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Liner Schedule Design under Port Congestion: A Container Handling Efficiency Selection Mechanism

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Abstract: Port congestion significantly impacts the reliability of container ship schedules. However, the existing research often treats vessel time in port as a random variable, failing to systematically consider the complex impact of port congestion on ship schedules. This study addresses the issue of container ship schedule design under port congestion. Vessel waiting times in ports are predicted and quantified by queueing theory, along with information on vessel schedules, cargo handling volumes, and available port operating time windows. We propose a mechanism for selecting container handling efficiencies for arriving vessels, thereby determining their in-port handling times. By jointly considering the uncertainty of vessel waiting and handling times in port, we establish a mixed-integer nonlinear programming model aimed at minimizing the total cost of liner transportation services. We linearize the model and solve it using CPLEX, ultimately devising a robust ship schedule. A simulation analysis is conducted on a real liner shipping route from Asia to the Mediterranean, revealing that extreme weather events, geopolitical conflicts, and other factors can lead to severe congestion at certain ports, necessitating timely adjustments to vessel schedules by shipping companies. Moreover, such events can impact the marine fuel market, prompting shipping companies to adopt strategies such as increasing vessel numbers and reducing vessel speeds in response to high fuel prices. Additionally, the container handling efficiency selection mechanism based on information sharing enables shipping companies to flexibly design liner schedules, balancing the economic costs and service reliability of container liner transportation.

Keywords: container liner shipping; liner schedule design; port congestion; container handling efficiency selection mechanism

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1. Introduction

Liner shipping handles over 80% of the transportation of finished products in world trade [1]. Liner vessels operate container transportation services along fixed routes, calling at designated ports in a predetermined sequence, adhering to a published schedule, and charging relatively fixed freight rates. Typically, liner companies announce their schedules for various regions 3–6 months in advance [2]. Shippers, such as global manufacturers, then plan their production and transportation schedules accordingly. However, recent years have seen a significant escalation in the frequency and severity of port congestion. For instance, the ports of Los Angeles and Long Beach in the United States have been grappling with prolonged and severe congestion since October 2020, with 109 vessels backlogged as of 9 January 2022 [3]. This discrepancy between planned and actual vessel arrivals and departures disrupts the delivery timelines of maritime shipments and undermines the reliability of vessel schedules. Consequently, unreliable schedules compel shippers to maintain substantial safety stocks, hindering the implementation of efficient just-in-time production plans and posing challenges to the security and stability of global supply chains [4].

Liner schedule punctuality reflects a liner company's capability in route planning, port cooperation, and risk management, serving as a crucial factor for shippers when selecting maritime service providers. Notteboom [5] investigated the primary reasons for low liner punctuality, finding that over 93.8% of schedule delays were related to port operations, with congestion causing 65.5% of the unexpected waiting time before cargo handling and port infrastructure capacity limitations accounting for 20.6%. Scholars [6,7] have extensively studied port congestion using methods like stochastic programming, robust optimization, and variational inequality model. However, since late 2023, events such as the significant container backlog at ports in Durban and Cape Town, drought-induced low water levels in the Panama Canal, and geopolitical conflicts in the Red Sea have led to frequent disruptions, maintaining the severity of global port congestion and undermining container liner schedule punctuality. The global mainline punctuality index released by the Shanghai Shipping Exchange in February 2024 stood at a mere 32.48%, plummeting by 16.16% compared to February 2023. Addressing the real challenges of port congestion while considering the uncertainties of vessel waiting and handling times in ports (where waiting time refers to the time ships spend at anchor after arrival, and handling time refers to the time spent handling containers by terminal cranes), as well as designing reliable liner schedules, presents a significant opportunity for liner companies to attract customers and seize market share.

On the one hand, port congestion primarily affects the waiting time for vessels before entering berths, known as the in-port waiting time. However, the existing literature often treats in-port waiting time as a uniform random variable [8–11]. In reality, vessel berthing patterns and transit times vary significantly across different ports. Queueing theory, recognized as a crucial analytical tool for congestion studies, has found applications in optimizing berth allocation and describing port performance, proving to be a highly effective method for quantifying port congestion. It allows for the estimation of critical parameters such as average vessel waiting time, queue length, and average number of vessels in port. Therefore, leveraging queueing theory models can effectively address the uncertainty of vessel in-port waiting time.

On the other hand, shipping companies and ports have also made collaborative efforts to address port congestion [12–14]. Before opening a new route, shipping companies engage in information sharing with port operators, including vessel details (especially vessel length and draft) and the range of container handling volumes. Port operators, considering port throughput capacity and expected handling requirements, share available container handling time windows and average handling efficiency data with liner companies and offer multiple options for container handling efficiency at a higher-than-normal level, along with additional charges for this "VIP" service. Then, shipping companies predict in-port waiting times due to port congestion, determine vessel arrival and departure times at each port, and select suitable container handling efficiency based on a comprehensive evaluation of their schedules, available port operation time windows, and container handling plans, thus determining the vessel's in-port operation time [15]. This mechanism for selecting container handling efficiency is a win-win strategy, enhancing the reliability and stability of shipping company schedules, while also optimizing port revenue and reducing congestion issues without compromising port operator interests. However, this will also make the vessel handling time in port uncertain, thus increasing the difficulty of the design of the shipping schedule.

Therefore, this study addresses the liner schedule design in the context of port congestion. It utilizes queueing theory to predict and quantify vessel waiting times in port, integrating vessel schedules, cargo handling volumes, and available port operation time windows; proposing a container handling efficiency selection mechanism for inbound vessels; and determining vessel handling times in port. By jointly considering the uncertainty of vessel waiting and handling times, a mixed-integer nonlinear programming model is developed, aiming to minimize the total cost of shipping services. The model undergoes linearization and is solved using CPLEX to devise a reliable shipping schedule. Finally, an experiment analysis is conducted on a real-world Asia-to-Mediterranean shipping route. Our study makes the following contributions:

(1) This paper addresses the issue of liner schedule design, considering the uncertainties of vessel waiting and handling times caused by port congestion. It determines the vessel deployment quantities on the route, vessel speeds on segments, and arrival and departure times at each call port. This comprehensive framework for handling port congestion can be extended to other types of emergencies, such as disruption.

(2) In maritime logistics, the existing literature often treats vessel time in port as a uniform random variable, overlooking significant differences in vessel berthing and container handling across different ports. This paper treats vessel operations upon arrival as a queueing system and employs queueing theory models to predict and quantify vessel waiting times in port. In fact, this model can also be applied to terminal operations and truck-to-port transfers.

(3) We propose a port handling efficiency selection mechanism. Numerical simulations confirm that this information-sharing mechanism not only enhances the flexibility and reliability of liner shipping services but also increases profitability for both liner companies and ports. This mechanism offers a new approach to collaboration and information sharing in maritime logistics.

(4) A simulation analysis on a real Asia-to-Mediterranean liner shipping route demonstrates that extreme weather events and geopolitical conflicts can cause severe congestion at certain ports. Liner companies must adjust vessel schedule promptly in response. Additionally, such events can impact the marine fuel market, necessitating strategies such as increasing vessel operations and reducing vessel speeds under high fuel prices.

The remainder of this paper is organized as follows. Section 2 presents related studies on liner schedule design, application of queueing theory in maritime operations, and port handling efficiency selection mechanism. Section 3 develops a mixed-integer nonlinear programming model. Section 4 linearizes the model and utilizes commercial solver CPLEX to solve. Section 5 describes computational experiments, illustrates the result analysis, and derives managerial insights. Section 6 provides conclusions.

2. Literature Review

This study primarily addresses the schedule design in maritime shipping, taking into account port congestion. By utilizing queueing theory models to quantify and predict vessel berth times and proposing a port handling efficiency selection mechanism based on port–harbor cooperation agreements, it aims to determine the deployment quantities of liners on routes, vessel speeds on various segments, and arrival and departure times at each call port. The following literature review delves into liner schedule design, the application of queueing theory in maritime operations, and the implementation of the port handling efficiency selection mechanism.

2.1. Liner Schedule Design

For liner companies, schedule design constitutes a mid-term tactical decision. At this stage, the ports of call along the route are predetermined, requiring the determination of service frequencies, vessel deployment quantities, planned arrival and departure times at each port, and estimated speeds for each segment [16]. Initially, scholars exploring schedule design typically studied it in conjunction with port operating time window constraints or cargo transshipment requirements, without considering uncertain factors like adverse weather or port congestion [17–19]. Fagerholt [20] introduced soft time windows for port operations in schedule design, imposing penalty costs on vessels unable to arrive at ports within the designated time, aiming to enhance schedule reliability and reduce transportation costs by controlling port time windows. Wang et al. [21] and Alharbi et al. [22], respectively, studied schedule optimization problems considering port time windows for single routes and liner shipping networks. Their findings indicate that port time windows, port handling efficiency, and fuel prices influence total transportation costs, vessel configurations, and schedule plans.

However, in reality, vessels encounter various unexpected situations, such as adverse weather and port congestion during navigation and port operations [23]. Notteboom [5] first explored the reasons for low schedule adherence rates, finding that over 93.8% of delays were related to port operations, and proposed strategies for reschedule port calls and port skipping to resume interrupted services. Scholars continuously attempt to incorporate uncertain factors into schedule design problems through modeling, simulation, and quantitative analysis to enhance the practical value of theoretical models. Song et al. [24] considered the joint tactical planning problem of vessel deployment quantity, planned maximum speed, and liner service frequency, optimizing vessel operating costs, service reliability, and vessel emissions under port time uncertainty. Wang and Meng [11,25] incorporated port-to-port transshipment time, sea accident time, and port time uncertainty into schedule design, modeling vessel speed as a function related to sea accident time to determine vessel arrival times at each port along the route and speeds for each segment. The uncertainty here is divided into two parts: uncertainty in waiting time caused by port congestion and uncertainty in container handling time. Subsequently, to improve the accuracy of schedule design, scholars often separately consider these uncertainties [26].

2.2. Application of Queueing Theory in Maritime Operations

Port congestion is a primary factor affecting liner schedule adherence. Existing studies generally treat the waiting time for vessels at ports as a random variable. For example, Zhang [10] used common mean and variance-based distribution functions to represent the uncertainty of port waiting times and incorporates liner reliability objectives into the schedule design. Wang et al. [11] assumed vessel waiting time and port operation time follow a truncated normal distribution and designed a robust liner schedule accordingly. However, these variables are closely related to the number of arriving vessels and the port's handling capacity [27–29]. Queueing theory, a key analytical tool for studying congestion, has been applied to optimize berth quantities and describe port performance [30].

As early as 1978, Edmond and Maggs [31] applied queueing theory to berth construction and investment decisions regarding cargo-handling equipment. Dragović et al. [32] utilized simulation and queueing theory to calculate parameters such as berth utilization, average number of ships in queues, and average waiting time for ships, thereby evaluating operational efficiency and processes at berth stages. Subsequently, some scholars optimized port berth quantities using real port data. El-Naggar [33] aimed to minimize total costs, optimizing berth quantities at the Port of Alexandria using queueing theory, confirming that the ship arrival pattern followed a Poisson distribution. Saeed and Larsen [34] utilized queueing theory to minimize ship waiting time costs and berth construction costs, assessing whether the berth quantity at the container terminal of the Port of Manila in the Philippines was sufficient. Zheng et al. [35] designed a three-stage optimization method based on queueing theory and cooperative game theory, exploring the optimal allocation and distribution of berth resources in multi-port regions.

2.3. Port Container Handling Efficiency Selection Mechanism

With the deepening integration of port and shipping, the relationship between port operators and liner companies has become closer [36,37]. Although the average container handling efficiency of the port is relatively fixed [38], due to the temptation of the liner enterprise to pay extra costs, the port operator can provide more efficient container handling efficiency for the arrival vessels. Liner companies can negotiate handling rates and higher efficiencies with container terminal operators to reduce the overall vessel turnaround time. Pasha et al. [39] suggested that, in tactical liner transportation decisions—such as determining service frequency, fleet deployment, optimizing vessel speed, and designing schedules—vessels can be serviced by various operators with different handling efficiencies upon arrival at the terminal. Choosing operators with higher handling efficiency reduces vessel loading and unloading time and fuel consumption but increases container handling costs.

Therefore, some scholars [21,40] have considered the impact of port and shipping cooperation agreements on liner schedule optimization design, proposing that ports can offer multiple optional operation time windows and container handling efficiencies to liner companies. Liu et al. [15] explored the influence of shipping companies requesting higher container handling efficiency from ports after paying certain costs on schedule design, aiming to reduce vessel turnaround time and fuel consumption by decreasing vessel speeds. Building upon this, Dulebenets et al. [41] proposed a more comprehensive collaborative mechanism where port operators can offer multiple options for arrival time windows, multiple start and end times for available time windows, and multiple handling efficiencies, calculating potential benefits for liner companies under comprehensive cooperation agreements. Yu et al. [42] examined how information sharing, communication, and feedback between shipping companies and terminal operators enable the sharing of updated vessel arrival times. This dynamic information exchange allows for the planning of critical berth and quay crane allocations based on the updated arrival times.

2.4. Summary

Research on schedule optimization design has evolved from deterministic to uncertain factors, with some scholars integrating uncertain factors with constraints such as operation time windows and acceleration strategies. As port container throughput continues to rise, vessel queueing at ports for service has become common. However, the existing literature often treats vessel time at port as a uniform random variable during the schedule design stage, overlooking the distinct vessel docking and container handling situations at different ports. Moreover, statistical studies indicate that vessel arrival at ports follows a Poisson distribution, and vessel berth occupation time generally follows a negative exponential distribution. Therefore, this paper studies vessel operations at ports as a queueing system, employing queueing theory to compute vessel waiting times at ports. Additionally, in the context of port and shipping collaborative development, the relationship between shipping companies and terminal operators has become closer, with increased transparency of information. Scholars propose that ports can offer multiple optional operation time windows and container handling efficiencies to liner companies under cooperation agreements to enhance the schedule reliability of container liners and promote the interests of multiple parties.

Considering these factors, this study fully takes into account the uncertainty of vessel waiting times and handling times at ports under real port congestion conditions, establishing a nonlinear mixed-integer programming model for schedule design. This aims to mitigate the adverse effects of port congestion on liner transportation operations and service quality.

3. Model Formulation

3.1. Problem Description

The current global port congestion significantly impacts liner schedule punctuality, necessitating consideration of port congestion uncertainty in schedule design. Vessels adhere to predetermined port call schedules, sequentially visiting port $i \in I, I = \{1, \ldots, N\}$ during each voyage, and they need to arrive at the port within designated time windows $[TW_i^{start}, TW_i^{end}]$ for operations. However, in the context of port congestion, vessel time in port is affected by waiting for berths and container handling, thus introducing significant uncertainty. To address the uncertainty of waiting for berths, we utilize queueing theory models to predict and quantify vessel berth times. By considering factors such as the daily vessel arrivals number, λ_i; available berths number, c_i; and average number of vessels served per berth, μ_i, we calculate the queue service intensity, ρ_i, at port i. Subsequently, we determine the actual wait times, θ_i^{wait}, for vessels at port i by comparing the relationship of queue times, θ_i^{queu}; arrival times, θ_i^{arr}; and available service windows.

To address the uncertainty of handling times, we propose a container handling efficiency selection mechanism. This allows vessels to choose one of several container handling efficiency options, H_i, at port i based on their specific needs. We introduce a decision variable, $x_{i,h}$, to represent vessel choices, determining container handling efficiency, $\pi_{i,h}$, at port i. Subsequently, considering container quantity, δ_i^{port}, and unit port service costs, $c_{i,h}^{hand}$, we calculate the total container handling fees that vessels need to pay at each port and determine vessel berth times, θ_i^{hand}, accordingly.

After addressing vessel berth and handling uncertainties, vessels decide their sailing speeds, v_i, based on segment distances, D_i; departure times, θ_{i-1}^{depart}, from previous ports; and vessel speed ranges, $[V^{min}, V^{max}]$, calculating sailing times, θ_i^{sail}, for each segment. Fuel costs, determined by unit fuel costs, c^{fuel}; fuel coefficients, α, γ; unit fuel consumption, φ_i; and segment distances, D_i, is incurred during voyages, in addition to and container inventory costs, c^{inv}, based on container quantities, δ_i^{seg}, and sailing times, θ_i^{sail}, per segment. If vessels fail to complete operations within specified time windows, we impose a penalty cost based on delay duration, θ_i^{late}, and unit penalty cost, c_i^{late}, aiming to enhance liner service quality.

Last, shipping companies need to deploy m vessels on the route, each incurring fixed operational costs, c^{ope}, per voyage to meet weekly service requirements. The number of vessels available for deployment has an upper bound, M. In summary, this study establishes an optimization model minimizing total costs by considering vessel operational, fuel consumption, container handling, inventory, and delay penalty costs.

3.2. Assumptions

(1) Vessel berth time is primarily determined by waiting and container handling, without considering time spent on other activities [26].

(2) The sequence and ports of call along the route are known, with liner companies offering weekly service frequencies [43].

(3) The study focuses solely on optimizing schedules for a single route, excluding transshipment issues [25].

(4) Similar vessel types are deployed on the route, sharing identical technical characteristics, and only the fuel consumption of main engines is considered [25].

(5) The import process of vessel arrivals at ports follow a Poisson distribution [44,45], while berth occupancy times adhere to an exponential distribution [46].

(6) It is assumed that vessels adhere to a first-come, first-served principle at ports. If vessels arrive before the available berth window, they must wait at anchor. Given predetermined port calls at a strategic level, liner companies exhibit patience in queueing without departing or diverting mid-route [47].

3.3. Symbol Specification

Before formulating the model for this problem, we list the notation as follows.

Indices and sets:

I: set of liner line call port or line segment; that is, the ith call port corresponds to the ith section, $i \in I$.

H_i: set of container handling efficiency in port i, $h \in H_i$.

Parameters:

N: number of ports of call.

c^{ope}: fixed operating costs $(USD/week)$.

c^{inv}: unit inventory cost of containerized goods $(USD/(TEU \times hour))$.

c_i^{late}: unit penalty cost of vessel delay in port i $(USD/hour)$.

c^{fuel}: unit fuel cost (USD/ton).

$c_{i,h}^{hand}$: vessel unit service cost when selecting container handling efficiency, h, at port i (USD/TEU).

$\pi_{i,h}$: container handling efficiency, h, at port i $(TEU/hour)$.
V^{min}: minimum vessel speed $(knot)$.
V^{max}: maximum vessel speed $(knot)$.
D_i: distance of segment i $(n\ mile)$.
TW_i^{start}: start time of port i's available time window $(hour)$.
TW_i^{end}: end time of port i's available time window $(hour)$.
α, γ: correlation coefficients of fuel consumption function.
δ_i^{seg}: number of containers transported by the vessel in segment i (TEU).
δ_i^{port}: number of containers handled at port i (TEU).
M: maximum number of vessels that can be deployed on the route $(ship)$.
λ_i: average number of vessels arriving at port i per day $(ship)$.
μ_i: average number of vessels serviced per day by a berth in port i $(ship)$.
c_i: number of berths available in port i.
Intermediate variables:
φ_i: Unit fuel consumption of the vessel in segment i $(ton/n\ mile)$.
θ_i^{arr}: Arrival time of the vessel at port i $(hour)$.
θ_i^{hand}: Container handling time of the vessel at port i $(hour)$.
θ_i^{depart}: Departure time of the vessel from port i $(hour)$.
θ_i^{sail}: Sailing time of the vessel in segment i $(hour)$.
θ_i^{late}: Delay time of the vessel at port i $(hour)$.
θ_i^{queu}: Queueing time of the vessel at port i $(hour)$.
θ_i^{wait}: Waiting time of the vessel at port i $(hour)$.
ρ_i: Queue service intensity of port i $(ship)$.
Variables:
v_i: speed of vessel on segment i $(knot)$.
$x_{i,h}$: 0–1 variable, 1 if the vessel selects the hth container handling efficiency, $\pi_{i,h}$, in port i; otherwise, 0.
m: Number of vessels deployed on the route $(ship)$.

Regarding the abbreviations used in this paper, TEU stands for Twenty-Foot Equivalent Unit, USD represents United States Dollar, and n mile denotes Nautical Mile. We use these abbreviations consistently throughout the text.

3.4. Mixed-Integer Nonlinear Programming Model

Based on the above description, the first liner schedule design model (M1) is established with the goal of minimizing the total costs of liner services as follows:

$$Min\ z = c^{ope} \cdot m + c^{fuel} \cdot \sum_{i \in I} D_i \cdot \varphi_i + \sum_{i \in I} \sum_{h \in H_i} c_{i,h}^{hand} \cdot \delta_i^{port} \cdot x_{i,h} + \sum_{i \in I} c^{inv} \cdot \delta_i^{seg} \cdot \theta_i^{sail} + \sum_{i \in I} c_i^{late} \cdot \theta_i^{late} \tag{1}$$

Equation (1) represents the objective function of the model, aiming to minimize the total costs of liner services. The first term pertains to vessel operating costs, which are contingent upon the number of vessels deployed along the route. The second term accounts for fuel costs, determined by vessel speed and distance traveled. The third term reflects port handling costs, influenced by container handling volumes and efficiency. The fourth term represents container inventory costs, primarily linked to the duration of containers on board. Lastly, the fifth term denotes penalties for vessel delays, contingent upon the duration of container delays.

3.4.1. Waiting Time Prediction Model Based on Queueing Theory

Initially, we quantify the queueing time for vessels waiting for berthing in port due to port congestion. When vessels randomly dock at any public berth at a port, their arrivals follow the $M/M/c$ multi-server queueing system. As this study involves multiple time points related to port availability windows, Figure 1 elucidates the interrelationships among these time points.

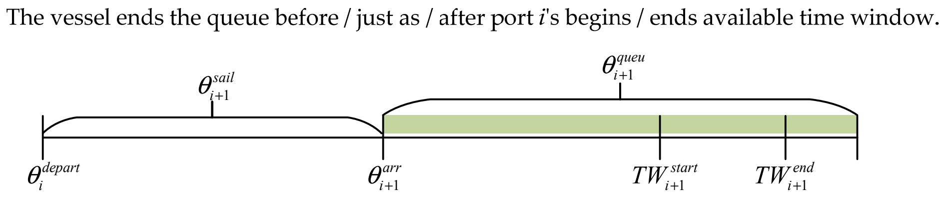

Figure 1. Queueing scenarios for ships arriving at the port.

In Figure 1, the vessel departs from port i at time θ_i^{depart}, after traveling for θ_{i+1}^{sail} units of time in segment $i+1$ (from port i to port $i+1$) and arriving at the port $i+1$ at time θ_{i+1}^{arr}. The start and end times of the available time window at port $i+1$ are TW_{i+1}^{start} and TW_{i+1}^{end}, respectively. As the queueing time for the vessel at port $i+1$ is uncertain and subject to variation within the green matrix, three scenarios can occur:

(1) The vessel completes queueing before the start of the available time window, TW_{i+1}^{start}, at port $i+1$. In this case, the waiting time for berthing is $TW_{i+1}^{start} - \theta_{i+1}^{arr}$.

(2) The vessel completes queueing precisely between the start time, TW_{i+1}^{start}, and the end time, TW_{i+1}^{end}, of the available time window at port $i+1$ (including these two time points). Here, the waiting time for berthing is θ_{i+1}^{queu}.

(3) Vessels complete queueing after the end of the available time window TW_{i+1}^{end} at port $i+1$. Despite being unable to avail port services at this point, there is still an opportunity cost for waiting, and the waiting time for berthing is θ_{i+1}^{queu}. Certainly, this scenario corresponds to $\rho_i > 1$; that is, the number of vessels exceeds the port's service capacity for the day, theoretically resulting in prolonged queueing. For instance, since October 2023, Durban Port on the South African East Coast has experienced extreme weather conditions and equipment malfunctions by the port operator, Transnet. This has led to over 100,000 containers being stranded, with more than 100 container vessels becoming stuck, causing significant delays in liner schedules. Experts estimated that the backlog would not be cleared until February 2024. Opting to skip the port would be the optimal choice for liner companies during such circumstances. However, as these events are extreme emergencies beyond schedule design considerations, this study does not account for them. Hence, we assume that vessels always arrive before the end of the available time window at the port.

The waiting time prediction model based on queueing theory is as follows:

$$\rho_i = \frac{\lambda_i}{c_i \cdot \mu_i}, \ \ \forall i \in I \tag{2}$$

$$P_i^0 = \left[\sum_{n=0}^{c_i - 1} \frac{1}{n!} \cdot \left(\frac{\lambda_i}{\mu_i}\right)^n + \frac{1}{c_i!} \frac{1}{1-\rho_i} \left(\frac{\lambda_i}{\mu_i}\right) \right]^{c_i - 1}, \ \ \forall i \in I, if \ \ \rho_i < 1 \tag{3}$$

$$P_i = \frac{1}{(c_i!) \cdot (1-\rho_i)} \left(\frac{\lambda_i}{\mu_i}\right)^{c_i} \cdot P_i^0, \ \ \forall i \in I \tag{4}$$

$$L_i^q = \frac{\left(\frac{\lambda_i}{\mu_i}\right)^{c_i} \cdot \rho_i}{(c_i!) \cdot (1-\rho_i)^2} \cdot P_i^0, \ \ \forall i \in I \tag{5}$$

$$\theta_i^{queu} = \frac{L_i^q}{\lambda_i}, \ \ \forall i \in I \tag{6}$$

$$\theta_{i+1}^{wait} = Max\left\{ TW_{i+1}^{start} - \theta_{i+1}^{arr}, \theta_{i+1}^{queu} \right\} \ \ \forall i \in I, 0 \le i < N \tag{7}$$

Formulas (2)–(7) quantify the vessel's berth waiting time at port i using queueing theory, with a progressive relationship between them. Equation (2) computes the queue service intensity, ρ_i, at port i. Furthermore, when the service intensity is $\rho_i < 1$, Equation (3) calculates the probability, P_i^0, of no vessels arriving at port i. Building upon Equations (3) and (4), we assess the probability, P_i, of the vessel needing to queue at port i when n vessels arrive and $n \geq c_i$. Equation (5) determines the queue length, L_i^q, of the vessel waiting for service at port i. Equation (6) computes the waiting time, θ_i^{queu}, of the vessel at port i. Additionally, Equation (7) calculates the waiting time of the vessel at port i. Given the closed-loop nature of liner shipping, this waiting time is contingent upon the departure time from the last port of the previous loop and the sailing time to the first port of the current loop.

3.4.2. Liner Schedule Design Model

For liner enterprises, schedule design involves determining vessel arrival and departure times of vessels at various ports along established routes or shipping networks. Before finalizing these times, liner companies need to gather information about available working hours, container handling capacity, vessel turnaround times at ports, distances and weather conditions along route segments, cargo volume between ports, and constraints related to vessel attributes and quantity on the route. Based on this information, parameters are set according to principles of minimizing costs or optimizing service. With these parameters, round-trip voyage times can be calculated. This refers to the time that vessels spend traveling from the originating port, visiting all ports in sequence, and returning to the originating port, encompassing both sailing and port waiting and operation times. The calculation formulas and constraints of the model are provided below.

$$\sum_{h \in H_i} x_{i,h} = 1, \ \ \forall i \in I \tag{8}$$

$$\theta_i^{sail} = \frac{D_i}{v_i}, \ \ \forall i \in I \tag{9}$$

$$\varphi_i = \frac{\gamma \cdot (v_i)^{\alpha - 1}}{24}, \ \ \forall i \in I \tag{10}$$

$$\sum_{i \in I} (\theta_i^{sail} + \theta_i^{wait} + \theta_i^{hand}) = 168 \cdot m \tag{11}$$

$$\theta_i^{depart} = \theta_i^{arr} + \theta_i^{wait} + \theta_i^{hand}, \ \ \forall i \in I \tag{12}$$

$$\theta_1^{arr} = \theta_N^{depart} + \theta_N^{sail} - 168 \cdot m \tag{13}$$

$$\theta_{i+1}^{arr} = \theta_i^{depart} + \theta_i^{sail}, \ \ \forall i \in I, i < N \tag{14}$$

$$\theta_i^{late} = Max\left\{\theta_i^{depart} - TW_i^{end}, 0\right\}, \ \ i \in I \tag{15}$$

$$\theta_i^{hand} = \sum_{h \in H_i} \frac{\delta_i^{port}}{\pi_{i,h}} x_{i,h}, \ \ \forall i \in I \tag{16}$$

$$m \leq M \tag{17}$$

$$V^{min} \leq v_i \leq V^{max}, \ \ \forall i \in I \tag{18}$$

Formula (8) indicates that the vessel must select one container handling efficiency scheme at each port. Formula (9) calculates the sailing time for each route segment. Formula (10) computes the fuel consumption per unit distance based on the vessel sailing speed for each segment. Formula (11) ensures that the deployed vessel quantity maintains a weekly port service frequency. The left side represents the total time for a vessel to complete one voyage, including sailing, handling, and waiting times, while the right side is the product of the total hours in a week ($168 = 7 \times 24(hour)$) and the vessel quantity, m, needed to be deployed. Formula (12) calculates the departure time of the vessel from port i based on arrival, waiting, and handling times. Formulas (13) and (14) determine the arrival times at the first port and subsequent ports. Formula (15) calculates the delay time for the vessel at each port. Formula (16) computes the handling time for the vessel at each port. Formula (17) restricts the deployed vessel quantity on the route in order to not exceed the maximum available vessel quantity. Formula (18) sets the constraint for vessel sailing speed.

4. Model Solving

The model's constraints (9) and (10) involve reciprocal and power functions of vessel sailing speed, $v_i, i \in I$, making it a mixed-integer nonlinear programming problem. To facilitate solving, the model needs linearization.

For Formula (9), we replace vessel speed, v_i, with its reciprocal, $v_i^{rec} = 1/v_i$, to linearize the constraint.

Regarding the model, the liner speed is a continuous variable within a certain range, meaning the unit fuel consumption calculated using Formula (10) can be any rational number within that range. Various methods have been proposed in the liner transportation literature to address this nonlinearity.

(1) Enumerative method: assuming constant vessel speeds for route segments, minimizing the total service cost to solve vessel schedule.

(2) Discretization method: discretizing vessel speed reciprocals, estimating fuel consumption for each discretized reciprocal speed, and simplifying vessel schedule into a mixed-integer linear problem.

(3) Dynamic programming method: simplifying vessel schedule into a shortest-path problem in a spatiotemporal network, with time as the horizontal axis (usually in days) and ports as the vertical axis.

(4) Customized method: substituting nonlinear fuel consumption functions with approximate functions (e.g., sets of tangent or secant lines) to simplify vessel schedule into a mixed-integer linear problem.

(5) Second-order cone programming method: transforming the original mixed-integer nonlinear schedule model into a mixed-integer second-order cone programming model [48].

Among these methods, dynamic programming, discretization, and customized methods usually more effectively approximate nonlinear fuel consumption functions [49]. This paper focuses on liner schedule design, a static problem determined before the ship's voyage. This static nature distinguishes it from dynamic scheduling problems that dynamic programming typically addresses. Therefore, dynamic programming is not suitable for our context. On the other hand, discretization can set appropriate speed precision for liner ships based on the shipping company's expectations and is frequently used in schedule design and ship scheduling. Thus, we employed the discretization method.

According to this method, vessel speed reciprocals are discretized into a finite set of values, $K = \{1, \ldots, e\}$. Let v_k^{val} be the reciprocal vessel speed at the discretization point, k. Then, $\varphi_k^{val} = \gamma (v_k^{val})^{-(\alpha-1)}/24$ denotes the unit fuel consumption when using the reciprocal vessel speed value at the discretization point, k. The degree of discretization, k, increases the precision of approximating the fuel consumption function but also increases the number of variables in the model, potentially leading to longer solution times. Further discussion on this is provided in the Section 5.2. We introduce a new parameter, $\beta_{i,k}$, which equals 1 if the vessel's fuel consumption value for segment i is estimated using the discretization point, k, and it is 0 otherwise. The impact of different discretization precisions (i.e., speed selection ranges) on the unit fuel consumption is illustrated in Figure 2.

Figure 2 shows the results for four different speed discretization scenarios. In each scenario, we selected one point with the same x-coordinate, where the reciprocal of the vessel speed, $1/v_i$, is 0.054750. The y-coordinates (unit fuel consumption φ_i) vary due to the different ranges of speed options available. In the first scenario (k = 5), the speed selection range includes only five options, resulting in an approximated unit fuel consumption coefficient, φ_i, of 0.173774. In contrast, in the fourth scenario (k = 20), with twenty speed options, the approximated unit fuel consumption coefficient, φ_i, is 0.172868, yielding a precision difference of 0.524%. Although this difference in precision is not significantly large, it can substantially affect the final fuel costs. For instance, consider the example from

Section 5.1, where the total voyage distance is 20,948 n miles, the vessel speed is assumed to be 20 knots, and the fuel price is 300 USD/ton. The calculated fuel cost difference between the first and fourth scenarios is USD 125,688. Given that the precision of the discretization method significantly impacts the quality and computation time of the final results, we provide a more detailed analysis in Section 5.4.

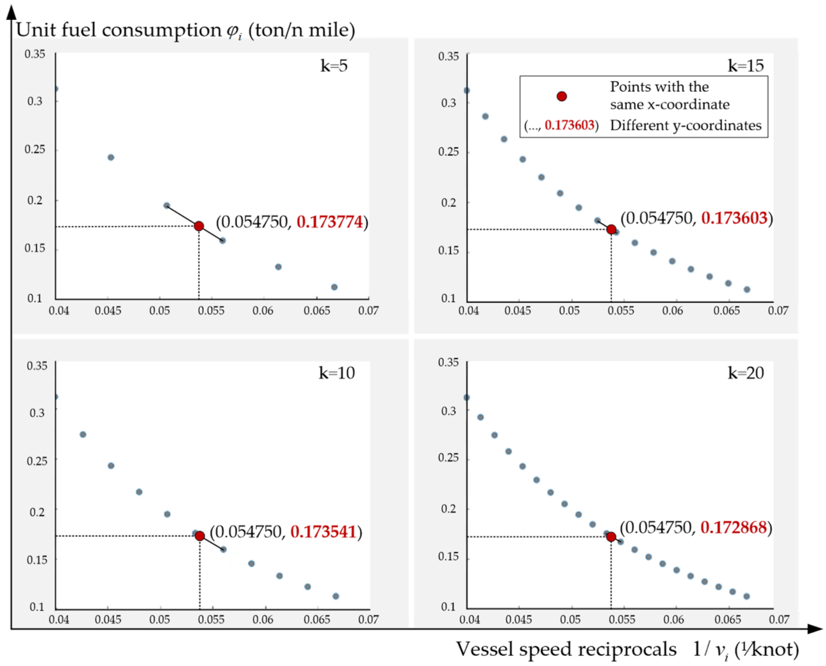

Figure 2. Effect of speed discretization precision on unit fuel consumption.

At this point, the original nonlinear model (M1) is converted to a mixed-integer linear programming model (M2):

Objective function:

$$Min\ z = c^{ope} \cdot m + c^{fuel} \cdot \sum_{i \in I} D_i \cdot \varphi_i + \sum_{i \in I} \sum_{h \in H_i} c_{i,h}^{hand} \cdot \delta_i^{port} \cdot x_{i,h} + \sum_{i \in I} c^{inv} \cdot \delta_i^{seg} \cdot \theta_i^{sail} + \sum_{i \in I} c_i^{late} \cdot \theta_i^{late} \tag{19}$$

Constraints:

Formulas (2), (3), (6)–(8), and (11)–(17).

$$\sum_{k \in K} \beta_{i,k} = 1, \ \ \forall i \in I \tag{20}$$

$$v_i^{rec} = \sum_{k \in K} \beta_{i,k} \cdot v_k^{val}, \ \ \forall i \in I \tag{21}$$

$$\varphi_i = \sum_{k \in K} \beta_{i,k} \cdot \varphi_k^{val}, \ \forall i \in I \tag{22}$$

$$\theta_i^{sail} = D_i \cdot v_i^{rec}, \ \ \forall i \in I \tag{23}$$

$$\frac{1}{V^{max}} \leq v_i^{rec} \leq \frac{1}{V^{min}}, \ \ \forall i \in I \tag{24}$$

Objective function (19) aims to minimize the total operational, fuel, port handling, container inventory, and vessel delay penalty costs. Formula (20) ensures selecting only one discretization point for estimating fuel consumption on each route segment. Formula (21) dictates the selection of discretization points for calculating vessel speed reciprocals on each route segment. Formula (22) computes vessel fuel consumption using the chosen discretization points on each route segment. Formula (23) calculates the sailing time for vessels on each route segment. Formula (24) imposes constraints on the vessel speed reciprocal values.

At this stage, M2 becomes a mixed-integer linear programming model, solvable using commercial solvers like CPLEX.

5. Numerical Experiments

5.1. Study Case

According to the existing liner shipping references [41], this study takes an Asia–Mediterranean container liner transport route as the research object. This route comprises 17 ports ($N = 17$), namely [1] Busan (535) → [2] Shanghai (87) → [3] Ningbo (915) → [4] Shekou (18) → [5] Hong Kong (1460) → [6] Singapore (4685) → [7] Jeddah (760) → [8] Suez (1842) → [9] Genoa (51) → [10] La Spezia (228) → [11] Fos-sur-Mer (185) → [12] Barcelona (164) → [13] Valencia (1973) → [14] Suez (760) → [15] Jeddah (4685) → [16] Singapore (1460) → [17] Hong Kong (1140) → [1] Busan. Port sequences are denoted within square brackets, and the distances between consecutive ports are provided in parentheses. For instance, [1] Busan (535) → [2] Shanghai (87) indicates that the liner's first port is Busan port, and the second port is Shanghai port, with a distance of 535 nautical miles between them.

Based on the research of Dulebenets [41], we determined the relevant cost coefficients, vessel coefficients, and fuel coefficients. Considering the practical factors, such as the fluctuation of the demand for container freight between the affiliated ports, the uniform distribution is used to randomly generate several ship arrival data [50]. Port operators allocate available operation time windows to vessels based on liner schedules. These time windows vary but generally do not exceed 3 days. Therefore, this study assumes time window durations ranging from 1 to 3 days, denoted by $(TW_i^{end} - TW_i^{start}) \sim U[24,72]$. The start time of the first port's (Busan port) time window is set at 0, with subsequent ports' window start times calculated based on the previous port's window end time, the distance between consecutive ports, and the vessel speed limit, denoted as $TW_{i+1}^{start} = TW_i^{end} + D_i/U[V^{min}, V^{max}]$. Additionally, we specified that the discretized decision variable, v_i, has 30 nodes. Specific parameter values are provided in Table 1.

Table 1. Relevant cost coefficients, vessel coefficients, and fuel coefficients.

Symbol	$c^{ope}(USD/ton)$	$[V^{min}, V^{max}](knot)$	$c^{inv}(USD/(TEU \times hour))$	$c^{fuel}(USD/ton)$	$M(ship)$
value	300,000	[15,20]	0.5	300	15
Symbol	$c_i^{late}(USD/hour)$	$\delta_i^{seg}(TEU)$	$\delta_i^{port}(TEU)$	$\mu_i(ship)$	c_i
value	$U[5000, 10,000]$	$U[8000, 15,000]$	$U[200, 2000]$	$U[2,3]$	$U[4,6]$
Symbol	$\lambda_i(ship)$	$TW_i^{end} - TW_i^{start}(hour)$	α	γ	-
value	$U[11,13]/U[4,8]$	$U[24,72]$	3	0.012	-

Assuming each port offers four selectable container handling rate schemes, the average handling efficiency is calculated using the formula $\pi_{i,h} = \overline{\pi_{i,h}} + \Delta\pi_{i,h}$, where $\overline{\pi_{i,h}}$ represents the average handling efficiency, and $\Delta\pi_{i,h}$ represents the variable handling efficiency for each port. Similarly, the unit container handling cost corresponding to each handling rate is calculated using the formula $c_{i,h}^{hand} = \overline{c_{i,h}} + \Delta c_{i,h}$, where $\overline{c_{i,h}}$ represents the average handling cost, and $\Delta c_{i,h}$ represents the variable handling costs for each port. Detailed container handling efficiency selection plans are listed in Table 2.

Table 2. Container handling efficiency selection plans.

<table>
<tr><th>Port</th><th>Shanghai</th><th>Ningbo</th><th>Singapore</th><th>Hong Kong</th><th>Others</th></tr>
<tr><td>$\overline{\pi_{i,h}}(TEU/hour)$</td><td colspan="4">{160,210,260,310}</td><td>{160,180,210,240}</td></tr>
<tr><td>$\Delta\pi_{i,h}(TEU/hour)$</td><td colspan="5">$U[0,10]$</td></tr>
<tr><td>$\overline{c_{i,h}}(USD/TEU)$</td><td colspan="5">{100,150,200,250}</td></tr>
<tr><td>$\Delta c_{i,h}(USD/TEU)$</td><td colspan="5">$U[0,10]$</td></tr>
</table>

5.2. General Results

Table 3 reveals that vessels on this route depart from Busan Port and navigate through numerous ports, including Shanghai Port, Ningbo Port, and so on, before returning to Busan Port, completing a round of liner shipping service totaling 1512 h (9 weeks). Thus, to maintain a weekly liner service frequency, at least nine vessels are required on this route, denoted by the decision variable, $m = 9$. This computation aligns with the requirement for the deployment of liner vessels.

Table 3. Schedule for container liner vessel round-trip voyages.

Sequence	Port	Segment (n Mile)	Speed (Knot)	Arrival Time (Hour)	Departure Time (Hour)
1	Busan	Busan → Shanghai (535)	21.53	0	36
2	Shanghai	Shanghai → Ningbo (87)	21.53	61	93
3	Ningbo	Ningbo → Shekou (915)	21.12	97	127
4	Shekou	Shekou → Hong Kong (18)	20.33	170	199
5	Hong Kong	Hong Kong → Singapore (1460)	21.97	200	233
6	Singapore	Singapore → Jeddah (4685)	21.97	300	330
7	Jeddah	Jeddah → Suez (760)	21.53	543	576
8	Suez	Suez → Genoa (1842)	21.53	611	638
9	Genoa	Genoa → La Spezia (51)	20.71	723	766
10	La Spezia	La Spezia → Fos-sur-Mer (228)	21.12	768	801
11	Fos-sur-Mer	Fos-sur-Mer → Barcelona (185)	21.12	812	835
12	Barcelona	Barcelona → Valencia (164)	20.33	844	877
13	Valencia	Valencia → Suez (1973)	19.95	885	916
14	Suez	Suez → Jeddah (760)	19.95	1015	1039
15	Jeddah	Jeddah → Singapore (4685)	21.12	1077	1105
16	Singapore	Singapore → Hong Kong (1460)	20.71	1326	1359
17	Hong Kong	Hong Kong → Busan (1140)	18.91	1430	1452
1	Busan	-	-	1512	-

5.3. Sensitivity Analyses

The specific schedule scheme and related costs within the planning horizon are influenced by factors such as the container handling efficiency selection mechanism, fuel price, port service intensity, and port time window interval length. To further analyze the impact of these factors on the schedule plan, numerical experiments were conducted using the previous example as a benchmark, and sensitivity analyses were performed for each category of factors.

5.3.1. Impact of Container Handling Efficiency Selection Mechanism

In the context of port–harbor information sharing, port operators can offer container handling services with more options and higher efficiency, provided a certain fee is charged. Therefore, we devised a mechanism for selecting container handling efficiency based on collaborative information sharing, taking into account the availability of port operation time windows. This mechanism addresses the liner schedule design problem considering berth operation time window constraints and optional container handling efficiency. To explore the impact of this mechanism on vessel operating costs, we conducted a rational comparative analysis. M2 incorporates the container handling efficiency selection mechanism, while M3 does not consider operation time window restrictions and optional handling efficiency. Detailed comparative analysis results are presented in Table 4.

Table 4. Comparison of costs of the container handling efficiency selection mechanism ($10^3(USD)$).

Model	Total Costs	Fuel Costs	Operational Costs	Handling Costs	Inventory Costs	Penalty Costs
M2	14,618	1404	2700	4390	5767	357
M3	15,476	1689	2700	4274	6124	689

According to Table 5, the container handling efficiency selection mechanism can reduce total costs by 5.54%, primarily seen in fuel and penalty costs. This is because the mechanism offers liner companies multiple optional time windows and handling efficiencies, enabling vessels to employ strategies like slow steaming during voyages. This flexibility allows for the reduction of fuel and penalty costs by selecting arrival times and handling efficiencies, demonstrating the scheme's high flexibility and robustness.

Table 5. The impact of port time window interval length on different performance indexes.

Time Window Interval	Total Costs $10^3(USD)$	Fuel Costs $10^3(USD)$	Handling Costs $10^3(USD)$	Penalty Costs $10^3(USD)$	Average Speed $(knot)$	Vessel Number $(ship)$
(18,24)	15,157	1542	4835	396	20.23	9
(24,30)	15,169	1540	4835	388	20.34	9
(30,36)	15,036	1544	4801	367	20.43	9
(36,42)	14,913	1559	4801	349	20.48	9
(42,48)	14,885	1570	4786	324	20.48	9
(48,54)	14,796	1576	4786	311	20.55	9
(54,60)	14,732	1578	4786	302	20.56	9

5.3.2. Impact of Fuel Price

Fuel costs are a significant component of liner company operational expenses, closely tied to international fuel prices. In recent years, volatile fluctuations in fuel prices have influenced parameters in liner schedules. Thus, this study conducts a sensitivity analysis on fuel prices, ranging from 300 to 600 (USD/ton). The impact of fuel price on operational costs and vessel deployment can be seen in Figure 3.

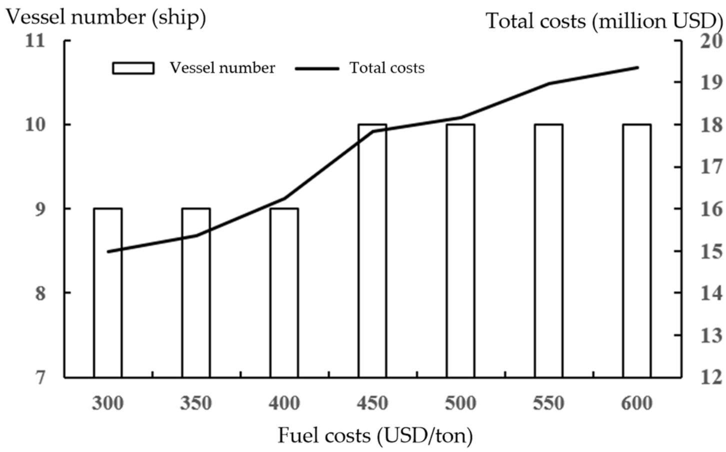

Figure 3. The impact of fuel price on operational costs and vessel deployment.

As Figure 3 depicts, an increase in fuel prices from 300 (USD/ton) to 600 (USD/ton) leads to a corresponding rise in operational costs. While within a certain range ($[300, 400]$ or $[450, 600]$), there is no change in the number of vessels deployed, overall, an increase in fuel prices results in more vessels being allocated to the route. This is because fuel consumption is exponentially related to vessel speed. A slight speed increase significantly boosts fuel consumption and costs. To mitigate this, vessels may employ slow steaming to reduce fuel consumption. However, this strategy prolongs shipping times, necessitating the deployment of more vessels to maintain weekly service frequency. Moreover, when fuel prices remain below 450 (USD/ton), liner companies deploy nine vessels on the route. Despite higher fuel costs due to relatively faster vessel speeds, the increase is lower than deploying an additional vessel; hence, there is no change in vessel allocation. Conversely, when fuel prices exceed 500 (USD/ton), the cost increase surpasses deploying an extra vessel, prompting companies to lower vessel speeds and increase vessel allocation.

Therefore, amid global events like extreme weather or geopolitical conflicts impacting the fuel market, liner companies should increase vessel numbers and reduce speeds when fuel prices are high.

5.3.3. Impact of Port Service Intensity

In the international container shipping market, route density and container demand are dynamic, leading to fluctuating vessel numbers at ports and consequently impacting vessel queueing times. This study quantifies port busyness using port service intensity and conducts a sensitivity analysis to explore its influence on schedule design. Assuming that other parameters remain constant, we analyze the sensitivity of schedule indicators when port service intensity varies from 0.5 to 1.5 times the baseline. The impact of port service intensity on vessel average speed and total costs can be seen Figure 4.

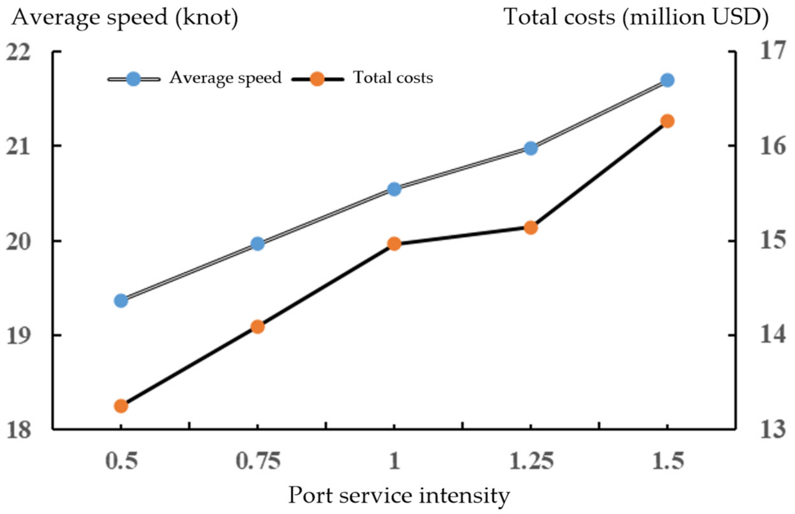

Figure 4. The impact of port service intensity on vessel average speed and total costs.

According to Figure 4, as port service intensity increases, more vessels arrive at ports, prolonging vessel queueing times. To maintain weekly service frequency, liner companies must increase vessel speeds to minimize sea transit time and expedite port service reception. Alternatively, they may opt for higher port handling efficiency to reduce port operation time. And both strategies result in increased overall operational costs.

5.3.4. Impact of Port Time Window Interval Length

In the operational phase of vessels, the duration of available port working time windows directly influences decisions regarding vessel speed and container loading/unloading efficiency. This, in turn, affects fuel costs and handling costs and ultimately leads to adjustments in schedule design. In this section, we analyze the impact of the length of port working time window intervals on schedule indicators, varying the window length uniformly from (18,24) to (54,60), as shown in Table 5.

According to Table 5, in all seven scenarios, the number of vessels deployed on the route remains at nine, indicating that the increase in the length of available port working time windows does not significantly affect vessel deployment. This is primarily because the length of available port working time windows mainly affects the current rotation of liner services and generally does not impact subsequent rotations unless the window is too small. However, other schedule indicators show some degree of variation. Specifically, vessel average speed gradually increases, leading to higher fuel costs, while handling costs and penalty costs decrease gradually. This is mainly because, with the extension of available port working time intervals, vessels have more time for container loading and unloading. Consequently, vessels opt for lower loading and unloading efficiency to reduce handling costs, and the increased loading and unloading time results in reduced sea transit time, thereby increasing vessel speed. Overall, the reduction in handling and penalty costs outweighs the increase in fuel costs, resulting in a downward trend in total service costs.

5.4. Performance of Solution Methods

Typically, the performance of solution methods is demonstrated through benchmark examples and comparisons with results from other researchers or algorithms. However, traditional studies often treat waiting time as a random variable without accounting for the systematic impact of port queueing mechanisms, such as the studies by Cheon et al. [8], Liu et al. [9], Zhang et al. [10], and Wang and Meng [11]. In contrast, our research uses observable ship arrival data and a queueing theory model to predict waiting times. This novel approach results in no direct comparability with existing studies, leading to a lack of benchmark data for comparison. In addition, since we employ the commercial solver CPLEX to solve the linearized model, the results obtained are exact optimal solutions. Therefore, we need to consider only whether our method can effectively solve problems on a realistic scale within ideal time.

In M1, the liner's sailing speed is a continuous variable within a range, which causes the commercial solver CPLEX to be unable to solve the problem directly. Therefore, in Section 4, we approximated the sailing speed by discretizing it into several points, $K = \{1, \ldots, e\}$, within the range. The choice of the number of points significantly affects CPLEX's solving time. To demonstrate the effectiveness of our solution approach and mitigate potential instability caused by the random nature of some parameters, we used the real-world case from Section 5.1. We varied the discretization levels from 5 to 50 points and generated 10 parameter scenarios. The solving times for these 10 scenarios are shown in Table 6.

Table 6. Model solving time under different examples.

Scenario	Discrete Points of the Fuel Function	Model Solving Time (s)	Scenario	Discrete Points of the Fuel Function	Model Solving Time (s)
1	5	0.1830	6	30	0.2085
2	10	0.1945	7	35	0.2275
3	15	0.1995	8	40	0.2385
4	20	0.2023	9	45	0.2586
5	25	0.2054	10	50	0.2878

Increasing the discretization levels from 5 to 50 resulted in a 57.3% rise in computational time, from 0.1830 s to 0.2878 s. The case we used was based on a real-world route scenario, and even at the highest discretization level of 50 points, the computational time remained under 1 s. This demonstrates that our solving method can effectively handle most real-world scenarios.

6. Conclusions

The uncertainty in vessel waiting and port handling times affects the stability of liner schedules. Current research typically treats vessel port times as random variables, failing to accurately predict port congestion based on observable factors, like vessel arrivals and port operations capacity. This study addresses the issue of liner schedule design under port congestion. We employ queueing theory models to describe vessel waiting time uncertainty and propose a container handling efficiency selection mechanism for arriving vessels to determine their port handling time. By jointly considering these two uncertainties, a robust liner schedule design model is established and solved using the CPLEX.

Numerical simulations on an Asia-to-Mediterranean liner route reveal that extreme weather events or geopolitical conflicts may cause severe port congestion, affecting vessel punctuality and requiring timely adjustments to vessel schedules. Additionally, such events impact the international maritime fuel market, prompting liner companies to consider strategies like increasing vessel operations and reducing vessel speed under high fuel prices. The container handling efficiency selection mechanism allows liner companies to flexibly design schedules, while balancing economic costs and service reliability.

Future studies can be conducted in the following areas. (1) Enhanced data accuracy: This study references Dulebenets' study [50] for simplifying vessel arrival data, employing queueing theory to predict port congestion and designing a robust liner schedule. However, real-world vessel arrivals are subject to uncertainties caused by events like COVID-19 or the Red Sea crisis, which can disrupt normal patterns. To further enhance the robustness of liner schedules, future work could involve using extensive port historical data to train queueing models for each port and utilizing automatic identification systems to track real-time vessel locations near ports, thus enhancing data accuracy. (2) Cooperative strategies among heterogeneous fleets: By coordinating schedules and operations, fleets can better manage uncertainties and optimize resource utilization, leading to a more resilient and efficient schedule. (3) Collaborative agreements between adjacent terminal operators: Such agreements can facilitate workload sharing and improve overall port efficiency, reducing congestion and delays. (4) Incorporation of carbon emission costs: Incorporating these costs into the scheduling model can help liner companies balance economic efficiency with environmental sustainability, promoting greener shipping practices.

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