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# Improved Double-Layer Soil Consolidation Theory and Its Application in Marine Soft Soil Engineering

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**Abstract:** Marine soft soil foundation is a double-layer foundation structure with a crust layer and soft substratum. Moreover, it is common that there are various forms of drainage. Accordingly, based on Terzaghi's consolidation theory and the continuous drainage boundary conditions theory of controllable drainage conditions, an improved double-layer soil consolidation theory considering continuous drainage boundary conditions was proposed. To improve the computational efficiency and accuracy, the Laplace transform and the Stehfest algorithm was used to deduce the numerical solution of the improved double-layer soil consolidation theory considering continuous drainage boundary conditions and to compile a computer program. Subsequently, the theory was validated and analyzed by the degenerated model of the perfectly permeable boundary conditions and the semi-permeable boundary conditions, respectively, which showed that this theory has higher accuracy. Simultaneously, the analysis of double-layer consolidation settlement under continuous drainage boundary conditions for marine soft soil foundation of Guangxi Binhai Highway was carried on. The result showed that the consolidation settlement calculated by the improved double-layer consolidation theory presented is basically consistent with the field measurement results, and that the correlation coefficient between them is higher. Accordingly, the research results can provide useful basic information for marine soft foundation engineering.

**Keywords:** marine soft soil; double-layer foundation; consolidation theory; drainage boundary

## 1. Introduction

Marine soft soil is widely distributed throughout the world, and the performance of the soil varies as a result of differences in geological origin, occurrence law, and composition. Therefore, the engineering characteristics that emerged from these factors reflect substantial spatial and temporal variability and regionality. For example, the Canadian coastline exceeds 200,000 km [1], and therefore marine soft soil represented by Leda soft soil is widely distributed in Canada, which has the significant characteristics of marine soft soil [2]. Norway's Drammen marine soft soil also has characteristics of marine soft soil, but Sangrey showed that the engineering properties of Drammen soft soil were different from those of Leda soft soil [2–4]. Other soft soils, such as London soft soil [5], Mexican soft soil [6], Japanese soft soil [7], Busan soft soil [8], Shenzhen soft soil, Shanghai soft soil and so on, all exhibit different marine soft soil characteristics [9–16]. The engineering characteristics of marine soft soil make the problem of consolidation and settlement of marine soft soil foundation complicated, but the one-dimensional consolidation theory by Terzaghi is not suitable for the analysis of complex marine soft soil foundation consolidation problems. The main reason for this is that Terzaghi's one-dimensional consolidation theory assumes that the drainage boundary is perfectly permeable or perfectly impervious, and the pore water pressure equations were as follows [17]:

$$u(0,t) = 0 \tag{1}$$

and:

$$u_t(h,t) = 0 \tag{2}$$

In reality, the sand layer of the top surface of the foundation treatment and the lower layer of the bottom surface are neither perfectly permeable nor perfectly impervious, and they are often somewhere in between. Previous studies showed that permeability of drainage boundary will have a major impact on the final calculation results [18,19]. In view of the unreasonable drainage boundary conditions of consolidation theory, Gray [20] took the lead in conducting research. Based on one-dimensional consolidation theory by Terzaghi, Gray [20] proposed a semi-permeable boundary theory, and the pore water pressure equation was as follows:

$$u_t(h,t) + hu(h,t) = 0 \tag{3}$$

Later, Schiffiman and Stein [21] conducted a detailed study on the one-dimensional consolidation problem with a semi-permeable boundary. Huang [22] discussed and promoted the semi-permeable boundary theory. While the one-dimensional consolidation theory of semi-permeable boundary is more practical than one-dimensional consolidation theory by Terzaghi, it is relatively simple, and it assumes that the soil layer is homogeneous elastomer without considering multi-layer soil. According to this, Xie [23] proposed a one-dimensional consolidation theory for a double-layer foundation with a semi-permeable boundary and solved and discussed it. The results showed that the semi-permeable boundary conditions and the stratification of soil can bring the foundation consolidation analysis closer to the actual situation [23]. Based on the research of Xie [23], Hu and Xie [24] studied the one-dimensional consolidation problem of the semi-permeable boundary under the gradual load of a multi-layer elastic foundation and systematically analyzed the layered soil. Hu and Xie [24] studied the effect of semi-permeable boundary conditions, soil properties, and loading rate of external load on pore pressure distribution and total average consolidation. However, the above scholars considered the foundation soil to be a homogeneous elastic soil layer and a static load [23,24]. For this reason, Fang et al. [25], Lin et al. [26], Wang et al. [27], Wang et al. [28], Cai et al. [29], Li et al. [30], Wang and Xia [31], and Zheng et al. [32] carried out consolidation analysis of viscoelastic soil layers under semi-permeable conditions and dynamic load. According to the one-dimensional consolidation equation of the semi-permeable boundary proposed by Gray [20], scholars carried out consolidation analysis under different loading modes and non-Darcy's law conditions [33,34]. However, the one-dimensional consolidation equation based on the semi-permeable boundary proposed by Gray [20] is difficult to solve and not easy to generalize. Combined with the above problems, Mei et al. [35] proposed and solved a one-dimensional consolidation equation for continuous drainage boundary in order to solve the contradiction between the boundary conditions and the initial conditions of the one-dimensional consolidation equation by Terzaghi. The pore water pressure equations by Mei et al. [35] are as follows:

$$u(0,t) = q(t)e^{-bt} \text{ (top drainage)}, \tag{4}$$

and

$$u(2h,t) = q(t)e^{-ct} \text{ (bottom drainage)}. \tag{5}$$

The continuous drainage boundary is a time-dependent interface boundary that varies between perfectly pervious and impervious conditions. The boundary pore pressure changes exponentially with time. By adjusting the parameters $b$ and $c$ related to the undisturbed soil, the drainage properties of the top and bottom drainage surfaces of the soil layer can be controlled. For example, $b$ can tend to infinity, and thus the top surface of the soil layer is undrained. Subsequently, scholars carried out research on continuous drainage boundary theory. Zong et al. [36] and Zheng et al. [37] established a generalized Terzaghi's consolidation theory under double-sided asymmetric continuous drainage boundary conditions. Cai et al. [38] developed a subroutine within ABAQUS program to verify the solution of the one-dimensional consolidation equation of the continuous drainage boundary. Wang et al. [39] showed that the continuous boundary has good applicability and extends to the semi-analytical solution of one-dimensional consolidation of unsaturated soil. Feng et al. [40] established the one-dimensional consolidation equation for the continuous drainage boundary and studied the contribution of a soil's self-weight stress. Sun et al. [41] established a general analytical solution for the one-dimensional consolidation of soil for the continuous drainage boundary under a ramp load. Zhang et al. [42] analyzed the excess pore water pressure and the average degree of consolidation under the continuous drainage boundary conditions and discussed the effect of the drainage capacity of the top surface, the smear effect, and the well resistance on consolidation. However, there are few reports on the one-dimensional consolidation theory with the continuous drainage boundary of double-layer soil and its application in marine soft soil engineering.

Domestic and foreign scholar have done a lot of research on the algorithm for the mathematical modeling of consolidated seepage. In 1949, Van Everdingen and Hurst [43] proposed the Laplace transform method. In 1968, Dubner and Abate [44] proposed the Laplace numerical inversion method. Subsequently, Durbin [45] improved the Dubner and Abate algorithm. In 1970, Stehfest [46,47] proposed the Stehfest algorithm because the algorithm is easy to program, has few parameters, has fast calculation, does not involve complex numbers, and has high stability. It has thus been widely used in engineering. It is worth mentioning that the Stehfest algorithm looks like an empirical formula, but it actually has a complicated mathematical background and theoretical derivation. Then, Crump [48] proposed the Crump algorithm, which is based on the Fourier series. Duffy [49] improved the Crump algorithm, avoiding the trigonometric function term. However, because the Crump algorithm does not put forward a method to determine the appropriate attenuation index and truncation term number, it is difficult to apply widely. Therefore, according to the advantages and disadvantages of the above algorithm, an appropriate method should be adopted to carry out the calculation according to the characteristics of the consolidation theoretical model.

Based on an analysis of the above literature, aiming at the characteristics of the marine soft soil double-layer foundation structure and complex drainage conditions, a numerical solution of the improved double-layer foundation consolidation theory considering continuous drainage boundary conditions is presented. In Section 2, the basic equations of the improved consolidation theory is introduced and deduced in detail, and the Laplace transform and Stehfest algorithm are applied in the derivation process, and the improved model is compiled into a program by this paper. In Section 3, the improved model is validated and analyzed by three examples in the literature. This section includes degradation analysis of perfectly permeable boundary conditions and semi-permeable boundary conditions by the improved model. Finally, in Section 4, the improved model is applied to an actual marine soft soil foundation project in Guangxi for application analysis. The settlement and consolidation degree of soft soil foundation are analyzed and compared with the measured data for verification. The conclusions can provide scientific guidance for consolidation analysis of marine soft soil foundation. They also have certain theoretical value and practical significance.

## 2. Improved Double-Layer Soil Consolidation Theory Considering Continuous Drainage Boundary Conditions

In addition to the load, the same basic assumptions as in the one-dimensional consolidation theory by Terzaghi were made. Equations (1)–(5) are the basic assumptions of one-dimensional consolidation theory by Terzaghi:

(1) The soil layer is homogeneous and fully saturated.
(2) Soil particles and water are incompressible.

(3) Water seepage and compression of the soil layer occur only in one direction (vertical).
(4) The seepage of water obeys Darcy's law.
(5) In the osmotic consolidation, the permeability coefficient and the compression coefficient of the soil are constants.
(6) The external load is applied at two levels of average speed.
(7) Additional stress of soil does not decrease with depth under the large area load of highway roadbed.

Based on the above assumptions, the double-layer soil consolidation equations considering continuous drainage boundary conditions were proposed. Figure 1a presents a simplified diagram of the theoretical calculation of an improved double-layered soil foundation consolidation considering continuous drainage boundary conditions. We take the ground table as the coordinate origin. In the figure, $q(t)$ is an arbitrary loading function. $h_1$, $E_{s1}$, and $k_1$ are the parameters of topsoil. $h_2$, $E_{s2}$, and $k_2$ are the parameters of subsoil. The load simplification is applied in two stages. $q_1$ and $q_2$ are the primary and secondary load increments, respectively, as shown in Figure 1b.

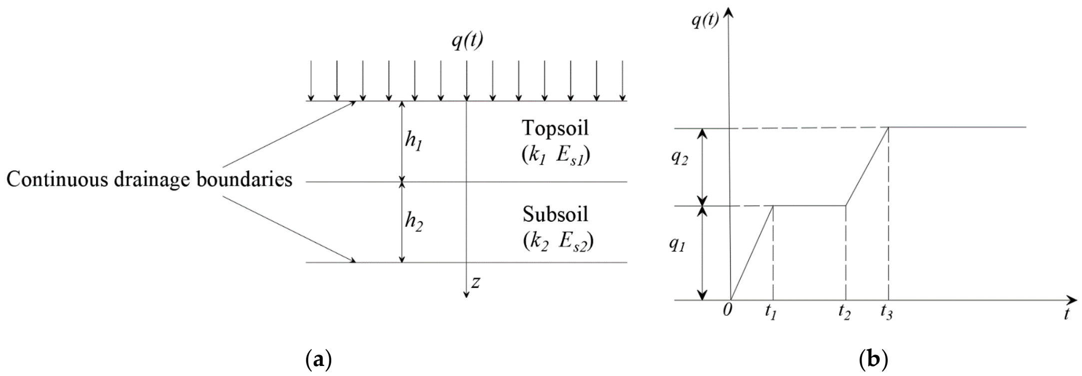


**Figure 1.** Schematic diagram: (**a**) Double-layer soil foundation calculation model, and (**b**) the curve of loading.

Therefore, one-dimensional consolidation differential equations for a double-layer soil foundation can be obtained:

$$c_{v1}\frac{\partial^2 u_1(z,t)}{\partial z^2} = \frac{\partial u_1(z,t)}{\partial t} - \frac{\partial q(t)}{\partial t}(0 \le z \le h_1) \tag{6}$$

$$c_{v2}\frac{\partial^2 u_2(z,t)}{\partial z^2} = \frac{\partial u_2(z,t)}{\partial t} - \frac{\partial q(t)}{\partial t}(h_1 \le z \le h_1 + h_2) \tag{7}$$

where $u_1$ and $u_2$ are the first and second layers of excess pore water pressure, respectively; and $c_{v1}$ and $c_{v2}$ are the first and second layers of consolidation coefficients, respectively. The latter are calculated by:

$$c_{v1} = \frac{k_1 E_{s1}}{\gamma_w},\ c_{v2} = \frac{k_2 E_{s2}}{\gamma_w}.$$

The pore pressures in Equations (6) and (7) can be expressed by the effective stress and converted into the following equations:

$$\frac{\partial \sigma_1'(z,t)}{\partial t} = c_{v1}\frac{\partial^2 \sigma_1'(z,t)}{\partial z^2}(0 \le z \le h_1) \tag{8}$$

$$\frac{\partial \sigma_2'(z,t)}{\partial t} = c_{v2}\frac{\partial^2 \sigma_2'(z,t)}{\partial z^2}(h_1 \le z \le h_1 + h_2) \tag{9}$$

The initial condition is:

$$\sigma_{1,2}'(z,0) = 0 \tag{10}$$

The boundary conditions are:

$$\sigma_1'(0,t) = q(t) - q(t)\exp(-bt) \tag{11}$$

$$\sigma_2'(h_1+h_2,t) = q(t) - q(t)\exp(-ct) \tag{12}$$

These boundary conditions are continuous drainage boundary conditions [35,36], and $b$ and $c$ are parameters related to soil drainage properties. They are interface parameters that reflect the drainage properties of the top and bottom drainage surfaces of the soil layer [41]. As shown in Table 1, the permeability of the drainage boundary can be controlled by adjusting parameters $b$ and $c$.

**Table 1.** Description of parameters $b$ (top surface) and $c$ (bottom surface).

| Parameter \ Value | 0 | (0, +∞) | +∞ |
|---|---|---|---|
| $b$ | impermeable | semi-permeable | perfectly permeable |
| $c$ | impermeable | semi-permeable | perfectly permeable |

The continuous condition between layers is:

$$\sigma_1'\big|_{z=h_1} = \sigma_2'\big|_{z=h_1} \tag{13}$$

The continuous flow condition is:

$$\frac{k_1}{\gamma_w}\frac{\partial\sigma_1'}{\partial z}\bigg|_{z=h_1} = \frac{k_2}{\gamma_w}\frac{\partial\sigma_2'}{\partial z}\bigg|_{z=h_1} \tag{14}$$

Equations (8)–(14) constitute the improved double-layer soil consolidation theory equation considering continuous drainage boundary conditions. A Laplace transform is performed on Equations (8)–(14):

$$s\overline{\sigma_1'}(z,s) - \overline{\sigma_1'}(z,0) = c_{v1}\frac{\partial^2\overline{\sigma_1'}(z,s)}{\partial z^2}(0 \le z \le h_1) \tag{15}$$

$$s\overline{\sigma_2'}(z,s) - \overline{\sigma_2'}(z,0) = c_{v2}\frac{\partial^2\overline{\sigma_2'}(z,s)}{\partial z^2}(h_1 \le z \le h_1+h_2) \tag{16}$$

$$\overline{\sigma_{1,2}'}(z,0) = 0 \tag{17}$$

$$\overline{\sigma_1'}(0,s) = \overline{q}(s) - \overline{q}(s+b) \tag{18}$$

$$\overline{\sigma_2'}(h_1+h_2,s) = \overline{q}(s) - \overline{q}(s+c) \tag{19}$$

$$\overline{\sigma_1'}\big|_{z=h_1} = \overline{\sigma_2'}\big|_{z=h_1} \tag{20}$$

$$\frac{k_1}{\gamma_w}\frac{\partial\overline{\sigma_1'}}{\partial z}\bigg|_{z=h_1} = \frac{k_2}{\gamma_w}\frac{\partial\overline{\sigma_2'}}{\partial z}\bigg|_{z=h_1} \tag{21}$$

The general solutions of Equations (8) and (9) obtained by Equations (15)–(17) are as follows in the Laplace transform domain:

$$\overline{\sigma_1'}(z,s) = A_{11}\exp(r_1 z) + A_{12}\exp(-r_1 z)(0 \le z \le h_1) \tag{22}$$

$$\overline{\sigma_2'}(z,s) = A_{21}\exp(r_2 z) + A_{22}\exp(-r_2 z)(h_1 \le z \le h_1+h_2) \tag{23}$$

Here:

$r_1^2 = \frac{s}{c_{v1}}, r_2^2 = \frac{s}{c_{v2}}$.

Bands Equations (18)–(21) substituted into Equations (22) and (23) yields the following:

$$\begin{cases} A_{11} + A_{12} = \overline{q}(s) - \overline{q}(s+b) \\ A_{21}e^{r_2(h_1+h_2)} + A_{22}e^{-r_2(h_1+h_2)} = \overline{q}(s) - \overline{q}(s+c) \\ A_{11}e^{r_1h_1} + A_{12}e^{-r_1h_1} = A_{21}e^{r_2h_1} + A_{22}e^{-r_2h_1} \\ \frac{k_1}{\gamma_w}\left(A_{11}r_1e^{r_1h_1} + A_{12}(-r_1)e^{-r_1h_1}\right) = \frac{k_2}{\gamma_w}\left(A_{21}r_2e^{r_2h_1} + A_{22}(-r_2)e^{-r_2h_1}\right) \end{cases} \tag{24}$$

where:

$$\overline{q}(s) - \overline{q}(s+b) = \int_o^{+\infty} q(t)\left(1 - e^{-bt}\right)e^{-st}dt,$$
$$\overline{q}(s) - \overline{q}(s+c) = \int_o^{+\infty} q(t)\left(1 - e^{-ct}\right)e^{-st}dt.$$

$q(t)$ can be expressed as follows (Figure 1b):

$$q(t) = \begin{cases} \frac{t}{t_1}q_1 & (0 \le t \le t_1) \\ q_1 & (t_1 \le t \le t_2) \\ q_1 + \frac{t-t_2}{t_3-t_2}q_2 & (t_2 \le t \le t_3) \\ q_1 + q_2 & (t_3 \le t) \end{cases} \tag{25}$$

According to Equations (24) and (25), $\overline{q}(s) - \overline{q}(s+b)$, $\overline{q}(s) - \overline{q}(s+c)$, $A_{11}$, $A_{12}$, $A_{21}$, $A_{22}$ can be obtained, see the Appendix A for details.

By putting $A_{11}$, $A_{12}$, $A_{21}$, and $A_{22}$ into Equations (22) and (23) and then performing Laplace inverse transformation, the solutions of consolidation Equations (6) and (7) can be obtained. However, for the complex Laplace solution, it is difficult to carry out the Laplace inverse transform. Then it needs to be solved by the numerical solution of Laplace inverse transform. According to the prior literature, the Stehfest algorithm has better stability and has the advantage of fewer computational parameters [50]. In this case, the Stehfest algorithm is used to write the corresponding program for numerical inversion of Equations (22) and (23). The Stehfest inversion equation is as follows:

$$f(T) = \frac{\ln 2}{T}\sum_{i=1}^{N} V_i\overline{f}\left(\frac{\ln 2}{T}i\right) \tag{26}$$

where $\overline{f}(s)$ is the Laplace function of $f(t)$, $\overline{f}(s) = L[f(t)] = \int_0^{\infty} f(t)e^{-st}dt$, and $V_i = (-1)^{N/2+i}\sum_{k=[\frac{i+1}{2}]}^{Min(i,N/2)} \frac{k^{N/2}(2k)!}{(N/2-k)!k!(k-1)!(i-k)!(2k-i)!}$, where $N$ must be a positive even number. Stehfest [46,47] recommended taking $N$ as between 4 and 32. Through repeated verification and reference to the relevant literature [50], we found that 8 was the best choice for $N$.

The Stehfest algorithm can be used to invert the numerical solution of the improved double-layer soil consolidation equation considering the continuous drainage boundary conditions. Then the total consolidation settlement of the double-layer soil can be calculated as follows:

$$S_t = \int_0^{h_1} \frac{\sigma_1'(z,t)}{E_{s1}}dz + \int_{h_1}^{h_1+h_2} \frac{\sigma_2'(z,t)}{E_{s2}}dz \tag{27}$$

The average consolidation degree of the double-layer soil is:

$$U = \frac{S_t}{S_\infty} = \frac{S_t}{\left(\int_0^{h_1} \frac{p(t)}{E_{s1}}dz + \int_{h_1}^{h_1+h_2} \frac{p(t)}{E_{s2}}dz\right)} \tag{28}$$

## 3. Improved Double-Layer Consolidation Theory Model Verification Analysis

### 3.1. Degradation Analysis of Perfectly Permeable Boundary Conditions by the Improved Model

Xie made remarkable contributions to the theoretical research of the double-layer consolidation model. Therefore, in order to verify the improved model, it is used the case in Xie's paper [51] to conduct the degradation analysis of perfectly permeable boundary conditions by the improved model. When $b$ tends to $\infty$ and $c$ tends to 0, the improved equations can be degraded to Xie's double-layer soil consolidation equation (single-sided permeability). In order to verify the conclusion, the corresponding program is compiled by this paper for calculation using the data for the examples of Xie's paper [51]. The parameters of example in paper [51] are shown in Table 2.

**Table 2.** Study data of perfectly permeable boundary conditions.

| Layer | Layer Thickness $h$ (m) | Permeability Coefficient $k$ ($10^{-8}$m/s) | Compression Modulus $E_s$ (MPa) |
|---|---|---|---|
| Topsoil | 1 | 1.014 | 8 |
| Subsoil | 9 | 2.028 | 4 |

Question 1: When the load is applied instantaneously, how long will it take for the average consolidation degree of the foundation to reach 60%?

Answer to question 1: According to the data in Table 2, analysis of the double-layer soil consolidation degree is carried out according to the corresponding program compiled by this paper. The results by the proposed model are compared with those of Xie's model. Figure 2 is the solution graph of Question 1. When the load is applied instantaneously, it takes 55 days to reach the average consolidation degree of 60% according to the proposed method, which is basically consistent with the calculation results of Xie's model. Further analysis shows that the relation curves between time and consolidation degree (the $t$–$U$ curve) obtained by the proposed method is slightly different from those of Xie's model. This difference shows that the solution by the proposed model is slightly larger in the early stage and is slightly smaller in the later stage. The analysis shows that this reason is based on the Stehfest algorithm. While the algorithm requires fewer parameters and provides higher accuracy, it also leads to some errors in the inversion data. According to Figure 2, the consolidation curve obtained by the proposed method is basically consistent with Xie's method.

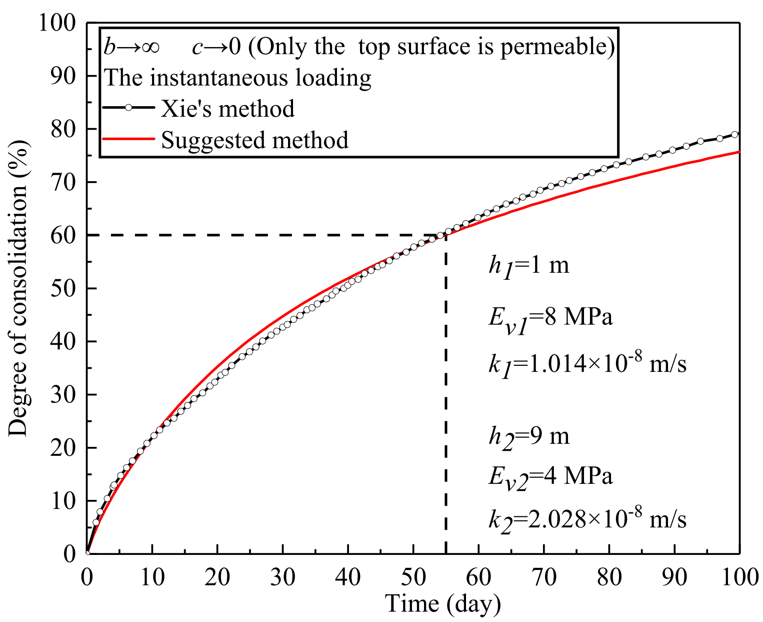


**Figure 2.** Consolidation curve for the instantaneous loading.

Question 2: When the single-stage constant-speed load is 70 days, what is the average consolidation degree of the foundation at 140 days?

Answer to question 2: Figure 3 is the solution graph of Question 2 and shows that when the single-stage constant-speed load is 70 days, the average consolidation degree of the foundation after 140 days is about 80%, which is consistent with Xie's solution. Similarly, the analysis of the whole curve shows that the solution by the proposed model is slightly larger in the early stage and is slightly smaller in the later stage. The specific reasons for this have already been explained as those above mention. Through the answers to Questions 1 and 2, the double-layer soil continuous drainage boundary consolidation theory based on the Stehfest algorithm is found to have higher accuracy.

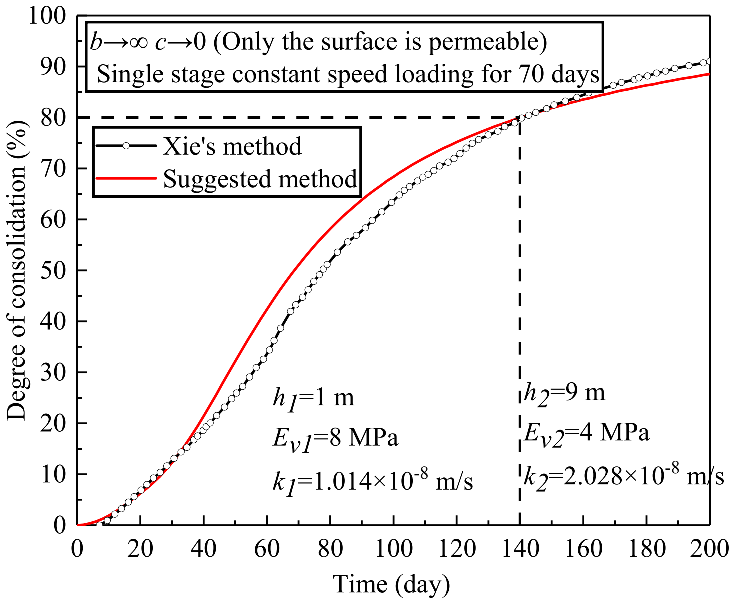


**Figure 3.** Consolidation curve for single-stage constant-speed loading.

### 3.2. Degradation Analysis of Semi-Permeable Boundary Conditions by the Improved Model

Gray [20] pioneered the study of the semi-permeable boundary of the consolidation theory model, but the stratum studied was homogeneous. On this basis, Xie [23] studied the theoretical model of double-layer soil consolidation of the semi-permeable boundary. Therefore, we again use the example in Xie's study [23] for analysis. The parameters of example in paper [23] are shown in Table 3.

**Table 3.** Study data of semi-permeable boundary conditions.

| Layer | Layer Thickness $h$ (m) | Permeability Coefficient $k$ ($10^{-9}$ m/s) | Compression Modulus $E_s$ (MPa) |
|---|---|---|---|
| Topsoil | 3 | 1 | 8 |
| Subsoil | 3 | 5 | 1.6 |

Question 3: Under the conditions of an impervious bottom and semi-permeable top, how long will it take for the average consolidation degree of the foundation to reach 70%?

Answer to question 3: According to the data in Table 3, analysis of the double-layer soil consolidation degree is carried out by using the corresponding program compiled by this paper. $c \to 0$ can simulate the bottom surface being impervious, but the semi-permeable boundary of top surface is a fuzzy concept. Through repeated debugging of $b$ value, it is found that semi-permeable boundary of the top surface can be better simulated when $b = 10$. The obtained results are shown in Figure 4. Using the method provided, the growth rate of the first 50 days is faster than that in Xie's method, and then the consolidation degree gradually became consistent with Xie's, indicating that the method provided is suitable for double-layer soil and the prediction has high accuracy. The time required for the proposed method and Xie's method to calculate the total average consolidation degree to 70% is 1115 and 1185 days, respectively, and the error is about 6%. The main reasons for the errors are as follows: (1) The proposed method is to control the permeability of the drainage boundary by adjusting parameters $b$ and $c$. However, the parameter values do not easily fully correspond to those in Xie's method. (2) The Stehfest algorithm requires fewer parameters and has higher accuracy, but it also has some errors in the inversion process.

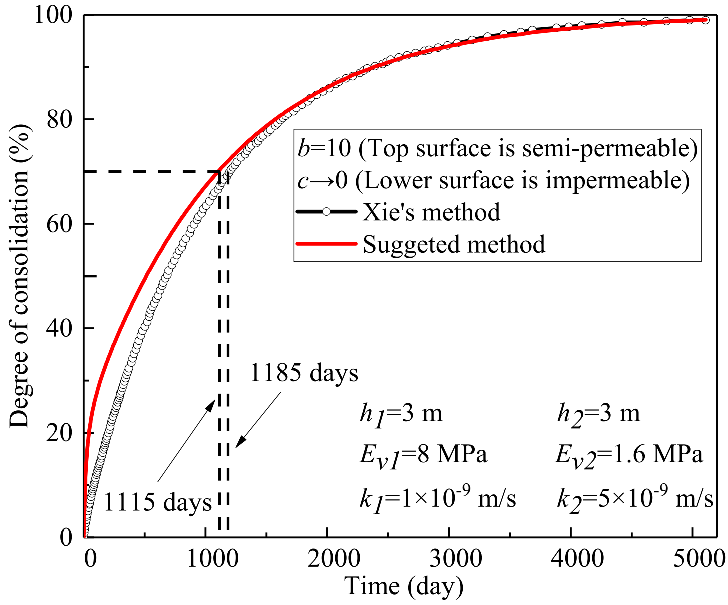


**Figure 4.** Consolidation curve for the semi-permeable boundary.

## 4. Engineering Case Analysis

### *4.1. Project Overview*

The Guangxi Binhai Highway is located along the coastline of Beibu Gulf (Figure 5). According to the on-site investigation, the completed survey and design section, the soft land base section exceeds 200 km, accounting for 70% of the total length of the route.

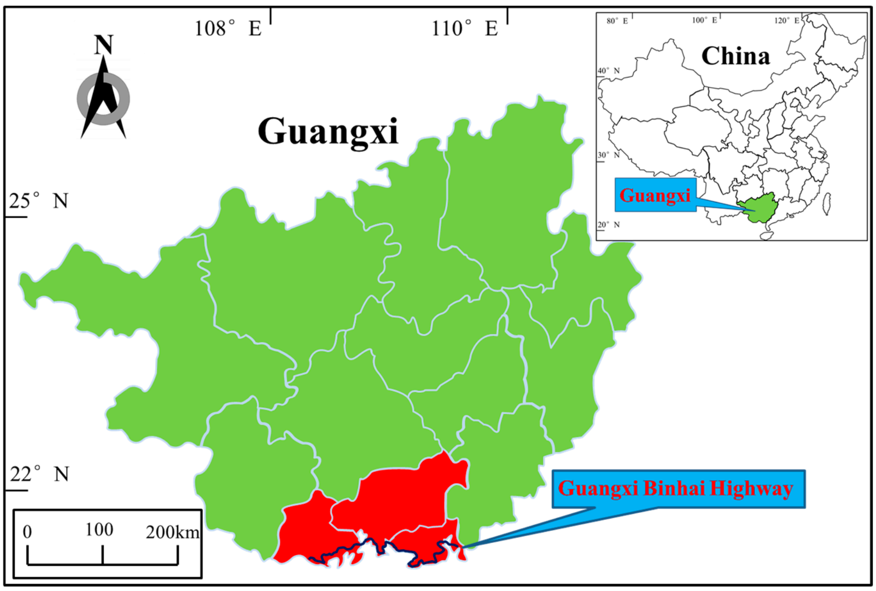


**Figure 5.** Location of the Guangxi Binhai Highway within China.

The Guangxi Binhai Highway starts from Dongxing City, passes through Fangchenggang, Qinzhou, and Beihai, and ends in Shankou. The main line is 314.2 km in length. The project relies on Xiniujiao town to Dafengjiang section of the Guangxi Binhai Highway, with a total length of 10.9 km. The road grade of the project is Grade I, and the width of the roadbed is 24.5 m (Figure 6). Geological research shows that the project section was originally a tidal zone between the high tide and low tide of the sea. After the construction of a flood control seawall, it was gradually reclaimed as paddy fields, shrimp ponds, or dry land. The surface is mostly distributed with typical coastal sedimentary soft soil or soft soil to saturate. Silt clay, silt, and fine sand are the main components, and the coarse sand and gravel sand are partially sandwiched between thin layers or lens bodies. Figure 7 is a picture of the excavation site of marine soft soil.

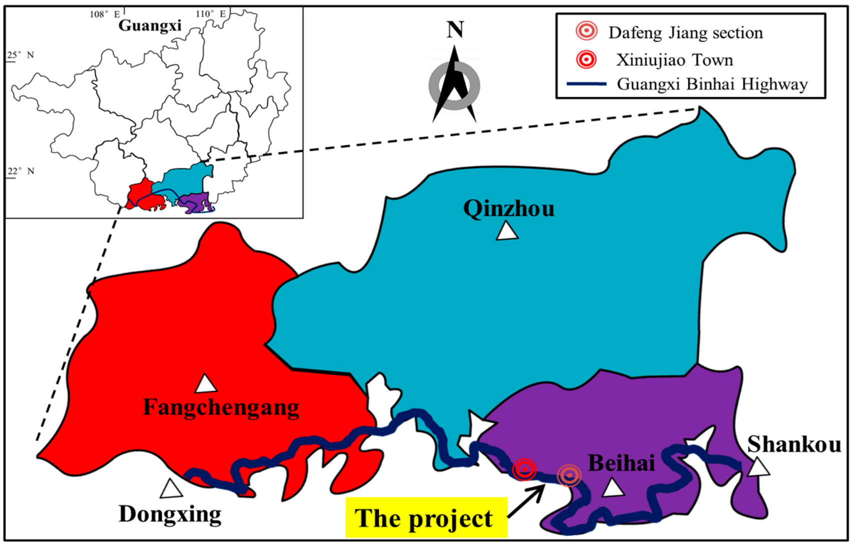


**Figure 6.** Relying on the engineering location map.

**Figure 7.** Site excavation of marine soft soil.

The consolidation settlement calculation is carried out selecting a section from K8+000 to K9+000. The water table level is 1 m below the original ground line. The upper layer of the section is a crust layer with a thickness of 2–3 m, the high liquid limit soft plastic sand-containing clay has a high compression modulus of 15–20 MPa, and the permeability coefficient is only $3 \times 10^{-6}$ to $4 \times 10^{-6}$ cm/s. The lower layer is a soft soil layer with a thickness of 5–10 m. The compression modulus of the soft soil containing sand is low, only 4.5–8.0 MPa, and the permeability coefficient is only $2 \times 10^{-6}$ to $3.5 \times 10^{-6}$ cm/s. The underlying bedrock in the soft soil layer is Indosinian granite. After discussion, it was decided to replace the crust layer with the middle-decomposed granite ($E_s$ = 1200 MPa, $k = 5 \times 10^{-2}$ cm/s). Figure 8 is a schematic diagram of the soft soil foundation treated by the replacement method. According to the data provided in the geotechnical engineering investigation report of the Soft Soil Foundation Treatment project of Guangxi Binhai Highway area (provided by Guangxi Communication Design Group Co., Ltd., Guangxi, China), in combination with in-situ test data and the laboratory soil test data sampled, the data obtained are shown in Table 4. The preloading was then carried out and monitored continuously for 205 days. The upper layer's drainage boundary is neither perfectly permeable nor perfectly impervious (actually, it is a semi-permeable boundary). Due to the strong dispersion of rock and soil, the consolidation settlement of this kind of soft soil foundation can be effectively predicted by adjusting the boundary parameters *b* and *c* in the proposed method.

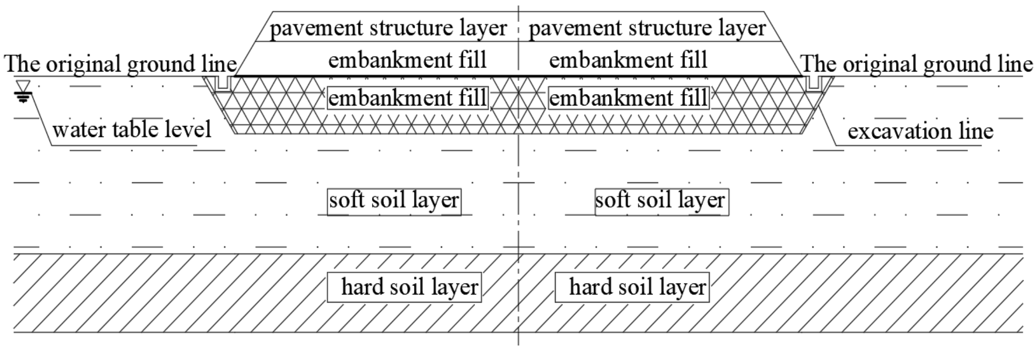


**Figure 8.** Schematic diagram of disposal of soft soil foundation by the replacement method.

**Table 4.** Soil parameters of subgrade settlement monitoring section.

| Layer | Layer Thickness (m) | Bulk Unit Weight (kN/m$^3$) | Water Content (%) | Liquid Limit (%) | Cohesion (kPa) | Friction Angle (°) | Compression Modulus (MPa) | Permeability Coefficient ($10^{-8}$m/s) | SPT |
|---|---|---|---|---|---|---|---|---|---|
| Crust layer | 3 | 18.8 | 20.6 | 35.7 | 25.2 | 18.7 | 15 | 3.8 | 7 |
| Soft soil layer | 7 | 16.7 | 45.2 | 36.7 | 4.5 | 8.6 | 7.0 | 3.0 | 6 |
| Filling in granite | - | 26 | - | - | 150 | 45 | 1200 | $5 \times 10^4$ | >50 |

### 4.2. Settlement and Consolidation Analysis

By making *b* equal to 100 and *c* tend to 0, it is better to simulate the consolidation settlement of the monitored section. The proposed method simplifies the actual load of multi-stage loading mode to the secondary load. The calculated settlement and measured settlement are plotted in Figure 9. The calculated settlement curve obtained by using the replacing soil is basically consistent with the development trend of the measured settlement, which indicates that the proposed method is suitable for analyzing soft soil foundations with a crust layer.

In the process of highway construction, an engineer is typically more concerned with the change in consolidation degree than the settlement of the foundation. The comparison between the calculated and the measured values of consolidation degree is shown in Figure 10. The calculated consolidation degree is found to be basically consistent with the measured consolidation degree. At 205 days, the calculated consolidation degree and the measured consolidation degree are approximately 55%.

The comparison of the calculated and measured results shows that the proposed method is reliable, and the calculated results are basically consistent with the measured results. Therefore, this proposed method is further used to calculate the settlement and consolidation degree of foundation before the replacement, compared with the foundation after the replacement, and study the improvement of foundation performance before and after the replacement. Here, for the convenience of analysis, the conditions of the first layer of soil are different before and after the replacement, and the other conditions are the same. The proposed method is used to calculate the difference in consolidation settlement before and after the replacement (Figure 11). The settlement of the foundation after the replacement is found to be greatly reduced, and the differential settlement is 21.5 mm after 205 days. Comparison of the change in consolidation degree of the foundation before and after the replacement (Figure 12) reveals that the consolidation degree of the foundation after the replacement is relatively high. As same as the settlement of the foundation, the difference in consolidation degree before and after the replacement becomes greater with increasing time. At 205 days, the consolidation degree differed by about 12%. As shown in Figures 11 and 12, the performance of the foundation is improved after the replacement.

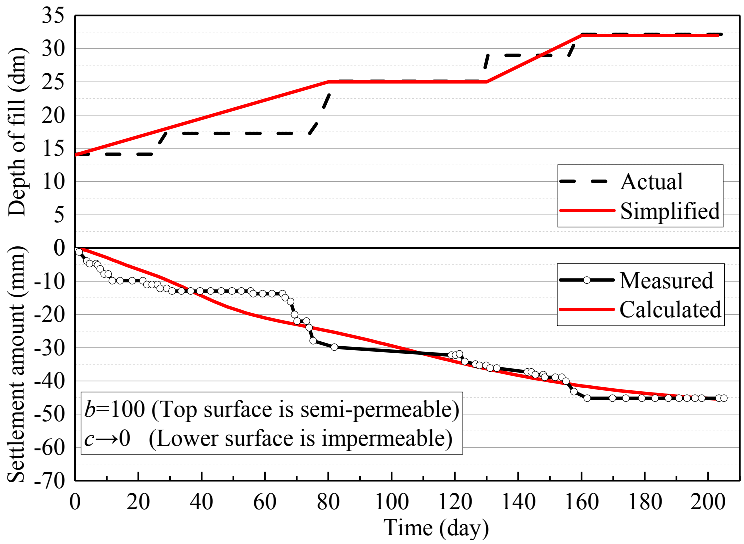


**Figure 9.** Comparison of measured and calculated values of settlement.

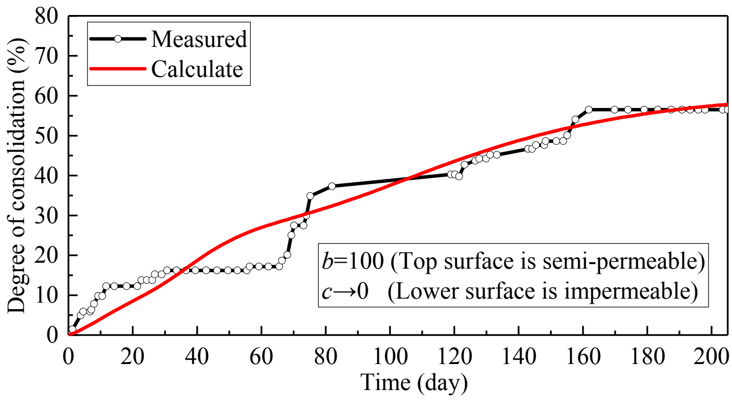


**Figure 10.** Comparison of the calculated and measured consolidation degrees.

As shown in Table 5, the measured final settlement after the replacement is 80 mm. The calculated settlement after the replacement is 78.7 mm, the error is about 1.6%. The settlement and consolidation degree are calculated for 205 days, the errors are about 0.7% and 2.3%, respectively. Comparison of the consolidation settlement before and after the replacement by calculated revealed that the final settlement after the replacement reduced by 68.9 mm, and the reduction rate is 87.5%. The consolidation settlement after the replacement of 205 days is reduced by 21.5 mm, and the reduction rate over 205 days is 47.3%. After the replacement, the consolidation degree increased by 12.4%, and the improvement rate increased by 21.5%, which further showed that the performance of the foundation after the replacement is greatly improved.

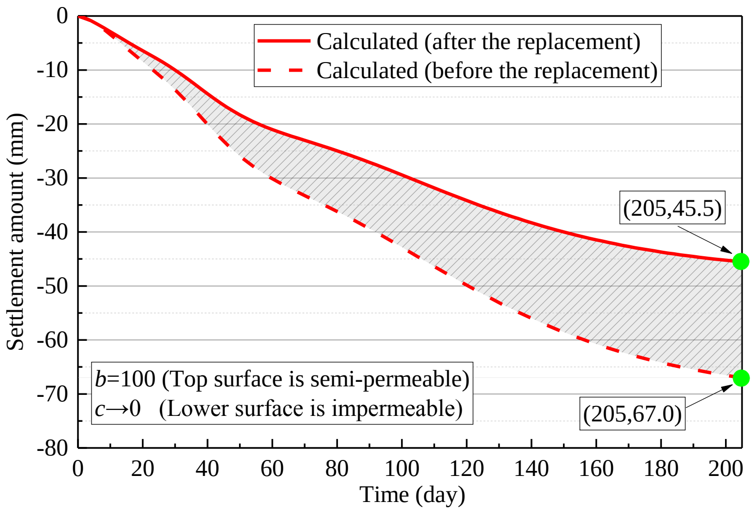


**Figure 11.** Comparison of the calculated settlement curve before and after the replacement.

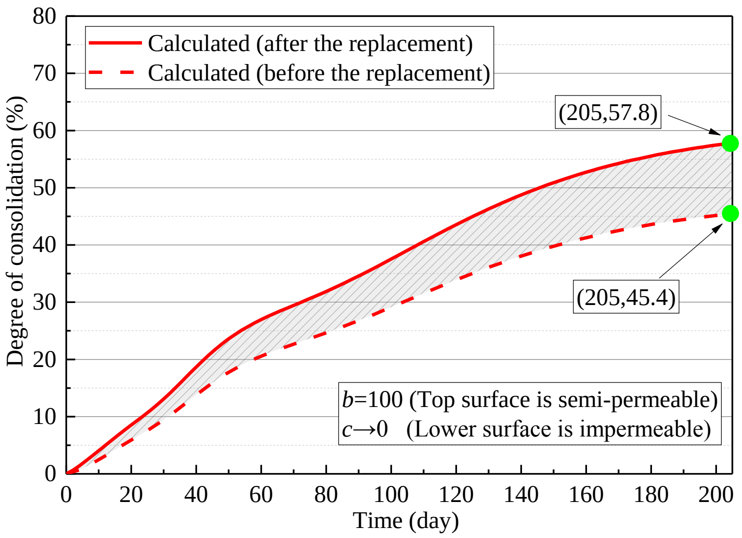


**Figure 12.** Comparison of the calculated consolidation curve before and after the replacement.

**Table 5.** Calculation parameters of the replacement foundation of marine soft soil in Guangxi.

| Data Source | Final Settlement (mm) | Settlement (mm) After 205 Days | Consolidation Degree (%) After 205 Days |
|---|---|---|---|
| Measured data | 80.0 | 45.2 | 56.5 |
| Calculated data (after the replacement) | 78.7 | 45.5 | 57.8 |
| Calculated data (before the replacement) | 147.6 | 67.0 | 45.4 |

## 5. Conclusions

Relying on marine soft soil foundation of complex drainage conditions, the continuous drainage boundary conditions are introduced into the double-layer soil consolidation theory model. Combined with the Laplace transform and Stehfest algorithm, the equations of the improved double-layer soil consolidation theory are deduced and solved. The degradation model of the theory is validated and analyzed by the perfectly permeable boundary conditions and the semi-permeable boundary conditions, respectively. Then, the consolidation analysis of coastal soft soil in Guangxi is carried out. We made some useful conclusions:

(1) Considering the complex drainage boundary conditions, combining the continuous drainage boundary conditions theory, the improved double-layer soil consolidation theory is derived by using the Laplace transform and Stehfest algorithm with high computational efficiency.
(2) Based on the improved double-layer soil consolidation theory, the perfectly permeable boundary case and the semi-permeable boundary case are used to verify this theory. The calculation results are basically consistent with Xie's solution, and it is concluded that the improved double-layer soil consolidation model proposed in the present paper had higher accuracy.
(3) The improved double-layer soil consolidation theory is used in an actual engineering case of marine soft soil in Guangxi. Compared with the measured data, the error of calculated data is 0.7–2.3%. The calculated results are basically consistent with the measured results, indicating that this theory is suitable for the analysis of consolidation and settlement of marine soft soil foundation with complex drainage conditions.
(4) It is difficult to quantitatively control the drainage parameters of the improved double-layer soil consolidation theory, which requires a large number of practical cases to determine the parameters. Only the double-layer foundation has been studied here, and soft foundation with more layers needs further study.

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## Nomenclature

| Symbol | Description | Units |
|---|---|---|
| $A_{11}$, $A_{12}$ | The undetermined constant in Equation (17) | [-] |
| $A_{21}$, $A_{22}$ | The undetermined constant in Equation (18) | [-] |
| $b$ | Related to the drainage property of topsoil | [-] |
| $c$ | Related to the drainage property of subsoil | [-] |
| $c_{v1}$, $c_{v2}$ | Consolidation coefficients of the first/second layer | [$m^2$/s] |
| $E_{s1}$, $E_{s2}$ | Compression modulus of the first/second layer | [Pa] |
| $\overline{f}(s)$ | Laplace function of $f(t)$, $\overline{f}(s) = L[f(t)] = \int_0^\infty f(t)e^{-st}dt$ | - |
| $f(t)$ | Time domain function of $\overline{f}(s)$ | - |
| $f(T)$ | An approximation of the inverse Laplace transform of $\overline{f}(s)$ | - |
| $h_1$, $h_2$ | Thickness of the first/second layer | [m] |
| $i$ | The process variable of accumulation equation | - |
| $k$ | The process variable of accumulation equation | - |
| $k_1$, $k_2$ | Permeability coefficients of the first/second layer | [m/s] |
| $N$ | Positive even number | [-] |
| $q(t)$ | Arbitrary loading function | [Pa] |
| $q_1$, $q_2$ | Load increments at the first/second levels | [Pa] |
| $S_t$ | Total consolidation settlement at time $t$ of foundation | [m] |
| $S_\infty$ | Final total consolidation settlement of foundation | [m] |
| $r_1$, $r_2$ | Intermediate variable, no real meaning | [/m] |
| $s$ | Complex variable involved in the Laplace transform | [/s] |
| $t$ | Time | [s] |
| $t_1$ | Loading time of first class load | [s] |
| $t_2$ | Starting time of the second stage load | [s] |

| Symbol | Description | Units |
| --- | --- | --- |
| $t_3$ | Completion time of the second stage load | [s] |
| $u(h,t)$ | Excess pore water pressure | [Pa] |
| $U$ | Average degree of consolidation of foundation | [-] |
| $u_1(h,t)$ , $u_2(h,t)$ | Excess pore water pressure of the first/second layer | [Pa] |
| $u_t(h,t)$ | Derivative of $u(h,t)$ with respect to $t$ | - |
| $V_i$ | Dependent on N only | [-] |
| $z$ | Foundation depth, from the ground up | [m] |
| $\gamma_w$ | Water unit weight, take $10^4$ N/m$^3$ | [N/m$^3$] |
| $\sigma'_1(h,t)$ , $\sigma'_2(h,t)$ | Effective stress of the first/second layer | [Pa] |
| $\sigma'_{1,2}(h,t)$ | Effective stress at any point in the foundation at any time | [Pa] |

## Appendix A

$$\begin{aligned}\overline{q}(s)-\overline{q}(s+b) &= \int_o^{+\infty} q(t)\left(1-e^{-bt}\right)e^{-st}dt \ = \int_0^{t_1} \tfrac{t}{t_1}q_1\left(1-e^{-bt}\right)e^{-st}dt + \int_{t_1}^{t_2} q_1\left(1-e^{-bt}\right)e^{-st}dt \\ &\quad + \int_{t_2}^{t_3}\left(q_1+\tfrac{t-t_2}{t_3-t_2}q_2\right)\left(1-e^{-bt}\right)e^{-st}dt + \int_{t_3}^{\infty}(q_1+q_2)\left(1-e^{-bt}\right)e^{-st}dt \\ &\quad = \tfrac{q_1}{t_1}\left(\tfrac{1-e^{-st_1}}{s^2}+\tfrac{e^{-(s+b)t_1}-1}{(s+b)^2}\right)+\tfrac{q_2}{(t_3-t_2)}\left(\tfrac{e^{-st_2}-e^{-st_3}}{s^2}+\tfrac{e^{-(s+b)t_3}-e^{-(s+b)t_2}}{(s+b)^2}\right)\end{aligned}$$

$$\begin{aligned}\overline{q}(s)-\overline{q}(s+c) &= \int_o^{+\infty} q(t)\left(1-e^{-ct}\right)e^{-st}dt \ = \int_0^{t_1} \tfrac{t}{t_1}q_1\left(1-e^{-ct}\right)e^{-st}dt + \int_{t_1}^{t_2} q_1\left(1-e^{-ct}\right)e^{-st}dt \\ &\quad + \int_{t_2}^{t_3}\left(q_1+\tfrac{t-t_2}{t_3-t_2}q_2\right)\left(1-e^{-ct}\right)e^{-st}dt + \int_{t_3}^{\infty}(q_1+q_2)\left(1-e^{-ct}\right)e^{-st}dt \\ &\quad = \tfrac{q_1}{t_1}\left(\tfrac{1-e^{-st_1}}{s^2}+\tfrac{e^{-(s+c)t_1}-1}{(s+c)^2}\right)+\tfrac{q_2}{(t_3-t_2)}\left(\tfrac{e^{-st_2}-e^{-st_3}}{s^2}+\tfrac{e^{-(s+c)t_3}-e^{-(s+c)t_2}}{(s+c)^2}\right)\end{aligned}$$

$$A_{11} = -\frac{k_1P_1r_1\left(e^{-h_1r_1-h_2r_2}-e^{h_2r_2-h_1r_1}\right)+k_2P_1r_2\left(e^{h_2r_2-h_1r_1}+e^{-h_1r_1-h_2r_2}\right)-2k_2P_2r_2}{k_1r_1\left(e^{h_1r_1+h_2r_2}+e^{h_2r_2-h_1r_1}-e^{h_1r_1-h_2r_2}-e^{-h_1r_1-h_2r_2}\right)+k_2r_2\left(e^{h_1r_1+h_2r_2}+e^{h_1r_1-h_2r_2}-e^{h_2r_2-h_1r_1}-e^{-h_1r_1-h_2r_2}\right)}$$

$$A_{12} = \frac{k_1P_1r_1\left(e^{h_1r_1+h_2r_2}-e^{h_1r_1-h_2r_2}\right)+k_2P_1r_2\left(e^{h_1r_1+h_2r_2}+e^{h_1r_1-h_2r_2}\right)-2k_2P_2r_2}{k_1r_1\left(e^{h_1r_1+h_2r_2}+e^{h_2r_2-h_1r_1}-e^{h_1r_1-h_2r_2}-e^{-h_1r_1-h_2r_2}\right)+k_2r_2\left(e^{h_1r_1+h_2r_2}+e^{h_1r_1-h_2r_2}-e^{h_2r_2-h_1r_1}-e^{-h_1r_1-h_2r_2}\right)}$$

$$A_{21} = \frac{k_1P_2r_1\left(e^{-h_1(r_1+r_2)}+e^{h_1(r_1-r_2)}\right)+k_2P_2r_2\left(e^{h_1(r_1-r_2)}-e^{-h_1(r_1+r_2)}\right)-2k_1P_1r_1e^{-(h_1+h_2)r_2}}{k_1r_1\left(e^{h_1r_1+h_2r_2}+e^{h_2r_2-h_1r_1}-e^{h_1r_1-h_2r_2}-e^{-h_1r_1-h_2r_2}\right)+k_2r_2\left(e^{h_1r_1+h_2r_2}+e^{h_1r_1-h_2r_2}-e^{h_2r_2-h_1r_1}-e^{-h_1r_1-h_2r_2}\right)}$$

$$A_{22} = \frac{k_1P_2r_1\left(e^{h_1(r_1+r_2)}+e^{h_1(r_2-r_1)}\right)+k_2P_2r_2\left(e^{h_1(r_2-r_1)}-e^{-h_1(r_1+r_2)}\right)-2k_1P_1r_1e^{(h_1+h_2)r_2}}{k_1r_1\left(e^{h_1r_1+h_2r_2}+e^{h_2r_2-h_1r_1}-e^{h_1r_1-h_2r_2}-e^{-h_1r_1-h_2r_2}\right)+k_2r_2\left(e^{h_1r_1+h_2r_2}+e^{h_1r_1-h_2r_2}-e^{h_2r_2-h_1r_1}-e^{-h_1r_1-h_2r_2}\right)}$$

where $P_1 = \overline{q}(s)-\overline{q}(s+b)$, $P_2 = \overline{q}(s)-\overline{q}(s+c)$.

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