Natural Science and Supernatural Thought Experiments

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Abstract: Religious notions have long played a role in epistemology. Theological thought experiments, in particular, have been effective in a wide range of situations in the sciences. Some of these are merely picturesque, others have been heuristically important, and still others, as I will argue, have played a role that could be called essential. I will illustrate the difference between heuristic and essential with two examples. One of these stems from the Newton–Leibniz debate over the nature of space and time; the other is a thought experiment of my own constructed with the aim of making a case for a more liberal view of evidence in mathematics.

Keywords: thought experiment; God; Newton–Leibniz; mathematical methods

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There is a long tradition of using religious notions in science. Einstein famously said that God does not play dice. Bohr countered with the remark that not only does God play dice but he also tosses them where they can't be seen. These, respectively, are picturesque ways of rejecting or embracing indeterminism in physics. God is a convenient metaphor, but is in no way essential to making Einstein's or Bohr's respective points. This is a rather trivial use of God in science. We get a little more serious when we encounter phrases such as "a god's eye point of view". In this case there is the suggestion of the existence of a fact of the matter that an omniscient being would grasp, even if we humans were not capable. It can help advance a philosophical debate in one direction or another, and even atheists find the metaphor useful in debate.

On the other hand, theological notions are sometimes invoked only to be mocked. The question "How many angles can dance on the head of a pin?" is taken to be the height of folly. The targets of derision were medieval scholastics, even though it turns out that as a matter of historical fact they did not entertain the issue.

Thought experiments involving God often aim at telling us something new about God. Other thought experiments involving God have a different aim: They use the idea of God to tell us something new about the world. What follows is an instance (actually two instances), of the latter. In other words, God is not the end but rather is a means to the end. I am going to discuss two thought experiments where God plays an important or even crucial function. The first of these is the thought experiment (or cluster of thought experiments) that played a role in the Newton–Leibniz debate on the nature of space and time. The second is a thought experiment of my own concerning the nature of legitimate mathematical methodology. Though both are effective in achieving their goals, they turn out to function in interestingly different ways.

In his *Principia*, Isaac Newton claimed "Absolute, true, and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. Relative, apparent, and common time is any sensible and external measure (precise or imprecise) of duration by means of motion; such a measure—for example, an hour, a day, a month, a year—is commonly used instead of true time." (Newton 1999, p. 408).

The claim about the nature of time was followed by a similar claim about the nature of space: "Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable. Relative space is any movable measure or dimension of this absolute space; such a measure or dimension is determined by our senses from the situation of the space with respect to bodies and is popularly used for immovable space, as in the case of space under the earth or in the air or in the heavens, where the dimension is determined from the situation of the space with respect to the earth." (Newton 1999, p. 408f).

What Newton called "absolute space and time" is now more adequately called "substantivalism", meaning that time and space are things in their own right, not dependent on anything else. The picture Newton painted is that space and time exist independently from everything else (except God, perhaps). It has nothing to do with the bodies that might or might not be in space or the events that might or might not happen in time; nor has it anything to do with how we measure and observe space and time, and so on. They are, according to Newton, "God's sensorium", which is something of a mystery. Newton thought of creation as God placing matter at particular locations in an otherwise empty space and this act of creation happened at a particular time.

Newton offered his famous bucket thought experiment as the justification for his belief in absolute space. We cannot detect our location in absolute space, nor can we detect our absolute velocity through space. Moreover, we cannot detect our location in absolute time. We can, however, detect absolute accelerations. It is, in fact, an everyday experience which you can feel when driving around a corner. This cannot be reduced to relative acceleration. Imagine two cars facing in opposite directions, initially stationary with respect to one another. They might start to move apart, gaining in relative velocity; they are accelerating relative to one another. So far the situation seems symmetrical. However, the passengers inside could notice a difference. In one car the passengers feel thrown back in their seats, while the others feel no such force. The former is the one that is undergoing genuine absolute acceleration.

The bucket experiment (Figure 1) is designed to show that absolute acceleration is detectable. We have three cases to consider in this experiment (which could be either a real experiment or a thought experiment). In stage I the bucket and the water are at rest with respect to one another. In stage II the bucket is turning relative to the water. In stage III the bucket and water are again at rest with respect to one another. We need to explain the difference between stage I where the surface of the water is flat and stage III where the surface is concave. Notice that we cannot do it by appeal to relative motion, since there is no relative motion of water to bucket in either stage I or III.
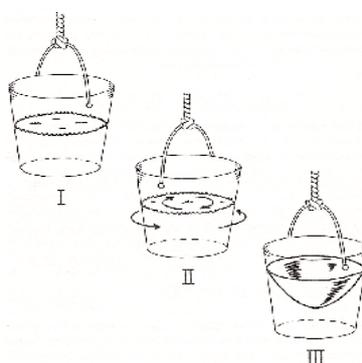
Figure 1. Newton's bucket thought experiment.

Newton's answer is this: In stage III (but not in I) the water and bucket are in absolute motion; they are rotating in absolute space. The crucial claim that Newton makes is that the best explanation—perhaps the only explanation—is absolute space. The water-bucket system is rotating with respect to space itself. Therefore, space exists (as a thing in its own right).

God played a huge role in Newton's thinking generally, but only a minor role in the actual arguments. This was not so with Gottfried Wilhelm von Leibniz, who rejected Newton's absolutism, using powerful theological thought experiments.

The correspondence between Samuel Clarke and Leibniz is one of the great scientific exchanges of all time. Clarke was a close associate of Newton, and it is widely believed Newton had a big hand in Clarke's side of the exchange. Leibniz and Clarke argued about many things including the principles of dynamics, miracles, and the nature of God, but the chief topic of interest was the nature of space and time. Clarke upheld Newton's absolutism while Leibniz attacked it and offered his relational account in its place. He put his relationalism succinctly: "I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences, as time is an order of successions". (Alexander 1977, letter III, Section 4)

In other words, objects are not in independently existing space; space is merely the order of their arrangement. Similarly, events are not in independently existing time; time is simply the order of events. There is a rough and ready test to see whether you are instinctively a Newtonian or a Leibnizian. Ask yourself: If there were no bodies, could there still be empty space? If there were no events, could there still be time? If you said yes to both of these, you are a Newtonian. If you find empty space or time where nothing happens to be nonsensical, then you are on the side of Leibniz. It is remarkable how intrenched these rival intuitive views can be.

Leibniz attempts to refute any sort of absolutist or substantivalist view with a remarkable argument based on what we now recognize as symmetry considerations. First he posits the *Principle of Sufficient Reason:* "Nothing happens without a reason why it should be so and not otherwise". He then points out that there can be no reason why the universe is where it is in absolute space and not at some other location, say, a meter to the right. This is sometimes called a "Leibniz shift" (Figure 2).

> ...if space were an absolute being, there would something happen for which it would be impossible there should be a sufficient reason. Which is against my axiom. And I prove it thus. Space is something absolutely uniform; and without the things placed in it, one point of space does not absolutely differ in any respect whatsoever from another point of space. Now from hence it follows, (supposing space to be something in itself, besides the order of bodies among themselves) that 'tis impossible there should be a reason, why God, preserving the same situations among bodies among themselves, should have placed them in space after one certain particular manner, and not otherwise; why everything was not placed the quite contrary way, for instance, by changing East into West. (Alexander 1977, p. III-5)

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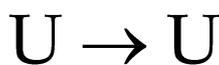
Figure 2. A Leibniz shift.

The two universes are completely symmetrical. There can be no reason to favour the creation of one over the other, so God would have no reason to choose. But God only acts for a reason, so like Buridan's ass he would be incapable of creating a universe. The existence of Newton's absolute space leads to this absurdity, so space cannot be as Newton thought. Since the universe was created, God could not have been faced with such a predicament. Note that this argument can be run considering absolute location (no difference between being located here or there) or considering absolute velocity (no difference between moving, say, west with a velocity v or a velocity $2v$, $3v$, etc.) Whether it also works for accelerations is questionable. Leibniz, unfortunately, did not address this case in any detail.

The problem with Newton's absolute space, as Leibniz sees it, is easily solved if we simply hold that the only thing that exists (and the only thing that matters) is the set of relations among bodies.

> But if space is nothing else, but that order of relation; and is nothing at all without bodies, but the possibility of placing them; then those two states, the one such as it now is, the

other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. But in truth the one would exactly be the same thing as the other, they being absolutely indiscernible... ([Alexander 1977](#), p. III-5.)

Exactly the same considerations can be raised about time. Why did God make the universe at the particular (absolute) time he did, when he could have made it a million years earlier or a day later? By the Principle of Sufficient Reason and the nature of absolute time (every instant is exactly like every other), God could have no reason for picking any particular time. Therefore, Newton's view must be wrong about time just as he was wrong about space. Leibniz's solution for time is the same as it was for space. It is also the solution of Augustine in his *Confessions*. Creation did not happen at some instant in pre-existing absolute time. Rather, time itself began with creation.

Leibniz offered a second argument against absolute space and time, an argument that rests on a different assumption, the *Principle of the Identity of Indiscernibles*: Two entities that have all their properties in common are identical (they are in fact one and the same object). For example, there cannot be two snowflakes that are exactly the same; there must be some difference, otherwise they would be one and the same snowflake. This principle is related to sufficient reason. If two snowflakes were indeed identical, then God would have no reason to place one here and the other there, rather than vice versa. To imagine two distinct universes, completely alike except that one is at a different place or is moving in a different direction in absolute space, is to suppose a difference that is not a genuine one, since all the points of space are the same. (The same can be said about time.) Consequently, by the Principle of the Identity of Indiscernibles, they must be the same universe.

To say that God can cause the whole universe to move forward in a right line, or in any other line, without making otherwise any alteration in it; is another chimerical supposition. For, two states indiscernible from each other, are the same state; and consequently, 'tis a change without any change. ([Alexander 1977](#), p. IV-13.)

Leibniz was never able to fully answer Newton's bucket thought experiment. However, he definitely had the upper hand when it came to location and velocity. Acceleration was his downfall. In addition, it remains the nub of the problem to the present day. However, I am not interested in adjudicating the Newton–Leibniz debate. I am, instead, using it to illustrate how theological arguments, in particular theological thought experiments, have played a very fruitful role in the history of science.

God played a huge and effective role in figuring out how the world works.

Leibniz's arguments are still effective today, but not because of God. The Principle of Sufficient Reason—God only acts on reason—turns out to be a symmetry principle. Put in current terms, every point of absolute space (and every point of absolute time) is exactly like every other point. This gives rise to all sorts of symmetries: Rotation symmetry (rotate the material universe any degree and nothing will have changed); translation symmetry (move the universe to the east, or any other direction, and nothing will have changed); reflection symmetry (flip the universe and nothing will have changed). In order to explain why the universe is located where it is in absolute space there would have to be some symmetry breaking operation, such as the presence of some cause. No such symmetry breaking cause exists.

Newton would have seen no problem with this, since God would have provided the cause, albeit supernatural. Additionally, he would have placed the universe in one place rather than any other simply on a whim. Newton's God could do *anything* she wanted. Leibniz's God, a paragon of rationality, is utterly incapable of acting on a whim to break a symmetry. However, the universe does exist. Hence, Leibniz's striking conclusion: It does not exist in space; space was created along with the material universe. Similarly, time, as we would now say, began with the Big Bang. The universe was not created in time. There are no symmetries to be broken.

For religious people in the 17th century, such as Newton and Leibniz, invoking God in the foundations of physics would have seemed philosophically natural and acceptable. However, we can

see with hindsight that Leibniz had some intuitive notion of symmetry at work in his thinking about space and time. It is fairly easy for us now to reconstruct his thinking in terms of modern symmetry and evaluate his argument in those modern terms. He comes off well, though the final word on the status of space and time is not yet written.

The role of God in Leibniz's thinking is *heuristic*. The reason I call it *heuristic*, and contrast it with *essential*, is that the idea of God was useful to Leibniz, but we can reconstruct the argument without appeal to God. Will this always be the case when thought experiments involve God? Can thought experiments involving God always be reconstructed without God? I think the answer is no, which might come as a surprise. I now want to present a thought experiment (one of my own), in which the role of God is *essential*. It involves the nature of mathematical evidence. I will say more about this distinction between heuristic and essential at the end.

There is a standard view of mathematics that says proofs are the one and only source of evidence and proofs are deductive derivations from first principles. (Proof sketches are acceptable, but it is understood that they could be fleshed out, if necessary.) This attitude has a long tradition and there is a comforting surety about it. Occasionally, however, there are voices in opposition, including one that should be particularly influential.

> If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics. The fact is that in mathematics we still have the same attitude today that in former times one had toward all science, namely we try to derive everything by cogent proofs from the definitions (that is, in ontological terminology, from the essences of things). Perhaps this method, if it claims monopoly, is as wrong in mathematics as it was in physics. (Gödel 1995, vol. III, p. 313)

I'm going to argue for the same conclusion as Gödel, but I will come at it in a very different way. Instead of trying directly to liberalize the notion of evidence in mathematics, I will assume certainty in physics. That is, I will assume that the first principles of quantum mechanics (QM) are just as certain as the Peano axioms (PA), the first principles of arithmetic. The consequence for what counts as legitimate mathematical methods will surprise.

Let's begin with a parable, a thought experiment involving God. God parts the clouds and says: "Verily, verily I say unto you, the principles of quantum mechanics are true". Imagine God as you will. I picture Athena, goddess of wisdom and patron of science. However, be sure to include her being omniscient and truthful. This means we can now know with certainty that quantum states are represented by vectors in Hilbert space; they evolve according to the Schrödinger equation; the Born rule will give us the right probabilities for measurement outcomes; and so on. We now have perfect confidence in the truth of the standard principles of QM, which until now were merely empirically well justified. In addition, we also know that anything we can derive from those first principles, such as Heisenberg's uncertainty principle or the spectrum of the hydrogen atom, is unquestionably true, since logic preserves truth.

So far, so good, but we have more questions for God to answer: Is QM complete, in the sense of implying yes or no to every QM question? If P is a consequence of QM, can we derive it in a feasible time? What is the relation of QM to other theories? Do chemistry and biology, for instance, reduce to QM or not? Other questions will readily come to mind.

We ask, but God won't answer. She smiles benignly then, alas, departs. (Athena frequently helped Odysseus out of a jam, then left him to fend for himself.) Suppose this is how things now stand with us. We know much with certainty, but we remain either ignorant or only mildly confident of much else in QM. How should we proceed?

Obviously, we should try to construct derivations for as many QM propositions as possible, that is, derivations from the known-to-be-certainly-true first principles. However, what about the rest? After some reflection we would probably continue as before. That is, we would continue with a combination of conjectures and experimental testing. Aside from the parts of QM that are clearly certain, it would

be business as usual. We would treat the rest of it as a typical empirical science to be investigated in typical empirical ways. We would continue to argue over what this involves, but details would be more or less the same. There will be experimental probing, hypothesis testing, the use of various statistical techniques, thought experiments, philosophical considerations, and so on.

We would continue to tackle many problems the way we do currently. For example, perhaps in a platonic sense there is a derivation of the details of protein folding from the first principles of QM, but no such derivation can be found by humans. Calculating the energy levels of highly complex objects is hopelessly difficult. U_{235} is a many-body problem that can't be exactly solved. Quantum field theory is a relativistic extension of QM, not derivable from it. What about dark energy? Is this even a QM problem? God is no help in answering these questions. We have to carry on as before using the same empirical techniques we have long used.

The upshot from all of this is that some physics is certain and some is not, and we will continue to learn about the latter in the same old empirical, fallible, inductive way. Why not demand certainty everywhere in QM? The argument for not doing this, if one is needed, is simple: Pre-God we have lots of justified but fallible beliefs involving QM. Then God tells us that part of this is not merely justified though fallible belief but is in fact certain knowledge. Great news. Do we abandon the remaining justified beliefs on the grounds that they are not certain? No, since their status as justified but fallible beliefs remains unchanged from what it was before God certified some of it. In that respect, nothing has changed. The fact that God guarantees some of it should not turn us into sceptics about the rest.

Of course, it is still debatable precisely what good scientific method is, but that is a detail that need not trouble us here. Most of QM remains fallible by anybody's lights and should be investigated empirically and inductively. We have certain knowledge of the first principles of QM and their deductive consequences. The rest of QM has the same inductive and empirical status as it had before God intervened. Does this have consequences for our knowledge claims elsewhere? Yes.

Let's turn to mathematics, where the common attitude is that much of it is certain knowledge (and we don't need God to tell us). I'll stick to an elementary part, basic arithmetic (numbery theory), which, for most of us, is probably as certain as anything could be. (If you like, you could assume that God tells us that the basic principles of arithmetic are certainly true.).

There is a common ideology that goes along with the general attitude about mathematics. Let's assume the Peano axioms (PA), which are a set of rules characterizing the natural numbers. PA says there is a number zero, and for each number there is a successor. Thus, one is the successor of zero; two is the successor of one, and so on. There are axioms for addition and multiplication, and for the principle of mathematical induction. These axioms are typically taken to be certainly true. Of course, there are people who claim to doubt them, but there are also people who claim to doubt the law of non-contradiction. In any case, PA is considered to be a safer bet than anything in the natural sciences (at least prior to God certifying QM).

A theorem may be asserted, according to the common ideology, if and only if there is a proof, and a proof is a derivation from the basic axioms. (In practice, as I mentioned above, a sketch of a derivation will suffice, but it is understood that full details could in principle be provided.) Nothing else should be believed, according to this ideology—a proof is the only evidence allowed.

All of this can be easily illustrated by a famous theorem, first proved in Euclid's *Elements*. The theorem follows from PA. *Prime numbers* are numbers that cannot be factored, that is, they cannot be divided by any number except one and themselves without remainder. The primes include: 2, 3, 5, 7, 11, 13, ... The rest are *composite numbers*, which are the product of primes. For instance, $4 = 2 \times 2$, $6 = 2 \times 3$, $8 = 2 \times 2 \times 2$, $9 = 3 \times 3$, $10 = 2 \times 5$, $12 = 2 \times 2 \times 3$, ..., $2093 = 7 \times 13 \times 23$, and so on. How many primes are there?

Theorem 1. *There are infinitely many prime numbers.*

Proof. Suppose there are only finitely many primes. Hence, there is a highest *p*. Let $q = (2 \times 3 \times 5 \times 7 \times \ldots \times p) + 1$. If *q* is a prime, then *p* is not the highest prime after all. If *q* is composite, then *q* is divisible by primes. But none of 2, 3, 5, . . . , *p* can divide *q*, since there is always a remainder of 1. Thus, some prime *r* must divide *q*. But *r* > *p*. Either way, *p* is not the highest prime. So, the initial assumption that there is a highest prime is false. Thus, there are infinitely many prime numbers. □

Now we have two interesting systems to think about, PA and QM. The first principles of PA and QM (post God) are both certain. They are analogous in that anything we can derive from either we can be sure is true. Additionally yet, we would treat them differently in a fundamental way. We would be happy to go beyond the certain first principles of QM and continue to use inductive methods to enlarge what we know about the physical realm. However, we have been reluctant to do the same with PA. Their epistemic situations are the same, so we should have the same epistemic outlook for each.

The parallel is obvious. In the quantum case (post God), we have three kinds of propositions: (1) QM principles and logical consequences that we can actually derive, (2) all other likely truths (or likely falsehoods) of quantum mechanics that we cannot either practically or in principle derive (or refute), but for which we have empirical support (or evidence to the contrary), and (3) propositions with no evidence either for or against them.

In the arithmetic case, we also have three kinds of propositions: (1) PA axioms and logical consequences we can derive from those axioms, (2) all other likely truths (or likely falsehoods) of arithmetic that we cannot either practically or in principle derive (or refute), but for which we have empirical support (or evidence to the contrary), and (3) propositions with no evidence either for or against them.

We treat QM (post God) and PA differently. How should we respond to this schizophrenic methodological attitude? Obviously we should follow the QM example and try to extend our mathematical knowledge by adding various inductive techniques to PA. This will have profound implications for mathematical practice, to which I will now turn. Things that are considered mere conjectures should be thought of as knowledge (although fallible, of course).

The twin primes conjecture will provide a good example of a more liberal way of proceeding. *Twin primes* are pairs of prime numbers of the form (*p*, *p* + 2). For instance, (3,5), (5,7), (11,13), (17,19), and so on. How many are there? This is an open problem in number theory in the sense that there is no standard proof that the number of twin primes is either infinite or finite. Number theorists have been attacking the problem for a long time without finding the answer.[1] It is possible, of course that the problem is unsolvable, in the sense that no proof exists either way. We know from Gödel's incompleteness theorem that such unsolvable problems exist. Euler, who is often quoted on this topic, wondered about the possibility. "Mathematicians have tried in vain to discover some order in the sequence of prime numbers but we have every reason to believe that there are some mysteries which the human mind will never penetrate". (1710, quoted in Simmons 1992, 276n3) Fortunately, we need not be that pessimistic.

Think of the gap between primes. For instance, the gap between five and the next prime seven is two; the gap between 11 and 13 is also two, while the gap between 13 and the next prime 17 is four, and so on. The apparent randomness of the primes will be reflected in the randomness of the size of the gaps. Since there are infinitely many primes, we can expect the number of gaps of size two to occur infinitely often. That means that primes of the form (*p*, *p* + 2) will occur infinitely often. In short, *the twin primes conjecture is true*. This is justified by rather simple but quite compelling inductive means.

The argument is easily generalized to prime pairs of the form (*p*, *p* + 4), (*p*, *p* + 6), and so on. There are infinitely many pairs of each of these, as well, since there will be infinitely many gaps of size four,

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[1] The literature on number theory, especially primes, is enormous. Extensive discussions can be found in (Hardy and Wright 2008; Ribenboim 1991; Shanks 1993).

size six, and so on. The moral to be drawn from this example is that inductive methods can provide legitimate evidence in mathematics more generally. The situation is as it would be in QM post God. (For more on this topic see Brown 2008, 2017.).

I want to stress that the foregoing argument significantly differs from other arguments for inductive methods in mathematics. Besides Gödel, who was quoted at the outset, lots of people (including me (Brown 2008, 2017)), have argued for similar conclusions. One of the simplest arguments for a more liberal methodology is the fact that the first principles cannot be proven (without begging the question), so it is in principle hopeless to demand that all our mathematical evidence be based on proofs. Another argument for mathematical fallibility is based on conceptual change. For instance, in the 18th century it was thought that all functions are continuous. The proof for this theorem was flawless. The concept of function, however, changed during the 19th century, so that now we take a function to be an arbitrary association between sets. This allows the radically discontinuous Dirichlet function $f(x)$, which equals one or zero, depending on whether x is rational or irrational.

The argument here is quite different than other urging more liberal standards of mathematical evidence in that it assumes that mathematics is in part certain. Specifically, the Peano axioms are taken to be as certain as anything. Additionally, so are its deductive consequences. The argument then follows the lesson of QM resulting from the God thought experiment, namely, that inductive methods should supplement the known-to-be-certainly-true first principles. This is why the God TE presented at the outset is important; it guarantees the analogy between mathematics and physics, which is the basis of the main argument.

God (in the form of an omniscient being) plays an essential role in this thought experiment. This is importantly different from the heuristic role that God plays in Leibniz's use of God to refute Newton's absolutism. The difference is simply this: An argument involving God *heuristically* can (in principle) be reconstructed without God. In the case of Leibniz, it can be turned into a nontheological argument involving symmetry. By contrast, an argument involving God *essentially* cannot be reconstructed without God. In our mathematical case, God (or some sort of supernatural entity that is omniscient) cannot be eliminated from the story that somehow gives us certain knowledge of a part of physics. God is *sine qua non*. I do not know whether anyone has attempted a taxonomy of thought experiment involving God, but the distinction between a *heuristic* and an *essential* use of God seems to me to be a potentially important one with interesting consequences for both epistemology and theology. I am sure that the notions of heuristic and essential will not exhaust the possibilities, but they do illustrate how powerful the very notion of a supernatural being can be to our better understanding of the natural world.

However, starting from an atheistic standpoint, as I do, I worry about the legitimacy of any thought experiment that makes fundamental use of a being who does not exist. In expressions such as "God does not play dice", the use of God involves no commitment to an actual supernatural being. It is a picturesque but non-literal way of expressing a belief in determinism in physics. Leibniz's use of God in thought experiments aimed at Newton's absolutism about space and time is much more serious. But ultimately it does not commit us to anything supernatural, since we can readily reconstruct Leibniz's argument using non-theological symmetry considerations. The trickiest case is the last one involving God's guarantee that the first principles of QM are true. Does embracing this thought experiment commit us to the possibility that such a being exists who could actually provide such a guarantee?

Of course, there is no God who guarantees the first principles of QM, and we cannot continue to take the QM first principles to be certain. The thought experiment has done its job and led us to a new way of viewing legitimate mathematical methods. Now we can treat the thought experiment like Wittgenstein's ladder. We climb up and see things in a new and better way. With our new perspective, we can toss the ladder away. We now see that even the first principles of QM and PA are fallible, as is all knowledge. The forfeiture of certainty in mathematics might seem like a genuine defeat, but the liberalization in what counts as evidence and the huge gain in real knowledge more than makes up for the loss of certainty.

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