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Mean Received Resources Meet Machine Learning Algorithms to Improve Link Prediction Methods

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Abstract: The analysis of social networks has attracted a lot of attention during the last two decades. These networks are dynamic: new links appear and disappear. Link prediction is the problem of inferring links that will appear in the future from the actual state of the network. We use information from nodes and edges and calculate the similarity between users. The more users are similar, the higher the probability of their connection in the future will be. The similarity metrics play an important role in the link prediction field. Due to their simplicity and flexibility, many authors have proposed several metrics such as Jaccard, AA, and Katz and evaluated them using the area under the curve (AUC). In this paper, we propose a new parameterized method to enhance the AUC value of the link prediction metrics by combining them with the mean received resources (MRRs). Experiments show that the proposed method improves the performance of the state-of-the-art metrics. Moreover, we used machine learning algorithms to classify links and confirm the efficiency of the proposed combination.

Keywords: social recommendation; link prediction; similarity measures; area under the curve; machine learning; regression models; network behaviors

Citation: Ayoub, J.; Lotfi, D.; Hammouch, A. Mean Received Resources Meet Machine Learning Algorithms to Improve Link Prediction Methods. *Information* **2022**, *13*, 35. https://doi.org/10.3390/info13010035

Academic Editors: Gabriele Gianini and Gennady Agre

Received: 11 November 2021
Accepted: 6 January 2022
Published: 13 January 2022

1. Introduction

Many real-world complex systems are represented by graphs, where nodes are entities such as universities, people or even proteins and edges represents the relations between the nodes. The most central property of these networks is their evolution, where edges are added and deleted over time. Link prediction (LP) is one of the most important tasks in social network analysis. It has many applications in various areas: spam E-mail [1]; disease prediction [2]; system recommendations [3]; and viral marketing [4]. The authors in [5] used a network-based technique to assess multimorbidity data and build algorithms for forecasting which diseases a patient is likely to develop in their research. A temporal bipartite network is used to describe multimorbidity data, with nodes representing patients and diseases, and a link between these nodes indicating that the patient has been diagnosed with the condition. The link prediction problem is defined as: let G(V,E) be an undirected graph, V is the set of nodes that refer to users of a social network (SN), E is the set of links or edges that represent relations between nodes. Given a snapshot of links at time t, could we infer the links that will be established at time $t + \Delta t$?

To solve this issue, authors have proposed various approaches. The authors in [6] have compared several similarity measures using AUC, and their results show that the Adamic Adar metric (AA) [7] performs best in four of the five datasets followed by Jaccard [8]; the difference between those two metrics is that (AA) [7] assumes that the rare common neighbors are heavily weighted; however, Jaccard [8] measures the probability that two nodes have a common neighbor. The authors in [9] have an proposed adaptive degree penalization metric (ADP). They proposed a generalized formula for the existing degree

penalization local link prediction method in order to provide the degree penalization level. Then, the penalization parameter for a network is predicted using logistic regression. In [10,11], we proposed a similarity metric based on the path depth from a source node to a destination node and their degrees. In addition, we found a strong correlation between the clustering coefficient and area under the curve. Machine learning algorithms were used to solve the link prediction problem where the enhancement was by 40% in the power grid dataset.

In [12], the authors solved the link prediction problem as a binary classification task, where they set the precision as an objective function, and then transform the link prediction problem into an optimization problem where the authors defined a feature set for each edge. This set contains state-of-the-art metrics, the used class label was $+1$ for existing links and -1 for non-existing links. In [13], the authors collected four different data, namely: node feature subset, topology features subset, social features subset (following or followed) and collaborative filtering for the voting feature subset, then they trained SVM, naive Bayes, random forest and logistic regression classifiers. In [14], the authors used the importance of neighborhood knowledge in link prediction that has been proven. As a result, they suggest extracting structural information from input samples using a neighborhood neural encoder.

In [15], the authors proposed a combination of the preferential attachment metric (PA) and (AA) to solve the link prediction problem using weights for each used metric and then obtained good accuracy for the GitHub dataset: $S_{comb}(x,y) = 0.7 * s_{PA}(x,y) + 0.3 * s_{AA}(x,y)$. The same idea was applied in [16] where the authors proposed common neighbor and centrality-based parameterized algorithm (CCPA): $S_{xy} = \alpha \times (\Gamma(x) \cap \Gamma(y)) + (1-\alpha) \times \frac{N}{d_{xy}}$, α is a parameter between $[0;1]$. It is used to control the weight of common neighbors and centrality, $\Gamma(x)$ represents the neighbors of node x, the fraction $\frac{N}{d_{xy}}$ represents the closeness centrality between the nodes x and y, where N is the number of nodes in the network, and d_{xy} is the shortest path between x and y.

As shown previously, recent decades have witnessed a tremendous growth in the amount of research seeking to provide precise predictions of links. Researchers have only focused on proposing new metrics and compared their results using area under the curve (AUC) against the state-of-the-art metrics. However, there is still a need for a unique method that enables the users of state-of-the-art metrics to improve their results in terms of accuracy. This paper proposes a solution to a link prediction (LP) problem based on the combination of state-of-the-art metrics and mean received resources (MRRs). This method is parameterized to grant full control to the user/system to give the importance to the link prediction metric or the MRRs. The main goal is to improve the area under the curve (AUC) of the state-of-the-art measures and any other local metric that could or will be proposed. We proved that the proposed combination has a meaningful effect on the results. Then, we used machine learning algorithms to classify the links. The results show the superiority of machine learning models whenever we add our proposed metric as an additional feature. Furthermore, we found that the decision tree performs best using the proposed metric.

To summarize, the principal contributions of this paper are:

1. We proposed a new parameterized link prediction metric that grants the user or system the full control of metric. Note that the proposed metric enhances the performance of the state-of-the-art metrics;
2. We compared the performance of the proposed metric against the state-of-the-art metrics using the AUC;
3. We studied the impact of using the parameter on each enhanced version of link prediction;
4. We studied the correlation between the parameter and the network features;
5. We used machine learning algorithms to confirm the efficiency of the proposed method.

This paper is organized as follows: in Section 2, we describe the state-of-the-art metrics and introduce the proposed metric. Section 3 presents the evaluation metric AUC and the datasets used to compare the proposed metric with the state-of-the-art metrics. In Section 4,

we report the results. In Section 5, we used machine learning algorithms to classify the links. We conclude our paper in Section 6.

2. Methods

In this section, we introduce the state-of-the-art metrics, particularly the local metrics, their merits and drawbacks. Then, we present the proposed metric which is based on the mean received resources and a local similarity metric.

2.1. Related Works

According to [17], the authors classified the link prediction approaches into three major categories. In the first approach, link prediction is solved using the dimensionality reduction, the second approach relies on probabilistic and maximum likelihood models (note that we cannot use the first and second category of approaches for large-scale networks because of their high computational cost), the last approach uses similarity-based methods, which is divided into three sub-categories, namely: global metrics, quasi-local metrics and local similarity metrics. The most used metrics are local metrics because of their reasonable computational coast and the high AUC results they provide; some of these metrics are designed for a specific domain (such as cosine similarity, which is used in information retrieval and text mining [18]).

In this work, we only focused on local metrics, also known as neighborhood-based metrics (see Table 1). Through this paper, we use $e_{x,y}$ to refer to the link between nodes x and y, $\Gamma(x)$ is the set of neighbors of x. $|\Gamma(x)|$ is the degree of node x (how many neighbors the node x has). ShortestPaths(x,y) is the set of all shortest paths between x and y.

Table 1. Neighborhood-based similarity metrics.

Metric Name	Equation	
Preferential attachment (PA) [19]	$PA(x,y) = \|\Gamma(x)\| \cdot \|\Gamma(y)\|$	(1)
Common neighbor (CN) [20]	$CN(x,y) = \|\Gamma(x) \cap \Gamma(y)\|$	(2)
Hub promoted (HP) [21]	$HP(x,y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{min\{\|\Gamma(x)\|, \|\Gamma(y)\|\}}$	(3)
Hub depressed (HD) [21]	$HD(x,y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{max\{\|\Gamma(x)\|, \|\Gamma(y)\|\}}$	(4)
Jaccard coefficient (JA) [8]	$JA(x,y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x) \cup \Gamma(y)\|}$	(5)
Leicht–Holme–Nerman (LHN) [22]	$LHN(x,y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x)\| \cdot \|\Gamma(y)\|}$	(6)
Parameter dependent (PD) [23]	$PD(x,y) = \frac{\Gamma(x) \cap \Gamma(y)}{(\Gamma(x) \cdot \Gamma(y))^{\delta}}$	(7)
Adamic/Adar (AA) [7]	$AA(x,y) = \sum_{z \in \|\Gamma(x) \cap \Gamma(y)\|} \frac{1}{log\|\Gamma(z)\|}$	(8)
Salton [24]	$Salton(x,y) = \frac{\|\Gamma(x) \cap \Gamma(y)\|}{\sqrt{\|\Gamma(x)\| \cdot \|\Gamma(y)\|}}$	(9)
Sorensen [25]	$Sorensen(x,y) = \frac{2.\|\Gamma(x) \cap \Gamma(y)\|}{\|\Gamma(x)\| + \|\Gamma(y)\|}$	(10)
Resource allocation (RA) [26]	$RA(x,y) = \sum_{z \in \Gamma(x) \cap \Gamma(y)} \frac{1}{\|\Gamma(z)\|}$	(11)

The neighborhood-based metrics presented in Table 1 rely on simple assumptions; for instance, the authors in [20] assumed that the more two nodes have common friends, the higher the probability is that they will be connected in the future. The authors in [19]

assumed that the more both nodes have friends, the higher the probability that they will be connected. This assumption follows the principle "The rich get richer" from the field of economics. In conclusion, we can notice that each author has a different point of view with respect to this problem, but the end point is to provide a metric that performs well in term of AUC. The motivation of this work was to propose a new approach that enables researchers to power up the AUC of the metrics. We propose a parameterized expression based on the mean received resources MRR and a local metric. Note that we can apply the proposed enhancement to all the existing local metrics by adjusting the combination parameter for any type of dataset.

2.2. The Proposed Metric

Let G (V, E) be a simple graph (no loop or multiple edges are allowed). Motivated by the RA [26] index, where the authors used the flow of resources transferred from a source node to a destination node through common neighbors; considering two non-connected nodes A and B, the node A can transfer some resources (we assume that A has only one resource to share) to target node B, that the common neighbors play the role of transmitters, the node A distributes the resource to all their neighbors, and every neighbor will do the same until the resource reaches the node B. We extended this metric to a global scale and considered the mean received resources from the source node through the shortest paths. Because neighborhood-based metrics could not capture global relations, we used MRR as the second criterion to enhance the precision of the link prediction metrics previously introduced in Table 1. We define the mean received resources as

$$MRR_{A,B} = \frac{1}{|Shortest\_Paths(A,B)|} \times \sum_{path \in Shortest\_Paths(A,B)} \left(\prod_{node \in path} \frac{1}{|\Gamma(node)|} \right) \quad (12)$$

The following example describes one limitation of the neighbor-based metrics (such as resource allocation) and shows the advantages of the mean received approach. Let G (V, E) be a graph, x and y are two unconnected nodes. On the first hand, if $|\Gamma(x) \cap \Gamma(y)| = 0$, then the resource allocation cannot capture the interaction between nodes x and y. On the other hand, the MRR will capture the interactions between x and y using the shortest paths. We clarify the problem with a simple graph of six nodes as shown in Figure 1. If we use the RA measure or any other neighbor-based metric presented in the previous section, then $s_{X,Y} = 0$ (see Figure 1). However, the use of the mean received resources provides the capture of the interactions between the two nodes. There are three shortest paths between the two nodes (x, y):

1. **The path through the nodes A and B:** $s_1 = \frac{1}{|X|} \times \frac{1}{|A|} \times \frac{1}{|B|} \times \frac{1}{|Y|} = \frac{1}{3} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{2}$
2. **The path through the nodes C and B:** $s_2 = \frac{1}{|X|} \times \frac{1}{|C|} \times \frac{1}{|B|} \times \frac{1}{|Y|} = \frac{1}{3} \times \frac{1}{8} \times \frac{1}{4} \times \frac{1}{2}$
3. **The path through the nodes C and D:** $s_3 = \frac{1}{|X|} \times \frac{1}{|C|} \times \frac{1}{|D|} \times \frac{1}{|Y|} = \frac{1}{3} \times \frac{1}{8} \times \frac{1}{3} \times \frac{1}{2}$

$MRR_{x,y} = \frac{1}{3} \times (s_1 + s_2 + s_3)$.

Each similarity measure captures different information data. The combination of these information data allows us to group them into a single equation and optimize the classification task. We power up each local measure (LM) presented in the previous subsection by combining them with Equation (12) using the weighted sum model (weighted combination [27]). We define the weighted combination as

$$PSI_{x,y} = \alpha \times sigmoid(LM(x,y)) + (1-\alpha) \times sigmoid(MRR(x,y)) \quad (13)$$

where α is a combination parameter, $\alpha \in [0;1]$. This parameter controls the contribution of each part of the equation; for some datasets, the MRR gives good results, but for others, the LM leads to higher prediction efficiency. Therefore, we used α to adjust the amount of contribution of each part. The MRR gives values in the range of $[0;1]$, however, other metrics such as CN provide values which are superior to 1. Therefore, we should normalize the local metric and MRR. To this end, we used the sigmoid function [28].

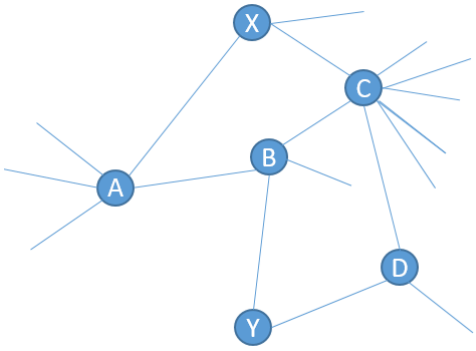


Figure 1. A simple undirected graph.

3. Evaluation

In this section, we introduce the methodology and the evaluation metric commonly used in the field. We define the datasets used to compare our metric against state-of-the-art metrics. We also describe the characteristics of each network.

3.1. Methodology

Each dataset is separated into two graphs: training G_T and probe G_P, which are distinct and non-overlapping. The training graph G_T is created by sampling the original graph G at random. G_P is formed by the remaining edges that are not included in G_T. Similarly, the set of edges in G_T refers to E_T, whereas those in G_P are referred to as E_P, i.e., $E = E_T + E_P$. It is essential to mention that E_T and E_P are mutually exclusive. However, the nodes in G_T and G_P may overlap. For our experiments, E_T represents 90% of the edges and E_P contains the remaining 10%. Because the graph G_T (and hence G_P) is generated at random, we repeat the trials 10 times to guarantee that the results were not acquired by coincidence. We generate G_T (and thus G_P) at random for each run. G_T was then used as an input of the algorithm, which produced the final graph G'. Then, we calculated AUC (see Section 3.2). The average values of the 10 runs were used to evaluate the proposed method. The value of the combination parameter was from the interval $[0, 1]$. We provide the average results for $\alpha = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

3.2. Evaluation Criterion

Let G(V,E) be a simple graph (loops and multi-edges are not allowed). We split our graph into $G_T(V, E_T)$ and $G_P(V, E_P)$. Note that the training set contains 90% edges and the test set contains 10% edges. $E = E_P \cup E_T$ and $E_P \cap E_T = O$. We used AUC to evaluate all metrics; AUC is defined as the probability that a link randomly chosen from E_P has a higher score than an edge randomly chosen from $\bar{E}$:

$$AUC = \frac{N' + 0.5 \times N''}{N} \tag{14}$$

N' is the number of times an edge from E_P and an edge from $\bar{E}$ have the same score. N'' is the number of times that the edges from E_P have a higher score than the edges from $\bar{E}$. N is the number of independent comparisons.

3.3. Datasets

Real-world datasets were used to evaluate the proposed and state-of-the-art algorithms. We selected eight popular real-world datasets to test the accuracy of our algorithm. Note that the closer the value of AUC is to 1, the better the metric will be. A brief description of each dataset is presented in Table 2:

- YeastS dataset [29,30] consists of a protein–protein interaction network being described and analyzed.

- Power Grid [31,32] is a network of the power grid for the western states of the United States of America, where edges represent a power supply line and nodes are either a generator, a transformator, or a substation.
- USAir [33] is the US air transportation network. The nodes represent airports, and links indicate routes.
- Florida [34]—in this network, the nodes are compartments and edges represent directed carbon exchange in the Florida bay.
- Football [35] represents the American football games between Division IA colleges during regular season in Fall 2000.
- Political network [31] is a directed network of hyperlinks between political blogs about politics in the United States of America. Note that we considered this network as undirected.
- Les Misérables [36] is an undirected network that contains co-occurrences of characters in Victor Hugo's novel 'Les Misérables'. The nodes represent a character and edges show that two characters appeared in the same chapter of the book.
- Zachary Karate Club [31] in this network, a node represents a member of the club (Zachary Club), and each edge represents a tie between two members of the club. The network is undirected.

Table 2. Network features.

Network	N	M	C	r	Average Degree	D	H	e
YeastS	2284	6646	0.134	−0.099	5819	4.37	2.84	0.233
Power Grid	4941	6594	0.08	0.003	2669	18,989	1.45	0.063
USAir	332	2126	0.625	−0.207	12,807	2738	3,463	0.406
Florida	128	2075	0.334	−0.111	32,421	1776	1237	0.622
Football	115	613	0.403	0.162	10.66	2508	1006	0.45
Political Network	105	440	0.481	−0.132	8.38	3092	1.41	0.396
Les Misérables	77	253	0.559	−0.163	6571	2651	1829	0.434
Zachary	34	77	0.485	−0.478	4529	2424	1668	0.489

Table 2 describes the characteristics of the networks, namely the number of nodes N, M is the number of edges, e the efficiency of the network [37], C and r are the clustering coefficient [32] and the assortative coefficient [38], respectively, (note that nodes with degree 1 are excluded from the calculation of the clustering coefficient), D is the diameter of the graph and H is the degree of heterogeneity [39].

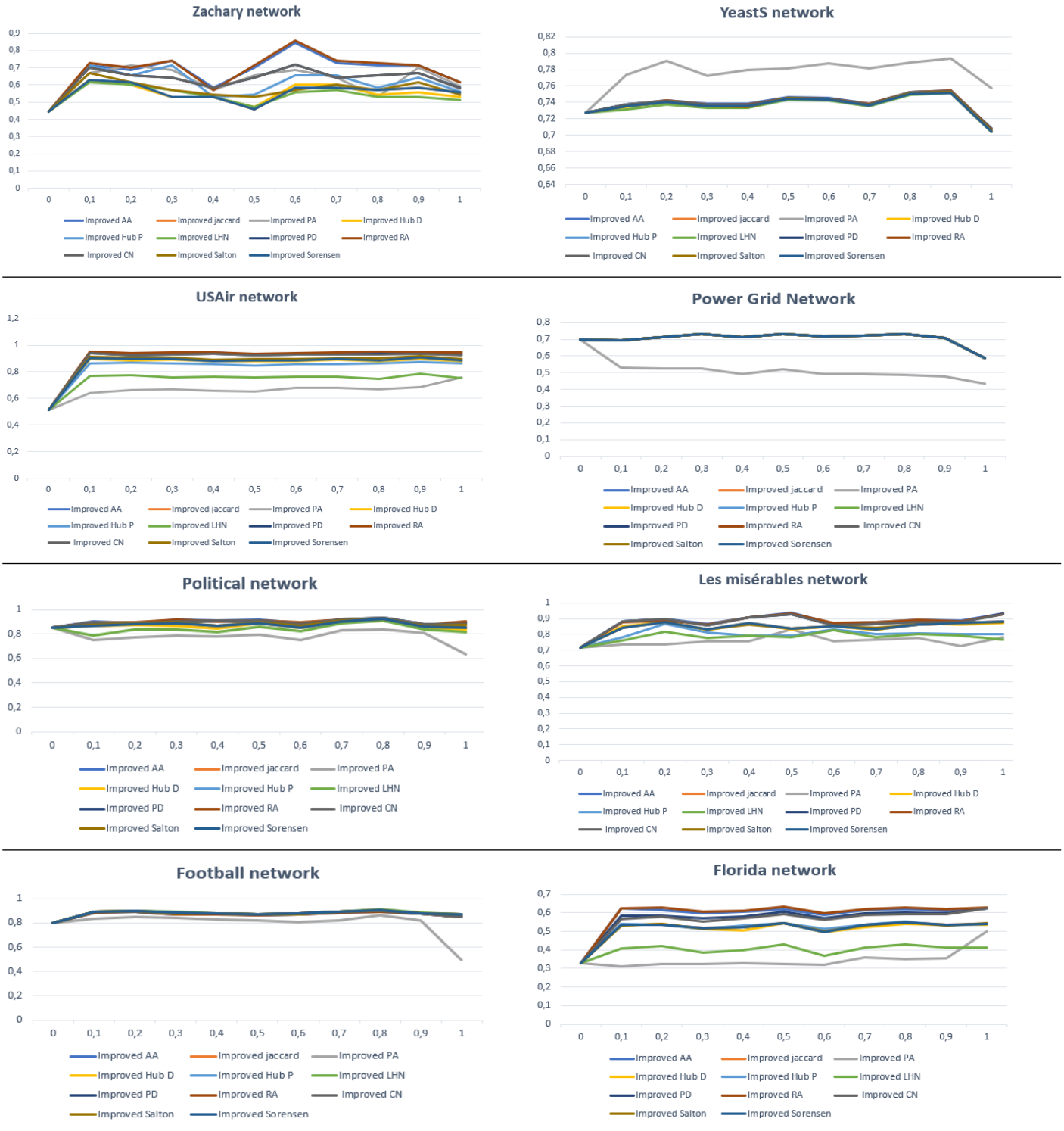
4. Results

In our study, we used 90% of the graph as a training set and 10% as a test set. We tested the different value of α from 0 to 1 with a step of 0.1 to show the impact of the contribution of MRR and LM on the AUC values.

In Table 3, we used Equation (13) with different values of α, from $\alpha = 0$ where the MRR provides the scores of links, to $\alpha = 1$ where the state-of-the-art metric defines the scores of links. The x axis defines the value of α, the y axis defines the AUC value. From Zachary's results, we can conclude that the best combination is between MRR and RA for a value of $\alpha = 0.6$. As we can notice, the curve is above all other combinations with a high AUC value. The results of YeastS highlight that MRR and AP combination outperforms all other combinations and gives good results (since the gap between the AP and MRR combination and other metrics curves is huge) for all values of α (the best $\alpha = 0.2$). From USAir results, we can notice that the curve of the combination between MRR and RA has a minor advantage over the curve of the combination of MRR and AA for $\alpha = 0.1$. Power grid results show that all combinations provide great results with no big difference except for CN when the accuracy decreases for $\alpha > 0$. At $\alpha = 0$, we obtain the best accuracy value. This proves that the only use of MRR can give promising results for some datasets. The results of political networks exhibit that both the CN and MRR combination and PD and MRR combination offer the best results in term of AUC, and the curves of other methods are very close except for the AP metric. The curve of Les Misérables

dataset shows that the PD metric performs very well in comparison with other metrics. Football curves make it clear that the majority of combinations have very close performance. Moreover, the test shows that the best combination for this dataset is the combination of LHN with MRR. From Florida dataset results, we can sum up that the combination of RA and MRR provides the best accuracy.

Table 3. x axis represents the values of α and the y axis represents the values of AUC using Equation (13).



We then compare our proposed metric Equation (13) against the state-of-the-art algorithms on eight datasets from different fields—using the maximum AUC found in Table 3.

From Table 4, we can draw the following conclusions: the improved version of the Jaccard metric offers a great enhancement in terms of accuracy. Furthermore, the average AUC value of the improved Jaccard is better than the average value of simple Jaccard by 6.7% in terms of AUC. The results of AA show that the improved version has refined the certainty of the algorithm for all datasets except for the Florida dataset. The improvement of the average value of AUC in all datasets was by 6.5% in the terms of AUC. We found that the improved version of AP amplifies the AUC results of the simple AP, for instance, the AUC of AP for Football dataset is 0.271 which is lower than pure chance, and the improved version reaches 0.864. For the Florida dataset, the overall improvement was by 6.18% on average, and the improved version of CN outperforms the simple version by 6.18%. The best improvement was 14% in the Power Grid dataset. The same conclusions can be drawn for the rest of algorithms, for the promoted hub the enhancement was 6.38%; for LHN, the enhancement was by 7.13%; RA was improved by 6.4%; and the Salton and Sorensen improvement was by 7.8% and 6.7%, respectively.

Table 4. AUC results of both simple metrics and improved metrics, cells in green represent the metrics having the highest AUC values for each dataset, and cells in red represent the metrics having the smallest AUC values for each dataset.

	Les Misérables	**Political Network**	**Football**	**USAir**	**Power Grid**	**Zachary**	**Florida**	**YeastS**
AA	0.895	0.902	0.819	0.941	0.59	0.643	0.625	0.715
Improved AA	0.938	0.925	0.89	0.947	0.733	0.843	0.624	0.754
jaccard	0.83	0.882	0.833	0.898	0.59	0.464	0.536	0.712
Improved jaccard	0.884	0.923	0.905	0.911	0.733	0.629	0.549	0.751
AP	0.74	0.695	0.271	0.87	0.446	0.721	0.743	0.771
Improved AP	0.834	0.852	0.864	0.756	0.7	0.714	0.501	0.794
Hub D	0.826	0.869	0.83	0.89	0.59	0.443	0.529	0.712
Improved Hub D	0.876	0.916	0.908	0.9	0.733	0.629	0.546	0.751
Hub P	0.814	0.886	0.831	0.874	0.582	0.564	0.542	0.713
Improved Hub P	0.868	0.916	0.905	0.876	0.733	0.714	0.553	0.751
LHN	0.787	0.848	0.832	0.778	0.584	0.45	0.4	0.711
Improved LHN	0.828	0.909	0.91	0.785	0.733	0.614	0.431	0.751
PD	0.878	0.89	0.832	0.929	0.585	0.536	0.608	0.714
Improved PD	0.934	0.932	0.902	0.942	0.733	0.721	0.622	0.753
RA	0.9	0.904	0.819	0.949	0.59	0.657	0.63	0.714
Improved RA	0.934	0.925	0.89	0.955	0.733	0.857	0.631	0.754
CN	0.882	0.893	0.82	0.928	0.59	0.593	0.621	0.714
Improved CN	0.93	0.932	0.902	0.941	0.733	0.721	0.622	0.753
Salton	0.834	0.886	0.833	0.905	0.584	0.479	0.539	0.712
Improved Salton	0.88	0.925	0.905	0.92	0.733	0.671	0.551	0.752
Sorensen	0.83	0.882	0.833	0.898	0.59	0.464	0.536	0.712
Improved Sorensen	0.884	0.923	0.905	0.911	0.733	0.629	0.549	0.751

In Figure 2, we calculate the mean of every row of Table 4 to obtain the average AUC on all datasets. This allows us to globally compare the performance of every algorithm on all datasets.

According to the results of Figure 2, we notice that for all algorithms, the improved version has a higher AUC average than the existing metrics. For the preferential attachment metric, the improved AP is superior by 6.8%. We can draw the same conclusion for the rest of metrics.

The experiment shows that the proposed metric outperforms the existing local metrics. Furthermore, it demonstrates that any local metric may be improved in terms of precision. As expected, our metric gives a higher score to links in the E_{test} against the links in $\bar{E}$. Then, the probability that a link exists in the graph G(V,E) is high compared to a link from $G(V, \bar{E})$.

For instance, in the Power Grid dataset, LM has $AUC = 0.59$ while the proposed weighted combination reached $AUC = 0.73$.

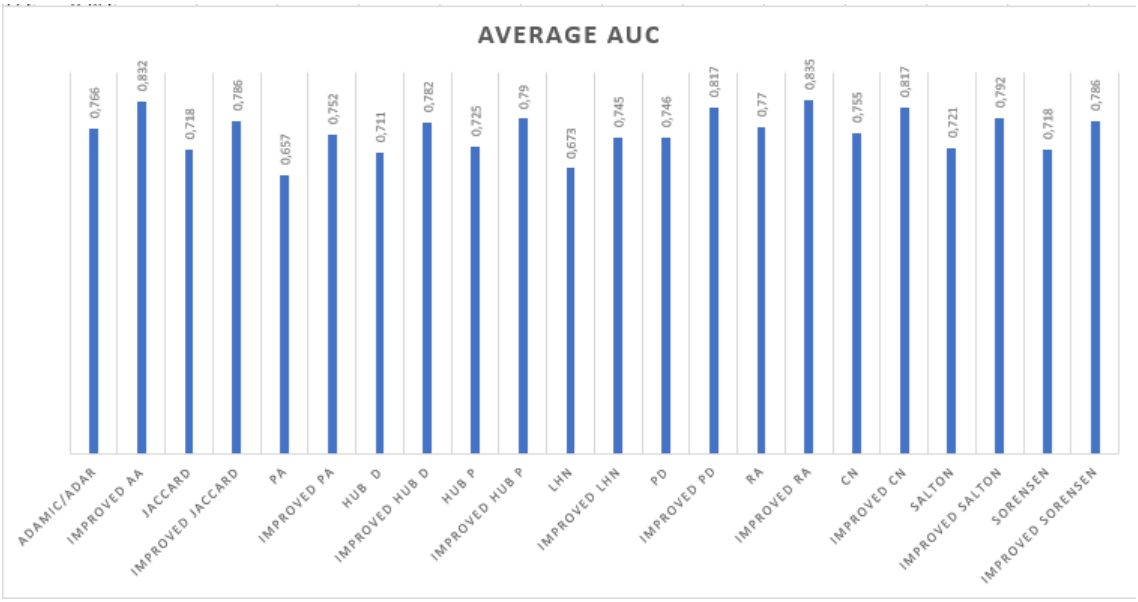


Figure 2. Average AUC of the algorithms.

From Figure 3, we can conclude that for the majority of datasets used to test the validity of our algorithm, the best $\alpha \in [0,5;1]$. Furthermore, we can notice that all the algorithms have the same *best* α for the same dataset, for instance, the *best* α is 0.9 for the YeastS network using all algorithms and 0.5 for all algorithms on Power Grid dataset. We then try to find a correlation between the *best* α and any network feature presented in Table 2.

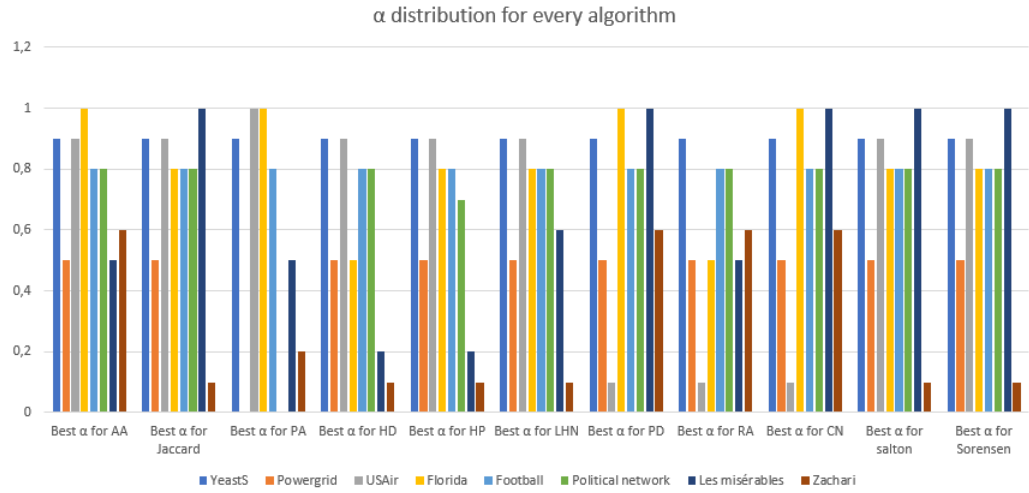


Figure 3. The α distribution on each dataset.

From Table 5 and using the rule of thumb, we can conclude that for AA and PA, we have a strong correlation between the *best alpha* and the average degree of the networks. For Jaccard, hub depressed, hub promoted, LHN and Sorensen, we have moderately strong correlation with r. For the metrics PD, CN and Salton, we have a moderately strong correlation with H. The RA index is the only metric to have a moderately strong correlation with C. Consequently, the parameter α can be written as a product of the network feature and a constant. For instance, $\alpha = average\_degree \times Constant$ for AA and PA.

Table 5. Correlation between the best α of each algorithm and the network features.

Correlation	Best α for AA	Best α for Jaccard	Best α for PA	Best α for HD	Best α for HP	Best α for LHN	Best α for PD	Best α for RA	Best α for CN	Best α for Salton	Best α for Sorensen
N	−0.347	−0.175	−0.328	0.097	0.052	−0.137	−0.183	0.113	−0.183	−0.183	−0.499
M	0.044	0.055	0.055	0.343	0.361	0.22	−0.12	0.1	−0.12	−0.12	0.055
C	0.034	0.143	0.123	−0.15	−0.209	−0.032	−0.229	−0.446	−0.229	−0.229	0.143
r	0.161	0.521	0.197	0.541	0.555	0.6	0.219	0.291	0.219	0.219	0.521
Average Degree	0.695	0.285	0.594	0.098	0.426	0.394	0.251	−0.237	0.251	0.251	0.285
D	−0.511	−0.277	−0.509	−0.048	−0.107	−0.218	−0.277	−0.078	−0.277	−0.277	−0.277
H	0.267	0.295	0.431	0.389	0.313	0.325	−0.547	−0.439	−0.547	−0.547	0.295
e	0.42	0.062	0.41	−0.234	−0.047	0.008	0.289	−0.124	0.289	0.289	0.062

5. Link Prediction Using Machine Learning Algorithms

In this section, we apply supervised learning algorithms to study the link prediction problem as a classification problem. We use random forest [40], k-nearest neighbors [41], support vector machine (SVM) [42], artificial neural network [43], and logistic regression [44]. Then, we compare the different supervised learning algorithms using the accuracy to evaluate their performance.

5.1. Methodology

To evaluate the performance of the proposed metric when modeling the link prediction as a classification task, we use the classification accuracy:

$$Accuracy = \frac{Number\ of\ correct\ predictions}{Total\ number\ of\ predictions\ made} \tag{15}$$

Let G(V, E) be an undirected and unweighted graph. In order to transform the link prediction problem into a binary task, we construct two sub-sets:

The first one contains the edges of E, the second one contains randomly chosen edges from $\bar{E}$. Then, we attribute a null value to the edges from $\bar{E}$ and 1 to those of E. We split them into test set and training set where the $test\ size = 10\%$. Finally, we train our classifier on the training set and then predict the test set. We use the Sklearn framework [45] to apply machine learning algorithms. Note that we use the best α of the improved RA for each dataset (see Figure 3).

5.2. Results

Table 6 shows the results of the KNN algorithm in two cases. The curve in blue represents the first case when we use only the state-of-the-art algorithms. The curve in orange represents the case in which we use the state-of-the-art algorithms along with the proposed metric. We can notice that the orange curve always has the highest accuracy, and we can conclude that the classification task becomes accurate when we add the PSI (Equation (13)) as an additional feature.

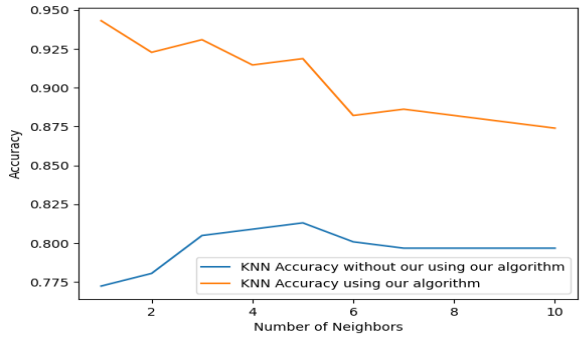
Table 7 shows the results of logistic regression. We obtain a higher orange curve when we apply PSI metric (Equation (13)) with the state-of-the-art algorithms as additional features of logistic regression. The curve in blue represents the case in which we only use the state-of-the-art algorithms. Note that for the Political Network, USAir and Zachary datasets, the logistic regression did not converge, and therefore we did not see a clear advantage.

Table 8 shows the results of the random forest algorithm in two cases. We can notice that the orange curve has greater accuracy compared to the blue curve. Note that the curve in blue represents the first case in which we only use state-of-the-art algorithms. The curve in orange represents the case in which we use the state-of-the-art algorithms along with the proposed metric.

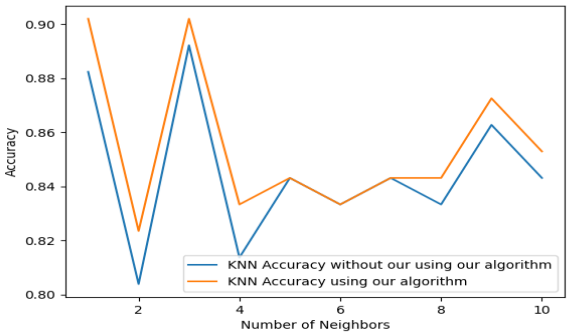
Table 9 shows the importance of each feature used in the random forest algorithm (see Table 8). We can confirm that the proposed metric (Equation (13)) contributes more than the other metrics to the decision process.

Table 6. The x axis represents the number of neighbors in the KNN model and the y axis is the accuracy.

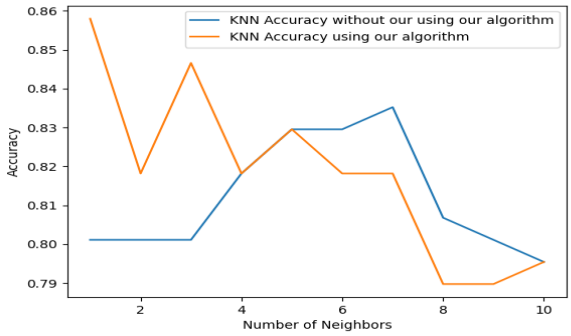
Football dataset



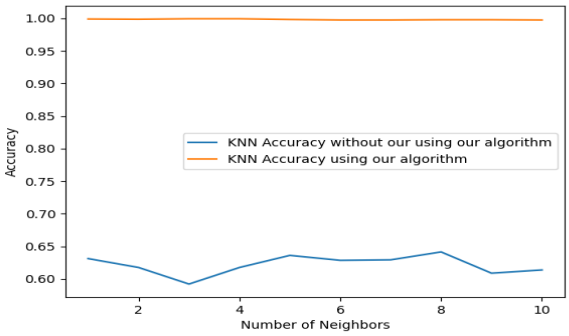
Les Misérables dataset



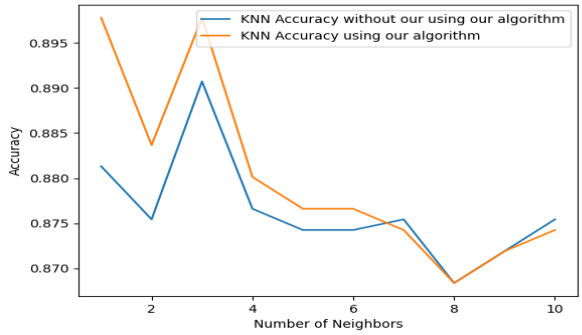
Political Network dataset



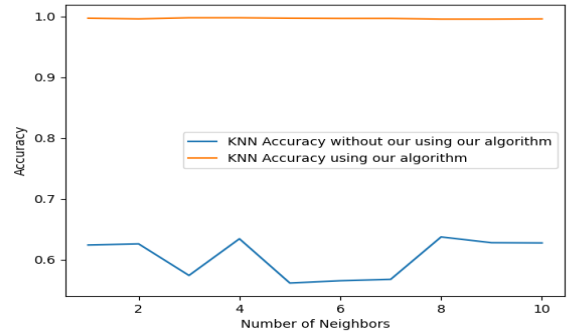
Power Grid dataset



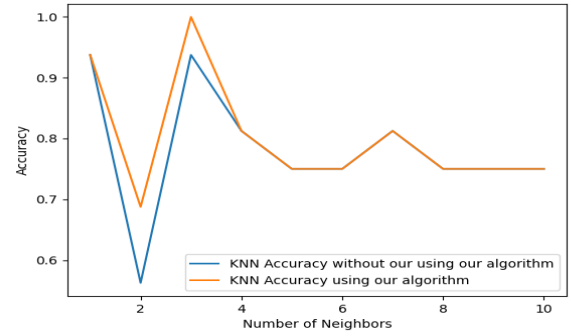
USAir dataset



YeastS dataset



Zachary dataset



Florida dataset

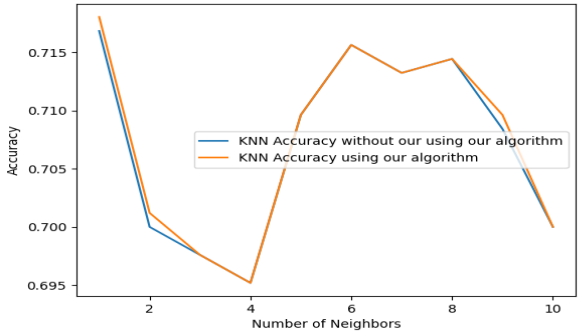


Table 7. The x axis represents the inverse of regularization strength. The y axis is the accuracy of the logistic regression model.

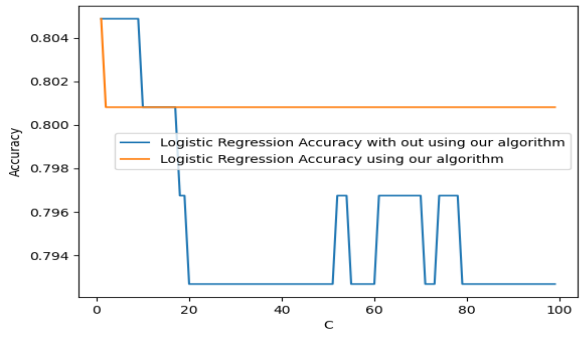
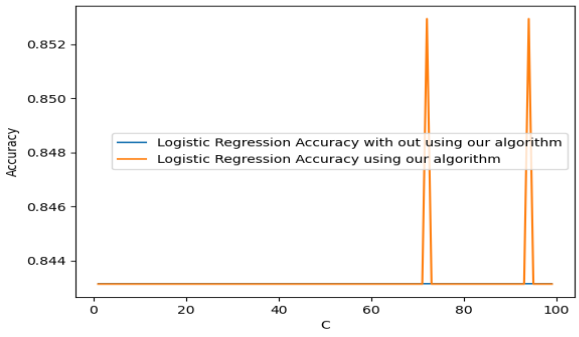
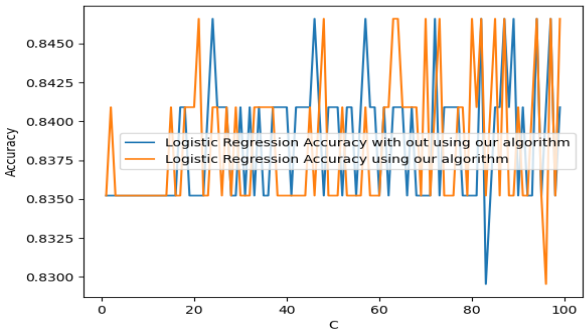
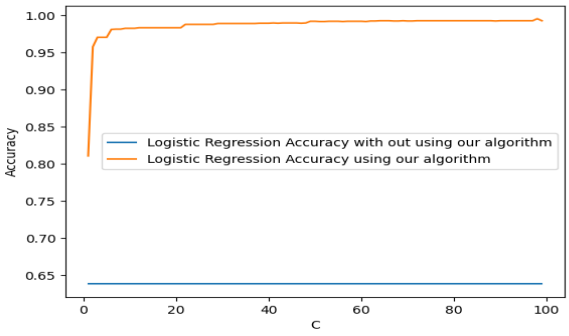
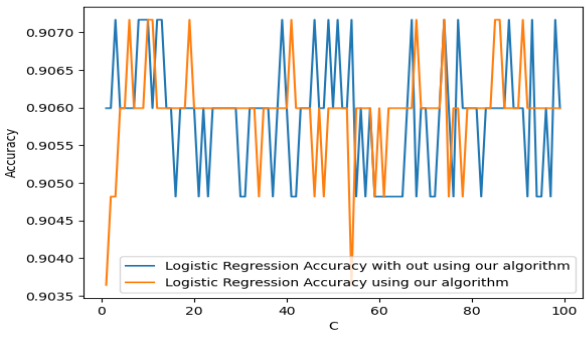
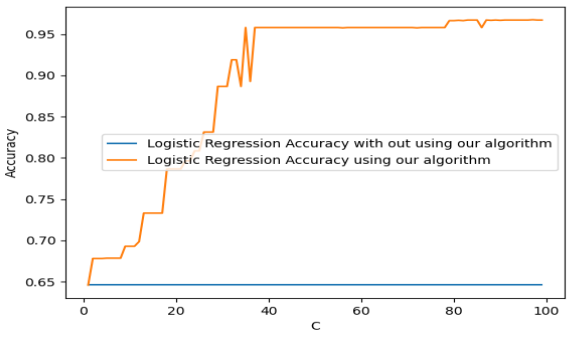
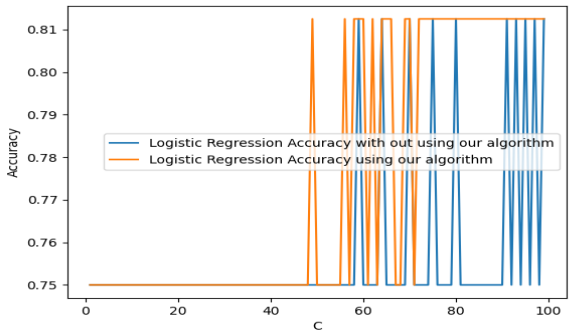
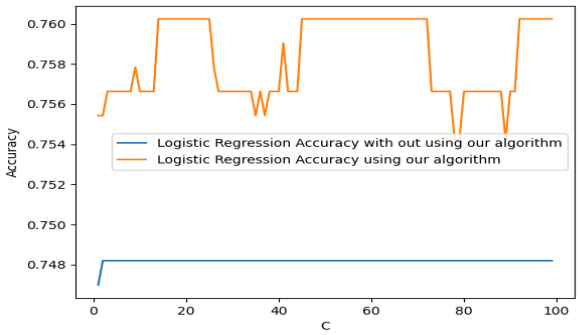
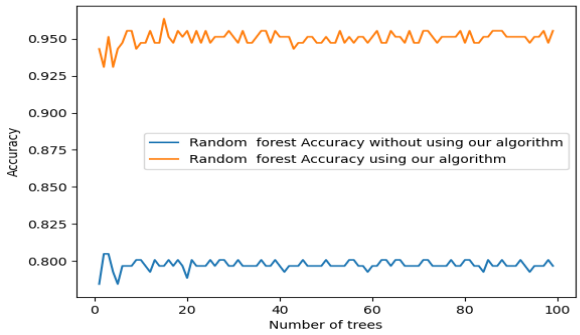
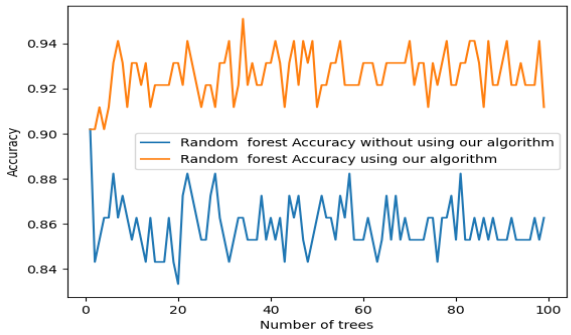
Football dataset	Les Misérables dataset
	
Political Network dataset	Power Grid dataset
	
USAir dataset	YeastS dataset
	
Zachary dataset	Florida dataset
	

Table 8. The x axis represents the number of trees. The y axis represents the accuracy of the random forest model.

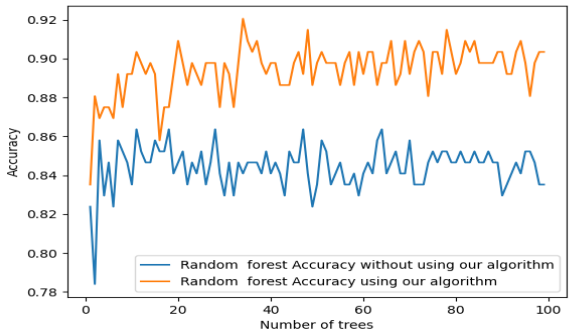
Football dataset



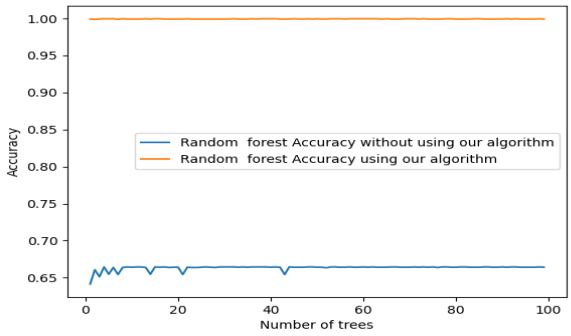
Les Misérables dataset



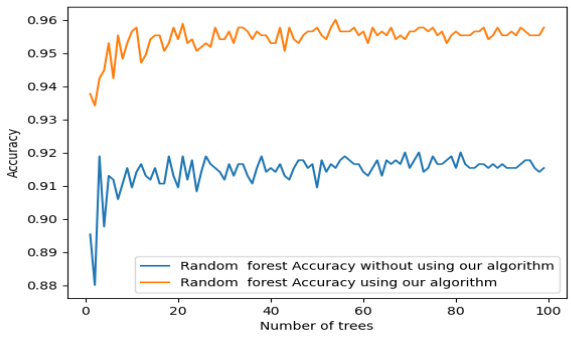
Political Network dataset



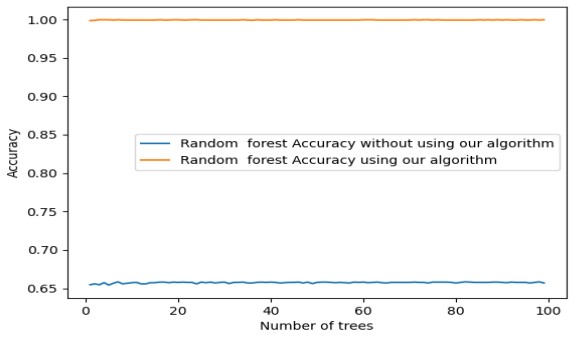
Power Grid dataset



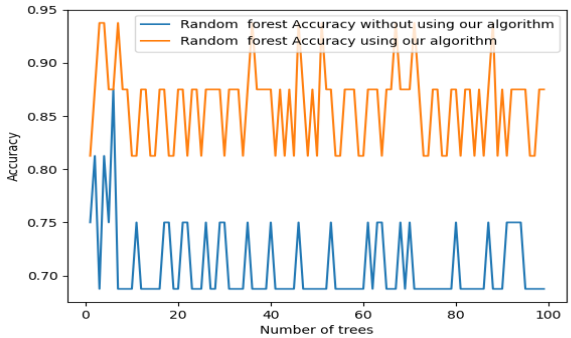
USAir dataset



YeastS dataset



Zachary dataset



Florida dataset

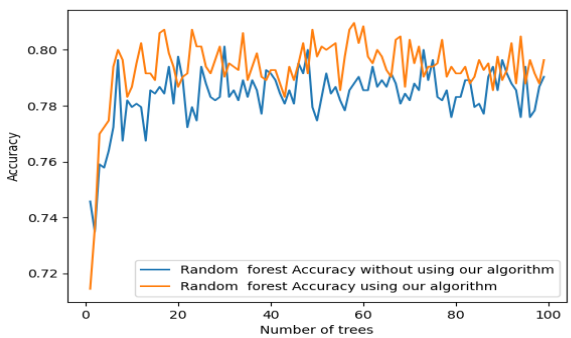
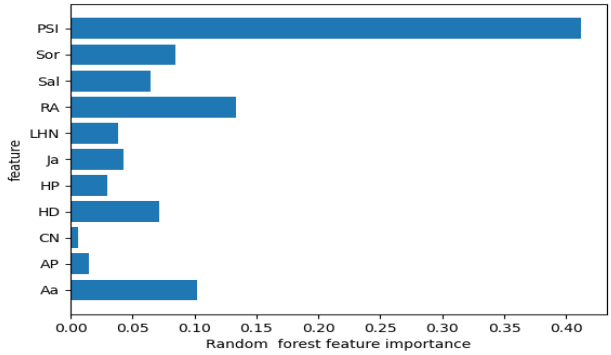
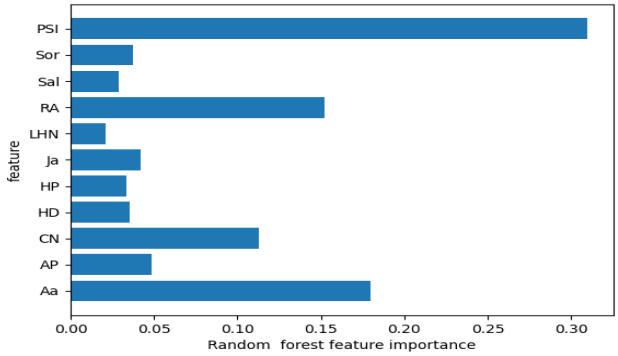


Table 9. Random forest feature importances.

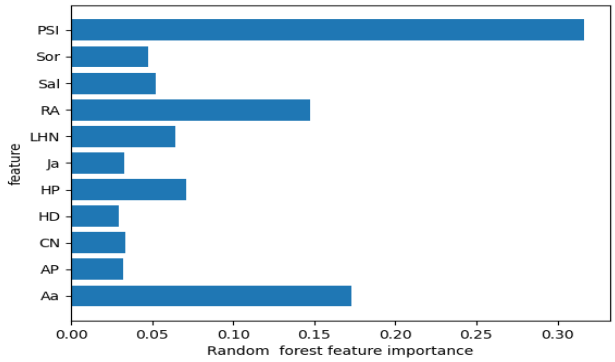
Football dataset



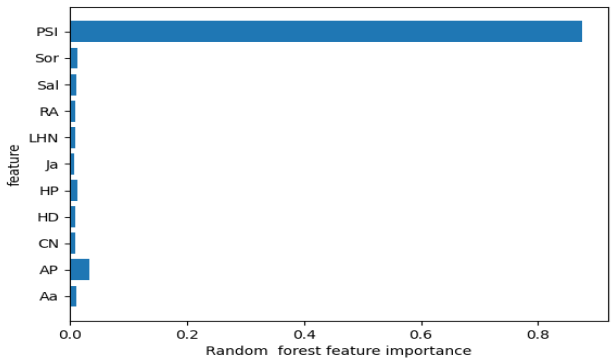
Les Misérables dataset



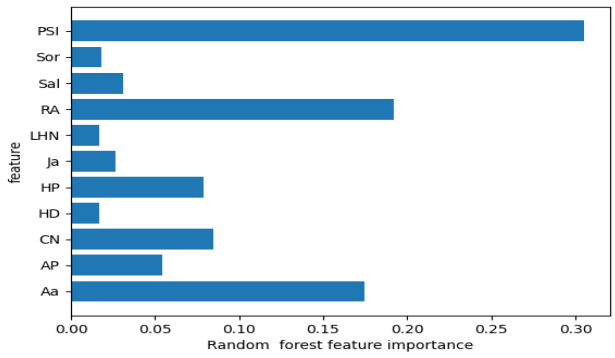
Political Network dataset



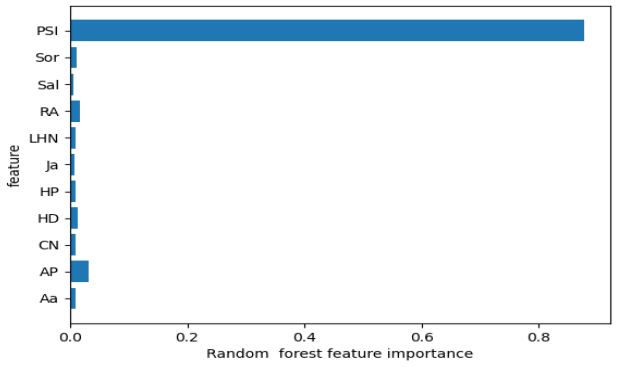
Power Grid dataset



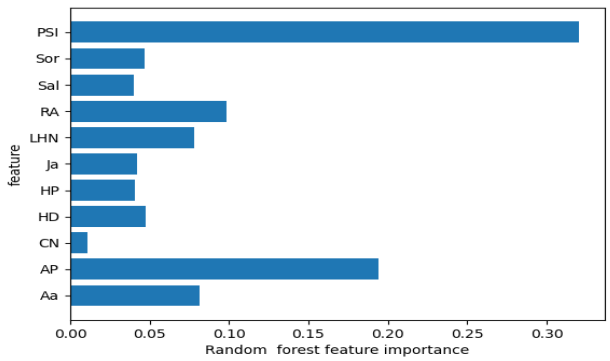
USAir dataset



YeastS dataset



Zachary dataset



Florida dataset

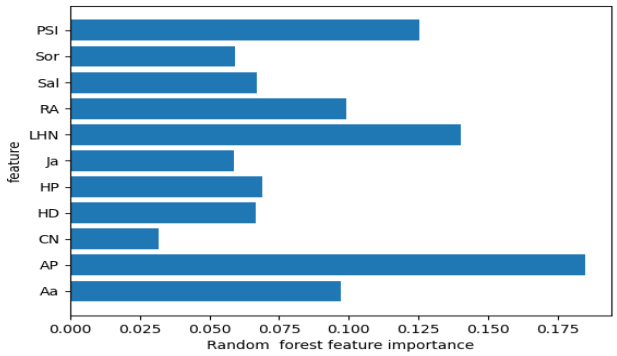


Table 10 represents the weights of every hidden unit. The blue area represents large positive values, while the white area represents negative values. For every dataset, if the color of cells are darker, this means that the used parameter is important to the process. We notice that, for most of the datasets, our proposed metric has the darkest row. Thus, it plays a role in the decision-making process.

Table 10. Results of the neural network.

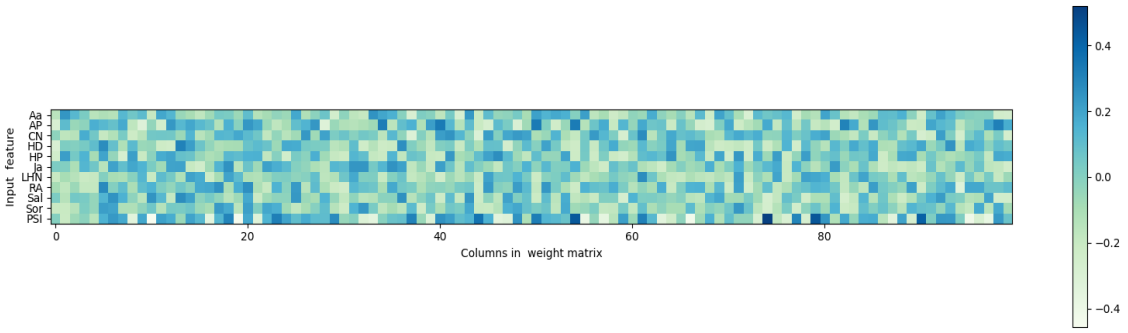
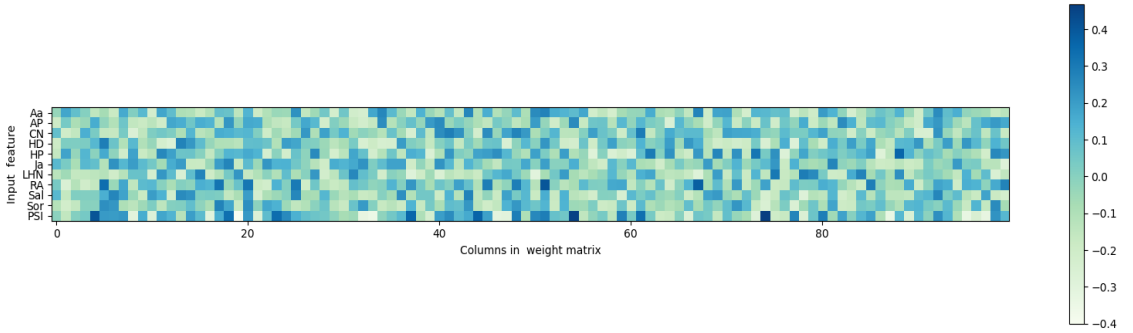
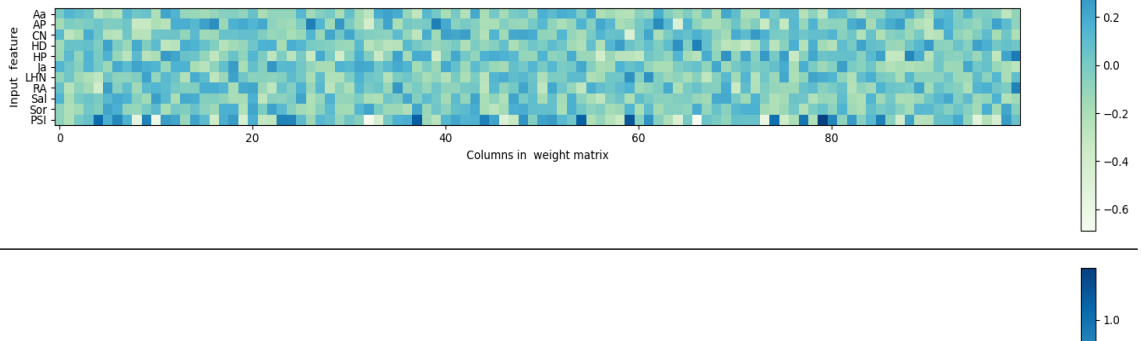
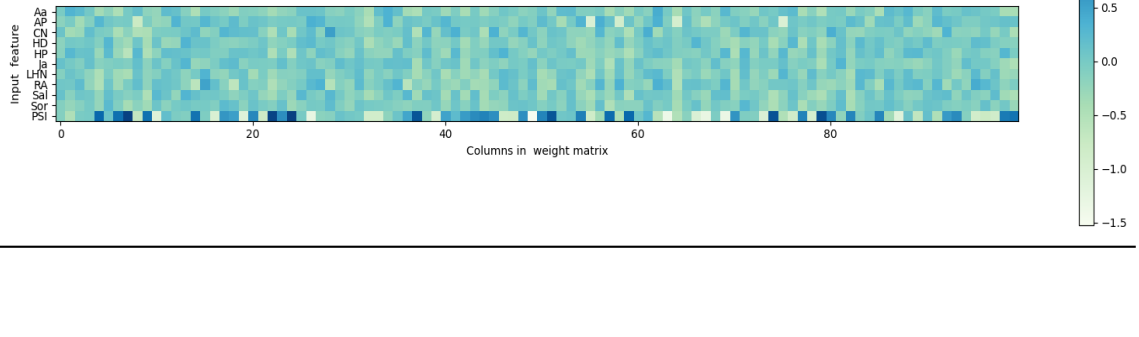
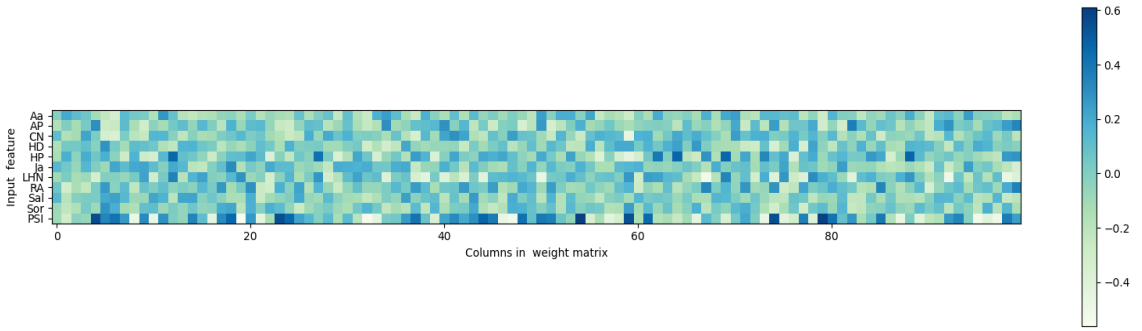
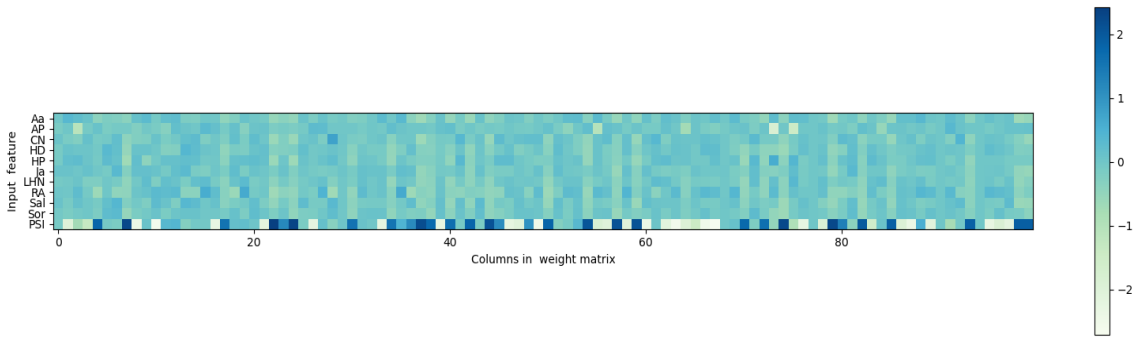
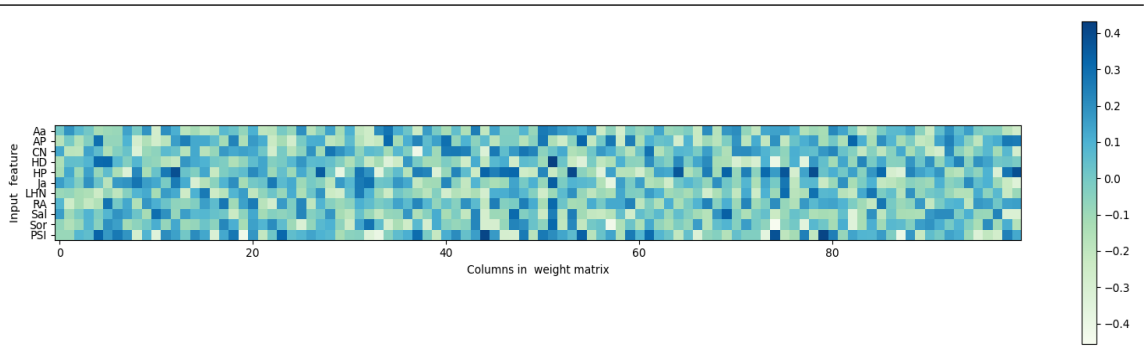
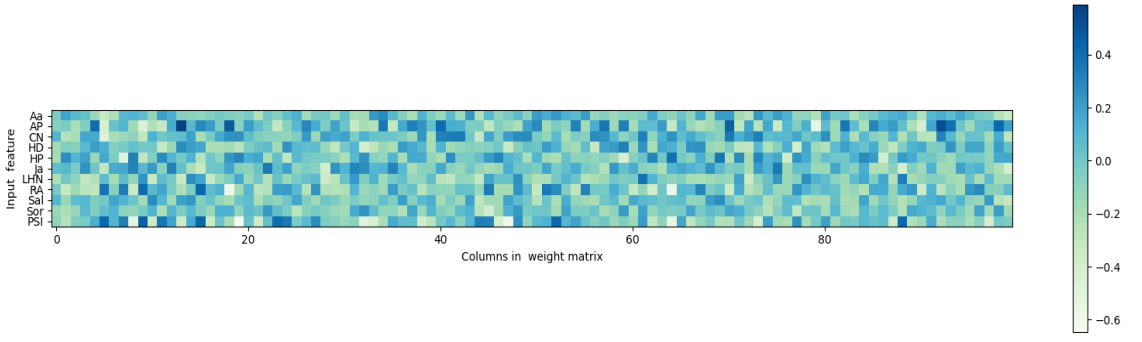
Football dataset	
Les Misérables dataset	
Political Network dataset	
Power Grid dataset	

Table 10. *Cont.*

USAir dataset	
YeastS dataset	
Zachary dataset	
Florida dataset	

From the Table 11, we can conclude that PSI (see Equation (13)) enhances the accuracy of KNN, especially for the Football, Power Grid and YeastS datasets. For the logistic regression algorithm, the best performance was for the Power Grid, Football, YeastS and Florida datasets. For the random forest model, we can notice that PSI enhanced the performance in all datasets; in addition, it further contributed to the decision process. We can draw the same conclusion for the neural network. Overall, decision tree has the best accuracy on all datasets. The results show that for all datasets, the performance increased when we used the PSI metric as an additional feature.

Table 11. Results of support vector machine (SVM), neural network and decision tree.

	Les Misérables	Political Network	Football	USAir	Power Grid	Zachary	Florida	YeastS
SVM with PSI	0.902	0.858	0.756	0.942	0.995	0.677	0.763	0.808
SVM without PSI	0.892	0.852	0.756	0.892	0.638	0.677	0.760	0.801
neural network with PSI	0.922	0.862	0.768	0.968	0.999	0.774	0.778	0.834
neural network without PSI	0.912	0.852	0.760	0.913	0.643	0.710	0.761	0.819
decision tree with PSI	0.971	0.875	0.959	0.969	0.998	0.903	0.740	0.985
decision tree without PSI	0.873	0.841	0.760	0.885	0.641	0.742	0.761	0.806

6. Conclusions

This paper presents a new parameterized metric for link prediction in social networks. We based the proposed metric on both mean received resources (MRRs) and a state-of-the-art local similarity measure (LM). The proposed metric is parameterized, and as a result, the system/user can adjust the importance of the factor under consideration. We tested the performance of the proposed metric using the AUC value on eight datasets from different fields, and we compared its results with 11 existing metrics.

The finding of this study shows that the proposed metric has a very high performance over the local similarity metrics in all datasets. It captures the interactions between unconnected nodes, even if they do not have common neighbors. Furthermore, we found a correlation between the parameter α and some networks' features. In addition that, we used machine learning algorithms to classify links. The results show that whenever we use the proposed metric as an additional parameter, the accuracy of any algorithm increases; also, we concluded that the decision tree algorithm has the best performance in terms of accuracy.

This study can be extended to various networks such as directed networks, weighted networks. In addition, because most real-world networks are highly sparse, with a small number of positive cases relative to negative examples, dealing with imbalanced datasets in link prediction may be a real challenge.

Author Contributions: Conceptualization, J.A.; Supervision, D.L. and A.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

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