

Dejan Brkić and Pavel Praks: Colebrook's flow friction explicit approximations based on fixed-point iterative cycles and symbolic regression. Computation

Eq. 3: An explicit approximation with one internal iterative cycle and with a fixed starting point

MS Excel:

$$\frac{1}{\sqrt{f}}: \quad C10 = C154 + 0.8686 * (C176 * (C176 + 4) - 5) / (4 * C176 + 2)$$

$$a: \quad C176 = C132 / ((\$A176 / 3.71) + (2.51 * C154 / C175)) \text{ where } Re: C175 \text{ and } \varepsilon: A176$$

$$b: \quad C154 = -2 * \text{LOG10}(C132)$$

$$c: \quad C132: 16.9 / C131 + \$A132 / 3.71 \text{ where } Re: C131 \text{ and } \varepsilon: A132$$

Matlab code:

```
x=16.9/2.51;
y0= 2.51*x./R+K./3.71;
x1 = -2*log10(y0);
z = y0./((K/3.71)+(2.51*x1./R));
ln11 = @(z) (z.*(z+4)-5)./(4*z+2);
x = x1+0.8686*ln11(z);
f=1./x.^2;
```

In the Matlab codes, the Reynolds number  $Re$  is noted as  $R$ , while the relative roughness of inner pipe surface  $\varepsilon$  as  $K$ .

Eq. 4: An explicit approximation with one internal iterative cycle and with a rational starting point given by Eq. 2

MS Excel:

$$\frac{1}{\sqrt{f}}: \quad C10 = C154 + 0.8686 * (C176 * (C176 + 4) - 5) / (4 * C176 + 2)$$

$$a: \quad C176 = C132 / ((\$A176 / 3.71) + (2.51 * C154 / C175)) \text{ where } Re: C175 \text{ and } \varepsilon: A176$$

$$b: \quad C154 = -2 * \text{LOG10}(C132)$$

$$c: \quad C132: C199 / C131 + \$A132 / 3.71 \text{ where } Re: C131 \text{ and } \varepsilon: A132$$

$$p_0: \quad C199: 2.51 * ((2600 * C198) / (657.7 * C198 + 214600 * C198 * \$A199 + 12970000) - 13.58 * \$A199 + (0.0001165 * C198) / (0.00002536 * C198 + C198 * \$A199 + 105.5) + 4.227)$$

where  $Re: C198$  and  $\varepsilon: A199$

Matlab code:

Note: for the rational starting point,  $x$  is given by Eq.2, i.e.

$$x = (2600 * R) / (657.7 * R + 214600 * R * K + 12970000) - 13.58 * K + (0.0001165 * R) / (0.00002536 * R + R * K + 105.5) + 4.227;$$

The rest of the Matlab code is unchanged.

Eq. 5: An explicit approximation with two internal iterative cycles and with a fixed starting point

MS Excel:

$$\frac{1}{\sqrt{f}}: \quad C10: C154 + 0.8686 * (C221 * (C221 + 4) - 5) / (4 * C221 + 2)$$

$$a: \quad C176 = C132 / ((\$A176 / 3.71) + (2.51 * C154 / C175)) \text{ where } Re: C175 \text{ and } \varepsilon: A176$$

$$b: \quad C154 = -2 * \text{LOG10}(C132)$$

$$c: \quad C132 = 18.15 / C131 + \$A132 / 3.71 \text{ where } Re: C131 \text{ and } \varepsilon: A132$$

$$d: \quad C221 = C132 / ((\$A221 / 3.71) + 2.51 * C198 / C220)$$

where  $Re: C220$ ,  $\varepsilon: A221$  and  $C198: C154 + 0.8686 * (C176 * (C176 + 4) - 5) / (4 * C176 + 2)$

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Matlab code:

```
x=16.9/2.51;
y0= 2.51*x./R+K./3.71;
x1 = -2*log10(y0);
z = y0./((K/3.71)+(2.51*x1./R));
ln11 = @(z) (z.*(z+4)-5)./(4*z+2);
x = x1+0.8686*ln11(z);
z = y0./((K/3.71)+(2.51*x./R));
x = x1+0.8686*ln11(z);
f=1./x.^2;
```

Eq. 6: An explicit approximation with two internal iterative cycles and with a rational starting point given by Eq. 2

MS Excel:

$\frac{1}{\sqrt{f}}$ : C10: C154+0.8686·(C221·(C221+4)-5)/(4·C221+2)  
a: C176 = C132/((A176/3.71)+(2.51·C154/C175)) where Re: C175 and ε: A176  
b: C154 = -2·LOG10(C132)  
c: C132 = C244/C131+A132/3.71 where Re: C131 and ε: A132  
d: C221 = C132/(\$A221/3.71+2.51·C198/C220)  
where Re: C220, ε: A221 and C198: C154+0.8686·(C176·(C176+4)-5)/(4·C176+2)  
p<sub>0</sub>: C244: 2.51·((2600·C243)/(657.7·C243 + 214600·C243·A244 + 12970000) - 13.58·A244 + (0.0001165·C243)/(0.00002536·C243 + C243 ·A244 + 105.5) + 4.227)  
where Re: C243 and ε: A244

Matlab code:

Note: for the rational starting point,  $x$  is given by Eq. (2). The rest of the Matlab code is unchanged.