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# Data-Driven Modal Decomposition Methods as Feature Detection Techniques for Flow Fields in Hydraulic Machinery: A Mini Review

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**Abstract:** Cavitation is a quasi-periodic process, and its non-stationarity leads to increasingly complex flow field structures. On the other hand, characterizing the flow field with greater precision has become increasingly feasible. However, accurately and effectively extracting the most representative vibration modes and spatial structures from these vast amounts of data has become a significant challenge. Researchers have proposed data-driven modal decomposition techniques to extract flow field information, which have been widely applied in various fields such as signal processing and fluid dynamics. This paper addresses the application of modal decomposition methods, such as dynamic mode decomposition (DMD), Proper Orthogonal Decomposition (POD), and Spectral Proper Orthogonal Decomposition (SPOD), in cavitation feature detection in hydraulic machinery. It reviews the mathematical principles of these three algorithms and a series of improvements made by researchers since their inception. It also provides examples of the applications of these three algorithms in different hydraulic machinery. Based on this, the future development trends and possible directions for the improvement of modal decomposition methods are discussed.

**Keywords:** data-driven; modal decomposition; cavitation; hydraulic machinery



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## 1. Introduction

Hydraulic machinery plays an increasingly crucial role in marine engineering, with complex fluid mechanics phenomena within it, including cavitation, turbulence, and various other phenomena. Cavitation and turbulence are characterized by complex occurrences like shock waves, detachment, and shedding. Therefore, extensive research and effective control of flow fields in hydraulic machinery are vital. In liquid flow, cavitation arises due to the formation of vapor nuclei, accompanied by significant vapor–liquid phase changes triggered by alterations in static pressure surrounding the local liquid resulting from the liquid flow [1]. This phenomenon is typical in high-speed flows and vortices, leading to flow field non-uniformity and disturbance. The essence of cavitation lies in phase change, where cavitation bubble formation involves the sudden transition of a thin layer of liquid molecules near the bubble wall into vapor molecules. Cavitation bubble disappearance occurs when vapor molecules inside the bubble pass through the bubble wall and return to liquid form, a vigorous process. Consequently, cavitating flow can induce flow instability, reduced system performance, material damage, and other issues, necessitating effective control and management in engineering design and practical applications. Since the 1970s and 1980s, cavitation technology has gained increasing recognition and development across various fields such as surface cleaning, cutting, environmental protection [2], materials science [3], and bioengineering [4], emerging as an advanced technology actively undergoing further development. Delaying cavitation occurrence and inhibiting cavitation shedding have long been focal points in hydraulic machinery research. Scholars worldwide have conducted extensive scientific research on cavitation development and the unsteady state mechanism it induces. They contend that the main cause of sheet cavitation shedding and cloud cavitation generation is the re-entrant jet flow formed by reverse pressure gradients. Thus, controlling this re-entrant jet flow can partially regulate cloud cavitation. Methods for controlling cavitation flow are categorized into active and passive control based on external energy input. Whether employing active or passive control methods, research on delaying and suppressing cloud cavitation has achieved its intended purpose to some extent.

Flow fields in hydraulic machinery represent complex phenomena. As prediction methods advance, accurately and swiftly extracting key information from extensive flow field data in hydraulic machinery has become imperative. To process these data more effectively, researchers have integrated various data-driven algorithms and developed diverse non-stationary flow field modal decomposition methods. Commonly employed methods include DMD (dynamic mode decomposition), POD (Proper Orthogonal Decomposition), and SPOD (Spectral Proper Orthogonal Decomposition).

DMD is a data-driven method employed to analyze dynamic systems by extracting their dynamic modes from time-series data. These modes describe the system's evolution behavior, approximating the spectrum of the Koopman operator through a matrix. The primary eigenvalues and eigenvectors of this matrix provide detailed information about the system's dynamic properties, including frequency, decay, growth, and flow patterns [5]. First proposed by Schmid [6] in 2010, DMD has found extensive applications across different fields such as fluid dynamics [7], power systems [5], and meteorology [8]. It involves matrix decomposition of time-series data to extract dynamic modes, which are then used to predict future system behavior. DMD relies on the linear mapping relationship between flow fields or systems at consecutive time instants. By constructing a snapshot matrix from the time-series data and performing Singular Value Decomposition (SVD) on it, one can obtain eigenvectors and eigenvalues. These eigenvectors represent the dynamic modes of the system, while the eigenvalues describe the growth rate and oscillation frequency of each mode.

POD is proficient in handling large datasets by decomposing the physical information in a flow field into different sets of signals. This decomposition aims to maximize energy retention by using the fewest basic functions. Widely applied across various fields including signal processing [9], machine learning [10], fluid dynamics [11], and structural dynamics [12], especially in fluid dynamics and structural dynamics, it is extensively used to analyze primary modes and structural features in flow field or vibration data. Presented by Lumley [13] in the introduction of turbulence, POD was independently discovered by different researchers in other fields [11]. Both POD and Principal Component Analysis (PCA) employed in statistics and machine learning rely on SVD, constituting data-driven dimensionality reduction techniques. The POD method involves selecting a smaller number of uncorrelated variables from a system with numerous interdependent variables through data dimensionality reduction, thereby revealing primary modes and structures within the system [14]. According to the sorting of eigenvalues, the larger the eigenvalue, the greater the contribution of the corresponding mode. By retaining a few larger eigenvalues and their corresponding eigenvectors, termed modes, most of the energy can be captured. Discarding the smaller orthogonal eigenvalues allows for the reconstruction of the original data, facilitating the analysis and prediction of changes in research problems. Two prevalent methods for solving POD are SVD-based POD and eigenvalue-based POD [15].

SPOD [16], an extension of the classical POD method expressed in the frequency domain, finds wide applications across various fields, particularly in fluid dynamics [17]. It utilizes spectral information to decompose signals or data and extract their spatial and temporal features. By transforming from the time domain to the frequency domain, SPOD achieves a decoupling of time and space, yielding a series of modes possessing both temporal and spatial orthogonality. While extending temporal orthogonality, SPOD retains all the features of POD, such as spatial orthogonality and optimal energy capture. Additionally, SPOD introduces a temporal constraint, enabling the explicit separation of phenomena occurring at multiple frequencies and energy levels. This facilitates a smooth transition from energy-optimal POD to spectral pure Fourier decomposition through adjustments to individual parameters [16]. The key algorithm of SPOD is the Welch method, which consistently and accurately estimates the average Cross-Spectral Density (CSD) from time series data [18].

Data-driven modal decomposition methods utilize machine learning and data reduction techniques to extract the main modes from flow field data, revealing the inherent structure and dynamic characteristics of the flow field. Experimental data of aerodynamic flow fields are often influenced by factors such as noise and disturbances. Data-driven methods can effectively handle these disturbances by employing appropriate data processing and feature extraction techniques, thereby improving the quality and reliability of the data. These methods not only reduce computational costs and enhance analysis efficiency but also uncover hidden information and patterns from large datasets, deepening our understanding of flow structures and mechanisms. In recent years, researchers have continuously improved and expanded modal decomposition methods to make them more applicable to various engineering problems. These enhanced methods provide new ideas and approaches for research and practice in the engineering field, offering important theoretical and practical significance for solving engineering problems.

## 2. Mathematical Principles and Enhancements

### *2.1. The Dynamic Mode Decomposition Method*

#### 2.1.1. The Mathematical Principles of the DMD

Before conducting the DMD analysis, the time series of the unsteady flow field needs preprocessing. Snapshots of $N$ moments from physical experiments or numerical simulations can be arranged in a sequence from the first to the $N$-th moment, denoted as $\{x_1, x_2, x_3, \cdots x_N\}$ Here, each column vector $x_i$ represents a snapshot of the flow field at the $i$-th moment, with a time interval of $\Delta t$ between any two consecutive snapshots. It is presumed that the flow field $x_{i+1}$ can be linearly mapped from the flow field $x_i$:

$$x_{\mathrm{i}+1} = Ax_{\mathrm{i}} \tag{1}$$

where $A$ is a system matrix of a high-dimensional flow field. However, due to the high dimension of $A$, it is difficult to calculate directly. It needs to be calculated in another way [7,19]. Using $X_1, X_2, X_3 \cdots X_N$ constructs snapshot matrix $X, Y$.

From the above formula:

$$Y = AX \tag{2}$$

where $Y = \{x_1, x_2, x_3 \cdots x_N\}; X = \{Ax_1, Ax_2, Ax_3, \cdots Ax_N\}$ the dynamic characteristics of the system are reflected in matrix $A$. The flow field at the $N$-th moment can be expressed using the snapshots of the flow field from the first to the $N-1$ moments. The matrix format is:

$$x_N = b_1x_1 + b_2x_2 + b_3x_3 + \ldots + b_{N-1}x_{N-1} = Xb + r \tag{3}$$

where $r$ is the residual vector, $b^T = \{b_1, b_2, b_3 \cdots b_{N-1}\}$

$$AX_1^{N-1} = X_2^N = X_1^{N-1}S + re_{N-1}^T \tag{4}$$

where Matrix $S$ is a companion matrix, which takes the following form:

$$S=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & b_1 \\ 1 & 0 & \cdots & 0 & 0 & b_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & b_{N-2} \\ 0 & 0 & \cdots & 0 & 1 & b_{N-1} \end{bmatrix} \quad (5)$$

The only unknown quantity is the $b$ matrix, so the companion matrix $S$ can be solved for $b$ with the smallest residual vector $r$.

$$r = x_N - X_1^{N-1} b \quad (6)$$

When $r$ is small, the eigenvalues of the companion matrix $S$ are approximately equal to those of $A$, and $S$ can be regarded as a low-dimensional representation of $A$. Therefore, the eigenvalues of $S$ can approximately represent the main eigenvalues of $A$. Performing an eigendecomposition on $S$:

$$S = TNT^{-1}, N = \mathrm{diag}(\lambda_1, \lambda_{2,} \cdots \lambda_{N-1}) \quad (7)$$

Provide the modes of DMD, $\delta_i$

$$\delta_i = X_1^{N-1} T_i \quad (8)$$

The snapshot of the flow field at any given moment can be represented using the preceding $m$ snapshots of the flow field:

$$\delta_i = \sum_{j=1}^{m} \lambda_j^{i-1} \delta_j \quad (9)$$

In the equation, $m$ represents the number of modes in DMD, $\lambda_j$ denotes the eigenvalues, and $\delta_j$ represents the column vectors of modes.

Assessing the predominant influence of a mode in the flow field involves considering the energy level within the dynamic mode, which serves as a crucial indicator. This energy level is typically quantified by the norm of the mode. $\delta_i$:

$$\|\delta_i\| = \sqrt{\sum_{i=1}^{n} \left|\delta_i\right|^2} \quad (10)$$

#### 2.1.2. The Improvements of the DMD

The traditional DMD method has limitations when dealing with large-scale data or nonlinear systems, such as sensitivity to data quality and noise. Therefore, further improvement and optimization of the DMD method are needed to enhance its applicability and accuracy, enabling it to better address the analysis requirements of various complex systems.

Continuously improving the DMD method's algorithm enhances its performance and applicability across various disciplines. In 2012, Chen et al. [20] introduced an optimized DMD method (opt-DMD) that iteratively calculates optimal eigenvalues and corresponding principal modes using a global optimization algorithm to minimize residuals. They tested its effectiveness on the fluid flow around a cylinder at low Reynolds numbers. The optimized DMD outperforms traditional DMD by better calculating the physical correlation frequency and being less sensitive to numerical values. In 2014, Jovanović et al. [21] developed a sparsity-promoting variant of standard DMD, aiming to eliminate unessential flow structures. This method, applied to various flow scenarios, successfully obtained low-dimensional representations of unsteady flows, capturing their primary features. In 2016, Kutz et al. [22] introduced multiresolution DMD (mrDMD), which combines DMD with multiresolution analysis to decompose complex systems into hierarchical structures of multiresolution time-scale components. Noack et al. [23] proposed a compromised mode decomposition method, recursive dynamic mode decomposition (RDMD), which combines key characteristics of POD and DMD, showing effectiveness in analyzing flow around cylinders. Proctor et al. [24] extended DMD to DMD with control effects (DMDc) for extracting low-order models from high-dimensional systems, demonstrating its efficacy in analyzing complex systems, including infectious disease data models. In 2017, Le Clainche et al. [25] extended DMD to higher order dynamic mode decomposition to address general periodic, quasi-periodic, and transient dynamics. Statnikov et al. [26] applied an optimized DMD for reduced-order analysis of turbulent wakes. In 2019, Erichson et al. [27] proposed a randomized algorithm for computing approximately optimal low-rank DMD, showing accuracy and efficiency in extracting coherent structures from large datasets. Azencot et al. [28] proposed a variational formulation for DMD, applicable to various problems. In 2022, Abolmasoumi et al. [29] introduced robust DMD (RDMD) with statistical and numerical robustness. In 2023, Rosenfeld et al. [30] addressed limitations in DMD by removing the Koopman operator, achieving theoretical objectives not realized in other contexts. To capture temporal variation, Ferre et al. [31] developed non-stationary dynamic mode decomposition (NS-DMD), accurately predicting temporal evolution in simulations. Anzaki et al. [32] proposed dynamic mode decomposition with memory (DMDm) to analyze time-series data, overcoming the constraints of existing DMD methods.

Since the introduction of modal decomposition methods, researchers have proposed numerous criteria for selecting dominant modes tailored to different flow field conditions. Two of the most common are the Schmid amplitude-based criterion and Tissot's criterion. The Schmid criterion [33] considers mode amplitude as a significant indicator of mode importance, derived from DMD modes, and is widely used in coherent structure analysis of flow fields. Tissot's criterion [34], on the other hand, is based on the energy contribution of modes, considering the overall contribution of each mode across the entire flow field snapshots, effectively selecting dominant modes.

In 2017, Kou and Zhang [35] proposed a criterion for selecting the most important modes from DMD techniques. Unlike the standard DMD approach, this criterion considers the evolution of each mode across the entire sampling space and ranks them based on their contributions across all samples. Applying this criterion to cases such as transient testing of the NACA0012 airfoil in supersonic flow reveals its ability to discern the key DMD modes within the flow field. In 2022, Wu et al. [36] combined the advantages of clustering methods and proposed a novel criterion for selecting dominant modes based on cluster analysis. This criterion integrates two traditional dominant mode selection criteria and automatically classifies different spatial conditions, holding significant relevance in the analysis of spatialized flows. Since the inception of this method, researchers have been continually refining it, with the improvement methods outlined in Table 1.

**Table 1.** Improvements of the DMD method.

| Improvement Method | Reference |
|---|---|
| Optimized DMD | Chen et al. [20] |
| A sparsity-promoting variant of the standard DMD algorithm | Jovanović et al. [21] |
| The multiresolution DMD | Kutz et al. [22] |
| Recursive dynamic mode decomposition | Noack et al. [23] |
| DMD with control | Proctor et al. [24] |
| Higher order dynamic mode decomposition | Le Clainche et al. [25] |
| A reduced-order analysis using optimized DMD | Statnikov et al. [26] |
| A randomized algorithm for computing the near-optimal low-rank DMD | Erichson et al. [27] |
| A regularization term for the forward and backward dynamics | Azencot et al. [28] |
| Robust dynamic mode decomposition | Abolmasoumi et al. [29] |
| Discarding the characteristic functions | Rosenfeld et al. [30] |
| Non-stationary DMD | Ferre et al. [31] |
| DMD with memory | Anzaki et al. [32] |
| The mode amplitude criterion | Schmid [33] |
| The mode energy criterion | Tissot [34] |
| A criterion for selecting dominant modes from DMD technique | Kou et al. [35] |
| The clustering method | Wu et al. [36] |

*2.2. The Proper Orthogonal Decomposition Method*

2.2.1. The Mathematical Principles of POD

The mathematical principles of the POD method are as follows [37]:

Let $u$ be the scalar field $\left\{ u^k \right\}$ on the space domain $\Omega$. To find a set of optimal basis function $\varphi$, that can better describe the known function, we project the known function $\left\{ u^k \right\}$ onto the basis function $\varphi$ and maximize the square of the projection, which can be expressed as follows.

$$\max \frac{\left\langle |(u,\varphi)|^2 \right\rangle}{||\varphi||^2} \tag{11}$$

In the above equation, $|\cdot|$ represents mode, $||\cdot||$ represents $L_2 - norm$, $\langle \cdot \rangle$ represents the averaging operator, and $(\cdot,\cdot)$ represents the inner product.

To solve such extremum problems, the variational method can be employed to search for solutions that satisfy the constraint $||\varphi|| = 1$ and maximize the objective function $\langle |(u,\varphi)| \rangle$. The variational problem is constructed using the Lagrange multiplier method as follows:

$$J[\varphi] = \left\langle |(u,\varphi)|^2 \right\rangle - \lambda \left( \|\varphi\|^2 - 1 \right) \tag{12}$$

The necessary condition for Equation (12) to reach the extreme value is that, for all variations $A$ (where $B$ is a scaling factor), they must all satisfy the following expression:

$$\frac{d}{d\delta} J[\varphi + \delta\varphi]|_{\delta=0} = 0 \tag{13}$$

From Equations (12) and (13), we can see that:

$$\frac{d}{d\delta} J[\varphi + \delta\phi]|_{\delta=0} = 2[\langle (u,\phi)(\varphi,u) \rangle - \lambda(\varphi,\phi)] = 0 \tag{14}$$

Using the commutativity of the inner product of the function, the above equation is expanded as follows:

$$\langle (u,\phi)(\varphi,u) \rangle - \lambda(\varphi,\phi) = 0 \tag{15}$$

where $\phi(x)$ represents any variable, and the basis functions to be sought must satisfy the following equation:

$$\int_\Omega \langle u(x)u(x') \rangle \varphi(x') dx' = \lambda \varphi(x) \tag{16}$$

The optimal POD basis functions can be found using Equation (16), with the mean cross-correlation function $R(x,x') = \langle u(x)u(x') \rangle$ being the crucial factor. The problem of finding the optimal basis function $\varphi(x)$ can be transformed into constructing the core matrix by any uncorrelated function on the domain $\Omega$ and solving the eigenvalue problem of the matrix. By solving this eigenvalue problem, the corresponding eigenfunction $\{\varphi_k(x)\}$ can be obtained. This set of basis functions will ensure that they are most similar to the known function $\{u_k\}$ in the average sense, and then the original function is reconstructed with the following set of eigenfunctions:

$$u^k(x) = \sum_{m=1}^{M} a_n^k \varphi_n(x) \tag{17}$$

In the above formula, $\varphi_n(x)$ is the mode, $a_n$ is the coefficient corresponding to the mode $\varphi_n(x)$, and $M$ is the number of samples.

In summary, POD is a method for data analysis and dimensionality reduction. It involves projecting the data onto a new coordinate system through linear transformation to extract their main features.

#### 2.2.2. The Improvements of POD

To enhance the practical application of the POD method in engineering, researchers continually explore methods to integrate it with other techniques or improve and extend its capabilities. This integrated approach aims to overcome POD's limitations in specific scenarios, thus increasing its effectiveness in engineering practice.

In 2000, Hall et al. [38] introduced a method for constructing reduced-order models (ROMs) to represent unsteady flow disturbances. These ROMs are built using basis vectors derived from perturbation frequency-domain solutions obtained through POD. The POD/ROM technique was successfully applied to simulate the unsteady aerodynamic and aeroelastic behavior of isolated transonic airfoils. In 2008, Kunisch et al. [39] proposed the Optimality Systems–POD (OS-POD) method to address challenges within the POD approach, particularly when basis elements are computed from reference trajectories with features significantly different from optimal control trajectories. In 2018, Himpe et al. [40] presented the hierarchical approximate POD (HAPOD) method, offering a universal and easily implementable approach based on a hierarchical structure of working nodes. Rigorous error estimates ensure the method's reliability, while numerical examples demonstrate its performance. In 2019, Yan et al. [41] introduced the structure-preserved POD method, simplifying problems while preserving solution integrity. They showcased its capability in reducing model order in frequency domain solvers and improving RLC device structures. In 2020, Kastian et al. [42] proposed the Adaptive Proper Orthogonal Decomposition (APOD) method, which can be applied to other model problems and projection-based model reduction methods. In 2021, Butcher et al. [43] introduced the Zonal Proper Orthogonal Decomposition (ZPOD) method, which decomposes the velocity field into zones before computing POD modes, enabling better identification of significant structures and features in each zone. Cavalieri et al. [44] proposed the Cross Proper Orthogonal Decomposition (CPOD) method to optimally decompose the trace of cross-covariance of flow fluctuations, accurately reconstructing Reynolds stresses and mean flow. In 2024, Long et al. [45] proposed a technique that combines Proper Orthogonal Decomposition (POD) with Conditional Deep Convolutional Generative Adversarial Networks (CDCGANs) to swiftly reconstruct the physical field within a boiler furnace. This approach aims at achieving rapid and precise reconstruction of temperature and velocity fields. Peng et al. [46] introduced the POD–Radial Basis Function (POD-RBF) method for modeling and orthogonal decomposition, enabling the construction of an online simulation model for Laser Powder Bed Fusion (LPBF), providing timely feedback to correct potential anomalies.

By integrating machine learning techniques like deep learning with POD, researchers can effectively extract features from data, improving system modeling, prediction, and control. This fusion better captures complex patterns and nonlinear relationships within the data, enhancing model accuracy and generalization capability.

In 2020, Rageh et al. [47] proposed an automated framework for damage detection utilizing Proper Orthogonal Decomposition (POD) and Artificial Neural Networks (ANNs) to identify stiffness degradation resulting from fatigue cracks. In 2021, Jacquier et al. [48] extended the concept of POD–Neural Network (POD-NN) to POD–Artificial Neural Network (POD-ANN) for the construction of a non-intrusive surrogate model. They applied this to flood prediction, yielding safer and broader predictions for inundation zones compared to conventional models. In 2023, Shi et al. [49] improved POD modal coefficient calculation by establishing a Deep Belief Network (DBN) model to enhance temperature field distribution prediction accuracy. Results showed a significant improvement in the POD model's generalization ability after enhancement with a DBN (POD-DBN). In 2024, Zhang et al. [50] proposed an improved POD–Galerkin reduced-order model using Long Short-Term Memory (LSTM) neural networks for flow field prediction around a two-dimensional cylinder. The addition of neural network correction terms effectively enhanced the reduced-order model accuracy compared to the original standard POD–Galerkin model. Zhao et al. [51] introduced a hybrid approach that combines Proper Orthogonal Decomposition (POD) with Deep Neural Networks (DNNs) to improve the interpretability and accuracy of flow and thermal field reconstruction. This method utilizes POD to extract fundamental features from the physical field and then formulates the reconstruction task as finding the optimal linear combination of dominant POD modes. By doing so, it enhances the performance of neural networks, particularly in large-scale and irregular domain problems. Ying et al. [52] developed a deep learning-based framework for predicting fluid dynamics on a Mantle-Undulated Propulsion Robot (MUPRo), proposing a multiple POD (MPOD) algorithm to identify fluid features near the undulated mantle. This model supports offline parameter planning for waterborne bio-inspired robots. The improvement methods for the POD are depicted in the following Table 2.

**Table 2.** Improvements of the POD method.

| Improvement Method | Reference |
|---|---|
| Reduced-order models | Hall et al. [38] |
| Optimality systems–POD | Kunisch et al. [39] |
| Hierarchical approximate POD | Himpe et al. [40] |
| Structure-preserved POD | Yan et al. [41] |
| Adaptive POD | Kastian et al. [42] |
| Zonal POD | Butcher et al. [43] |
| Cross POD | Cavalieri et al. [44] |
| POD deep convolutional generative adversarial networks | Long et al. [45] |
| POD–Radial Basis Function | Peng et al. [46] |
| Further improvement of POD-ANN method | Rageh et al. [47] |
| POD–Artificial Neural Network | Jacquier et al. [48] |
| POD–Deep Belief Network | Shi et al. [49] |
| Improvement of POD–Galerkin reduced-order model | Zhang et al. [50] |
| A hybrid method based on POD and DNNs | Zhao et al. [51] |
| A multiple POD | Ying et al. [52] |

### *2.3. The Spectral Proper Orthogonal Decomposition Method*

#### 2.3.1. The Mathematical Principles of SPOD

SPOD usually involves a Fourier transform on the data, converting time-domain signals into frequency-domain representations. This process allows for the extraction of spatial modal shapes and energy distributions across frequencies, unveiling hidden structures and vibration characteristics within the data. The mathematical principles behind SPOD are as follows [53]:

The vector $u_k \in R^N$ represents the current state of $u(x,t)$ at time $t_k$ on a discrete set of points in the spatial domain Ω. The total length $N$ is equal to the number of grid points $n_x$ multiplied by the number of variables of concern $n_{\mathrm{var}}$. Taking an equal time interval $\Delta t$, $t_k + 1 = t_k + \Delta t$, to form a $M$-dimensional spatiotemporal matrix at $M$ time points.

$$U = [u_1, u_2, \ldots, u_M] \in R^{N\times M} \tag{18}$$

The Welch method is used to process the data, and the snapshot matrix $U$ is segmented into multiple overlapping block matrices.

$$U^{(n)} = [u_1^{(n)}, u_2^{(n)}, \ldots, \mathrm{u}_{N_{\mathrm{freq}}}^{(n)}] \in R^{N\times N_{\mathrm{freq}}}, 1 \le n \le N_a \tag{19}$$

In the formula, $N_{freq}$ is the number of snapshots for each block matrix, and $N_a$ is the total number of block matrices.

The $k$-th snapshot in the $n$-th block is defined as:

$$u_k^{(n)} = u_{k+(n-1)(N_{freq}-N_{over})} 1 \le k \le N_{freq} \tag{20}$$

In the equation, $N_{over}$ is the number of overlapping snapshots between two consecutive block matrices.

Next, the Discrete Fourier Transform (DFT) is applied to each of the partitioned blocks as well as a window function being applied to each block matrix to alleviate spectral leakage. The resulting snapshot matrices are denoted as:

$$\hat{U}^{(n)} = \left[\hat{u}_1^{(n)}, \hat{u}_2^{(n)}, \cdots, \hat{u}_k^{(n)}, \cdots, \hat{u}_{N_{freq}}^{(n)}\right] \tag{21}$$

The $k$-th snapshot in the $n$-th block $u_k^{(n)}$ is defined as:

$$\hat{u}_k^{(n)} = \frac{1}{\sqrt{N_{freq}}} \sum_{j=1}^{N_{freq}} w_j u_j^{(n)} e^{-i2\pi(k-1)[\frac{j-1}{N_{freq}}]} \tag{22}$$

In the equation, the weight $w_j$ represents the node values of the window function.

The correlation matrix $A_{f_k}$ with frequency $f_k$ is defined as:

$$A_{f_k} = \frac{\Delta t}{sN_a} \sum_{n=1}^{N_a} \hat{u}_k^{(n)} \left(\hat{u}_k^{(n)}\right)^* \tag{23}$$

where $s = \sum\limits_{j=1}^{N_{freq}} w_j^2$

The snapshot vector of the same frequency $f_k$ corresponding to each block is extracted into a new block and reorganized as:

$$\hat{U}_{fk} = \sqrt{k}\left[\hat{u}_k^{(1)}, \hat{u}_k^{(2)}, \hat{u}_k^{(3)}, \cdots \hat{u}_k^{(N_a)}\right] \tag{24}$$

where $k = \Delta t/(sN_a)$; then, the correlation matrix $C_{f_k}$ at the reconstructed frequency is:

$$C_{f_k} = \hat{U}_{f_k} \hat{U}_{f_k}^* \tag{25}$$

The eigenvector corresponding to the $C_{f_k}$ eigenvalue of the matrix is the SPOD mode:

$$C_{f_k} W \Phi_{f_k} = \Phi_{f_k} \Lambda_{f_k} \tag{26}$$

where the positive-definite Hermitian matrix $W \in \mathbb{C}^{N \times N}$ considers the weight $w(x)$ and the numerical quadrature of integrals on discrete grids, and $\Phi_{f_k}$ is the eigenvector matrix corresponding to the eigenvalue matrix $\Lambda_{f_k}$.

It should be noted that, in practice, the number of block matrices $N_a$ is usually much smaller than the length $N$ of the snapshot vector. To improve the calculation speed, the $N \times N$ matrix can be converted into an $N_a \times N_a$ matrix:

$$\hat{U}_{f_k}^* W \hat{U}_{f_k} \Theta_{f_k} = \Theta_{f_k} \widetilde{\Lambda}_{f_k} \tag{27}$$

where $\Theta_{f_k}$ is the eigenvector matrix corresponding to the eigenvalue matrix $\widetilde{\Lambda}_{f_k}$.

The eigenvectors corresponding to these non-zero eigenvalues can be recovered exactly as:

$$\widetilde{\Phi}_{f_k} = \hat{U}_{f_k} \Theta_{f_k} \widetilde{\Lambda}_{f_k}^{-1/2} \tag{28}$$

The matrix $\widetilde{\Phi}_{f_k}$ calculated from eigenvalues represents the $k$-th mode of SPOD. These modes are sorted by their corresponding eigenvalue $\widetilde{\Lambda}_{f_k}$, which characterizes the energy magnitude of each order of the modes.

#### 2.3.2. The Improvements of SPOD

The SPOD method efficiently obtains spatiotemporal single-frequency modes without encountering mode selection issues. Utilizing SPOD reduces sensitivity to numerical noise while capturing low-rank behaviors within the flow, facilitating a deeper understanding of flow mechanisms. Despite its success in data analysis and simulation, classical SPOD has room for improvement, especially when handling large spatial datasets, which can increase computational complexity, particularly in real-time or online applications. Enhancing the SPOD method is necessary to improve its efficiency, accuracy, and applicability.

In 2019, Schmidt et al. [53] introduced a new SPOD algorithm, capable of updating SPOD gradually as new data become available. Such algorithms are often termed learning, instant, or online algorithms. The algorithm's efficacy was demonstrated using large eddy simulation (LES) turbulent jet data and high-speed camera data from a stepped spillway, exhibiting minimal error and demonstrating computational efficiency and practicality. In 2022, Blanco et al. [54] developed an algorithm based on time data shift to enhance SPOD convergence, applying it to the Ginzburg–Landau system. Compared to the standard method, this approach significantly improved modal convergence with a smaller spectral footprint. Zhang et al. [55] combined extended POD with SPOD to create a new method termed extended SPOD (ESPOD) for correlating flow structures and surface pressures.

Nekkanti et al. [56] proposed a data completion method called the Gappy SPOD method to reconstruct flow data in damaged or missing areas. This method was verified through numerical simulations of cylindrical laminar flow and turbulent cavity flow, recovering 97% and 80% of the original data, respectively, surpassing traditional methods. In 2024, Ethan Brothers [57] enhanced the Gappy SPOD method for application to original PIV data. This improvement includes a detailed notch search method and processing of missing regions in all snapshots. It was observed that in regions with few missing elements, this method typically outperforms Gappy SPOD or GPOD. The improvements made by researchers to the SPOD method over the years have been summarized, as outlined in Table 3.

**Table 3.** Improvements of the SPOD method.

| Improvement Method | Reference |
|---|---|
| Streaming algorithm for SPOD | Schmidt et al. [53] |
| Improved convergence of the SPOD<br>SPOD through time shifting | Blanco et al. [54] |
| Extended SPOD | Zhang et al. [55] |
| Gappy SPOD | Nekkanti et al. [56] |
| Improvement of Gappy SPOD | Brothers, Ethan [57] |

### 2.4. Comparison

Before the introduction of the DMD method, the most frequently utilized method for flow field modal analysis was POD [58], which was also the first among these three commonly used modal decomposition methods. The DMD, POD, and SPOD methods can effectively extract the primary features of the flow field and facilitate the simplification of complex flow field calculations, thereby reducing computational costs. However, due to differences in principle, their application is also limited.

The essence of the POD method lies in approximating high-dimensional systems as low-dimensional ones [59]. Widely applied in fluid mechanics, it addresses various flow problems, including trailing edge shedding vortex of NACA airfoils [60], stall problems of wind turbine airfoils [61], evolution laws and feature extraction of supersonic tail jet flow fields [62], and complex aerodynamic configuration optimization design of compressor cascades [59]. It exhibits strong performance in modal decomposition and effectively extracts the main flow structures. However, it may struggle to capture coherent structures in flow states with low energy and frequencies close to the dominant one.

In the POD method, mode selection relies solely on the energy of each mode [63]. Conversely, the DMD method, rooted in Koopman analysis, decomposes the unsteady flow field into characteristic flow modes with single frequencies and fixed growth rates, aiding in the analysis of complex high-dimensional flow fields [64]. When developing the DMD method, a key challenge is capturing as many flow characteristics as possible through a small number of modes to accurately reconstruct and predict the flow field. Essentially, this entails selecting the main modes. However, the DMD method is a feature extraction technique based on the frequency angle flow field; hence, its modal sorting criterion is not unique [36]. Nonetheless, the original flow field can be reconstructed by identifying the first few modes with a large energy ratio [65]. The DMD method is particularly suited for signals with clear frequencies, simple means, and pulsations, making it advantageous in analyzing dynamic linear and periodic flows [7]. However, it is worth noting that in fluid cavitation problems, further study and discussion are needed regarding the applicability of the DMD method [36].

The POD method identifies coherent structures in turbulence based on modal energy levels, without imposing temporal scale restrictions. As a result, it may not capture flow structures at single frequencies, potentially encompassing coherent structures across various time scales. In contrast, the DMD method can generate modes with single frequencies and growth/decay characteristics, yet the absence of a universal mode sorting criterion poses challenges in identifying dominant modes. The SPOD method combines features of both POD and DMD, enabling the extraction of spatiotemporal coupled single-frequency coherent structures and capturing low-rank behavior in the flow. Consequently, SPOD finds widespread application. For instance, Jiang et al. [17] employed SPOD to study flow patterns in the rotor blade tip region of a compressor, as well as to analyze the unsteady flow field in a liquid ring pump ejector [66]. SPOD offers insights into system dynamics in the frequency domain and exhibits applicability to nonlinear systems. However, it may necessitate greater computational resources and data preprocessing efforts.

In the future, as the demand for analyzing dynamic behavior in complex systems grows, these methods will undergo further refinement and expansion, offering greater potential for scientific research and engineering applications. In practical scenarios, methods can be chosen based on the system's nature and data characteristics, or combined for comprehensive analysis, yielding more accurate and thorough results.

## 3. Application

### 3.1. Application of DMD

As an effective tool for analyzing flow structures and identifying modes, the DMD method holds significant importance in studying cavitation phenomena. By decomposing cavitating flow field data using DMD, key dynamic modes can be identified, aiding in understanding the evolution of cavitation and revealing crucial dynamic characteristics within the flow field. This offers a novel approach to comprehending and controlling cavitation phenomena. Table 4 presents various applications of the DMD method in the field of hydraulic machinery.

**Table 4.** Application of the DMD method in hydraulic machinery.

| Application Field | Specific Application | References |
|---|---|---|
| Hydrofoil | The unsteady flow field around a pitching airfoil was investigated using the DMD method. | Mariappan et al. [67] |
| | The velocity field of the unsteady cavitating flow around the NACA66 airfoil was decomposed using the DMD method, further obtaining the dynamic characteristics of the flow field and the structural features of cavitation. | Xie et al. [19] |
| | The unsteady cavitating flow over the Clark-Y hydrofoil was numerically investigated by DMD using an improved PANS model and a simplified Zwart–Gerber–Belamri cavitation model based on the R-P equation. | Qiu et al. [68] |
| | Using the DMD method, the cavitating flow field of the NACA0015 hydrofoil was analyzed. | Wu et al. [36] |

**Table 4.** *Cont.*

| Application Field | Specific Application | References |
|---|---|---|
| Pump | The transient velocity field of unsteady two-phase flow in a helical axial flow pump was decomposed by DMD. | Zhang et al. [69] |
| | DMD was utilized to explore the intricate transient behavior of two-phase flow within a multiphase pump operating at inlet gas volume fractions (GVFs) of 10% and 20%. | Liu et al. [70] |
| | To delve deeper into the gas–liquid flow characteristics of a three-stage multiphase pump, the method of DMD and reconstruction was introduced. | Liu et al. [71] |
| | The velocity distribution and oscillation characteristics of the volute under nominal and low flow-rate conditions were obtained using DMD | Li et al. [72] |
| | The DMD method was applied to analyze pressure fluctuations within the volute, taking into account the unsteady flow conditions in centrifugal pumps with varying trailing edge shapes. | Song et al. [73] |
| | The DMD method was employed to decouple and reconstruct the flow in the centrifugal pump. | Yu et al. [74] |
| | The complex non-stationary flow in centrifugal pumps with varying Inlet Gas Volume Fractions (IGVFs) was analyzed using numerical simulation and the DMD method. | Zhang et al. [75] |
| Turbine | FFT and DMD methods are used to analyze the dynamics of the near wake region. | Wu et al. [76] |
| Pump turbine | The DMD method was employed to investigate both incipient and critical cavitation of a model pump turbine, accurately extracting runner characteristics in pump mode under cavitation conditions. | Wu et al. [77] |
| | In this paper, DMD is used for the first time to decompose and reconstruct the tip leakage vortex (TLV) in a mixed flow pump operating as a turbine at pump mode. | Han et al. [78] |
| Propeller | The transient eddy current structure obtained by LES is analyzed by DMD, which expands understanding of propeller wake dynamics. | Zhi et al. [79] |
| | The wake dynamics of a pump-jet propulsor (PJP) and a ducted propeller (DP) were investigated using DMD analysis, to understand the influence of the pre-swirl stator on the PJP system. | Zhao et al. [80] |

### 3.2. Application of POD

The complexity and variability of cavitating flow fields make comprehensive analysis challenging. POD can be utilized to extract the dominant feature modes within cavitating flow fields, aiding in revealing the primary structures and variations within the flow field. By decomposing the flow field data using POD, important flow structures such as shock waves, vortices, etc., within the cavitating flow field can be identified and analyzed, providing crucial support for a deeper understanding of the flow field. As an efficient data reduction technique, POD can effectively separate flows of different scales within the flow field and extract coherent structures for analysis, thereby achieving the goal of model simplification [14]. The applications of POD in the fields of hydraulic machinery are illustrated in Table 5 below.

**Table 5.** Application of the POD method in hydraulic machinery.

| Application Field | Specific Application | References |
| --- | --- | --- |
| Hydrofoil | This study aims to uncover how the wedge-type cavitating-bubble generator (WCG), a passive control method, affects the cloud cavitation dynamics of the NACA 66 hydrofoil, utilizing the POD method to extract the dominant flow structures. | Hong et al. [81] |
| | PIV was used to study the vortex structures of a hydrofoil with leading-edge tubercles, compared to a standard hydrofoil. PIV velocity field data from water tunnel tests were analyzed using the POD technique. | Wei et al. [82] |
| | Proposed the use of an inlet V-groove to investigate cavitating flow around NACA66 hydrofoil and applied POD to study the coherent structures of cavitation. | Jia et al. [83] |
| | The implementation of POD theory analyzed the cavitating flow around the NACA0015 hydrofoil, delving into the fundamental mechanisms of the hydrofoil's reattachment jet behavior and pressure gradient mechanism. | Yu et al. [84] |
| Pump | The POD method was used to decompose and reconstruct the flow field at the tongue plate of the centrifugal pump. | Lu et al. [85] |
| | The utilization of the POD method further elucidated the intricate relationship between the shape of centrifugal pump blades and their corresponding hydraulic performance, uncovering the impact of optimized blade shapes on flow solutions. | Zhang et al. [86] |
| | The POD method elucidates the emergence and evolution of the predominant unsteady flow structures within a vanless centrifugal pump impeller. | Liao et al. [87] |
| | To examine the unsteady flow field evolving in a centrifugal pump, the POD method is utilized to separate and reconstruct the coherent flow structures. | Chen et al. [88] |
| | Given the intricacy of the two-phase flow field within the liquid ring pump, the POD method is utilized to decompose the transient two-phase flow field within the pump. | Guo et al. [89] |
| | The POD method is used to analyze the TLV structure in axial flow pump. | Fei et al. [90] |
| | The spatiotemporal characteristics of multiscale flow structures in the diffuser of a water jet pump were obtained through statistical analysis and the POD method. | Zhang et al. [91] |
| Turbine | The POD is applied to the antisymmetric and symmetric components of the turbulent fluctuating velocity field in the draft tube to distinguish the dynamics of azimuthal instabilities. | Litvinov et al. [92] |
| | The study employed the POD method to investigate the unsteady cavitating spiral vortex, extracting dominant modes and frequencies, thus providing insights for enhancing the design and performance of hydraulic turbines. | Stefan et al. [93] |
| Pump turbine | The coherent structures within the intricate flow field in the runner area of the pump turbine were isolated and analyzed employing the Finite-Time Lyapunov Exponent (FTLE) and POD methods. | Guang et al. [94] |
| | Utilizing the POD method, they investigated the frequency characteristics and spatial intensity distribution of the stall cell in the pump turbine. | Yang et al. [95] |
| | The simplified turbine model underwent POD analysis to examine individual modes. | Skripkin et al. [96] |

**Table 5.** *Cont.*

| Application Field | Specific Application | References |
|---|---|---|
| Propeller | Applying the POD method for marine propeller shape optimization has been validated in the case of the INSEAN-E779A propeller. | Gaggero et al. [97] |
| | The POD method is applied to the non-steady wall pressure field to analyze the unsteady flow characteristics and its associated hydroacoustic emission, utilizing both POD methodology and experimental approaches. | Witte et al. [98] |
| | The method of combining POD with Wavelet Transform is employed to investigate how the dominant structures mutually influence each other. | Nargi et al. [99] |
| | The POD method can identify the dominant flow structures, providing a quantitative means to analyze the flow mechanism. | Wei et al. [100] |

### 3.3. Application of SPOD

The characteristics of cavitating flow field data typically involve large-scale and high-dimensional features, which traditional POD methods may struggle to effectively handle. The SPOD method, incorporating snapshot techniques and spectral analysis, offers an effective approach for dealing with large-scale datasets and extracting the principal structures and modes of variation within the flow field. SPOD not only extracts the dominant modal features within the flow field but also enables the analysis of correlations between modes. By analyzing the interplay and coupling between different structures revealed by the correlations between modes, SPOD provides deeper insights into understanding and modeling the flow field. Some applications of the SPOD method in the fields of hydraulic machinery are summarized in Table 6 below.

**Table 6.** Application of the SPOD method in hydraulic machinery.

| Application Field | Specific Application | References |
|---|---|---|
| Hydrofoil | The SPOD method was introduced to study the interaction between Internal Solitary Waves (ISWs) and hydrofoil ships. This method provides comprehensive frequency-domain flow field information and principal frequency modes. | Zou et al. [101] |
| Pump | The coherent structure of the noise characteristic signals induced by cavitation in the centrifugal pump was established using the SPOD method. | Lu et al. [102] |
| Turbine | The turbulent coherent structure in the draft tube of the bulb turbine was identified by the SPOD of the velocity field to correlate the change in its topological structure with the decrease in efficiency. | Buron et al. [103] |
| | To extract the dominant structure of the endwall flow field and its unsteady behavior, the SPOD method is used to analyze and compare the PIV measurement and numerical results | Donovan et al. [104] |
| | The study utilized the SPOD method to decompose the modal structures of vertical-axis turbine wakes into different frequencies. | Wang et al. [105] |

### 3.4. The Combined Application of Modal Decomposition Methods

A single mode decomposition method may not be able to fully capture all the features of a flow field, so researchers often combine multiple mode decomposition methods to analyze complex flow fields. By combining multiple methods, researchers can obtain a more comprehensive and accurate description of the flow field, allowing them to better understand the complex dynamic behavior and physical mechanisms of the flow. This integrated analytical approach helps to provide deeper insights and more precise simulations for the study of complex flow phenomena such as vortices and turbulence. Table 7 lists applications where multiple modal decomposition methods are combined for flow field analysis.

**Table 7.** The combined application of modal decomposition methods.

| Application Field | Specific Application | References |
|---|---|---|
| Hydrofoil | The dominant coherent structures around a Clark-Y hydrofoil were identified using the POD and DMD methods. Additionally, the DMD method was capable of predicting transient cavitating flows. | Liu et al. [63] |
| | The dynamic characteristics of sheet/cloud cavitation under flow–structure interaction were investigated on an improved NACA66 hydrofoil. | Liu et al. [106] |
| | The DMD and POD methods were employed to extract coherent structures in the cavitating flow around the ALE-15 hydrofoil. | Liu et al. [107] |
| Pump | The morphology and evolutionary characteristics were investigated using the SPOD and DMD methods. | Li et al. [108] |
| | DMD and SPOD are utilized to decouple the complex coherent structure of high-speed jets in liquid ring pump ejectors. | Jiang et al. [109] |
| | The POD and SPOD methods were introduced to analyze the complex spatiotemporal evolution of the flow field in the liquid ring pump injector. | Jiang et al. [110] |
| Propeller | The utilization of POD and DMD to identify the dominant modes in the physics of propeller wake instability has further enhanced our understanding of the inception mechanisms under heavy loading conditions. | Wang et al. [111] |
| | To decompose the wake field, the POD and DMD methods were used. | Shi et al. [112] |

## 4. Conclusions and Outlook

This paper introduces the basic theories and related advancements of three data-driven modal decomposition methods. It discusses the strengths and weaknesses of each method and summarizes their applications as cavitation characteristic detection techniques in hydraulic machinery. In cavitation flow fields, modal decomposition methods show great potential for development. Considering the current development status, there are several future trends:

1. The DMD method can seize the dynamic characteristics of cavitating flow fields and provides relatively accurate modal decomposition results in effect. It not only identifies the primary vortex structures but also reveals their spatiotemporal evolution patterns. The modes obtained by DMD have single frequencies and growth rates, thus offering significant advantages in analyzing the dynamics of linear and periodic flows. Since the development of DMD is based on linear dynamic assumptions, future research in DMD algorithms will focus on how to select appropriate observation quantities and integrate high-precision nonlinear system identification techniques for better analysis of nonlinear problems.
2. The POD method also plays a significant role in analyzing cavitating flow fields. By performing spatial modal decomposition on flow field data, it obtains the primary vibration modes within the flow field, revealing the spatial structure of cavitation phenomena and the distribution pattern of energy. With increasing demand for artificial intelligence, there has been relatively little research, both domestically and internationally, on the application of the POD method in data-driven approaches, particularly in deep learning. In the future, the combination of the POD method with deep learning techniques could lead to more effective methods for flow field data analysis and mode extraction.
3. The SPOD method expands the understanding of cavitating flow fields. By performing spectral decomposition on flow field data, it extracts the primary frequency components within the flow field, the structures on other modes complement and reinforce the dominant mode structure, making the flow field information more realis-

tic. However, the computational complexity of SPOD is typically high, and further improvements in data processing techniques are needed to enhance the applicability and accuracy of the SPOD method for experimental or simulation data in the future.

The data-driven modal decomposition methods, namely DMD, POD, and SPOD, show promising prospects in cavitation flow fields. They enhance our understanding of the dynamic characteristics of cavitation phenomena and provide crucial theoretical support for controlling and optimizing cavitating flow fields.

Future research directions involve optimizing modal decomposition methods to better suit complex flow fields, exploring the combined use of various modal decomposition methods to acquire comprehensive flow field information, and integrating numerical simulations with experimental studies to delve deeper into the mechanisms and influencing factors of cavitation phenomena. This will offer a deeper understanding and guidance for controlling and applying cavitating flow fields.

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