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Large-Scale Group Decision-Making Method Using Hesitant Fuzzy Rule-Based Network for Asset Allocation

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Abstract: Large-scale group decision-making (LSGDM) has become common in the new era of technology development involving a large number of experts. Recently, in the use of social network analysis (SNA), the community detection method has been highlighted by researchers as a useful method in handling the complexity of LSGDM. However, it is still challenging to deal with the reliability and hesitancy of information as well as the interpretability of the method. For this reason, we introduce a new approach of a Z-hesitant fuzzy network with the community detection method being put into practice for stock selection. The proposed approach was subsequently compared to an established approach in order to evaluate its applicability and efficacy.

Keywords: alternatives selection; large scale; networked rule-based; fuzzy sets; Z-numbers; community detection method; explainable artificial intelligence

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1. Introduction

The trust value of decisions made decreases when many people are involved in the decision-making process, thus impacting the presentation of information negatively [1]. Notably, social network analysis (SNA) can establish the significance passed from one person to another, as well as other linkages between them, by showing the links and relationships established through individual interactions [2] in any substantial portion of the population ranging from a group of people to a significant segment of a nation. The community detection method, which is one of the approaches of SNA, can reduce the dimension of large-scale group decision-making to a convenient and understandable calculation in order to obtain the weights of individual decision makers (DMs) and partitions based on the centrality indexes that can reflect the importance of DMs. The formulation's extension of hesitant fuzzy sets can be used to determine the allocation or standards of options involved with the decision-making challenges faced [3]. A hesitant fuzzy set is subsumed in the formulation, as it helps with the insufficiency of available information. Evidently, hesitant fuzzy sets in decision-making allow decision makers to incorporate several possible values into an evaluation, which facilitates the proficiency of decision-making [4]. The hesitant fuzzy set can also effectively handle the haziness of a decision maker's judgments over alternatives in terms of attributes. A generalization of the fuzzy set, which is known as a hesitant fuzzy set, allows the membership degree of an element to be expressed as a number of potential values between 0 and 1 [5]. Recently, the use of hesitant fuzzy sets has been successfully applied in the fields of renewable energy [6], safety and healthcare, and the food industry [7]. It is best described in situations where people are reluctant to render their choices over artifacts in the decision-making process.

Z-numbers denote an approach used in the fuzzy theory, which provides data on people through the dependability of decision-making and is useful for providing ambiguous evidence. According to Kang, Wei, Li, and Deng, 2012 [5], Zadeh established the Z-number as a fuzzy set that can capture the dependability of decisions made through the confidence level of decision makers. The Z-number contains two parts in an ordered pair of fuzzy numbers. The first element constitutes the ratings, whereas the second element entails the decision makers' level of confidence in the ratings. Z-numbers are used to execute decision makers' evaluations, as the decision makers will decide on their level of confidence in the ratings made, thereby making the decision more reliable.

The benefits of fuzzy systems in terms of explainability, interpretability, and transparency have been emphasized in recent years. By depicting the arrangement of measures as a node and the network as links, Gegov introduced the fuzzy network, which is a networked rule-based system that contains the interior structure of the demonstrating network [6]. A fuzzy network also has the ability to be direct and precise, which is highly important when trying to make a better judgment. Although fuzzy networks have the same capabilities as other types of fuzzy systems with rule bases, they are also acknowledged as a novel approach in fuzzy systems owing to the straightforwardness and precision of the application's structure. These two characteristics are crucial for better decision-making and are often emphasized [7]. In order to illuminate the transparency that has received less attention in the paradigm of complex systems, fuzzy networks are used. Adel, Teh, and Raja [8] claimed that a system that has coherently applied transparency denotes a model that can accurately represent the relationship between the input and output applied. In order to accommodate more information in the decision-making process, the suggested approach incorporates social network analysis with a fuzzy network approach to highlight transparency.

According to Bonchi, Castillo, Gionis, and Jaimes [9], social network analysis is a significant and vital means of network analysis. Using the theoretical instrument of social network analysis, Perez, Mata, and Chiclana in 2014 [10] carried out a study to look at the links that bind people, groups, organizations, or communities. Moreover, in order to accommodate the idea of connected relationships among a group of people, social network analysis is helpful. Additionally, it is believed that people are linked to and interconnected with one another globally. As asserted by Canright and Engo-Monsen [11], social network analysis plays a crucial role in revealing the links and alliances formed by individuals and further confirming the context in which each individual interacts with others. Furthermore, the interactions occurring between individuals generate value for the information received; thus, they are useful for decision makers in making decisions.

2. Z-Hesitant Fuzzy Network with Social Network Analysis (Z-HFN SNA)

In this section, Z-HFN SNA is demonstrated in steps. Firstly, the community detection method (CDM) is carried out in Steps 1–3 and Steps 4–8 implying the TOPSIS method for computing the closeness coefficient (CC). Meanwhile, Steps 9–14 are the steps involved in the fuzzy network approach. The formulation steps are as follows:

Step 1: Using Pajek software, distinguish the network structure linked by experts to find the degree centralities, $C_D(d_\rho)$ of experts, ρ. Normalize the degree centralities, $(C'_D(d_\rho))$, and the expertise levels of experts as evaluated by the experts themselves.

$$C'_D(d_\rho) = \frac{C_D(d_\rho)}{\sum_{\rho=1}^{t} C_D(d_\rho)} \tag{1}$$

Step 2: Detect the partitions among the large-scale decision makers. Calculate the fusion of degree centrality, $C_F(d_\rho)$, and the expertise level in the fusion centrality, $C'_E(d_\rho)$, using the following normalized degree centralities:

$$C_F(d_\rho) = \varphi(C'_D(d_\rho)) + (1-\varphi)(C'_E(d_\rho)) \quad (2)$$

where φ is defined as the importance of the degree of the relation between two centralities that are set by the experts between 0 and 1.

Step 3: Compute the partitions weights, (ω_ρ). Calculate the weight of each node that represents the experts' and the partitions' weights according to the partitions grouped using Pajek in the community detection method. The value of the weight of node, $v_\rho \ (v_\rho \in C_t, 1 \le \rho \le r)$, is calculated as follows:

$$\omega_\rho = \frac{C_F(d_\rho)}{\sum_{\rho=1}^{s} C_F(d_\rho)} \quad (3)$$

where there are s nodes being clustered into ρ groups under the community detection method as r nodes are acquired in the community, $C_t(1 \le t \le \rho)$. The distance between a group and the entire network indicates the weight of the group, with a group's closeness to the entire network reflecting its higher weight. The mean fusion centrality for all DMs is accommodated at the center of the entire network, whereas the weight of the group, determined as the sum of the fusion centralities of its members, is similar to the center of the group. The fusion centrality, (C_F), of the whole network is calculated as follows:

$$C_F = \frac{\sum_{\rho \in N} C_F(d_\rho)}{N} \quad (4)$$

where N is defined as the total number of nodes as a whole in the network, $N = \{v_1,\ v_2,\ v_3,\ \ldots\ v_N\}$. Subsequently, calculate the fusion centrality of each group:

$$C_F^t = \frac{\sum_{\rho \in Z_t} C_F(Z_\rho)}{Z_t} \quad (5)$$

where Z_t is defined as the number of nodes in the tth group, and C_F and Z_ρ stand for nodes in Z_ρ in the group, C_F. Using the fusion centrality of the whole network, C_F, and fusion centrality of each group, C_F^t, calculate the weight of each group, ω_t, in the Formula (6). The relationship between a group's weight and its distance from the entire network can be shown by measuring the distance of the fusion centrality between each group and the network as a whole.

$$\omega'_t = \frac{\omega_t}{\sum_{\rho=1}^{t} \omega_t} \quad (6)$$

$$\sigma_k = \frac{\theta_k}{\sum_{i=1}^{k} \theta_k} \text{For } k = 1,\ 2,\ \ldots\ K \quad (7)$$

$$\omega_t = \frac{1}{|C_F^t - C_F|} \quad (8)$$

Step 4: Construct the decision matrices. Specifically, use the information in Table 1 to translate the ratings of alternatives into fuzzy numbers and build decision matrices. The implementation of Z-numbers in the fuzzy network approach typically requires additional reliability in the decisions made according to the alternatives delivered by decision makers in reference to each criterion. Thus, decision makers are advised to apply the linguistic terms that represent reliability, as shown in Table 2, in order to signify their confidence in the decisions made on the alternatives.

Table 1. Linguistic terms for the ratings of alternatives.

Linguistic Term	Fuzzy Number
Very poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium poor (MP)	(1, 3, 5)
Fair (F)	(3, 5, 7)
Medium good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very good (VG)	(9, 10, 10)

Table 2. Linguistic terms for the reliability of each criterion.

Linguistic Term	Rank	Fuzzy Number
Strongly unlikely	1	(0.00, 0.00, 0.10)
Unlikely	2	(0.00, 0.10, 0.25)
Somewhat unlikely	3	(0.10, 0.25, 0.40)
Neutral	4	(0.25, 0.40, 0.55)
Somewhat likely	5	(0.40, 0.55, 0.70)
Likely	6	(0.55, 0.70, 0.85)
Strongly likely	7	(0.70, 0.85, 1.00)

The hesitant fuzzy set is incorporated into the decision matrix, which can be expressed as follows:

$$H_{ij,k} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \tag{9}$$

where H_{ik} is the decision matrix in accordance with i alternatives, the jth attribute, and k decision makers. Hesitant fuzzy elements, h_{mn}, stand for possible decisions on the alternatives of the decision matrix delivered by the decision makers. Specifically, a hesitant fuzzy set of the ith alternative, A_i, on x_k is given by

$$A_i = \{< x_j, h_{A_i}(x_j) > | x_j \in X\}, \tag{10}$$

where $h_{A_i}(x_j) = \{\gamma | \gamma \in h_{A_i}(x_j), 0 \leq \gamma \leq 1\}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$

$h_{A_i}(x_j)$ is the possible membership degrees of the ith alternative, A_i, under the jth attribute, x_j, and it can be interpreted as HFE, h_{ij}. In simple words, decision makers are allowed to contribute several opinions to an alternative, according to an attribute.

Step 5: Assign the weight and normalize the decision matrices. Incorporate the weight into decision matrices accordingly by multiplying the importance of criteria to each decision matrix, $h_{A_i}(x_j)$, and normalize the membership function by dividing each value into the maximum values. The same weighting and normalization processes are applied for all sub-criteria.

$$\text{Normalized } h_{A_i}(x_j) = \frac{h_{A_i}(x_j)}{\max(h_{A_1}(x_1), h_{A_1}(x_2), \ldots, h_{A_m}(x_n))} \tag{11}$$

where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

Step 6: Retrieve the positive ideal solution (PIS) and negative ideal solution (NIS) for each alternative.

$$A^+ = \{\mathrm{x}_j, \max\langle h_{A_i}(x_j) \rangle, \mathrm{i} = 1, 2, \ldots, m\} \tag{12}$$

$$= \left\{ \begin{array}{c} \left\langle x_1, \left(\left(h^1_{A_1}\right)^+, \left(h^2_{A_1}\right)^+, \ldots, \left(h^l_{A_1}\right)^+ \right) \right\rangle \\ \times \left\langle x_2, \left(\left(h^1_{A_2}\right)^+, \left(h^2_{A_2}\right)^+, \ldots, \left(h^l_{A_2}\right)^+ \right) \right\rangle \\ \times \ldots \left\langle x_n, \left(\left(h^1_{A_m}\right)^+, \left(h^2_{A_m}\right)^+, \ldots, \left(h^l_{A_m}\right)^+ \right) \right\rangle \end{array} \right\} \tag{13}$$

$$A^- = \left\{ \mathrm{x}_j, \min\left\langle h_{A_i}(x_j) \right\rangle, \mathrm{i} = 1, 2, \ldots m \right\} \tag{14}$$

$$A^- = \left\{ \begin{array}{c} \left\langle x_1, \left(\left(h^1_{A_1}\right)^-, \left(h^2_{A_1}\right)^-, \ldots, \left(h^l_{A_1}\right)^- \right) \right\rangle \\ \times \left\langle x_2, \left(\left(h^1_{A_2}\right)^-, \left(h^2_{A_2}\right)^-, \ldots, \left(h^l_{A_2}\right)^- \right) \right\rangle \\ \times \ldots \left\langle x_n, \left(\left(h^1_{A_m}\right)^-, \left(h^2_{A_m}\right)^-, \ldots, \left(h^l_{A_m}\right)^- \right) \right\rangle \end{array} \right\} \tag{15}$$

Step 7: Determine the distance, d_i, of each alternative from the PIS and NIS using the hesitant fuzzy Euclidean distance.

$$d_i^+ = \sum_{j=1}^{m} d\left(h_{A_i}(x_j), h(x_j)^+\right) w_j \tag{16}$$

$$= \sum_{i=1}^{n} w_j \sqrt{\frac{1}{l} \sum_{\lambda=1}^{l} \left| h_{A_i}(x_j) - \left(h(x_j)^{\sigma(\lambda)} \right)^+ \right|^2} \tag{17}$$

$$d_i^- = \sum_{j=1}^{m} d\left(h_{A_i}(x_j), h(x_j)^-\right) w_j \tag{18}$$

$$= \sum_{i=1}^{n} w_j \sqrt{\frac{1}{l} \sum_{\lambda=1}^{l} \left| h_{A_i}(x_j) - \left(h(x_j)^{\sigma(\lambda)} \right)^- \right|^2} \tag{19}$$

$$i = 1, 2, \ldots n$$

Step 8: Compute each alternative's relative closeness coefficient ($\mathrm{CC_i}$).

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \tag{20}$$

θ resembles the influence degree of the kth partition. By allocating the normalized influence degree to each correlation coefficient of the alternatives, $ICC_{i,k}$, in line with the category of criteria, the procedure is carried out

$$ICC_{i,k} = \theta_k \times CC_{i,k} \tag{21}$$

according to $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, K$.

Next, $ICC_{i,k}$ is normalized as shown in the following equation in order to ensure that the values between 0 and 1 are achieved:

$$NICC_{i,k} = {}^{ICC_{i,k}}/_{\max_j ICC_{i,k}} \tag{22}$$

According to $j = 1,\ 2, \ldots,\ m$ and $k = 1,\ 2, \ldots,\ K$

Subsequently, the level of alternative performance is calculated by translating the normalized influenced closeness coefficient into linguistic terms.

Step 9: Based on DM opinions and NICC coefficient values, construct the antecedent and consequent matrices for the category systems. We can determine the antecedent matrix of each category, D and F, for each partition, k, the opinions of all DMs, and A_i for each possibility with regard to each criterion, as shown in Equations (21) and (22):

$$D_k = \begin{bmatrix} d_{11,k} & d_{12,k} & \cdots & d_{1m,k} \\ d_{21,k} & d_{22,k} & \cdots & d_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ d_{e1,k} & d_{e2,k} & \cdots & d_{em,k} \end{bmatrix} \tag{23}$$

$$F_k = \begin{bmatrix} f_{11,k} & f_{12,k} & \cdots & f_{1m,k} \\ f_{21,k} & f_{22,k} & \cdots & f_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ f_{e1,k} & f_{e2,k} & \cdots & f_{em,k} \end{bmatrix} \tag{24}$$

$$\text{for } k = 1, 2, \ldots, K$$

where $d_{em,k}$ and $f_{em,k}$ are the linguistic terms representing the opinions of decision makers for category D and F. The consequent matrices are defined as in Equations (19) and (20).

$$\vartheta_k = DL[\vartheta_{1,k}\ \vartheta_{2,k} \cdots \vartheta_{m,k}] \tag{25}$$

$$\tau_k = FL[\tau_{1,k}\ \tau_{2,k} \cdots \tau_{m,k}] \tag{26}$$

$$\text{as } k = 1, 2, \ldots, K.$$

where ϑ_k and τ_k are linguistic terms that represent the output of the category systems based respectively on the values of $NICC_{i,k}$. The D subsystem consists of K decision matrix rules presented in the rule base in Equation (21).

$$\text{If } D_k = \begin{bmatrix} d_{11,k} & d_{12,k} & \cdots & d_{1m,k} \\ d_{21,k} & d_{22,k} & \cdots & d_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ d_{e1,k} & d_{e2,k} & \cdots & d_{em,k} \end{bmatrix}, \text{ then } \vartheta_k = [\vartheta_{1,k}\ \vartheta_{2,k} \cdots \vartheta_{m,k}] \tag{27}$$

$$\text{Formatrices } k = 1,\ 2,\ \ldots,\ K.$$

The NICC equation indicates the difference between each alternative amid the fuzzy positive initial solution (FPIS) that represents the compromise solution and FNIS with the closest consensus solution value of 1, as the FNIS represents the worst possible solution. In other words, NICC values nearer to 1 result in the most exclusive coefficients among the alternatives. The scalar is then interpreted into linguistic terms under the value with the biggest membership degree, and can best be described in if-then rules:

$$rule\ 1:\ if\ D_1\ is\ d_{11,k} \cdots\ and\ D_e\ is\ d_{e1,k},\ then\ DL_1\ is\ \vartheta_{1,k}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$rule\ m:\ if\ D_1\ is\ d_{1m,k} \cdots\ and\ D_e\ is\ d_{em,k},\ then\ DL_m\ is\ \vartheta_{m,k}$$

where DL is the D level of alternatives for $i = 1, 2, \ldots m$ and $k = 1, 2, \ldots K$. The same application of rule bases is applied to the F subsystem, which consists of the following K decision matrix rules:

$$\text{If } F_k = \begin{bmatrix} f_{11,k} & f_{12,k} & \cdots & f_{1m,k} \\ f_{21,k} & f_{22,k} & \cdots & f_{2m,k} \\ \vdots & \vdots & \ddots & \vdots \\ f_{e1,k} & f_{e2,k} & \cdots & f_{em,k} \end{bmatrix}, \text{ then } \tau_k = [\tau_{1,k}\ \tau_{2,k} \cdots \tau_{m,k}] \quad (28)$$

$$\text{Formatrices } k = 1,\ 2, \ldots,\ K.$$

The NICC is then interpreted into linguistic terms under the value that has the biggest membership degree, and is best described in if-then rules as follows:

$$rule\ 1:\ if\ F_1\ is\ f_{11,k} \cdots\ and\ F_e\ is\ f_{e1,k},\ then\ FL_1\ is\ \tau_{1,k}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$rule\ m:\ if\ F_1\ is\ f_{1m,k} \cdots\ and\ F\ is\ f_{em,k},\ then\ FL_m\ is\ \tau_{m,k}$$

Step 10: Construct the antecedent matrices and consequent matrices for the alternative system (AS). The AS antecedent matrices are based on the category levels, DL, $\vartheta_{m,k}$, and FL, $\tau_{m,k}$, which represent the outputs of the category systems. Every ordered list of inputs corresponds to the computed degrees of identical alternatives using n different types of criteria. Therefore, the AS antecedent matrices, G, are of size $n \times m{\cdot}m$. For example, under the same matrices and rule bases of two inputs, DL and FL, the antecedent matrices, G_k, in the size of $2 \times m$ are as follows:

$$G_k = \begin{matrix} DL \\ FL \end{matrix} \begin{bmatrix} \vartheta_{1,k} & \vartheta_{2,k} & \cdots & \vartheta_{m,k} \\ \tau_{1,k} & \tau_{2,k} & \cdots & \tau_{m,k} \end{bmatrix} \quad (29)$$

$$\text{For } k = 1, 2, \ldots, K$$

Step 11: Derive the consequent matrices of the alternative system.

i. Calculate the average of weighted NICCs according to each category. The aggregation of weighted NICCs, $\delta_{i,k}$, is divided by the number of main categories, n, to reflect the equivalent value of each of the two subsystems in a weighted mean:

$$\delta_{i,k} = \frac{NICC_{i,k}^{D} \times \left(\frac{e}{e+f}\right) + NICC_{i,k}^{F} \times \left(\frac{f}{e+f}\right)}{n} \quad (30)$$

$$\text{for } i = 1, 2, \ldots,\ n \text{ and } k = 1, 2, \ldots,\ K.$$

ii. Subsequently, normalization is required to ensure that the NICC, $\delta\ _{i,k}$ is stated between 0 and 1 by adapting the following formula:

$$N\delta\ _{i,k} = \frac{\delta\ _{i,k}}{\max\limits_{i} \delta\ _{i,k}} \quad (31)$$

$$\text{for } i = 1, 2, \ldots, n \text{ and } k = 1, 2, \ldots, K.$$

iii. Translate the normalized affected closeness coefficient into linguistic terms to create the ensuing matrices for the alternative system (AS). Next, the K for AS consequent matrices, in this case of size $1 \times m$ instead of $1 \times m{\cdot}m$, is as follows:

$$N\delta_k = AL[N\delta_{\ 1,k} N\delta_{\ 2,k} \cdots \ N\delta_{\ m,k}] \tag{32}$$

$$\text{for } k = 1{,}2, \ldots, K$$

iv. K matrices represent decision rules that are elaborated on in terms of the alternative system in which AL stands for alternative level.

$$If\ G_k = \begin{matrix} DL \\ FL \end{matrix} \begin{bmatrix} \vartheta_{1,k} & \vartheta_{2,k} & \cdots & \vartheta_{m,k} \\ \tau_{1,k} & \tau_{2,k} & \cdots & \tau_{m,k} \end{bmatrix},\ then\ N\delta_{\ k} = AL[N\delta_{\ 1,k} N\delta_{\ 2,k} \cdots N\delta_{\ m,k}] \tag{33}$$

$$\text{for } k = 1, 2,\ \ldots,\ K$$

This is best described with if-then rules, as follows:

$$rule\ 1\text{: } if\ DL\ is\ \vartheta_{1,k}\ and\ FL\ is\ \tau_{1,k},\ then\ AL\ is\ N\delta_{1,k}.$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$rule\ m\text{: } if\ DL\ is\ \vartheta_{m,k}\ and\ FL\ is\ \tau_{m,k},\ then\ AL\ is\ N\delta_{m,k}.$$

$$\text{for } k = 1{,}2, \ldots, K.$$

Step 12: Build the generalized Boolean matrix of the overall system.

i. The rules developed based on the category systems, DS, FS, and AS, are used by converting the rules into decision matrices based on the evaluation of K decision makers in order to produce a generalized Boolean matrix for each alternative:

$$\begin{matrix} & & & \Theta_{1,k} & \cdots & \vartheta_{m,k} \\ d_{11,k} & \cdots & d_{1m,k} & 1 & \cdots & 0 \\ & \ddots & & \vdots & \ddots & \vdots \\ d_{e1,k} & \cdots & d_{em,k} & 0 & \cdots & 1 \end{matrix} \tag{34}$$

The Boolean matrix's row and column labels represent every potential permutation for the output's DS rule base of linguistic terms.

$$\begin{matrix} & & \tau_{1,k} & \cdots & \tau_{m,k} \\ f_{11,k} & f_{1m,k} & 1 & \cdots & 0 \\ & & \vdots & \ddots & \vdots \\ f_{e1,k} & f_{em,k} & 0 & \cdots & 1 \end{matrix} \tag{35}$$

where the FS rule base's linguistic phrases for the output are all feasible permutations for the row and column labels of the Boolean matrix.

ii. Conduct vertical merging between the generalized Boolean matrices of DS and FS in order to form a generalization of the Boolean matrix:

$$\begin{array}{ccc|ccc} & & & \vartheta_{1,k} & \cdots & \vartheta_{m,k} \\ & & & \tau_{1,k} & \cdots & \tau_{m,k} \\ d_{11,k} & \cdots & d_{1m,k} & & & \\ f_{11,k} & \cdots & f_{1m,k} & 1 & \cdots & 0 \\ & \vdots & & \vdots & \ddots & \vdots \\ d_{e1,k} & \cdots & d_{em,k} & 0 & \cdots & 1 \\ f_{e1,k} & \cdots & f_{em,k} & & & \end{array} \tag{36}$$

iii. Construct the generalized Boolean matrix *AS* with regard to the decision makers' evaluations:

$$\begin{array}{c|ccc} & N\delta_{1,k} & \cdots & N\delta_{m,k} \\ \vartheta_{1,k}\tau_{1,k} & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \vartheta_{m,k}\tau_{m,k} & 0 & \cdots & 1 \end{array} \tag{37}$$

The overall system is interpreted in the generalized Boolean matrix of *m* alternatives according to the evaluation of *K* decision makers, as follows:

$$\begin{array}{ccc|ccc} & & & N\delta_{1,k} & \cdots & N\delta_{m,k} \\ d_{11,k} & \cdots & d_{1m,k} & & & \\ f_{11,k} & \cdots & f_{1m,k} & 1 & \cdots & 0 \\ & \vdots & & \vdots & \ddots & \vdots \\ d_{e1,k} & \cdots & d_{em,k} & 0 & \cdots & 1 \\ f_{e1,k} & \cdots & f_{em,k} & & & \end{array} \tag{38}$$

Step 13: Set up the rules for the alternatives based on the system's generalized Boolean matrix.

rule 1: *if DL is* $\vartheta_{1,k}$ *and* … *and DL is* $\vartheta_{1,k}$, *and FL is* $\tau_{1,k}$, … *and FL is* $\tau_{1,k}$, *then AL is* $N\delta_{\ 1,k.}$

$$\vdots \quad \vdots \quad \vdots$$

rule m: *if DL is* $\vartheta_{1,k}$ *and* … *and DL is* $\vartheta_{1,k}$, *and FL is* $\tau_{1,k}$, … *and FL is* $\tau_{1,k}$, *then AL is* $N\delta_{\ 1,k.}$

Step 14: Derive the final score for each alternative.

Multiply the influence multiplier with the average aggregate membership value of the consequent part of the previous n_j rules to obtain the final score, φ_i, for each alternative, *j*.

$$\varphi_i = \frac{\sum_{Rule\ 1}^{n}\sum_{k=1}^{K} N\delta_{i,K}\cdot\left(NICC_{i,k}^{D} + NICC_{i,k}^{F}\right)}{n\cdot K} \tag{39}$$

$$\text{For } i = 1, \ldots m$$

Finally, the final scores, φ_i, of the alternatives are arranged in descending order to acknowledge the ranking of the alternatives. Better alternatives can be acknowledged to score the highest in the final score after the arrangement of all alternatives.

3. Asset Allocation

Investors consider stock market analysis carefully to ensure that their investments increase in value. Decisions are drawn from various sources, including social networks that include extensive levels of interactions and inputs. This has drawn a lot of attention to the development of big data and social computing. In this section, a case study takes into account 33 decision makers (DM) from a Facebook page, each with a different level of expertise, to evaluate 30 stocks from Bursa Malaysia Kuala Lumpur Composite Index (KLCI) companies to invest according to the assigned attributes, including the market value firm (MVF), return on equity (ROE), debt to equity (D/E), current ratio (CR), market value to net sales (MV/NS), and price per earning (P/E). In this study, the stocks were assigned as alternatives, $A = \{S1, S2, S3, \ldots, S33\}$, with respect to six attributes, and their weights, w, were determined. The alternatives were considered unknown to the decision makers, and only the values of the attributes of each stock were taken into account.

Step 1: Determine the network structure of the LGDM problem.

The decision makers are linked to the network structure based on their expertise level and propensity towards risk interaction and investment behavior towards each other. For example, the edge between DM1 and DM5 is denoted as $e(DM1, DM5)$.

Step 2: Detect partitions in large-scale DMs.

The 33 DMs can be classified into five partitions by running the community detection method via the Pajek 5.13 software package. The five partitions are shown in Table 3.

Table 3. Decision makers (DMs) according to partition.

Partition	Decision Makers (DMs)
***P*1**	DM1, DM3, DM8, DM12, DM15, DM25, DM33
***P*2**	DM2, DM11, DM16, DM18, DM19, DM20
***P*3**	DM4, DM6, DM14, DM22, DM24, DM28
***P*4**	DM5, DM7, DM9, DM13, DM17, DM21, DM23, DM27, DM30, DM32
***P*5**	DM10, DM26, DM29, DM31

Step 3: Calculate the weights for the nodes and partitions.

The node weight vectors, w, for the 33 DMs who belong to different clusters, are shown in Tables 4–8. Similarly, the partition weight vectors, w, for the five partitions are shown in Table 9.

Table 4. Node weights for Partition 1.

w_1	w_3	w_8	w_{12}	w_{15}	w_{25}	w_{33}
0.155634	0.111008	0.144849	0.122164	0.088695	0.211045	0.166605

Table 5. Node weights for Partition 2.

w_2	w_{11}	w_{16}	w_{18}	w_{19}	w_{20}
0.194151	0.096832	0.165069	0.204062	0.145735	0.194151

Table 6. Node weights for Partition 3.

w_4	w_6	w_{14}	w_{22}	w_{24}	w_{28}
0.155903	0.130246	0.142857	0.246358	0.207871	0.116765

Table 7. Node weights for Partition 4.

w_5	w_7	w_9	w_{13}	w_{17}	w_{21}	w_{23}	w_{27}	w_{30}	w_{32}
0.104396	0.126873	0.141858	0.10452	0.10452	0.10477	0.171454	0.037338	0.104271	0.104271

Table 8. Node weights for Partition 5.

w_{10}	w_{26}	w_{29}	w_{31}
0.17533	0.385609	0.193539	0.245521

Table 9. Partition weights for the entire network.

***P*1**	***P*2**	***P*3**	***P*4**	***P*5**
0.167123	0.480993	0.171617	0.13861	0.041656

Step 4: Create the decision matrices for the five partitions.

Apply HFN to construct the decision matrices for the five partitions. After being converted to fuzzy numbers, the evaluations of the five expert groups are applied in the form of HFS.

Step 5: Calculate the positive initial solution (PIS) and the negative initial solution (NIS).

Determine the hesitant fuzzy PIS (A^+) and NIS (A^-) separately according to the decision makers:

$A^+ =\{\langle 0.9, 1, 1, 1\rangle, \langle 0.9, 1, 1, 1\rangle, \langle 0.7667, 0.9333, 0.9333, 1\rangle, \langle 0.9, 1, 1, 1\rangle, \langle 0.9, 1, 1, 1\rangle, \langle 0.9, 1, 1, 1\rangle\}$

$A^- = \{\langle 0.0333, 0.1333, 0.1333, 0.3\rangle, \langle 0, 0.0333, 0.0333, 0.1667\rangle, \langle 0, 0.0333, 0.0333, 0.1667\rangle, \langle 0, 0.0333, 0.0333, 0.1667\rangle, \langle 0.0333, 0.1667, 0.1667, 0.3667\rangle, \langle 0.3667, 0.5667, 0.5667, 0.7667\rangle\}$

Step 6: Compute the separation measures for each alternative.

The distances, δ^+ and δ^-, are calculated according to each cost and benefit criterion as in Table 10. Alternatives A_i from the A^+ and A^- are determined using Equations (18)–(20).

Table 10. Separation measures of stocks from A^+ and A^-.

Stock	**Benefit Criteria**		**Cost Criteria**	
	$\delta+$	$\delta-$	$\delta+$	$\delta-$
S1	0.8792	0.3974	0.3616	0.1051
S2	0.4486	0.8389	0.3052	0.1643
S3	0.9257	0.3552	0.3777	0.0937
S4	0.1461	1.1354	0.2615	0.2107
⋮	⋮	⋮	⋮	⋮
S30	0.7166	0.5599	0.3535	0.1169

Step 7: Calculate the relative closeness to the ideal solution. The relative closeness coefficients ϕ_i alternative A_i are calculated and the result is shown is Table 11.

Table 11. Closeness coefficients, Φ, of stocks.

Stocks	ΦB	ΦC
S1	0.3113	0.2251
S2	0.6516	0.3499
S3	0.2773	0.1988
S4	0.8860	0.4462
⋮	⋮	⋮
S30	0.4386	0.2485

Step 8: Compute the normalized influence closeness coefficient (NICC) and the influence closeness coefficient (ICC).

Using the information in Table 11, incorporate the weights as the influence degree of each partition. To generate NICCs, the influence degree of each partition to the closeness coefficient of alternatives is calculated according to Equation (18) until Equation (20).

Step 9: The rule base for the benefit system (BS) and the cost system (CS) is constructed based on the NICC calculated. The NICC obtained is converted into linguistic terms in order to form the antecedent and consequent matrices of both the BS and CS, as performed in Equations (21)–(26).

$$NICC^{B}_{1,1} = 0.3266 = R$$

$$NICC^{C}_{1,1} = 0.2325 = B$$

$$M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} \lambda_{1,1} & , & \lambda_{2,1} & , & \lambda_{3,1} & \cdots & \lambda_{30,1} \\ \psi_{1,1} & , & \psi_{2,1} & , & \psi_{3,1} & \cdots & \psi_{30,1} \end{bmatrix}$$
$$= \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & , & R & \cdots & R \\ B & , & R & , & B & \cdots & R \end{bmatrix}$$

$$\text{Rule 1}: If\ BL_1\ is\ R\ and\ \cdots and\ CL_1\ is\ B,\ then\ AL_1\ is\ N\xi_{1,k}.$$

$$\vdots \qquad \vdots \qquad \qquad \vdots \qquad \vdots$$

$$\text{Rule } m: If\ BL_{30}\ is\ \lambda_{30,k}\ and\ \cdots and\ CL_m\ is\ \psi_{m,k}, then\ AL_m\ is\ N\xi_{m,k}.$$

Step 10: Build the antecedent matrices of the alternative system (AS). The antecedent matrices, M_k, of the alternative system (AS) of each DM, k, are constructed based on the benefit level (BL) and cost level (CL), which are the outputs of the benefit system (BS) and cost system (CS), respectively, based on the opinion of G1.

$$M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} \lambda_{1,1} & , & \lambda_{2,1} & , & \lambda_{3,1} & \cdots & \lambda_{30,1} \\ \psi_{1,1} & , & \psi_{2,1} & , & \psi_{3,1} & \cdots & \psi_{30,1} \end{bmatrix}$$
$$= \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & , & R & \cdots & R \\ B & , & R & , & B & \cdots & R \end{bmatrix}$$

The AS consequent matrices are derived as follows:

i. The calculation of the aggregation, $\xi_{j,1}$, of weighted $NICC^B$ and $NICC^C$ is as follows:

$$\xi_{1,1} = \frac{NICC^{B}_{1,1} \times \left(\frac{e}{e+f}\right) + NICC^{C}_{1,1} \times \left(\frac{f}{e+f}\right)}{2}$$

$$= \frac{0.3266 \times \left(\frac{4}{2+4}\right) + 0.2325 \times \left(\frac{2}{2+4}\right)}{2}$$

$$= 0.2952$$

ii. The normalization of the values of $\xi_{j,k}$ to confirm that their values lie between [0, 1] is as follows:

$$N\xi_{1,1} = \frac{\xi_{1,1}}{\max_j \xi_{j,1}} = \frac{0.2952}{0.5}$$

$$= 0.5905 = \mathrm{G}$$

iii. The values of $N\xi_{1,1}$ are converted into linguistic terms.

The AS consequent matrix, N_1, for G_1 is constructed based on the values of $N\xi_{j,1}$ or each alternative, j, as follows:

$$\text{If } M_1 = \begin{matrix} BL \\ CL \end{matrix}\begin{bmatrix} R & , & G & , & R & \cdots & R \\ B & , & R & , & B & \cdots & R \end{bmatrix}, \text{ then } N_1 = AL[G, R \cdots R].$$

This can best be interpreted in the following rule bases:

Rule 1: *if BL is R*, *and CL is B*, *then AL is G*.

Rule 2: *if BL is G*, *and CL is R*, *then AL is R*.

Rule 2: *if BL is G*, *and CL is R*, *then AL is R*.

Step 11: The derived rules from the BS, CS, and AS are presented as Boolean matrices. The resulting Boolean benefit system matrix for S1 is displayed below:

$$\begin{array}{cccccc}
 & 1 & 2 & 3 & 4 & 5 \\
1111 & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & 0 & \vdots & \vdots & \vdots \\
2222 & \vdots & 0 & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
5555 & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & 0 \\
6233 & \vdots & 0 & 1 & \vdots & 0 \\
6234 & \vdots & 0 & 1 & \vdots & 0
\end{array}$$

The Boolean cost system matrix for S1 was generated, as shown below:

	1	2	3	4	5
11	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
16	⋮	1	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
22	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
25	⋮	1	⋮	⋮	⋮
26	⋮	⋮	1	⋮	⋮

Vertical merging was projected to combine the BS-generalized Boolean matrices with the CS-generalized Boolean matrices to create a generalized Boolean matrix.

	11	···	22	···	31	32	33	34	35
5555/55	0	0	0	0	0	0	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	0	0	⋮
6233/16	⋮	⋮	⋮	⋮	⋮	1	0	0	⋮
6233/25	⋮	⋮	⋮	⋮	⋮	1	0	0	⋮
6233/26	⋮	⋮	⋮	⋮	⋮	⋮	1	0	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	0	⋮
6234/16	⋮	⋮	⋮	⋮	⋮	1	⋮	0	⋮
6234/25	⋮	⋮	⋮	⋮	⋮	1	⋮	0	⋮

The AS Boolean matrix for S1 was evaluated as follows:

	1	2	3	4	5
11	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
22	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
32	⋮	⋮	1	⋮	⋮
33	⋮	⋮	1	⋮	⋮

	1	2	3	4	5
5555/55	0	0	0	0	0
⋮	⋮	0	⋮	⋮	0
⋮	⋮	0	⋮	⋮	0
6233/16	⋮	⋮	1	⋮	⋮
6233/25	⋮	⋮	1	⋮	⋮
6233/26	⋮	⋮	1	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮
6234/16	⋮	⋮	1	⋮	⋮
6234/25	⋮	⋮	1	⋮	⋮
6234/26	⋮	0	1	⋮	⋮

The rules for stock S1 were generated in reference to the Boolean matrix derived:

Rule 1:	6233/16/3	6233	16	3	R
Rule 2:	6233/25/3	6233	25	3	R
Rule 3:	6233/26/3	6233	26	3	R
Rule 4:	6234/25/3	6234	25	3	R
Rule 5:	6234/26/3	6234	26	3	R

Five rules were obtained, which can be interpreted according to the linguistic terms on the level of rating, as follows:

Rule 1: If B_1 is G, and B_2 is P and B_3 is MP and B_4 is MP and C_1 is VP and C_2 is G, then S_1 is R.

Rule 2: If B_1 is G, and B_2 is P and B_3 is MP and B_4 is MP and C_1 is P and C_2 is MG, then S_1 is R.

Rule 3: If B_1 is G, and B_2 is P and B_3 is MP and B_4 is MP and C_1 is P and C_2 is G, then S_1 is R.

Rule 4: If B_1 is G, and B_2 is P and B_3 is MP and B_4 is N and C_1 is P and C_2 is MG, then S_1 is R.

Rule 5: If B_1 is G, and B_2 is P and B_3 is MP and B_4 is N and C_1 is P and C_2 is G, then S_1 is R.

Step 12: Derive the final scores and ranks. The ranking positions for all 30 stocks considered in this case study are defined based on the principle that the higher the final score, the better the ranking position.

4. Analysis of Results

For the validation of the proposed Z-HFN-SNA, the authors considered the established TOPSIS method [12] for comparison. This method was applied to find the final ranking of the stocks from the case study in Section 3, which was then compared with the performance of the novel Z-HFN SNA. As observed from the final rank obtained in Table 12 and scores calculated under Spearman analysis [13] in Table 13, the novel method, namely the Z-hesitant fuzzy network with social network analysis (Z-HFN SNA) approach, outperformed the established LGDM method since the novel approach could imply more decisions from DMs, which were divided into five divisions using the community detection method under SNA. The use of tentative fuzzy valuations increases the dependability and usefulness of the DMs' judgments in making decisions. Additionally, the fuzzy network added to the methodology provided transparency, as intermediate variables were used to translate inputs into outputs. The implementation of SNA in this approach also seems effective in adapting the model of relationships among a group of people. The study has demonstrated that when analyzing a network, having more networks in a personal network yields better outcomes. In fact, it is essential to consider how the nodes are connected or not connected, as well as the destinations of the networks that are created [14].

Table 12. Comparison of the ranking of stocks according to the proposed and established methods.

STOCK	RANKING		
	ACTUAL	Z-HFN SNA	Z-HFN
S1	30	28	25
S2	9	17	18
S3	21	25	26
S4	2	6	3
S5	24	30	30
S6	23	24	27
S7	18	13	15
S8	14	9	6
S9	12	18	16
S10	4	4	2
S11	1	1	12
S12	8	11	14
S13	13	10	13
S14	11	7	1
S15	20	29	29
S16	5	16	17
S17	27	27	28
S18	19	15	10
S19	7	14	9
S20	6	3	11
S21	10	5	4
S22	15	20	21
S23	22	21	19
S24	16	12	5
S25	17	8	7
S26	26	23	23
S27	28	19	24
S28	25	26	20
S29	3	2	8
S30	29	22	22

Table 13. Spearman rho correlation.

Methods	Spearman Rho Correlation
Proposed method: Z-HFN SNA	0.817
Established method: Z-HFN	0.712

5. Conclusions

This study has proposed a novel approach by enhancing rule-based fuzzy networks' capabilities for use in large-scale decision-making. The Z-HFN SNA considers experts' experiences, influence, and knowledge while making decisions. By explicitly considering all subsystems and interactions, the suggested strategy simultaneously increases transparency, notably in the decision-making process. However, due to the shifting financial conditions and limited time and sources, such as volunteer decision makers in the approach, there is a limit in testifying to the approach's effectiveness across a range of studies and at various times. The strategy can also be improved by combining other fuzzy approaches such as type-2 fuzzy numbers and implementing an Analytic Hierarchy Process (AHP) in the approach. By utilizing actual ranks as a benchmark in comparison to the approach, the performance of the suggested method can be validated.

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References

1. Liu, Y.; Liu, J.; Hong, Z.Y. A Multiple Attribute Decision Making Approach Based on New Similarity Measures of Interval-valued Hesitant Fuzzy Sets. *Int. J. Comput. Intell. Syst.* **2018**, *11*, 15–32. [CrossRef]
2. Aliev, R.; Memmedova, K. Application of Z-Number Based Modelling in Psychological Research. *Comput. Intell. Neurosci.* **2015**, *2015*, 760403. [CrossRef] [PubMed]
3. Tiryaki, F.; Ahlatcioglu, M. Fuzzy stock selection using a new fuzzy ranking and weighting algorithm. *Appl. Math. Comput.* **2005**, *170*, 144–157. [CrossRef]
4. Xia, M.; Xu, Z. Hesitant fuzzy information aggregation in decision making. *Int. J. Approx. Reason.* **2011**, *52*, 395–407. [CrossRef]
5. Kang, B.; Wei, D.; Li, Y.; Deng, Y. A method of converting Z-number to classical fuzzy number. *J. Inf. Comput. Sci.* **2012**, *9*, 703–709.
6. Yaakob, A.M.; Serguieva, A.; Gegov, A. FN-TOPSIS: Fuzzy Networks for Ranking Traded Equities. *IEEE Trans. Fuzzy Syst.* **2017**, *25*, 315–332. [CrossRef]
7. Wang, L.; Rodriguez, R.M.; Wang, Y.-M. A dynamic multi-attribute group emergency decision making method considering experts' hesitation. *Int. J. Comput. Intell. Syst.* **2018**, *11*, 163–182. [CrossRef]
8. Lahsasna, A.; Ainon, R.N.; Wah, T.Y. Enhancement of transparency and accuracy of credit scoring models through genetic fuzzy classifier. *Maejo Int. J. Sci. Technol.* **2010**, *4*, 136–158.
9. Bonchi, F.; Castillo, C.; Gionis, A.; Jaimes, A. Social Network Analysis and Mining for Business Applications. *ACM Trans. Intell. Syst. Technol.* **2011**, *2*, 1–37. [CrossRef]
10. Perez, L.G.; Mata, F.; Chiclana, F. Social network decision making with linguistic trustworthiness–based induced OWA operators. *Int. J. Intell. Syst.* **2014**, *29*, 1117–1137. [CrossRef]
11. Canright, G.S.; Engo-Monsen, K. Introducing Network Analysis. In *Telektronikk Network Analysis*; Lehne, P.H., Ed.; Telenor ASA: Oslo, Norway, 2008; Volume 104, p. 418.

12. Yaakob, A.M.; Shafie, S.; Gegov, A.; Abdul Rahman, S.F. Z Hesitant Fuzzy Network Model with Reliability and Transparency of Information for Decision Systems. *Int. J. Comput. Intell. Syst.* **2021**, *14*, 176. [CrossRef]
13. Sedgwick, P. Spearman's rank correlation coefficient. *BMJ* **2014**, *349*, g7327. [CrossRef] [PubMed]
14. Fox, W.P.; Everton, S.F. Mathematical modeling in social network analysis: Using TOPSIS to find node influences in a social network. *J. Math. Syst. Sci.* **2017**, *3*, 531–541.