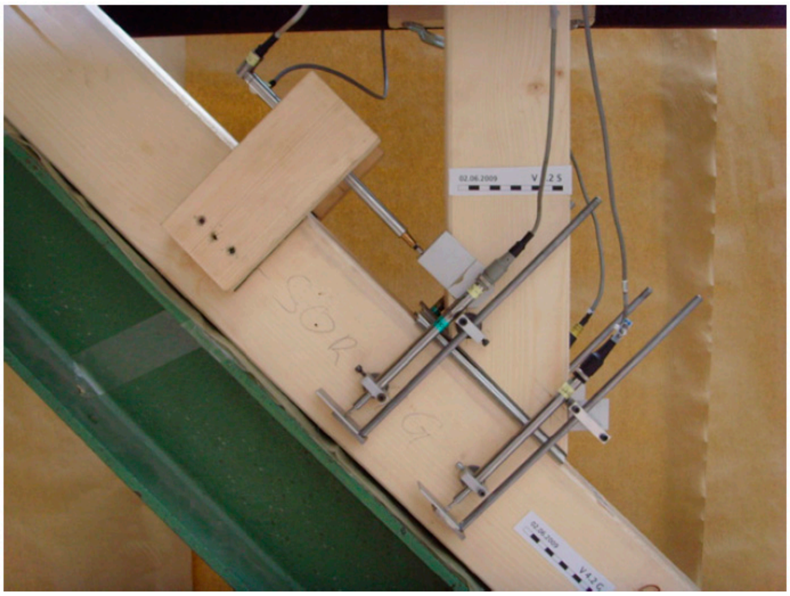

Figure 11. Arrangement of samples in the testing frame; reprinted from [36], copyright (2013), with permission from Elsevier.

A paper discussing traditional Chinese timber structures, Yue [37], also considers structural joints in wooden structures. Various types of joints were presented, and the selected joints were subjected to mechanical tests: wedged, corner cross, half penetrate, bread and sink wedged. Since not much detail is given about the FEM analysis performed using the ANSYS software, a comparative analysis of the experimental results remains the paper's main contribution.

Arciszewska-Kedzior et al. [38] conducted an experimental and numerical analysis of the lapped scarf joint. Such a joint was commonly used to repair damaged beams in historic structures. The numerical model of the lapped scarf joint is shown in Figure 12.

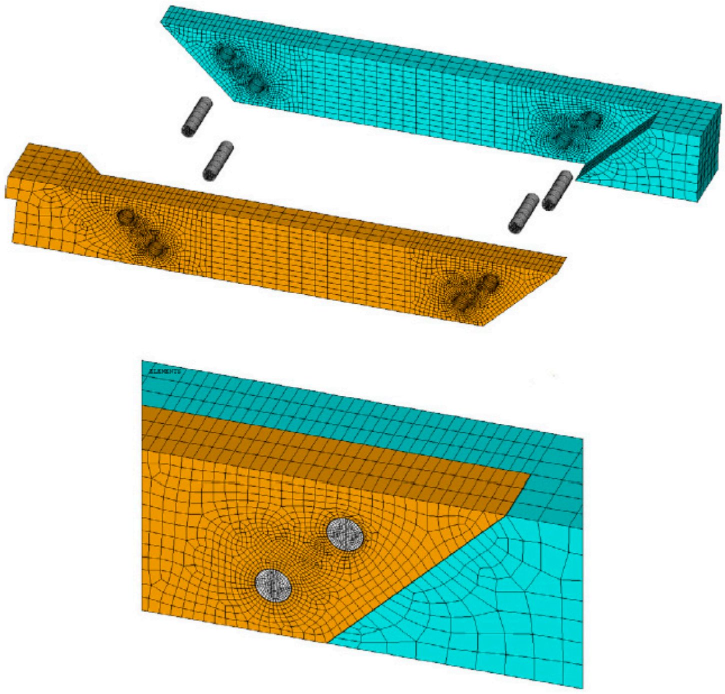

Figure 12. Numerical models of a four-dowel joint (**top**) and detail on the contact surface and dowels (**bottom**); reprinted from [38], copyright (2015), with permission from Elsevier.

First, a three-point bending test was performed to evaluate the yield load and stiffness of the timber beams with various lapped scarfed joints, from which the corresponding force–displacement curves were obtained. The numerical model was generated and analysis was performed in ANSYS software using volume finite elements. In addition to validation of the experimental results, numerical models were used for sensitivity analysis [38]. The number of dowels, the location of the joints along the beam, and the length of the joint were considered.

Other authors contributed as well. The flexural behaviour of wooden dovetail mortise and tenon joints was analyzed by Chen et al. [39], again via experimental and numerical analysis using the ABAQUS software. Milch et al. [40] conducted a numerical assessment of full-scale truss structures, reconstructed using traditional all-wooden joints, applying ANSYS software. Fajman and Maca [41] conducted an experimental and numerical analysis of the stiffness of scarf joints with dowels using the ANSYS software.

A systematic study of a section of the Vasa warship replica was conducted by Afshar et al. [42], with the aim of identifying the behaviour of structural joints. A selected section of the ship was modeled in detail: planking, frame, ceiling, futtock, rider (knee), transverse beams and support column. Fresh oak was the building material, and the corresponding material properties were introduced into the FEM model. Expensive non-linear FEM analysis was carefully simplified by defining a fully bonded contact. As such, the FEM model showed a higher stiffness than anticipated from the experimental data. Normal and tangential stiffness penalty factors were applied to compensate for that. Figure 13 shows the model, with loading and boundary conditions in the case of bending–compression loading.

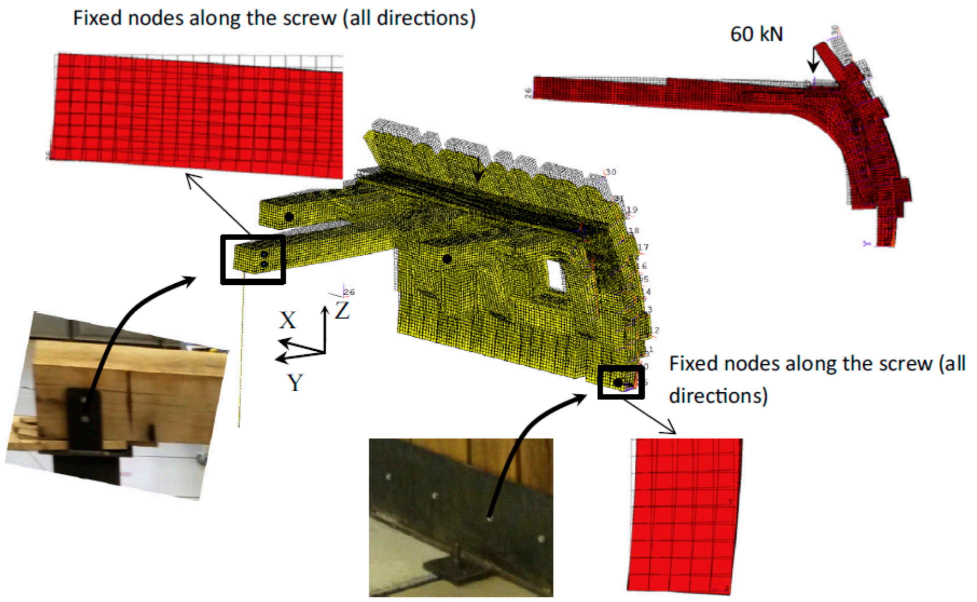

Figure 13. Boundary conditions for bending–compression loading; reprinted from [42], copyright (2017), with permission from Elsevier.

Afshar et al. [43] extended the study and conducted full-scale finite element modeling and analysis of the warship Vasa. The methodological approach and preliminary results were presented. The FEM model of the Vasa and a drawing of the longitudinal cross-section are shown in Figure 14. Prior to generating the geometry of the FEM model, a CAD model was created that required considerable effort, not only in the modeling process but also for onboard measurements. The purpose of the analysis was to capture the ship's response to current and potential future support solutions. ANSYS software was used for the analysis.

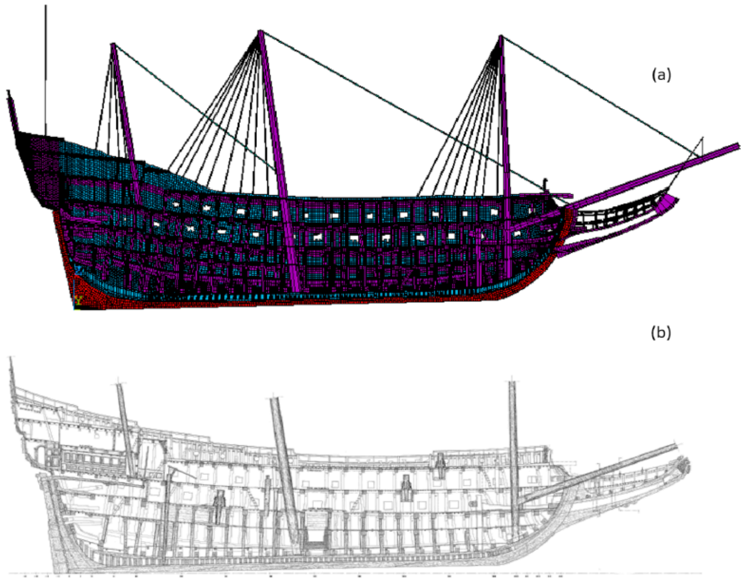

Figure 14. Longitudinal cross-section of the ship, comparing (**a**) FE model with (**b**) drawings provided by the Vasa Museum; reprinted from [43], copyright (2021), with permission from Elsevier.

Helfman et al. [44] used linear static finite element analysis to evaluate and compare the strength of shell-first and frame-based types of ships, and to verify whether mechanical factors contributed to the historical transition in ship construction. For this purpose, the Ma'agan Mikhael shell-first and Dor 2001/1 frame-based ships were modeled, due to similarities in their length and beam, although the Dor 2001/1 has approximately 40% greater displacement, as shown in Figure 15. However, comprehensive loading is considered for both hogging and sagging sailing conditions: hydrostatic load, gravity (acting on ship and cargo), as well as wind and torsion effects. Together with global model analysis, two local models were analyzed using FEA; the first was used to verify if the assumption of modeling the ship hull as a contiguous body is valid, while the second was a multiple frames partial model, loaded with a set of simple, multidirectional loads. While the structural analysis lacks both precision and the data required for the potential repetition of the analysis, the approach to the problem was consistent, and the study conclusions indicate a consistent and significant advantage of the shell-first method over the frame-based method.

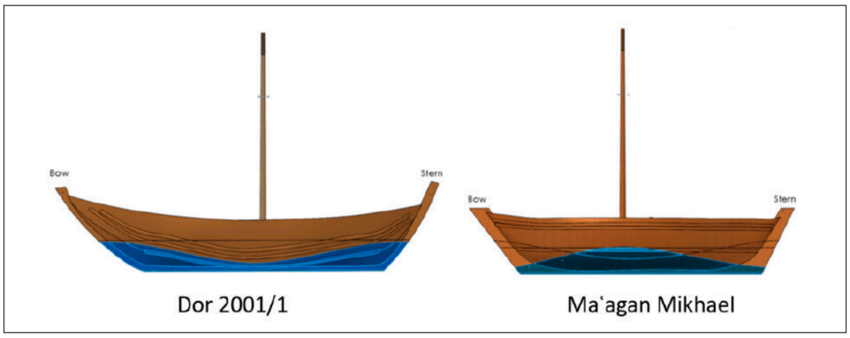

Figure 15. Stillwater hogging and sagging treatment: Dor 2001/1 sagging; Ma'agan Mikhael hogging; reprinted from [44], courtesy of the authors, 2023.

Additional details about the aforementioned analysis are provided in the following paper by Helfman et al. [45]. Here, three critical interdependent factors are studied: the number of transverse frames, the number of longitudinal reinforcements and their relative location. Three main ship hull reinforcement schemes were examined and the corresponding rigidity was determined, as shown in Figure 16. It has been shown that the insertion of longitudinal reinforcements contributes to the structural integrity of a frame-based hull, reducing the von Mises stress in all experimental load regimes.

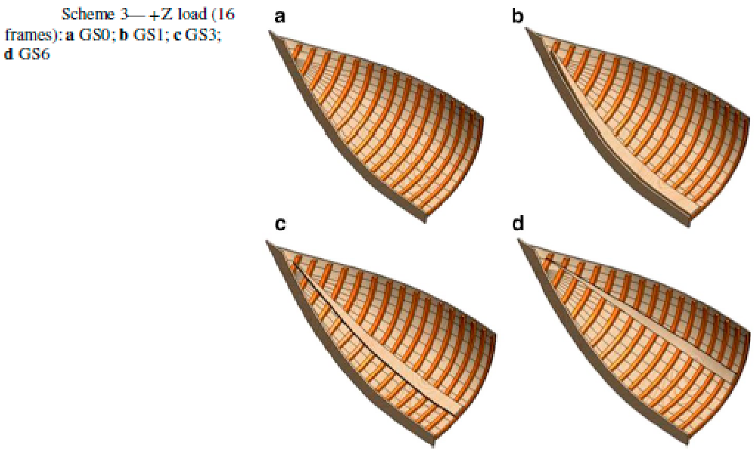

Figure 16. Frames of ship-strengthening scheme no. 3–16 plus different generic stringers locations; Variants a, b, c and d indicate location of generic stringer (GS)—no stringer (**a**), stringer at strake 1 (**b**), stringer at strake 3 (**c**) and stringer at strake 6 (**d**), respectively; reprinted from [45], under open-access CC-BY 4.0 license, 2023.

Recently, Eliav and Helfman [46] used finite element analysis to examine the feasibility and weight benefit of trireme construction with laced planks instead of mortise and tenons. A 36.9 m long Olympia trireme was used as a reference, as shown in Figure 17. On the basis of the performed finite element analysis, the authors concluded that the laced hull in hogging load condition is seaworthy, with a weight reduction of almost 50%. The shear-bearing capacity of laced joints was not examined, and instead, the emphasis was placed on sensitivity analysis. Different elaborated assumptions were made in performing the analysis regarding the type of wood, payload, and hypozomata rope tension load. Maximum compression stress determined in the keel showed that the laced design holds a reserve factor of 2.7, which is higher than the EN 1995-1-1:2004 Eurocode 5 requirement of 2.3 for the permanent load and of 1.8 for the short-term load [47].

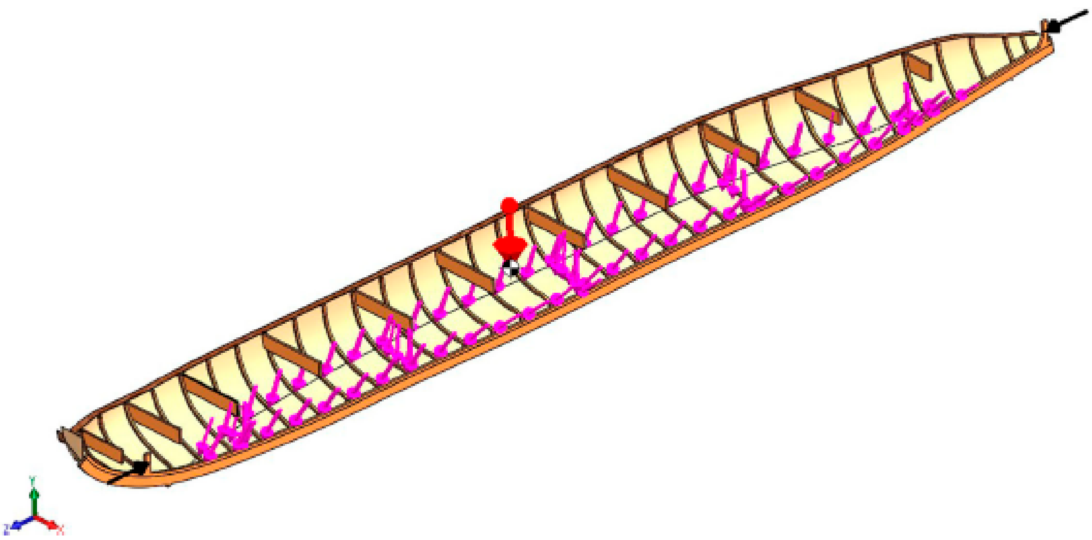

Figure 17. Payload distribution and hypozomata tension vectors (authors' CAD depiction); reprinted from; Arrows indicate load applied: red for the light ship weight, pink for the distributed weight of man, cargo and equipment and black for horizontal hypozomata tension [46], copyright (2022), with permission from Taylor & Francis.

Site formation processes are examined and studied by Foecke et al. [48] on battleship USS Arizona. The authors generated a 25 m long midship section finite element model consisting of 57,000 finite elements, representing a rather detailed structural mesh. The loads (own weight and encrustation weight) and material properties were fixed, while the density of the elements was increased to induce an increase in the stress that normally occurs due to corrosion thinning of structural elements. The linear corrosion rate was assumed when modeling the corrosion effects. As the finite elements were removed from the model once the stress exceeded a critical value, it was possible to visualise the decomposition process. Structural analysis was performed to simulate the deterioration of the wreck from years 1980 to 2240, when 90% of thickness loss is expected, and when large parts of the ship structure will collapse. Furthermore, analysis revealed that the rate of decomposition of the structure's exposed parts had tripled in comparison to the parts buried below harbor's floor. It is worth nothing that oil-containing spaces are situated below the harbor floor.

4.3. Simulation of Sailing, Capsizing and Sinking of the Ship

With advances in available computational power and software packages, it has become possible to apply numerical calculations to a wide range of problems, including those of ship sailing, capsizing, and sinking. Modeling each of these events is not a simple task, but it can provide valuable insights into the ship's response to environmental loads. Palmer [49] described the windward sailing capabilities of ancient vessels, suggesting that windward sailing ability was much less widespread than is commonly assumed. Computational fluid dynamics (CFD) is a numerical method available for testing hypotheses involving the interaction of structure and fluid, such as a ship on waves and in the wind.

Lasher and Flaherty [50] performed CFD analysis of a square-rigged sailing vessel's survivability. To validate the developed CFD model, wind tunnel tests and full-scale experiments were performed. The US Brig Niagara was used as a case study. The finite volume method employing the SIMPLE technique for determining pressure was used by the FLUENT software. Predicted and measured heel angles were compared for various sailing scenarios.

Ciortan and Fonseca [51] carried out a numerical simulation of the sails of the 16th century Portuguese nau, Nossa Senhora dos Martires. After the ship's excavation in 1996, it became possible to reconstruct its hull, but also the above-water structure, including the masts, yards and sails, as well as the latter's aerodynamics. The authors used the well-known Star-CCM+ CFD software to explore the combination of wind incidence angles and yard angles on the ship's sailing performance, as shown in Figure 18.

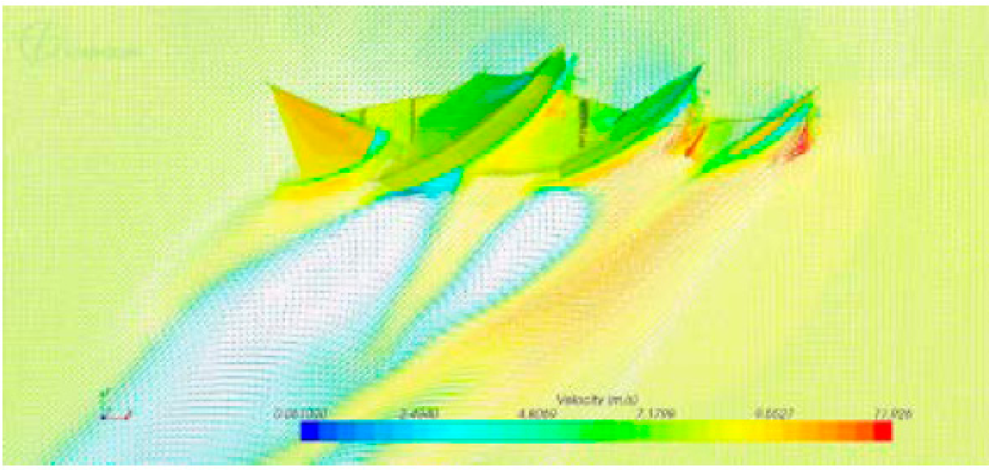

Figure 18. Velocity vectors around the hull and sails in a horizontal plane 15 m above the free surface; beta = 60 degrees and gamma = 35 degrees; reprinted from [51], courtesy of the authors, 2023.

Stettler and Thomas [52] performed the flooding and structural forensic analysis of the sinking of the RMS Titanic. Both the flooding and structural failure of a ship are complex events. Therefore, the authors separated the analysis. Using GHS (General Ship Hydrostatics) software, the authors estimated the flooding sequence. Structural analysis was performed using the well-known MAESTRO structural analysis software for hull failure and detailed stress analysis. A global FEM model of the Titanic was created to estimate the stress distribution, but also list the uncertainties that are almost inevitable in such a complex analysis. The presented flooding calculation method is applicable to ancient ships, although they have a much simpler structural arrangement.

More complex environmental conditions were considered in Kery [53]. The paper recounts three ships that sunk in heavy weather, and one suffered serious cracking (namely the Olympia, the sistership of the Titanic and Britannic). Numerical simulation of such events begins with the retrieval and interpretation of wind and wave data, followed by consideration of vessel properties such as weight, trim, stability, etc. These data act as input to software such as Orcaflex or Wasim (part of the Sesam software package) and other software that can evaluate the response of the ship to the waves until the sinking ends. Even if the details of the specific analysis are not presented, the paper can serve as a methodological guideline with the general theoretical background needed to perform such a complex analysis.

Hydrodynamic analysis of traditional vessels of Kerrala was conducted by Subbaiah et al. [54], using the Shipflow software. A study of the ideal draft, wave pattern, frictional and wave-making resistance were studied and compared. Rudan and Radić Rossi [55] performed a numerical simulation of a sinking ship, and Kyrenia was used in this case study. The analysis was divided into two phases; in the first phase, the movement of a ship with cargo exposed to side waves was analyzed. The simulation was performed in the LS-DYNA software package in the time domain, using the arbitrary Langrangian–Eulerian (ALE) method and fluid–structure interaction (FSI) coupling. As a result of this simulation, the movement of the ship, as well as its cargo, was recorded. In order to reduce the computational cost, the analysis was performed as a 2D simulation, capturing the degrees of freedom of roll, weight and sway, as expected in the lateral motion of the ship in waves. In the second phase, these movements were superimposed on a much more complex 3D model of the ship containing 48 freely moving amphorae, each in contact with the other and with the ship itself. However, when the limited motions that came from the first phase ended, i.e., when the ship went under the surface of the water as a result of gradual flooding, the model was left to sink freely to the seabed (gravity, residual buoyancy, sea current and contact with the seabed and between all bodies were unconstrained). Figure 19 shows a fine mesh model of the ship and amphorae in its resting position on the seabed. As a result of the simulation, the final position of the cargo on the seabed was determined. Depending on the simulated scenario, archaeologists expect to find this arrangement of cargo at the shipwreck site.

Figure 19. The sunken ship in the resting position on the sloping seabed (**left**) and the final position of the ship's cargo—the hull has been removed from the view (**right**). Different colors indicate different parts (or structures, such as ship, amphorae or sea bottom), and each part is in contact with all the nearby parts; reprinted from [55], courtesy of the authors.

Fawsitt and Hobberstad [56] performed a similar motion analysis of a surface ship (phase one in [55]) using the CFD method and Simerics software. The case study was the hypothetical model of Barcode 02 ship, the largest and most complex ship excavated in the old harbour of Oslo. The authors claimed that the results were not conclusive, but at the same time showed that CFD tools are available to maritime archaeologists and can help interpret ship performance. Figure 20 shows the model and movement of the ship in question on the sea's surface.

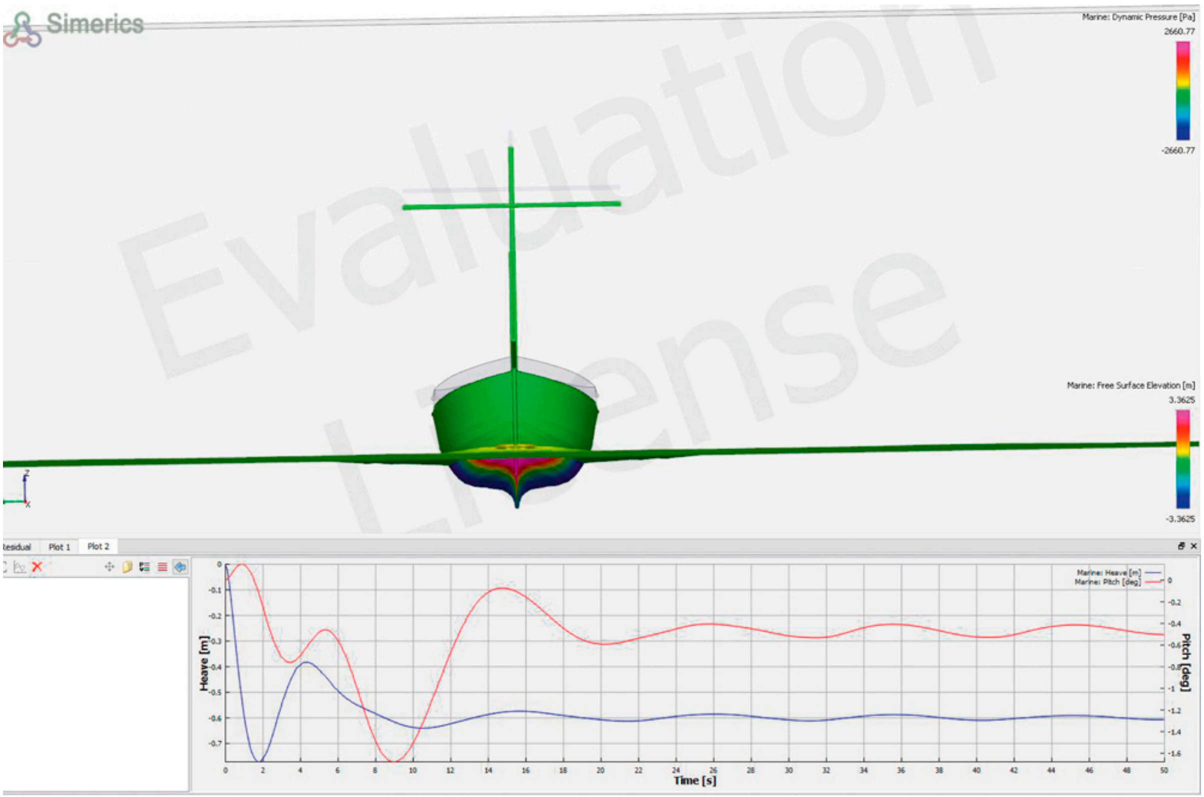

Figure 20. An example of Simerics graph output showing heave and pitch; reprinted from [56], under open-access CC-BY 4.0 license, 2023.

4.4. Shipwreck Forensics

Shipwreck forensics is the state-of-the-art analysis of the physical processes that affect shipwreck sites, such as sedimentation and erosion. While it is essentially a standard CFD method used for this purpose, the complexity of modeling and analysis is imposed due to the following principal reasons:

- It is necessary to know the situation at the site in detail: status of the shipwreck, position and orientation, bathymetry, etc.;
- It is necessary to know environmental conditions, mainly sea currents;
- Validation of the results is difficult, since any change at the site that may confirm the results occurs only over time, and sometimes over extended periods.

Smyth and Quinn [57] discussed the role of CFD in understanding the formation process of shipwreck sites. The aim of the research was a better understanding of hydro- and sediment dynamics, which can serve as an aid in decision-making about the preservation of a site. Figure 21 illustrates the model and outcome of the analysis—velocity streamlines at the shipwreck site in the considered scenario. Fluid velocity is affected by the shape of the seabed, including shipwreck and sea currents. The same fluid movement then affects the appearance or disappearance (or both in time) of sedimentation on the sea floor. The open-source software OpenFOAM was used to perform the simulation on a 153 × 143 × 20 m computational domain.

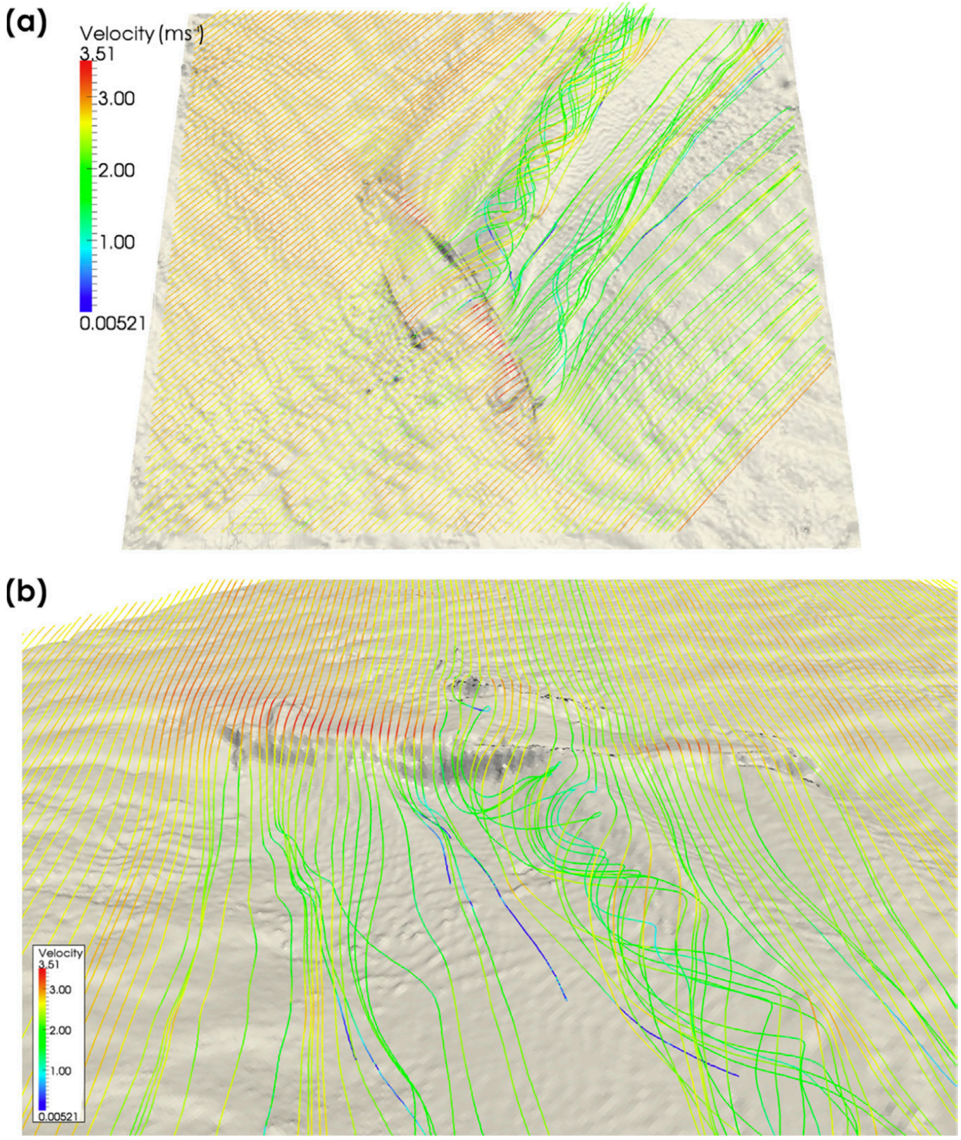

Figure 21. Three-dimensional streamline patterns around the wreck site just above the bottom as simulated in the CFD model (**a**) seen from the south and (**b**) the enlarged view from the north, in the wreck's lee; reprinted from [57], copyright (2014), with permission from Elsevier.

Quin and Smyth [58] extended their research to the processes and patterns of flow, erosion and deposition at shipwreck sites, again using the OpenFOAM CFD solver. The authors explored variations in hull orientation and resulting flowlines, vortex formation, development of low-pressure zones, and other results.

Fernandez-Montblanc et al. [59] conducted another CFD study on an underwater archaeological site. However, they focused on the wave-induced oscillatory flow and its interaction with the hull structure. They approached the problem by setting up a two-phase 2D computational model. The study case was the Fougueux ship, wrecked after the Battle of Trafalgar, southwest of the Iberian Peninsula, in the Gulf of Cadiz. The depth of the sea in the site varies from 5 to 10 m, so it is exposed to the action of waves. The InterFoam solver and SST K-Omega turbulence model were used in the analysis.

Majcher et al. [60] conducted a CFD study of the evolution of complex shipwreck sites under the influence of tides. Two wrecks were investigated: SS W.M. Barkley and HMS Vanguard, both located in the Irish Sea. Oceanographic data for a one-year period were collected and used in the study. Repeated hydrographic surveys were conducted in 2015 and 2019 in order to obtain data for the characterisation of the site in a four-year interval.

The structural changes of shipwrecks and geomorphological changes were also considered. OpenFOAM was used to perform the analysis using up to 13.4 million cells. Figure 22 shows the flow patterns for one analyzed scenario.

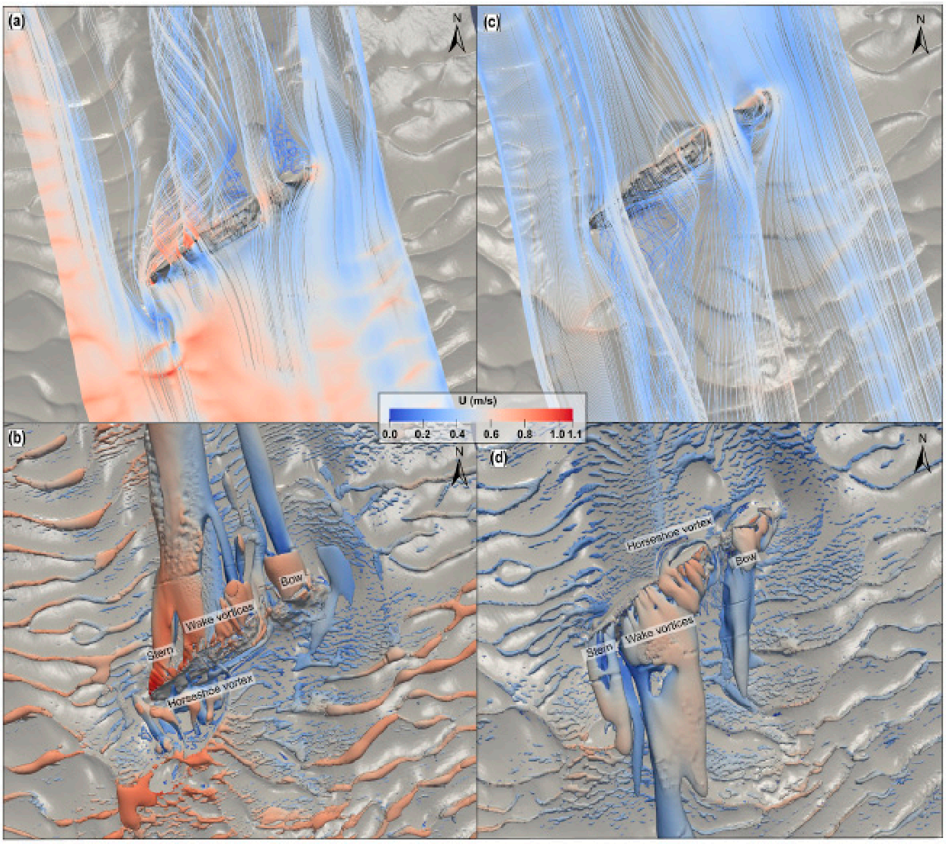

Figure 22. Simulated flow patterns for SS W.M. Barkley site: (**a**) flow streamlines and (**b**) Q iso-surface (Q = 0.0001) under the NNW current; (**c**) flow streamlines and (**d**) Q iso-surface under the SSE current. Reprinted from [60], under open-access CC-BY 4.0 license, 2023.

4.5. Ship Hull Form Variation

The design of ancient ships was driven by experience obtained from testing the ship's performance in real life. As such, there were various designs of ships, so reconstructing the shape of the hull and structure of a ship is a demanding task—even more so if ship remains are scarce. State-of-the-art software, towing tank experiments and evaluation of the performance of ship reconstructions can provide additional insight into the function of the ship's performance as a whole or its individual details.

Murray et al. [61] performed an experimental investigation of the origin and development of the waterline ram. The Trireme Trust's 1:20 scale model of Olympias was fitted with two different bow types: (a) a control bow and (b) a cutter bow, (Figure 23).

The conducted experiments confirmed the possibility that the cutwater bow enabled an increase in the speed of the vessel due to the more efficient hydrodynamics of the ship's hull, i.e., reduction in the hull's resistance to the formation of waves.

Jerat et al. [62] presented the results of research on the uncertainty of the reconstruction of the hull of an ancient ship, as well as its impact on sailing characteristics. The fourth-century Greek merchant ship Kyrenia was used as a case study. The hypothetical hull width was taken into account, as well as variations of ±5% and ±10% of the original hull width, leaving all other dimensions of the ship unchanged. The original resistance of the width of the ship's hull was estimated using the Holtrop method and CFD calculations, as shown in Figure 24.

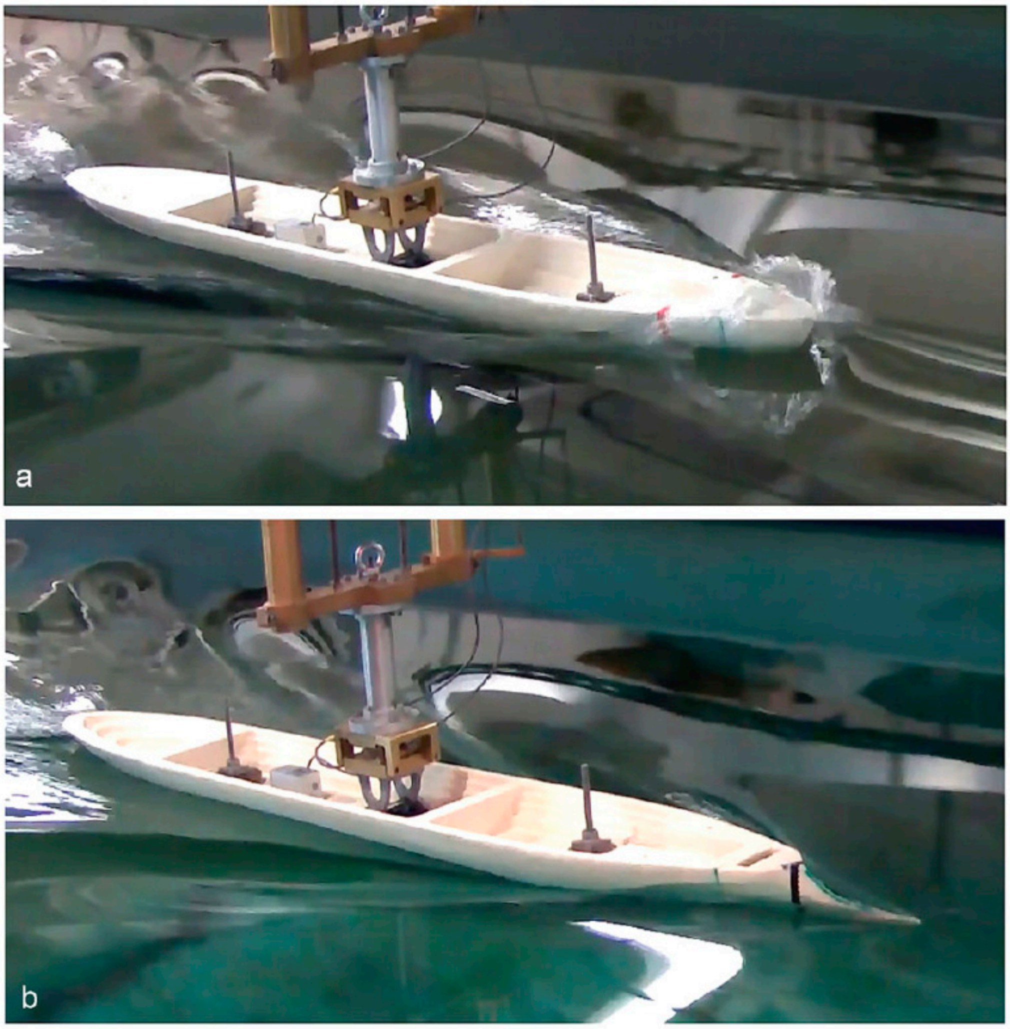

Figure 23. Control bow showing the increase in waves at the equivalent of 10 knots (top); cutwater bow showing wave attenuation at the equivalent of 11 knots (bottom); (**a**) indicate control bow, (**b**) indicate cutwater bow; reprinted from [61], under open-access CC-BY 4.0 license, 2023.

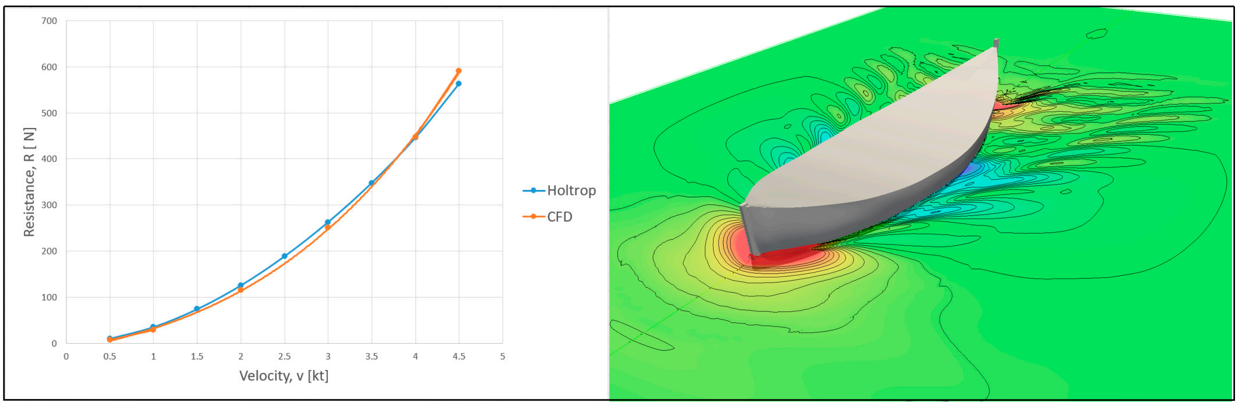

Figure 24. Comparison of the results of calculation of resistance to still water using the Holtrop method and CFD analysis of the original hull form (**left**) and a perspective view of CFD simulation of resistance to still water with surface elevation (**right**). Colors indicate surface wave elevation; reprinted from [62], courtesy of the authors, 2023.

Encouraged by the good matching of the ship resistance results obtained by the two methods, the Holtrop method was applied to analyze the ship resistance for the four variations of ship hull width. The results showed that a 10% wider ship could carry about 2.3 tons more cargo (about 10% more). The difference of the calculated ship resistance between the narrowest (−10%) and the widest (+10%) hull form is 13% at 4.5 knots [62].

5. Discussion

A review of papers on the application of numerical reconstruction in maritime archaeology showed an increasing interest in applying contemporary numerical modeling and analysis of different historical structures.

The strength of the amphorae has been studied using the finite element method for the past roughly 20 years. The research focuses on the determination of amphora material properties and related material strength, and on modeling realistic loads and boundary conditions. Amphorae were commonly made of locally available material, having different chemical compositions, consequently leading to variation in material strength. Therefore, the material properties need to be known first. Simple tests, such as tensile and three-point bending tests, can provide sufficient information about the material's elastic properties. However, in order to study crack propagation, either local or global, additional information about the material is needed, depending on the input required for a few material models available in the state-of-the-art software packages, such as LS-Dyna, Abaqus and Ansys. The study of an amphora's strength is useful in considering the process of evolution of its shape or the function of its structural parts, such as spikes. It should be noted that the assembly of a large number of amphorae in ships is difficult to model, in part also since attempts to reproduce the load on amphorae using a replica are scarce. Future research should aim to determine the parameters needed for the application of more complex material models. Once these parameters are known and the results validated, numerous hypotheses can be tested, such as the role of the spike, amphorae shape evolution, etc.

Another widely used historical material is wood. Many types of wood are used in building the complex structure of the wooden ship, including structural details. Research may be divided into two directions: the study of global ship behaviour (global strength) and local structure (fatigue, stress distribution and concentration). In both cases, specific wood properties should be known for applying a simple or more complex material model. Another difficulty in the numerical reconstruction of wooden structures is modeling structural details. Due to the strong orthotropic behaviour of the wood, prismatic volume finite elements are required to control the wood strength axis (transversal in two directions and longitudinal). Despite these limitations, a successful comparative analysis is made, minimizing the effects of assumptions and simplification in modeling. Rather complex numerical models are presented (such as in the Vasa example [32,42,43]). The common elastic property of the wood is applied, which is a reasonable assumption if the elastic modulus is correctly assumed. Many open research topics remain, despite the commendable efforts reviewed, such as modeling the ageing of the wood, modeling critical structural details (masts, joints), modeling and analysis of repairs, etc.

A review of the papers dealing with the effects of fluids, water and air on a ship's sailing performance and capsizing and sinking revealed the need for a fluid–structure interaction type of numerical analysis. This is a rather complex type of multi-physics analysis: the strength of the structure and the effect of fluids are analyzed simultaneously. Commonly, reasonable simplification entails studying structures, such as ship and cargo, as rigid bodies interacting with the fluid. Such analysis can provide information on sailing performance, cargo movement on the ship's decks and the potential spread of the cargo on the ship's bottom, consequences of ship flooding, etc. As such, these analyses require interdisciplinary expertise already at the level of engineering, making them costly and consequently rare. However, strong collaboration between engineers and archaeologists can be expected in the future, as numerical tools will become more sophisticated and computing power increases. The efficiency of sails shall be studied more thoroughly in the future, considering the reach of history and different solutions in the development of ship propulsion by the force of wind. Another interesting topic is the reconstruction of the wreck site, based on initial assumptions and the action of the laws of physics for the ship in fluid (sailing) or the fluid in the ship (sinking, capsizing, flooding).

Several authors performed a sophisticated analysis of the environmental conditions affecting a wreck site over the long term. Due to the complexity of the problem, only a few groups of researchers consistently model and analyze such problems: the deterioration of a wreck and the environmental changes around it due to the passage of time. While the analysis itself, commonly performed by the CFD method, is difficult but manageable, the main challenge remains the validation of the model and the results, since the changes are slow and occur over the long term. The two main aspects of wreck forensics are the need to observe and measure at least two or more historical situations at the site, with a reasonable time span between two observations, and proper modeling of hydrodynamic phenomena (sea currents) and geomorphic phenomena (movement of seafloor materials). The result of such analysis usually forecasts the end of the degradation process, but in the future, a "reverse engineering" analysis can be proposed – that is, an understanding of the historical events based on the situation at the wreck site.

Finally, problems related to the reconstruction of ship replicas can be efficiently considered using numerical analysis tools. Not many researchers studied this problem, although it seems inevitable that the reconstruction of hull lines, often based on limited information, may not be perfect. At the same time, if replicas are made, hull lines will directly affect the ship's sailing performance. Currently CFD analysis serves as a well-known and reliable method for testing ship resistance and seakeeping of a number of different hull line variants.

A review of the papers revealed a general need for the precise use of technical terms, as well as a precise description of all the parts of structural analysis. For example, the phrase “stress load” is found in Foecke et al. [48], while load, such as weight, is actually something that induces stress in the structure. Another example is use of “digital modeling”, such as in Hein and Kilikoglou [23]. Technically, digital modeling is the generation of any kind of a model in the digital domain, such as a CAD model or virtual reality model. Numerical modeling is the generation of a finite element mesh used for structural analysis and, as such, is clearly distinguished.

Numerical reconstruction still requires strong interdisciplinary cooperation between archaeologists, engineers and other experts. While it may be safely assumed that more sophisticated tools will become available in the future, enabling even more sophisticated analyses, it seems fair to say that there is no simple black box tool for analyzing historical structures and their performance.

6. Conclusions

State-of-the-art numerical methods are widely applied when considering contemporary shipbuilding tasks. The most commonly used numerical methods are FEM, which is mainly used to estimate the elastic response of the ship’s structure to a given load, and CFD, which is mainly used to estimate the ship’s resistance. However, these tools can be combined, and there are other numerical methods for solving rather complex problems such as hydroelasticity, seakeeping, ship collisions, etc.

Essentially, there is no significant difference between ancient and modern ships. Although the size, shape, materials, propulsion and other characteristics of these ships may differ significantly, they share a common requirement: the ship must sail safely and efficiently.

Numerical reconstruction in maritime archaeology provides insight into the structural and seafaring efficiency of ancient ships, and the interest in using state-of-the-art numerical tools in this field of research is evidently growing. A wide range of applications for these tools is presented, from structural analysis to complex fluid-structure interaction analysis.

Although it is clear that numerical reconstruction is capable of providing valuable new results in maritime archaeology, several reasons limit its application. The main reasons are the complexity of the software to be used, the expensive calculation that requires expensive hardware and time, and the need for educated and qualified experts capable of properly modeling the problem, controlling the calculation and correctly interpreting the results.

Wider application of numerical reconstruction in maritime archaeology would require tools specifically designed for use in archaeology, robust and reliable black box tools that allow for relatively fast and reasonably accurate results for most common tasks. Until this is done, numerical reconstruction in maritime archaeology will remain an interdisciplinary task shared between experts in archaeology and engineering, as well as other complementary research fields.

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References

1. McKinnon, J.F. Archaeology and the Emergence of Fields: Maritime. In *Encyclopedia of Global Archaeology*; Smith, C., Ed.; Springer: New York, NY, USA, 2014; pp. 414–420.
2. Gibbins, D.; Adams, J. Shipwrecks and Maritime Archaeology. *World Archaeol.* **2001**, *32*, 279–291. [CrossRef]
3. Liu, W.K.; Li, S.; Park, H.S. Eighty Years of the Finite Element Method: Birth, Evolution, and Future. *Arch. Comput. Methods Eng.* **2022**, *29*, 4431–4453. [CrossRef]
4. Zwick, D. Conceptual Evolution in Ancient Shipbuilding: An Attempt to Reinvigorate a Shunned Theoretical Framework. In *Interpreting Shipwrecks: Maritime Archaeology Approaches*; Adams, J., Johan, R., Eds.; The Highfield Press: Southampton, UK, 2014; pp. 46–71. ISBN 978-0-9926336-3-9.
5. Twede, D. The Packaging Technology and Science of Ancient Transport Amphoras. *Packag. Technol. Sci.* **2002**, *15*, 181–195. [CrossRef]
6. Molland, A.F. *The Maritime Engineering Reference Book: A Guide to Ship Design, Construction and Operation*; Butterworth-Heinemann: Oxford, UK, 2008; ISBN 978-0-7506-8987-8.
7. Zienkiewicz, O.C.; Taylor, R.L.; Zhu, J.L. *Finite Element Method: Its Basis and Fundamentals*, 7th ed.; Elsevier Ltd.: Amsterdam, The Netherlands, 2013; ISBN 978-1-85617-633-0.
8. Budynas, R.; Sadegh, A. *Roark's Formulas for Stress and Strain*, 9th ed.; McGraw Hill: New York, NY, USA, 2020; ISBN 978-1-260-45375-1.
9. Versteeg, H.; Malalasekera, W. *An Introduction to Computational Fluid Dynamics: The Finite Volume Method*, 2nd ed.; Pearson: London, UK, 2007; ISBN 978-0-13-127498-3.
10. Jiao, J.; Huang, S.; Guedes Soares, C. Viscous Fluid–Flexible Structure Interaction Analysis on Ship Springing and Whipping Responses in Regular Waves. *J. Fluids Struct.* **2021**, *106*, 103354. [CrossRef]
11. Knapp, A.B.; Demesticha, S. *Mediterranean Connections Maritime Transport Containers and Seaborne Trade in the Bronze and Early Iron Ages*, 1st ed.; Routledge: New York, NY, USA, 2019; ISBN 978-1-62958-354-9.
12. Kilikoglou, V.; Vekinis, G. Failure Prediction and Function Determination of Archaeological Pottery by Finite Element Analysis. *J. Archaeol. Sci.* **2002**, *29*, 1317–1325. [CrossRef]
13. Young, T. *A Course of Lectures on Natural Philosophy and the Mechanical Arts*; Joseph Johnson: London, UK, 1807; Volume II.
14. Jones, D.R.H.; Ashby, M.F. Elastic Moduli. In *Engineering Materials 1*; Elsevier: Amsterdam, The Netherlands, 2019; pp. 31–47. ISBN 978-0-08-102051-7.
15. Tsantini, E.; Jiménez-Piqué, E.; Montana, G.; Randazzo, L. Strength of Pre-Roman Amphorae: Comparison of the Different Types. *J. Archaeol. Sci. Rep.* **2015**, *2*, 405–417. [CrossRef]
16. Müller, N.S.; Vekinis, G.; Kilikoglou, V. Impact Resistance of Archaeological Ceramics: The Influence of Firing and Temper. *J. Archaeol. Sci. Rep.* **2016**, *7*, 519–525. [CrossRef]

17. Radić Rossi, I.; Senjanović, I.; Rudan, S.; Indof, J. Podrijetlo i Funkcija Šiljatoga Dna Amfora. *Pril. Inst. Arheol. Zagreb.* **2004**, *21*, 91–105.
18. Hein, A.; Georgopoulou, V.; Nodarou, E.; Kilikoglou, V. Koan Amphorae from Halasarna—Investigations in a Hellenistic Amphora Production Centre. *J. Archaeol. Sci.* **2008**, *35*, 1049–1061. [CrossRef]
19. De Francesco, A.M.; Graziano, T.; Andaloro, E.; La Marca, A.; Colelli, C.; Crisci, G.M.; Barrese, E.; Bocci, M. Archaeometric Characterization of Amphorae and Bricks of Imperial Age Found in a Roman Villa near the Luzzi Town (Cosenza, Calabria, Italy). *Period. Mineral.* **2011**, *80*, 217–230. [CrossRef]
20. Hein, A.; Kilikoglou, V. Breaking Pots—Simulating Design Failures of Transport Amphorae by Using the Finite Element Method (FEM). In Proceedings of the 1st CAA GR Conference, Rethymno, Crete, Greece, 6 March 2014; pp. 184–187.
21. Sviličić, Š.; Rudan, S.; Radić Rossi, I. Analysis of Amphorae Resistance to Ruptures and Cracks. *Archaeol. Maritima Mediterr. Int. J. Underw. Archaeol.* **2022**, *19*, 141–166. [CrossRef]
22. Hein, A.; Vekinis, G.; Kilikoglou, V. Modeling of Biaxial Flexure Tests of Transport Amphorae with the Finite Element Method: Fracture Strength, Deformation and Stress Distribution. *Results Eng.* **2022**, *15*, 100508. [CrossRef]
23. Hein, A.; Kilikoglou, V. Digital Modeling of Function and Performance of Transport Amphorae. *Int. J. Ceramic. Eng. Sci.* **2020**, *2*, 187–200. [CrossRef]
24. Uršić, J. *Čvrstoća Broda, I. Dio*; Sveučilišna Naklada d.o.o.: Zagreb, Croatia, 1991; ISBN 86-7819-001-9.
25. Gardiner, R.; Lambert, A. *Steam, Steel and Shellfire: The Steam Warship 1815–1905*; Conway Maritime Press: London, UK, 1992; ISBN 0851775640.
26. Fujita, Y.; Fujimoto, R.; Hagiwara, T. Shell Strength Tests and Structural Analysis of Large Wooden Ships. *J. Soc. Nav. Archit. Jpn.* **1990**, *167*, 215–227. [CrossRef]
27. Jensen, K. Documentation and Analysis of Ancient Ships. Ph.D. Thesis, Technical University of Denmark, Lyngby, Denmark, 1999.
28. Mackerle, J. Finite Element Analyses in Wood Research: A Bibliography. *Wood Sci. Technol.* **2005**, *39*, 579–600. [CrossRef]
29. Levy, R.; Dawson, P. Using Finite Element Methods to Analyze Ancient Architecture: An Example from the North American Arctic. *J. Archaeol. Sci.* **2009**, *36*, 2298–2307. [CrossRef]
30. Guan, Z.W.; Zhu, E.C. Finite Element Modelling of Anisotropic Elasto-Plastic Timber Composite Beams with Openings. *Eng. Struct.* **2009**, *31*, 394–403. [CrossRef]
31. Stoyanov, S.; Mason, P.; Bailey, C. Smeared Shell Modelling Approach for Structural Analysis of Heritage Composite Structures—An Application to the Cutty Sark Conservation. *Comput. Struct.* **2010**, *88*, 649–663. [CrossRef]
32. Garza, C.; Dabbagh, A. Finite Element Analysis of the Vasa's Bottom Structure. Master's Thesis, University of Skovde, Skovde, Sweeden, 2011.
33. del Coz Díaz, J.J.; García Nieto, P.J.; Lozano Martínez-Luengas, A.; Suarez Domínguez, F.J.; Domínguez Hernández, J. Non-Linear Numerical Analysis of Plywood Board Timber Connections by DOE-FEM and Full-Scale Experimental Validation. *Eng. Struct.* **2013**, *49*, 76–90. [CrossRef]
34. Invernizzi, S.; Bertolini-Cestari, C.; Fioravanti, M.; Chiabrera, E. Numerical Modeling and Assessment of the Ebe Schooner-Brig. *Int. J. Archit. Herit.* **2012**, *6*, 453–477. [CrossRef]
35. Melisan, J. Strength Analysis of Traditional Ships in Efforts To Improve Sea Transportations Safety in Indonesia. *Int. J. Eng.* **2012**, *12*, 878–883.
36. Koch, H.; Eisenhut, L.; Seim, W. Multi-Mode Failure of Form-Fitting Timber Connections—Experimental and Numerical Studies on the Tapered Tenon Joint. *Eng. Struct.* **2013**, *48*, 727–738. [CrossRef]
37. Yue, Z. Traditional Chinese Wood Structure Joints with an Experiment Considering Regional Differences. *Int. J. Archit. Herit.* **2014**, *8*, 224–246. [CrossRef]
38. Arciszewska-Kędzior, A.; Kunecký, J.; Hasníková, H.; Sebera, V. Lapped Scarf Joint with Inclined Faces and Wooden Dowels: Experimental and Numerical Analysis. *Eng. Struct.* **2015**, *94*, 1–8. [CrossRef]
39. Chen, C.; Qiu, H.; Lu, Y. Flexural Behaviour of Timber Dovetail Mortise–Tenon Joints. *Constr. Build. Mater.* **2016**, *112*, 366–377. [CrossRef]
40. Milch, J.; Tippner, J.; Sebera, V.; Kunecký, J.; Kloiber, M.; Navrátil, M. The Numerical Assessment of a Full-Scale Historical Truss Structure Reconstructed with Use of Traditional All-Wooden Joints. *J. Cult. Herit.* **2016**, *21*, 759–766. [CrossRef]
41. Fajman, P.; Máca, J. Stiffness of Scarf Joints with Dowels. *Comput. Struct.* **2018**, *207*, 194–199. [CrossRef]
42. Afshar, R.; van Dijk, N.P.; Bjurhager, I.; Gamstedt, E.K. Comparison of Experimental Testing and Finite Element Modelling of a Replica of a Section of the Vasa Warship to Identify the Behaviour of Structural Joints. *Eng. Struct.* **2017**, *147*, 62–76. [CrossRef]
43. Afshar, R.; Alavyoon, N.; Ahlgren, A.; Gamstedt, E.K. Full Scale Finite Element Modelling and Analysis of the 17th-Century Warship Vasa: A Methodological Approach and Preliminary Results. *Eng. Struct.* **2021**, *231*, 111765. [CrossRef]
44. Helfman, N.; Cvikel, D.; Nishri, B. A Comparative Structural Analysis of Shell-First and Frame-Based Ship Hulls of the 1st Millennium AD. *Nav. Eng. J.* **2018**, *130*, 91–103.
45. Helfman, N.; Nishri, B.; Cvikel, D. Finite Element Analysis of Shell-First and Longitudinally Reinforced Frame-Based Wooden Ships. *J. Marit. Archaeol.* **2019**, *14*, 291–309. [CrossRef]
46. Eliav, J.; Helfman, N. Lightweight Construction of an Athenian *Trireme*: A Feasibility Study. *Int. J. Naut. Archaeol.* **2022**, *51*, 187–194. [CrossRef]

47. *EN 1995-1-1:2004*; Eurocode 5: Design of Timber Structures—Part 1-1: General-Common Rules and Rules for Buildings. CEN: Brussels, Belgium, 2004.
48. Foecke, T.; Ma, L.; Russel, M.A.; Conlin, D.L.; Murphy, L.E. Investigating Archaeological Site Formation Processes on the Battleship USS Arizona Using Finite Element Analysis. *J. Archaeol. Sci.* **2010**, *37*, 1090–1101. [CrossRef]
49. Palmer, C. Windward Sailing Capabilities of Ancient Vessels. *Int. J. Naut. Archaeol.* **2009**, *38*, 314–330. [CrossRef]
50. Lasher, W.C.; Flaherty, L.S. CFD Analysis of the Survivability of a Square-Rigged Sailing Vessel. *Eng. Appl. Comput. Fluid Mech.* **2009**, *3*, 71–83. [CrossRef]
51. Ciortan, C.; Fonseca, N. Numerical Simulations of the Sails of a Xvith Century Portuguese Nau Marine 201. In Proceedings of the International Conference on Computational Methods in Marine Engineering, Lisbon, Portugak, 28 September 2011; pp. 278–289.
52. Stettler, J.W.; Thomas, B.S. Flooding and Structural Forensic Analysis of the Sinking of the RMS Titanic. *Ships Offshore Struct.* **2013**, *8*, 346–366. [CrossRef]
53. Kery, S. Marine Forensics: The Art and Science of Simulating Ships In Storm Conditions. In Proceedings of the Interservice/Industry Training, Simulation, and Education Conference (I/ITSEC) 2017, Orlando, FL, USA, 27 December 2017.
54. Subbaiah, B.V.; Thampi, S.G.; Rambabu, N.; Mustafa, V.; Akbar, M.A. Characterization and CFD Analysis of Traditional Vessels of Kerala. *Int. J. Innov. Technol. Explor. Eng.* **2020**, *9*, 132–149. [CrossRef]
55. Rudan, S.; Radić Rossi, I. Numerička Simulacija Potonuća Broda na Temelju Arheoloških Zapisa. *Archaeol. Adriat.* **2020**, *14*, 129–157. [CrossRef]
56. Fawsitt, S.; Hobberstad, L.C. Computational Fluid Dynamics: Floating a Digital Barcode 02. *Archaeonautica* **2021**, *21*, 325–330. [CrossRef]
57. Smyth, T.A.G.; Quinn, R. The Role of Computational Fluid Dynamics in Understanding Shipwreck Site Formation Processes. *J. Archaeol. Sci.* **2014**, *45*, 220–225. [CrossRef]
58. Quinn, R.; Smyth, T.A.G. Processes and Patterns of Flow, Erosion, and Deposition at Shipwreck Sites: A Computational Fluid Dynamic Simulation. *Archaeol. Anthropol. Sci.* **2018**, *10*, 1429–1442. [CrossRef]
59. Fernández-Montblanc, T.; Izquierdo, A.; Quinn, R.; Bethencourt, M. Waves and Wrecks: A Computational Fluid Dynamic Study in an Underwater Archaeological Site. *Ocean Eng.* **2018**, *163*, 232–250. [CrossRef]
60. Majcher, J.; Quinn, R.; Smyth, T.; Plets, R.; McGonigle, C.; Westley, K.; Sacchetti, F.; Coughlan, M. Using Difference Modelling and Computational Fluid Dynamics to Investigate the Evolution of Complex, Tidally Influenced Shipwreck Sites. *Ocean Eng.* **2022**, *246*, 110625. [CrossRef]
61. Murray, W.M.; Ferreiro, L.D.; Vardalas, J.; Royal, J.G. Cutwaters Before Rams: An Experimental Investigation into the Origins and Development of the Waterline Ram: An Experiment in Ancient Cutwater Hydrographics. *Int. J. Naut. Archaeol.* **2017**, *46*, 72–82. [CrossRef]
62. Jerat, G.; Rudan, S.; Zamarin, A.; Rossi, I.R. Uncertainty in the Reconstruction of the Ancient Ship Hull and Its Impact on Sailing Characteristics. In Proceedings of the 16th International Symposium on Boat and Ship Archeology, Zadar, Croatia, 26 September–1 October 2021.