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# Computing Spatiotemporal Accessibility to Urban Opportunities: A Reliable Space-Time Prism Approach in Uncertain Urban Networks

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**Abstract:** Space-time prism (STP) is a comprehensive and powerful model for computing accessibility to urban opportunities. Despite other types of accessibility measures, STP models capture spatial and temporal dimensions in a unified framework. Classical STPs assume that travel time in street networks is a deterministic and fixed variable. However, this assumption is in contradiction with the uncertain nature of travel time taking place due to fluctuations and traffic congestion. In addition, travel time in street networks mostly follows non-normal probability distributions which are not modeled in the structure of classical STPs. Neglecting travel time uncertainty and disregarding different types of probability distributions cause unrealistic accessibility values in STP-based metrics. In this way, this paper proposes a spatiotemporal accessibility model by extending classical STPs to non-normal stochastic urban networks and blending this modified STP with the attractiveness of urban opportunities. The elaborated model was applied on the city of Isfahan to assess the accessibility of its traffic analysis zones (TAZs) to Kowsar discount retail markets. A significant difference was found between the results of accessibility values in normally and non-normally distributed networks. In addition, the results show that the northern TAZs had larger accessibility level compared to the southern ones.

**Keywords:** space-time prism; accessibility analysis; time geography; transportation planning; urban planning

## 1. Introduction

Accessibility is a key concept in urban planning, transportation geography, and other related fields. This notion is a multi-dimensional construct, including individual, temporal, network, and land-use components [1,2]. There are different interpretations for accessibility from the opportunity attractiveness [3] to the connectivity level of different locations [4] to the ease of a place to be reached by different individuals in a certain geographical area [5]. However, after the paradigm shift from mobility to accessibility and by emerging sustainable transportation theories, this term is often defined as the easiness of participating in different activities [6–9].

Accessibility has been broadly applied to a lot of applications, such as: modeling land-use and transportation interactions [10], evaluating transportation network performance [11], appraising social equity and segregation [12–15], analyzing activity-travel pattern [16,17], analyzing coverage of health care services [18,19], etc.

Over the years, different models have been developed to quantify accessibility. There are numerous ways for classification of these models. Malekzadeh [20] summarized accessibility measures in five levels, namely, distance-based, cumulative-based, gravity-based, utility-based, and space-time models.

Cascetta et al. [5] classified accessibility metrics into three categories. The first category distinguishes opportunity-based from the utility-based measures. The second category classifies models in behavioral and non-behavioral branches. Ultimately, at the third category, level of data aggregation is applied for categorizing accessibility measures (e.g. individual-based disaggregate vs. place-based aggregate models). Other ways of classifying accessibility metrics can be found in [21–23].

Distance-related measures are generally calculated by enumerating opportunities or points of interest located in a given distance from a certain origin [20]. Different approaches have been proposed for calculating the distance between an origin and a destination: Euclidian distance, topological distance, network travel time distance, and dynamic single/multi-modal network travel times [2]. Although distance-related measures are easy to use and relatively simple for data gathering, these forms of accessibility only include the distance parameter and ignore the other accessibility components. For example, travelers' behavior and differences between distributed opportunities in an area are disregarded. Furthermore, distance-related models are not able to capture the interactions between land-use and transportation systems.

Cumulative-based measures, also named contour-based or isochrone metrics, are elaborated based on the notion of encountering the number of opportunities available within a fixed and pre-defined travel cost (e.g., time). There are three main branches of cumulative-based measures: 1) fixed opportunity models measuring the overall cost of reaching a fixed number of activities, 2) fixed impedance models counting all opportunities within a certain time or cost, and 3) fixed population models calculating the ratio of average number of accessible opportunities to the population within a fixed travel time [24,25]. Similar to distance-related measures, cumulative-based models are simple to apply to various travel modes, but these measures have several drawbacks. The first is that cumulative models assume that the system throughput is equal to the performance of the whole system, and the throughput represents the benefit obtained by an individual as well [26,27]. Another shortcoming is the sensitivity of cumulative-based measures to the determined geographic areas. For instance, if time or distance slightly varies, the number of reachable activities may increase significantly [25]. In addition, these measures neglect the effects of persons' preferences on the value of accessibility [9].

Gravity-based measures, first introduced by the groundbreaking work of Hansen [3], moved the previous models forward by considering the assumption that all available opportunities obtained from distance- and cumulative-based measures are not equally accessible. In this type of measures, available activities are weighted based on some variables such as: Attractiveness and travel expenses [28,29]. There are three main variants of the gravity-based models which encounter competition effect and spatial constraints in supply and/or demand areas. The first is origin-constrained approach established on the basis of calculating the demand likelihood of destination  $j$  and dividing the supply values reachable from origin  $i$  [30–32]. The second is destination-constrained approach developed on the foundation of dividing the total supply at an origin by the overall number of demand in origin zones to incorporate the demand potential in the measure (see: [24,33]). The third is double-constrained approach elaborated according to a balancing factor which ensures that flows from an origin to a destination be equal to the specific number of opportunities in the origin [2,28,34]. Albeit gravity-based measures are able to incorporate land-use and network components of accessibility in practice, there are several major criticisms on these models. The first is that the results of gravity-based measures are highly related to the impedance function, particularly when the decay functions are empirically specified for evaluation purposes [1,28]. The second is that gravity models presume that the calculated accessibility of a zone is identical for all residents of that zone, while it is obvious that the level of accessibility strongly depends on the physical and personal traits [35]. However, distance-, cumulative- and gravity-based measures are aggregate in nature and are unable to describe the intricacy of an individual's preferences and the space-time restrictions [36,37]. Moreover, these traditional metrics are not suitable for modeling accessibility in multi-purpose and multi-stop trips [38].

Utility-based measures proposed by Ben-Akiva and Lerman [39] are developed based on the random utility theory in which the probability of choosing a certain alternative by an individual

depends on the utility of all alternatives [40]. The underlying assumption of measuring utility-based accessibility is that people get benefits when they reach an available place during a particular travel time [5]. As individuals perceive the benefits of activities differently, the log-sum approach can be applied to calculate the expected maximum utility, on the basis of a considered choice set. This type of accessibility is capable of incorporating both individual characteristics and competition effect into the model's structure. In utility-based models, the utility function is related to the activity attractiveness and duration of activity participation directly and travel time indirectly [38,41].

Space-time measures created by Hägerstrand [42] and then operationalized by Lenntorp [43] attempt to capture the spatiotemporal facets and the constraints imposed on a person by transportation and activity systems [44]. These constraints are: (1) Capability constraints which encompass physical and psychological requirements to engage in an activity, (2) coupling constraints which enfold spatiotemporal requirements that make participation in different activities with other people possible, and (3) authority constraints which include laws that limit an individual's access to opportunities [20,37]. Space-time measures were constituted around the central concept of space-time prism in which if the origin and destination, available time for moving, and the maximum moving speeds from origin to destination are given, the space-time prism will delimit all reachable locations and will determine the remaining available time to spend at each opportunity [45–49]. There are two types of space-time prisms: Punctiform prism which works based on the Euclidian distance of between origin, opportunities, and destination [9,47], and network-based prism that is compatible with the topology of the road networks [14] (See Appendix A for complementary information about network-based space-time prisms).

Space-time measures are appropriate models to explain the accessibility at the individual level [9,20]. Reviewing the way of constructing basic forms of space-time prisms shows that in the basic forms of prisms, it is presumed that travel times are deterministic and do not have any variation. Therefore, average travel time or free-flow travel times are considered as the representative of the network costs (e.g. [41,50–52]). This presumption is in contradiction with the fact that travel times in the road networks are highly stochastic because of interruptions and fluctuations [53,54]. In this way, several studies have been conducted in the field of geographic information science to put uncertainty in the structure of space-time prisms. For instance, Kuijpers et al. [55] studied uncertainty of anchor points in a space-time prism and developed algorithms for computing network-based space-time prisms on the basis of probabilistic anchor regions. Kobayashi et al. [56] applied uncertainty concept to elaborate methods for assessing the error propagation in space-time prisms. Liao et al. [57] developed a model on the basis of super-networks which incorporates time uncertainty and space-time prism for activity-travel scheduling. Chen et al. [58] used the notion of reliable space-time prism to put the travel time uncertainty in the traditional space-time prisms as people in their decisions adopt risk-taking behavior when they are faced with travel time uncertainty and treat it in the form of reliability [59,60]. In these works, it was assumed that travel times obey normal distribution, while in real conditions travel times follow non-normal distribution types, such as exponential and log-normal distributions [57,58,61]. Regarding non-normal and complex distributions, travel time of a network cannot be analytically obtained [62] and using simulation techniques (i.e., Monte Carlo Simulation) becomes inevitable.

In short, considering pros and cons of the above-mentioned measures and due to the importance of developing person-based metrics [63,64] and comprehensiveness of space-time prism-based accessibility models [9], this paper aims to develop a spatiotemporal accessibility model for tackling the question of how accessibility to urban opportunities could be calculated in uncertain networks? However, responding to this question entails addressing the precedent question of how space-time prisms could be constructed in stochastic networks with non-normal travel time distributions? This way, Section 2 outlines model development. Section 3 offers numerical computation and discussion, including model implementation in the study area and comparison of the method with other existing ones. Lastly, the conclusions and future works are given in Section 4.

## 2. Model Development

Let  $G = (E, V, \Psi)$  be a directed graph that contains a set of edges  $E$ , a set of vertices  $V$ , and a set of movements  $\Psi$ . Each edge  $e_{ij} \in E$  has a tail vertex  $i$  and a head vertex  $j$  that are in  $V$ . Travel time of each edge is a random variable denoted by  $C_{ij}$  with mean  $\bar{t}_{ij}$  and standard deviation  $\sigma_{ij}$ . Successors and predecessors of  $i$  are  $S(i) = \{j : e_{ij} \in E\}$  and  $P(i) = \{w : e_{wi} \in E\}$ , respectively.  $\Psi$  is the set of possible movements between vertices, for example,  $\Psi_{ijk}$  is a movement from  $e_{ij}$  to  $e_{jk}$ . In other words, movement  $\Psi_{ijk}$  shows that there is a feasible edge rooted in  $i$  between vertex  $j$  and vertex  $k$ . In this regard, the model of non-normal reliable network-based space-time prism can be formally presented in the following sub-sections. Table 1 lists the notations for the parameters and variables.

**Table 1.** Parameters used in model development.

Symbol	Definition
$r$	Index of trip origin, $r \in V$
$s$	Index of trip destination, $s \in V$
$t_r$	Departure time from origin ( $r$ )
$t_s$	Arrival time to destination ( $s$ )
$b$	Travel time budget ( $b = t_s - t_r$ )
$Q$	Set of individuals or set of population groups
$q$	Index of an individual or a population group
$\alpha$	On-time arrival probability depicting risk-taking behavior of a traveler
$\alpha_q$	On-time arrival probability of individual or population group $q \in Q$
$t_i^a$	The earliest arrival time to vertex $i \in V$
$t_i^b$	The latest departure time from vertex $i \in V$
$x_i$	$x$ co-ordinate of vertex $i \in V$
$y_i$	$y$ co-ordinate of vertex $i \in V$
$K$	Set of all intermediate opportunities or points of interest $k$
$Z$	Set of traffic analysis zones (TAZs) in a geographic area
$z, \acute{z}$	Indices of TAZs
$CI$	Set of confidence interval including $\alpha_n$
$P_z$	Population of TAZ $z \in Z$
$\hat{P}_z^{\alpha_q}$	Portion of population group $q$ in TAZ $z$ conducting their trip at confidence interval $\alpha_q$
$O_k$	Attractiveness of opportunity $k \in K$ . Area could represent attractiveness of an opportunity [65–69].
$O_{max}$	Maximum attractiveness of opportunities, $\max_{k \in K}(O_k)$
$B(k)$	A binary variable with value of 1 if activity $k$ be in space-time prism, and 0 otherwise.
$L^{ij}$	Set of paths start from vertex $i \in V$ and end to vertex $j \in V$
$L^{w,ij}$	Set of paths from vertex $w \in V$ to vertex $j \in V$ passing through edge $e_{ij}$
$u$	Index of a path in $L$ or $L^{w,ij}$
$l_u^{ij}$	Path $u \in L^{ij}$ from vertex $i \in V$ to vertex $j \in V$
$l_u^{w,ij}$	Path $u \in L^{w,ij}$ from vertex $w \in V$ to vertex $j \in V$ passing through edge $e_{ij}$
$l_{NDM}^{ij}$	Non-dominated path from vertex $i \in V$ to vertex $j \in V$
$l_{NDM}^{r,ij}$	Non-dominated path from vertex $w \in V$ to vertex $j \in V$ passing through edge $e_{ij}$
$F_{l_u^{ij}}^{-1}(\alpha)$	Value of inverse cumulative density function of $l_u^{ij}$ at confidence interval $\alpha$

### 2.1. Non-Normal Reliable Network-Based Space-Time Prism Model

Considering origin, destination, departure time, arrival time, and on-time arrival probability, non-normal reliable potential path area (NNRPPA) can be represented by Equation (1). NNRPPA is a set of edges that travel time of the non-dominated (shortest) path from origin to destination going through, smaller or equal to  $b$ .

$$NNRPPA(\alpha, r, s) = \bigcup \left\{ e_{ij} \in E \mid \exists l_u^{rs} : e_{ij} \in l_{NDM}^{rs}, F_{l_u^{rs}}^{-1}(\alpha) \leq b \right\} \tag{1}$$

To construct non-normal reliable space-time prism (NNRNTP), edges in NNRPPA should be labeled by the earliest departure and the latest arrival times. Hence, for each  $e_{ij} \in NNRPPA(\alpha)$  Equation (2) is used for marking vertices and computing spatial extent of each edge.

$$\left\{ \left( x_i, y_i, t_i^a = t_r + F_{iNDM}^{-1}(\alpha) \right), \left( x_j, y_j, t_j^a = t_r + F_{jNDM}^{-1}(\alpha) \right), \left( x_i, y_i, t_i^b = t_s - F_{iNDM}^{-1}(\alpha) \right), \left( x_j, y_j, t_j^b = t_s - F_{jNDM}^{-1}(\alpha) \right) \right\} \quad (2)$$

### 2.2. Solution Algorithm

Finding non-dominated paths is required for solving Equation (1) and Equation (2). Here,  $l_u^{rs}$  dominates  $l_v^{rs}$  when  $F_{l_u^{rs}}^{-1}(\alpha) < F_{l_v^{rs}}^{-1}(\alpha)$ . Due to non-linearity of  $F_{l_u^{rs}}^{-1}(\alpha)$ , the optimum principle of Bellman is violated in uncertain urban networks and unlike the classical network-based prisms, additive methods like Dijkstra algorithm cannot be applied for constructing NNRPPA [70]. Furthermore, as the probability distribution of edges may be non-normal, the distribution of an uncertain path could not be obtained analytically, and using computational methods and simulation techniques is inevitable [71]. Given a confidence interval, Monte Carlo simulation (MCS) is a powerful and acceptable method for computing inverse value of cumulative density function (CDF) of a path comprised of edges with non-normal distributions [62,72]. The main procedure of MCS for calculating this value is as follows [73]:

- Step 1: Adopt a large number (i.e.,  $N = 1000$  or  $10000$ ).
- Step 2: Determine the confidence interval ( $\alpha$ ).
- Step 3: Generate  $N$  random samples from the probability distribution of each edge in the path.
- Step 4: Sum up values generated in Step 3.
- Step 5: Sort values obtained in Step 4.
- Step 6: Select element  $\alpha \times N$  from the matrix achieved in Step 5. This element is inverse value of CDF of the path at confidence interval  $\alpha$ .

As an example, consider a path that consists of five edges with uniform distribution and parameters given in Table 2. Using MCS, travel time of the path would be 55.773 at confidence interval 80%. Table 3 depicts the process and the results of MCS for the indicated example.

Table 2. Distribution parameters.

Edge	First Parameter	Second Parameter
U1	5	7
U2	10	11
U3	15	13
U4	18	20
U5	4	6

Table 3. Result of Monte Carlo Simulation for the given example (The highlighted cell is travel time of the path at confidence interval 80%).

Random Samples Taken from Distributions in Table 2					Summation	Sorting
U1	U2	U3	U4	U5		
6.284	10.620	13.847	19.058	5.049	54.860	52.477
5.442	10.600	13.378	19.659	5.945	55.026	53.748
6.674	10.172	14.192	19.717	5.420	56.177	54.257
6.942	10.090	13.023	19.578	4.623	54.257	54.860
6.692	10.255	14.820	18.635	4.582	54.986	54.986
6.011	10.858	14.358	18.904	5.700	55.833	55.026
5.557	10.911	13.977	19.504	5.823	55.773	55.569
6.493	10.699	14.878	18.219	5.278	55.569	55.773
5.473	10.725	13.548	18.219	4.510	52.477	55.833
6.914	10.229	13.886	18.539	4.177	53.748	56.177

Having a method for calculating the travel time of non-normally distributed paths, two steps are proposed for constructing NNRNTP.

*Step 1:* In this step, all reliable shortest paths spanning from the origin to the other vertices satisfying the budget constraint are calculated. First, edges emanating from the origin are identified, and their travel time is calculated by MCS. Second, feasible ones regarding time budget constraint are stored in scan eligible (*SE*) and path (*L*) sets. Third, elements in *SE* are selected one by one and new paths are constructed according to the allowed movements. Fourth, for each new constructed path, dominance condition is performed on paths with the same ending vertices in *SE* and *L*. If the new path is a non-dominated path, all dominated paths in *SE* and *L* will be removed, and this path will be added to *SE* and *L*. This procedure continues until  $SE = \emptyset$ . The result of this step is called non-normal reliable forward cone (NNRFC), which is executed through Algorithm 1.

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**Algorithm 1.** Inspired from [58]

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**Inputs:**  $r, b$ , and  $\alpha$

**Outputs:** NNRFC

**Step 1.** Initialization

Set  $NNRFC = \emptyset$

For each edge  $e_{ij}$  in  $E$ :

Create forward path  $l_u^{rj} = \emptyset$

End for

For each link  $e_{rj}$  emanating from  $r$

Create forward path  $l_R^{rj} = e_{rj}$

Calculate  $F_{l_R^{rj}}^{-1}(\alpha)$  for the constructed path

Set  $SE = SE \cup \{l_R^{rj}\}$

Set  $L_R^{rj} = L_R^{rj} \cup \{l_R^{rj}\}$

End for

**Step 2.** Path selection

If  $SE = \emptyset$

Stop

Else

Select  $l_R^{rj}$  at the top of  $SE$  and set  $SE - \{l_R^{rj}\}$

End if

**Step 3.** Path extension

For each allowed movement in  $\Psi_{jk} \in \Psi$  from  $j$  in  $l_R^{rj}$

Create new path  $l_R^{r,jk} = l_R^{rj} \oplus e_{jk}$

Calculate  $F_{l_R^{r,jk}}^{-1}(\alpha)$  by MCS

If  $F_{l_R^{r,jk}}^{-1}(\alpha) \leq b$

If  $l_R^{r,jk} \notin FC$

Set  $SE = SE \cup \{l_R^{r,jk}\}$

Set  $L_R^{rj} = L_R^{rj} \cup \{l_R^{r,jk}\}$

Else

Check dominance condition for  $l_R^{r,jk}$

If  $l_R^{r,jk}$  is non-dominated path

Set  $SE = SE \cup \{l_R^{r,jk}\}$

Set  $L_R^{rj} = L_R^{rj} \cup \{l_R^{r,jk}\}$

Remove dominated paths in  $SE$  and  $L_R^{rj}$

End if

End if

End if

End for

Set  $NNRFC = L_R^{rj}$

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*Step 2:* In this step, all reliable paths from the vertices of the network to the destination with travel time smaller or equal to  $b$  are determined and connected to the paths generated in the first step. The output of this step is non-normal reliable backward cone (NNRBC) integrated with NNRFC. This procedure is implemented through Algorithm 2, which is a combination of Algorithm 1 in reverse with integration operator in path extension.

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**Algorithm 2.** Backward cone calculator.

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**Inputs:**  $s, b, \alpha$ , and NNRFC

**Outputs:** NNRBC

**Step 1.** Initialization

Set  $NNRBC = \emptyset$

For each link  $e_{ij}$  in  $E$ :

Create  $l_u^{is} := \emptyset$

End for

For each link  $e_{is}$  emerging into  $s$

If  $l_R^{ri} \neq \emptyset$

For each  $l_R^{ri} \in NNRFC$

Integrate  $l_R^{ri}$  with  $e_{is}$ ,  $l_R^{r,is} = l_R^{ri} \oplus e_{is}$

Calculate  $F_{l_R^{r,is}}^{-1}(\alpha)$  by MCS

End for

If  $\min(F_{l_R^{r,is}}^{-1}(\alpha)) \leq b$

Set  $SE := SE \cup \{l_R^{is}\}$

Set  $L_R^{is} := L_R^{is} \cup \{l_R^{is}\}$

End if

End if

End for

**Step 2.** Path selection

If  $SE = \emptyset$

Stop

Else

Select  $l_R^{is}$  at the top of  $SE$  and set  $SE - \{l_R^{is}\}$

End if

**Step 3.** Path extension

For each allowed movement in  $\Psi_{ki} \in \Psi$  create  $l_R^{ks} = e_{ki} \oplus l_R^{is}$

If  $l_R^{ri} = l_R^{rk} \oplus e_{ki} \in NNRFC \neq \emptyset$

For each  $l_R^{ri} \in NNRFC$

Integrate  $l_R^{ri}$  with  $l_R^{ks}$ ,  $l_R^{rs} = l_R^{ri} \cup l_R^{ks} = l_R^{ri} \oplus e_{ki} \oplus l_R^{is}$

Calculate  $F_{l_R^{rs}}^{-1}(\alpha)$  by MCS

End for

If  $\min(F_{l_R^{rs}}^{-1}(\alpha)) \leq b$

If  $l_R^{ks} \notin L_R^s$

Set  $SE = SE \cup \{l_R^{ks}\}$

Set  $L_R^{is} = L_R^{is} \cup \{l_R^{ks}\}$

Else

Check dominance condition for  $l_R^{ks}$

If  $l_R^{ks}$  is non-dominated path

Set  $SE = SE \cup \{l_R^{ks}\}$

Set  $L_R^{is} = L_R^{is} \cup \{l_R^{ks}\}$

Remove dominated paths in  $SE$  and  $L_R^{is}$

End if

End if

End if

End if

End for

Set  $NNRBC = L_R^{is}$

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### 2.3. Accessibility Model

Having a method for constructing NNRPPA and NNRNTP, feasible locations an individual could attend are identified given on-time probability, time budget, origin, and destination. As combining magnitude or attractiveness of opportunities with reachable places can model accessibility [68,74], reliable space-time accessibility measure is defined through Equation (3), where  $A_k^{qrs}(\alpha)$  is the accessibility of a person or population group  $q$  conducting a trip with origin  $r$  and destination  $s$  to opportunity  $k$  located in graph  $G$  at confidence interval  $\alpha_q$ . In doing so, accessibility of a person or a population group to a set of opportunities denoted by  $A^{qrs}(\alpha_q)$  could be represented through Equation (4).

$$A_k^{qrs}(\alpha_q) = O_k \times B(k); O_k = \frac{O_k}{O_{max}}; q \in Q \tag{3}$$

$$A^{qrs}(\alpha_q) = \sum_{k \in G} O_k \times B(k); O_k = \frac{O_k}{O_{max}}; q \in Q \tag{4}$$

Considering [75], Equation (5) is applied for aggregating the accessibility of population groups to a set of opportunities in TAZ level, where  $A^{zz}$  is the accessibility of people going from TAZ  $z$  to TAZ  $\acute{z}$ . It should be noted that people with origin  $z$  and destination  $\acute{z}$  are categorized into different groups on the basis of confidence interval. This way,  $q$  shows population groups.

$$A^{zz} = \sum_{k \in K} \sum_{q \in Q} (P_z^{\alpha_q} \times P_z \times A_k^{qzz}(\alpha_q)); P_z = \sum_{q \in Q} (P_z^{\alpha_q} \times P_z). \tag{5}$$

### 3. Numerical Computation and Discussion

Access to high quality and healthy foods is crucial for city residents, particularly for low- and middle-income families. [68,76]. In this way, the municipality of Isfahan constructed 12 Kowsar discount retail markets to supply the needs of people in the city of Isfahan, Iran. In doing so, assessing the accessibility of TAZs of Isfahan was taken into account to test the applicability of the model in a real-world problem. Figure 1 depicts the study area along with distribution of the markets.

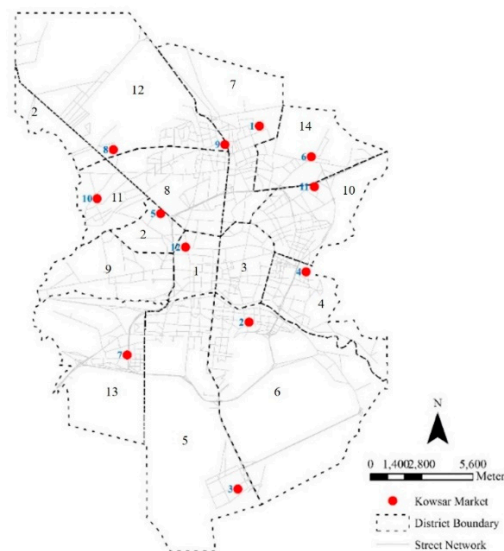


Figure 1. Study area.

Briefly, inputs of the model are: (1) A trip chain showing the travel origin and destination (location of the fixed activities), (2) time budget, (3) on-time arrival probability for conducting the selected trip



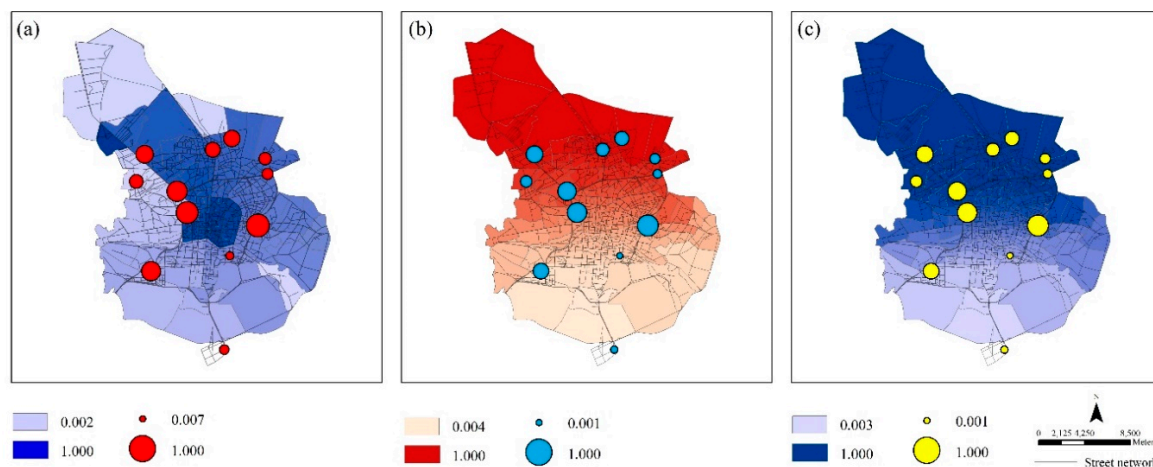
chain, (4) travel time of edges in the study area, and (5) area of the intermediate opportunities along with their locations. These items were set as below:

- Trip chain *Home* → *Kowsar market* → *Home* was defined due to the portion of home-based trips among the other trips (about 90% of all trips).
- Modeling period was considered the time window between 5:00 p.m. and 8:00 p.m., including the afternoon peak hour.
- Time budget was set to 60 minutes, following previous studies [36].
- Area of markets was considered as the indicator of opportunity attractiveness (Table 4).
- On-time arrival probability was defined 99% for simulating extreme conditions, which gives minimum value or lower bound of accessibility.
- Travel time distributions were set to normal, exponential, and log-normal distributions as travel time uncertainty in urban street networks is commonly defined by pre-allocated distributions [62]. Normal distribution was selected to provide a ground for comparing the results of the non-normal model with the conventional outputs. Additionally, log-normal distribution was taken into account because this distribution could model travel time in a more realistic manner [77].

**Table 4.** Attractiveness of Kowsar markets (standardized values).

Opportunity_ID	1	2	3	4	5	6	7	8	9	10	11	12
Attractiveness	0.37	0.09	0.18	1	0.33	0.02	0.54	0.65	0.26	0.28	0.20	0.48

To construct NNRNTP, normal reliable NTP (NRNTP), and measure accessibility of TAZs to Kowsar markets, algorithms given in the previous section were implemented using MATLAB programming language. Results were visualized by ESRI ArcGIS, the main analytical software in transportation (GIS-T) and urban planning. Figure 2 illustrates the accessibility level of each market and the accessibility of TAZs.



**Figure 2.** Accessibility of markets and traffic analysis zones (TAZs) (normalized values). (a) Accessibility with normal reliable space-time prism (NRNTP). (b) Accessibility with exponential non-normal reliable space-time prism (NNRNTP). (c) Accessibility with log-normal NNRNTP.

A *t*-test was performed on the outputs to test the difference between accessibility of TAZs in normally and non-normally distributed networks. Generally, null and alternative hypotheses of this test are equality and inequality of means between two groups, respectively. *T*-test checks whether the null or the alternative hypothesis could be rejected at a certain confidence interval. In addition, this test is used when the number of samples is more than 30. Thus, hypotheses of this test for analyzing accessibility in the study area are elaborated as below:

- Null hypothesis ( $H_0$ ): There is no statistically meaningful difference between the average of TAZ accessibility values in normally and non-normally distributed networks.
- Alternative hypothesis ( $H_1$ ): There is a statistically meaningful difference between the average of TAZ accessibility values in normally and non-normally distributed networks.

The results of this test confirmed the difference between accessibility level of TAZs in normally and non-normally distributed networks (Table 5).

**Table 5.** *t*-test for analyzing the difference between the accessibility of TAZs generated by the models.

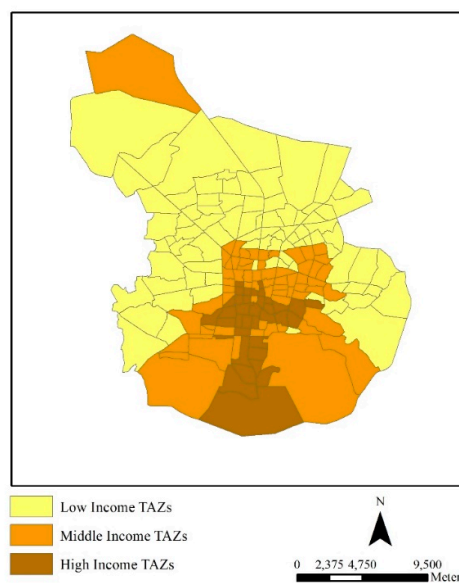
<i>t</i> -test in 95% of Confidence Interval for Accessibility Value of TAZs				
Distribution type	Accessibility with exponential NNRNTP		Accessibility with log-normal NNRNTP	
	<i>t</i> value	Sig. (2-tailed)	<i>t</i> value	Sig. (2-tailed)
Accessibility with NRNTP	8.354	0.000	6.254	0.000
Accessibility with log-normal NNRNTP	-7.938	0.000	-	-

Moreover, to examine the difference between NNRNTP and NRNTP, size of constructed space-time prisms for each TAZ was recorded and compared using a *t*-test. Prism size is the area of space-time polygons calculated by Equation (2) for each edge in the potential path area. Table 6 presents the results of the *t*-test. It shows a meaningful difference between the size of NNRNTPs and NRNTP.

**Table 6.** *t*-test for analyzing the difference between the size of NNRNTP and NRNTP.

<i>t</i> -test in 95% of confidence interval				
	Size of NNRNTP with exponential distribution		Size of NNRNTP with log-normal distribution	
	<i>t</i> value	Sig. (2-tailed)	<i>t</i> value	Sig. (2-tailed)
Size of NRNTP	3.233	0.003	7.085	0.000

Moreover, it could be observed that northern TAZs had larger access to the markets than the southern ones. In addition, comparing accessibility values achieved by NNRNTPs with the income of people living in TAZs portrayed that low-income TAZs had larger values than the others (Figure 3).



**Figure 3.** Income distribution in the city of Isfahan.

#### 4. Summary and Conclusion

In this study, a model was developed for quantifying accessibility to urban opportunities. This model incorporates network-based space-time prisms that could be implemented on stochastic networks with non-normal travel time distributions.

Firstly, a computational method was introduced for calculating reliable space-time prisms under the aforementioned conditions. Secondly, an accessibility measure was elaborated by combining the proposed space-time prism with the magnitude of urban opportunities. Finally, the accessibility model was applied in a study area under the extreme condition illustrating the lower bound of accessibility level regarding normal, exponential, and log-normal distributions.

Results demonstrated the applicability of the proposed model and statistical meaningfulness of difference between outputs of normal and non-normal methods. Results also illustrated the distribution of accessibility throughout the city of Isfahan where northern TAZs had larger access to the markets in comparison with southern ones.

For future studies, application of the introduced model in other cases for analyzing different urban phenomena is recommended. Another extension to this research is considering the correlation between link travel times in urban networks and modeling this effect in the structure of space-time prisms and accessibility metrics.

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#### Appendix A. Classical Network-Based Space-Time Prisms

Equations for constructing conventional space-time prisms are presented below, where  $PPA$  is potential path area,  $t_{RN}^{rx}$  is time of the shortest path from origin  $r$  to  $x$ ,  $t_{RN}^{xs}$  is time of the shortest path from  $x$  to destination  $s$ ,  $b$  is time budget,  $FC$  is the set of places reachable from origin at  $t$ ,  $BC$  is the set of places that can access destination during  $t$ , and  $NTP$  is network-based space-time prism.

$$PPA = \{x | t_{RN}^{rx} + t_{RN}^{xs} \leq b\} \quad (A1)$$

$$FC(t) = \{x | t_{RN}^{rx} \leq t - t_r, t \leq t_s\} \quad (A2)$$

$$BC(t) = \{x | t_{RN}^{xs} \leq t_s - t, t \geq t_r\} \quad (A3)$$

$$NTP(t) = FC(t) \cap BC(t) \quad (A4)$$

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