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The Equilibrium Scour Depth around a Pier under the Action of Collinear Waves and Current

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Abstract: In this paper, a mathematical equation is developed for the equilibrium scour depth considering an arbitrary shape of the scour hole around a pier under the action of collinear waves and current. A power-law current velocity profile is assumed for the purpose of the analysis. The equilibrium scour depth is obtained by equating the work done by the flowing fluid while interacting with the pier under the action of the collinear waves and the current and the work done by the total volume of the sediment particles removed from the scour hole, respectively. The equilibrium scour depths predicted by the model show good agreement with the experimental and numerical results available in the literature.

Keywords: analytical solution; energy balance; waves and current; scour; arbitrary shape of scour hole

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1. Introduction

Several complex phenomena takes place within the boundary layer adjacent to the sand bed when a vertical pier, exposed to the atmosphere, is placed on a seabed or a riverbed. For shallow and intermediate water depths of a marine environment, the scour is influenced by both the waves and the current whereas in the deep waters the scour is influenced by steady currents only (Myrhaug et al. [1]). The complex flow structure around a pier poses a serious question on the dependability of the structure. Hence, accurate assessment of the combined interaction of the waves and the current with the pier is of utmost importance. The main factors prompting the formation of the scour around a pier are the horseshoe vortices and the downflow created at the stoss side of the pier, the vortex shedding at the rear side of the pier, and the streamline contraction along the side of the pier edges, respectively (Dey et al. [2]). Among the above-mentioned parameters, the horseshoe vortices and the streamline contraction are reported to be the main factors responsible for causing the scour under steady current (Dey et al. [2]). On the contrary, the vortex shedding is identified as the key element influencing the scour under the action of waves only when Keulegan-Carpenter number (KC) lies between 6 and 100 (Sumer et al. [3], Kobayashi and Oda [4]). KC number is expressed as

$$KC = \frac{U_m T}{D} \tag{1}$$

where U_m is the maximum wave-influenced orbital velocity, T is the wave period, and D is the diameter of the pier, respectively. Sumer et al. [5] observed no vortex shedding when $KC < 6$ for a circular shaped pier; KC = 3 and 4 for 45° placements of the square piers, and KC = 10 and 11 for 90° placements of the square piers, respectively. The scour under the combined action of the waves and the current are significantly different from the scour development due to the action of only the waves or the current. Under collinear waves and current, the velocity at the front side of the pier increases significantly in comparison to the velocity at the rear side of the pier. This velocity difference elevates the magnitude of the favourable pressure gradient leading to an increased scour depth. Ninomiya et al. [6] reported that the equilibrium scour depth under the combined action of the waves and the current is larger than the equilibrium scour depth formed under the influence of steady currents only. Wang and Herbich [7] investigated the combined action of the waves and the current on a single pier. They did not incorporate KC number in their study. They derived an expression for the velocity under combined waves and current. Further, they reported a correlation coefficient of 0.939 after comparing the computed values of the wave-current velocity with the measured values, respectively.

The studies related to scour around a pier are extensively covered in, but are not limited to, [8–22]. References [23–27] explored the phenomenon of the scour formation based on numerical models and advanced computational techniques.

Considering only the waves, Sumer et al. [3] investigated the scour around a circular pier at the Technical University of Denmark. Based on their experimental data and the data provided by Das [28] and Kawata and Tsuchiya [29], they proposed an empirical equation for the prediction of the equilibrium scour depth (Equation (2)). The term d_s in Equation (2) is the equilibrium scour depth. Since the equation is based on the experimental datasets obtained from laboratory flume studies, the model is appropriate for small scale studies under regular wave conditions.

$$\frac{d_s}{D} = 1.3\,\{1 - exp[-0.03(KC - 6)]\} \tag{2}$$

Later, Sumer and Fredsøe [30] extended the work of Sumer et al. [3] considering the action of the irregular waves and the current. They expressed the combined velocity of the waves and the current (u_{cw}) as $u_c/(u_c + u_w)$, where u_c is the velocity of the current and u_w is the velocity of the waves, respectively. The experiments were conducted with sand of median diameter (d_{50}) equal to 0.16 mm comprising the bed. For the co-directional waves and current, a mean flow depth of 39 cm was adopted by them. The final results were reported in terms of the non-dimensional equilibrium scour depths for three different values of KC number, namely, 4, 8, and 26, respectively. Further investigations include the study by Mutlu Sumer [16] which was based on the data of Sumer and Fredsøe [30]. The study showed that the empirical equation suggested by Sumer et al. [3] can be used for irregular waves too, when the KC number is calculated as $KC_{rms} = u_{rms}T_p/D$, where u_{rms} is the root mean square value of the wave influenced velocity magnitude, and T_p is the peak time period, respectively.

Myrhaug and Rue [31] illustrated an approach to calculate the scour depth around a vertical pile under long-crested random waves. Waves were considered as a stationary Gaussian narrow-band random process. The results were validated against the data of Sumer et al. [3]. Myrhaug et al. [1] modified the work of Myrhaug and Rue [31] by incorporating the combined effect of the non-linear random waves and current. In this study, they used the empirical equation given by Sumer and Fredsøe [30]. Cataño-Lopera and García [32] also examined the combined effect of the waves and the current in their study around a pier. They observed that the scour depth at the front side of the pier is lesser than the scour depth at the rear side of the pier. Cataño-Lopera and García [32] considered zero-intersection wave period T_z, while Myrhaug et al. [1] used the peak time period T_p for the calculation of the KC_{rms}.

Ong et al. [33] proposed a stochastic technique for the estimation of the scour depth under the waves and the current in a wave dominant flow. They have determined the scour depth for $0 \leq u_c/(u_c + u_w) \leq 0.4$. For the estimation of angular frequency ω, they used the dispersion relationship of ω as $\omega = ku_c cos\sigma + (gktanhkh)^{0.5}$, where k is the wave number, σ is the direction of the wave propagation, g is the acceleration due to gravity, and h is the water depth, respectively. They found that a three-dimensional non-linear wave results in a larger scour depth than two-dimensional non-linear waves at higher values of the KC number. It is due to the fact that at higher values of KC number, the wave set-down (wave-influenced reduction in the mean water depth before the waves break) effect is less in three-dimensional non-linear waves when compared to the two-dimensional non-linear waves (Forristall [34]).

Qi and Gao [35] presented a physical model of the scour depth under the combined action of the waves and the current. They observed a higher stream velocity inside the boundary layer when the waves and the current are streaming along the same direction and the velocity is lesser when the waves and the current are flowing in the opposite direction. Chen and Li [36] and Chen and Li [37] considered the scour formation under the combined action of the waves and the current in a large-scale flume 5 m wide, 8 m deep, and 450 m long, respectively. They presented the scour depth results at five different locations around the pier.

From the available literature on the scour studies, it is apparent that not too many analytical models are available for scour related studies under the simultaneous influence of the waves and the current. In the present study, the authors propose an analytical model of the equilibrium scour depth under the action of the collinear waves and current. This model is a significant development over the previous model by Gazi and Afzal [38], where the action of the waves were not considered. Here, the equilibrium scour depth is obtained by equating the work done by the flowing fluid while interacting with the pier under the action of the collinear waves and current and the work done by the total volume of the sediment particles removed from the scour hole, respectively. Finally, the non-dimensional scour depth values estimated using the proposed analytical model are verified with the experimental and numerical results reported by Sumer and Fredsøe [30] and Quezada et al. [39], respectively.

2. Combined Wave and Current Velocity

In offshore engineering and naval architecture, the linear wave theory (mostly treated as airy wave theory) is considered as the core of surface wave modeling. Applying Bernoulli's dynamic equation to the free surface and using the method of separation of variables, the expression for velocity potential ϕ of a wave over an impermeable, horizontal and rigid seabed located at a depth h from the free surface is written as Equation (3) (Lamb [40]),

$$\phi = \frac{ag}{\omega}\frac{\cosh k\left(h+z\right)}{\cosh\left(kh\right)}\sin\left(kx-\omega t\right) \tag{3}$$

where ϕ is the velocity potential of the wave field, k is the wave number given by $\frac{2\pi}{L}$, L is the wavelength, T is the time period of the oscillation of the waves, ω is the wave angular frequency equal to $\frac{2\pi}{T}$, a is the wave amplitude, and h is the local water depth, respectively. For a given set of wave parameters, ϕ is a function of spatial co-ordinates x and z at a given time t. Further, the origin is taken at the free surface. A detailed description of the schematic view of the formation of the equilibrium scour depth is shown in Figure 1.

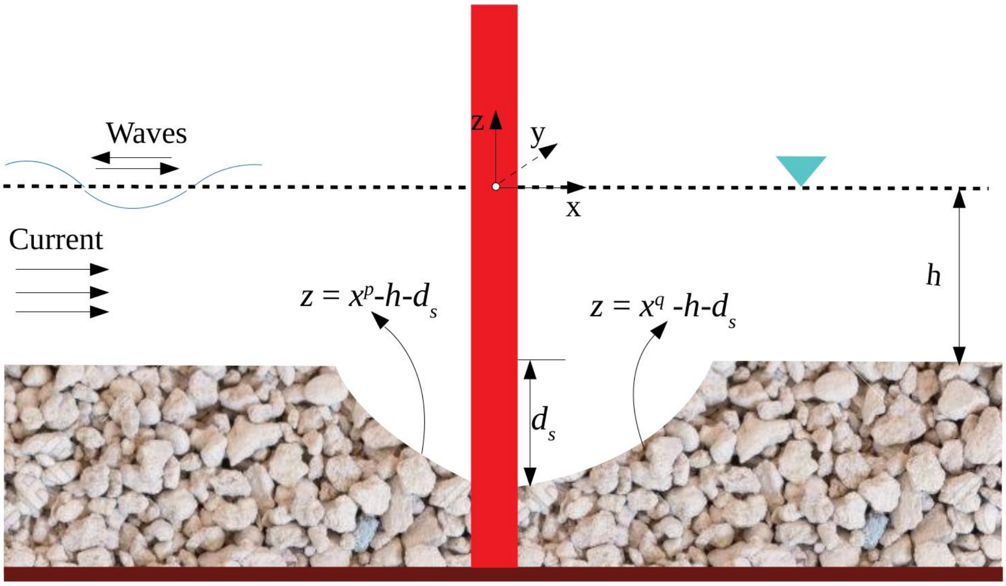

Figure 1. Schematic view of the formation of the equilibrium scour depth (d_s) around a pier due to the collinear waves and current.

The wave velocity in the streamwise direction is obtained by differentiating Equation (3) with respect to x and is given by Equation (4) as follows:

$$u_w = \frac{\partial \phi}{\partial x} = \frac{agk}{\omega} \frac{\cosh k\,(h+z)}{\cosh\,(kh)} \cos\,(kx - \omega t) \tag{4}$$

The surface waves raise the bed shear stress which cause the loosening of the sediment particles on the bed. This loosening of sediment particles result in increased upward seepage forces on the sand bed making the particles highly vulnerable to the destabilising forces responsible for setting the particles into motion. Ultimately the loosened sediment particles are transported by the tidal current ([35,41,42]). The velocity of the tidal currents become zero at the seabed and is maximum at or near the water surface. In the present model, it is assumed that the current velocity obeys the power-law velocity profile (Equation (5)) given by Soulsby and Humphery [43] and Romanoff and Soares [44] as follows:

$$u_c = \left(\frac{z+h}{h}\right)^{\frac{1}{n}} U \tag{5}$$

where n is an exponent valid for any integer value in the closed domain [6,10] (Dey [45]), and U is the depth averaged velocity of the current, respectively. In order to investigate the action of the collinear waves and the current on the pier surface, the combined velocity (u_{cw}) is expressed as the sum of the wave velocity and the current velocity (Equation (6)) as:

$$\begin{aligned} u_{cw} &= u_c + u_w \\ &= \left(\frac{z+h}{h}\right)^{\frac{1}{n}} U + \frac{agk}{\omega} \frac{\cosh k\,(h+z)}{\cosh\,(kh)} \cos\,(kx - \omega t) \end{aligned} \tag{6}$$

3. Estimation of the Scour Depth

In this section, an analytical equation capable of estimating the scour depth around a pier under the action of collinear waves and current is developed. The equation is obtained assuming the energy balance approach as outlined by Gazi and Afzal [38], Hafez [46] but incorporating the effect of waves, which was not considered in the previously mentioned works. When a pier is placed in a flowing fluid (case of oceans and/or river) the flow gets disturbed, and the fluid does work on the surface of the pier. The work done by the flowing fluid on the pier is expressed by Equation (7) given below.

$$\begin{aligned} W_{in} &= \int_{-h}^{0} \left[\frac{\rho h b u_{cw}^2}{\left(1-\frac{b}{B}\right)^2} \right] dz \\ &= \frac{\rho h b}{\left(1-\frac{b}{B}\right)^2} \left[\left(\frac{agk}{\omega}\right)^2 \frac{\cos^2(kx-\omega t)}{\cosh^2(kh)} \int_{-h}^{0} \cosh^2 k(h+z)\, dz \right. \\ &\quad + 2U \left(\frac{agk}{\omega}\right) \frac{\cos(kx-\omega t)}{\cosh(kh)} \int_{-h}^{0} \cosh k(h+z) \left(\frac{z+h}{h}\right)^{\frac{1}{8}} dz \\ &\quad \left. + U^2 \int_{-h}^{0} \left(\frac{z+h}{h}\right)^{\frac{2}{8}} dz \right] \end{aligned} \tag{7}$$

Simplification of Equation (7) after performing the integration results into Equation (8) as follows,

$$\begin{aligned} W_{in} &= \frac{\rho h b}{\left(1-\frac{b}{B}\right)^2} \left[\left(\frac{agk}{\omega}\right)^2 \frac{\cos^2(kx-\omega t)}{\cosh^2(kh)} \left\{ \frac{\sinh(2kh)}{4k} + \frac{h}{2} \right\} \right. \\ &\quad \left. + 2U \left(\frac{agk}{\omega}\right) \frac{\cos(kx-\omega t)}{\cosh(kh)} \zeta + \frac{4hU^2}{5} \right] \end{aligned} \tag{8}$$

where, $\zeta = \frac{128h^{\frac{9}{8}}k^2 cosh(kh) + k^{\frac{7}{8}}\left(64\Gamma\left(\frac{17}{8},kh\right) - 9\Gamma\left(\frac{1}{8}\right)\right) + (-k)^{\frac{7}{8}}\left(64\Gamma\left(\frac{17}{8},-kh\right) - 9\Gamma\left(\frac{1}{8}\right)\right)}{144h^{\frac{1}{8}}k^2}$, b is the width of the pier, B is the channel width or distance between two piers, ρ is the density of flowing fluid, and Γ is the gamma function.

Following Gazi and Afzal [38], the scour profiles at the front and the rear side of the pier are expressed by Equations (9) and (10) given below.

$$z = x^p - h - d_s \tag{9}$$

$$z = x^q - h - d_s \tag{10}$$

where, $z = x^p - h - d_s$ and $z = x^q - h - d_s$ represent the scour hole pattern at the upstream and the downstream section of the pier for any real values of p and q, respectively. The bed particles are set into motion when the total force on the pier due to the flowing fluid becomes higher than the critical tractive force on the sediment particles adjacent to the pier. The movement of the bed materials away from the vicinity of the pier results in the formation of the scour.

The total volume of the sediments carried away from the pier surface is given by $V_T = V_1 + V_2$, where V_1 and V_2 correspond to the volumes of the sediment particles carried away from the upstream and the downstream ends of the pier. In the present study, the top surfaces of the scour holes at the upstream and the downstream ends are assumed to be circular in shape and the radii r_1 and r_2 (Figure 2) of the top surfaces of the scour holes at the upstream and the downstream ends are expressed as $r_1 = x = (z + h + d_s)^{\frac{1}{p}}$ and $r_2 = x = (z + h + d_s)^{\frac{1}{q}}$, respectively. Utilizing these expressions for r_1 and r_2, V_1 and V_2 are estimated as,

$$V_1 = \int_{-h-d_s}^{-h} \pi r_1^2 dz = \frac{\pi p}{2+p} d_s^{\frac{2+p}{p}} \tag{11}$$

$$V_2 = \int_{-h-d_s}^{-h} \pi r_2^2 dz = \frac{\pi q}{2+q} d_s^{\frac{2+q}{q}} \quad (12)$$

where d_s is the equilibrium scour depth.

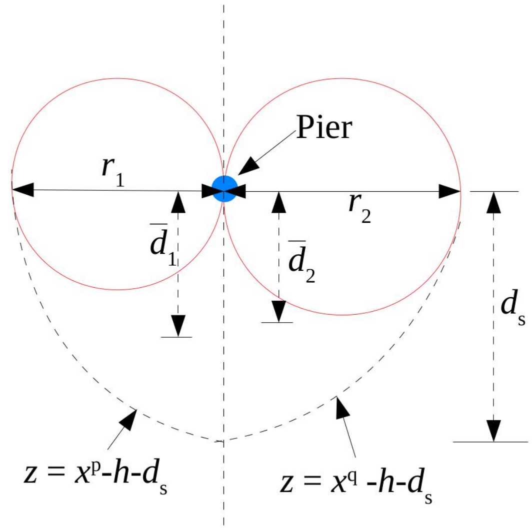

Figure 2. op view of the scour hole around a circular pier. r_1 and r_2 are the radii of the top surface of the scour holes at the front and the rear side of the pier; $\bar{d_1}$ and $\bar{d_2}$ are the distance of the centroids of the area bounded within the pier and the respective scour profiles at the upstream and the downstream portion of the pier, measured from the initial bed level, respectively.

V_T is obtained by adding the expressions for V_1 and V_2 (Equations (11) and (12)) and finally assumes the form as given by Equation (13) below.

$$V_T = \frac{\pi p}{2+p} d_s^{\frac{2+p}{p}} + \frac{\pi q}{2+q} d_s^{\frac{2+q}{q}} \quad (13)$$

The submerged weight (W_s) of the total bed materials removed from the neighbourhood of the pier to develop a scour hole of volume V_T is given by Equation (14) shown below,

$$W_s = \left[\frac{\pi p}{2+p} d_s^{\frac{2+p}{p}} + \frac{\pi q}{2+q} d_s^{\frac{2+q}{q}} \right] (1-\eta)(\gamma_s - \gamma) \quad (14)$$

where η is porosity of the sediment bed, γ is the unit weight of the flowing fluid, and γ_s is unit weight of the sediment particles, respectively.

The distance of the centroids measured from the top of the scour holes ($\bar{d_1}$ and $\bar{d_2}$) are calculated in terms of d_s as represented by Equations (15) and (16) given below.

$$\bar{d_1} = \frac{3p+1}{(4p+2)} d_s \quad (15)$$

$$\bar{d_2} = \frac{3q+1}{(4q+2)} d_s \quad (16)$$

Now, the work done by the bed material, while getting removed from the bed, at equilibrium is estimated as,

$$W_{out} = \left[\frac{\pi p(3p+1)}{(2+p)(4p+2)} d_s^{\frac{2p+2}{p}} + \frac{\pi q(3q+1)}{(2+q)(4q+2)} d_s^{\frac{2q+2}{q}} \right] (1-\eta)(\gamma_s - \gamma) \quad (17)$$

Utilizing the concept of energy balance, Equations (8) and (17) are equated to obtain the following non-linear equation,

$$Ad_s^{\frac{2p+2}{p}} + Md_s^{\frac{2q+2}{q}} - E = 0 \tag{18}$$

where
$A = \frac{\pi p(3p+1)}{(2+p)(4p+2)}(1-\eta)(\gamma_s - \gamma)$,
$M = \frac{\pi q(3q+1)}{(2+q)(4q+2)}(1-\eta)(\gamma_s - \gamma)$, and
$E = \frac{\rho h b}{(1-\frac{b}{B})^2}\left[\left(\frac{agk}{\omega}\right)^2 \frac{\cos^2(kx-\omega t)}{\cosh^2(kh)}\left\{\frac{\sinh(2kh)}{4k} + \frac{h}{2}\right\} + 2U\left(\frac{agk}{\omega}\right)\frac{\cos(kx-\omega t)}{\cosh(kh)}\zeta + \frac{4hU^2}{5}\right]$

Equation (18) is a non-linear equation of the scour depth d_s. The value of p and q are chosen based on the shape of the scour hole around the pier. For given values of p and q, Equation (18) can be solved using a suitable numerical technique.

4. Validation and Discussion

The proposed analytical model for the equilibrium scour depth is validated with the experimental data provided by Sumer and Fredsøe [30]and the numerical results reported by Quezada et al. [39], respectively. Since the present model is developed for the collinear waves and current, it can be safely assumed that the flow hydrodynamics around the pier varies only along the x and z directions and the variations along the y direction are not taken into consideration. While validating the present model, it is further assumed that the profiles of the scour holes upstream and downstream of the pier are identical, i.e. $p = q$, respectively.

Table 1 shows the comparison of the non-dimensional scour depth (d_s/D) as estimated by the present model and the non-dimensional scour depth reported by Sumer and Fredsøe [30] for collinear waves and currents when the wave propagation and the current are following the same direction. The comparisons are made for two pier diameters, namely, D = 90 mm and 30 mm, respectively. Other parameters used for the prediction of the equilibrium scour depth utilizing the present equation include the unit weight of water (γ) = 9810 N/m^3, unit weight of the bed material (γ_s) = 19,999 N/m^3, porosity of the bed material (η) = 0.4, and mass density of the water (ρ) = 1000 kg/m^3. Based on the experiments performed by Sumer and Fredsøe [30] a mean water depth (h) of 39 cm is chosen for the purpose of validation. The peak frequencies of the waves for different experimental runs are used to obtain the corresponding time periods. Using the relationship $\omega = 2\pi/T$, the angular wave frequencies corresponding to different wave periods are evaluated. Further, the wavelengths at different time periods are determined using the wave dispersion relationship. The wave numbers (k) are obtained from the values of the corresponding wavelengths. Finally, the wave amplitudes (a) for different experimental runs are obtained using the following equation (Equation (19)),

$$\frac{U_m}{a} = \frac{\omega}{\sinh(kh)} \tag{19}$$

where U_m is the maximum bed orbital velocity for a given wave. Equation (19) is based on the linear wave theory. As evident from Table 1 the non-dimensional scour depths predicted by the proposed analytical model shows excellent likeness with the non-dimensional scour depth observed by Sumer and Fredsøe [30] at different KC numbers.

Table 1. Comparison of the calculated non-dimensional scour depth (d_s/D) with the corresponding values obtained from the experimental work of Sumer and Fredsøe [30].

Sl No.	Diameter of the Pier D (m)	Peak Frequency of Waves S^{-1}	Bed Orbital Velocity U_m (m/s)	Un Disturbed Depth Averaged Current Velocity U (m/s)	Keulegan Carpenter Number KC	Sumer and Fredsøe [30]'s Data. (d_s/D)	Proposed Model's (d_s/D)
1	0.09	0.4	0.157	0.0	4	0.06	0.07
2	0.09	0.4	0.157	0.085	4	0.11	0.12
3	0.09	0.4	0.157	0.157	4	0.56	0.53
4	0.09	0.4	0.157	0.29	4	0.83	0.80
5	0.09	0.32	0.231	0.0	8	0.11	0.10
6	0.09	0.32	0.231	0.13	8	0.5	0.48
7	0.09	0.32	0.231	0.231	8	0.78	0.76
8	0.09	0.32	0.231	0.459	8	1.06	1.01
9	0.03	0.32	0.242	0.0	26	0.83	0.88
10	0.03	0.32	0.242	0.13	26	1.33	1.31
11	0.03	0.32	0.242	0.242	26	1.50	1.50
12	0.03	0.31	0.242	0.449	26	1.67	1.64
13	0.03	-	-	0.459	-	1.21	1.25
14	0.03	-	-	0.449	-	2	2.07

It is noteworthy that the model is capable of capturing any arbitrary profile of the scour hole for two different possible situations pertaining to the collinear waves and current, viz., waves following the current and waves opposing the current, respectively. This is achieved by varying the values of p and q in the upstream and the downstream scour profiles. Sumer and Fredsøe [30] did not mention anything related to the profile of the scour holes in their study. While validating the results predicted by the present model, it is observed that the equilibrium scour depths observed by Sumer and Fredsøe [30] are best represented by scour profiles for which the values of p and q lies between 0.7 and 2.

The non-dimensional scour depths under co-directional waves and current as obtained from the numerical investigations by Quezada et al. [39] are further utilized for the validation of the proposed model. Table 2 presents the comparison between the non-dimensional scour depths as found from the proposed analytical model and the numerical model illustrated by Quezada et al. [39]. To exactly represent the situation adopted by Quezada et al. [39], the calculations are performed considering a vertical pier of diameter 0.2 m placed in a channel of width 1 m and carrying the flow at a depth of 0.5 m, respectively. The values of the unit weight of the water, unit weight of the bed material, bed-porosity and the mass density of the water mentioned earlier remains unchanged. However, only those scenarios which pertain to the combined effect of the co-directional waves and current are considered for the purpose of the validation of the present model. The analytical model described in the present paper agrees well with the numerical model of Quezada et al. [39] for collinear waves and current when the values of p and q lies in the interval [0.9, 2], respectively.

Table 2. Comparison of the calculated non-dimensional scour depth (d_s/D) with the corresponding values obtained from the numerical studies of Quezada et al. [39].

Sl No.	Diameter of the Pier D (m)	Wave Time Period (S)	Bed Orbital Velocity U_m (m/s)	Un Disturbed Depth Averaged Current Velocity U (m/s)	Keulegan Carpenter Number KC	Quezada et al. [39]'s Study (d_s/D)	Proposed Model's (d_s/D)
1	0.2	1.4	0.12	0.23	0.86	0.251	0.25
2	0.2	1.4	0.19	0.22	1.31	0.304	0.31
3	0.2	2	0.28	0.24	2.76	0.132	0.13
4	0.2	3	0.31	0.1	4.61	0.085	0.08

The model developed in the present study is a generalized analytical model for the prediction of the equilibrium scour depth under the action of collinear waves and current. This model is based on the principle of energy balance and has the inherent advantage of dealing with any arbitrary profile of the scour hole, which can be achieved by varying the values of p and q (Equations (9) and (10)), respectively. Further, the model is not limited to only a particular type of fluid or bed material. By changing the values of the unit weight of the fluid, the unit weight of the sediment particles, and the bed porosity in Equation (18), the model can incorporate any flowing fluid and bed material, respectively.

Additional assessments based on more field informations are recommended for further validation and applicability of the proposed model. The concept of energy balance can be applied to develop analytical models for the prediction of the equilibrium scour depths around other hydraulic structures like abutments, groins, sluice gate openings and many more, under the action of the waves and the current.

Further research work has to be carried out to extend the capability of the present model in capturing the complete three dimensional flow hydrodynamics around any hydraulic or coastal structures under the combined action of the waves and the current with a non-zero angle between them.

5. Conclusions

A mathematical equation of the equilibrium scour depth around a pier under collinear waves and current is presented in this paper. Assuming that the current velocity profile obeys the power law, the combined velocity for the collinear waves and current is obtained by adding the wave velocity and the current velocity, respectively. The present model is an extension to the model developed by Gazi and Afzal [38] and has the ability of capturing any arbitrary scour profile under collinear waves and current. The results obtained using the proposed equation show very good agreement with the results of Sumer and Fredsøe [30] and Quezada et al. [39], respectively. However, further validations of the proposed equation based on more experimental and field data is recommended.

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