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A Novel Mutual-Coupling Dipole Model Considering the Interactions between Particles

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Abstract: The interactions between two or more particles and the calculation of the local electric field are widely applied in many fields, such as those of insulation, biology, medicine, and microfluidics. The dipole approximation model, which is a classical electric field calculation method, has been widely used in many fields to solve for the local electric field in a multi-particle system, but it does not consider the interactions between particles; as a result, it is easily limited by the calculation situation, and it generates a large calculation error when the distance between particles is small. Based on the physical essence of an interaction between two particles, a concept of the mutual-coupling dipole moment caused by the interactions between particles is defined for the first time. Moreover, by combining the calculation process of the dipole moment and the electric field of polarization, a novel mutual-coupling dipole model considering the interactions between particles is proposed in this paper, and analytical expressions of the local electric field that consider the interaction between two particles are obtained, thus compensating for the large error in the electric field calculation caused by the dipole approximation model when the distance between particles is small. In this paper, a mutual-coupling dipole model considering particle interactions is proposed. This model can effectively reflect the interactions between particles when the distance between particles D/R is less than 0.6 and accurately calculate the local electric fields of the particles. These results can be effectively used to investigate the interactions between particles and the control of particles in electric fields in many fields, such as in the calculation of the insulation of mixed dielectrics, the microscopic transport of medicines, the control of bio-cells and micro-fluids in electric fields, and environmental governance.

Keywords: mutual-coupling dipole model; a two-particle system; electric field distribution; interaction of particles

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1. Introduction

Research on interactions between particles and the control of particles in electric fields is an important research area that has been widely applied in many fields, such as in mixed dielectric insulation, microscopic transport of medical drugs, control of bio-cell electric fields, micro-fluid electro-control, and environmental control.

In the field of mixed dielectric insulation, some scholars have found that weather can affect the insulation performance of high-voltage insulation systems to a certain extent. For example, wind–sand and haze environments have a large impact on the corona, breakdown voltage, and ion flow field of transmission lines. The main reasons are the polarization of solid particles, particle charging, and interactions of particles distorting the local electric field, thus affecting the discharge characteristics of mixed dielectrics [1]. Similarly, the roof insulation system of a train exposed to a two-phase particle mixture consisting of haze or sand dust also induced insulation failure due to the electric field distortion caused by solid particles. Discharge in an electrostatic precipitator [2] and an electrostatic sprayer were also used to study the discharge phenomenon of a gas and solid mixture and a gas and liquid mixture; this was the key to calculating the local electric field in the space of the dust collector and studying the charging behavior of the particles. In addition, electrocoagulation technology is an effective control method for removing PM2.5, in which the charging of particles is the key to achieving particle electrocoagulation, and the charging process of particles is closely related to the local electric field around them and the interactions between them [3]. The key to studying the above problems is to obtain a fast and accurate solution for the local electric field under particle interactions in mixed dielectrics.

In the field of nano-materials, particle-to-particle interactions can be used to enhance or modify the properties of the materials. For example, nano-particles are added to transformer oil to form nano-modified transformer oil [4]. The key to improving the insulation level of a transformer is the dispersion of nano-particles in oil, which is closely related to the local electric field distribution of the nano-particles and the interactions between the nano-particles. In the field of microfluidics, ionic surfactants are used to control the attachment and peeling of substrates, thereby inhibiting the interactions between liquids and substrates [5]. The separation and enrichment of micro- and nano-scale materials can be achieved by using the relationship between the charge characteristics and electrophoretic behavior of microscale and nano-scale substances combined with the target distribution of the electric field of a circular electric field and the regulation of the distribution of an electric field on microscale and nano-scale substances [6]. In these fields, obtaining a fast and accurate solution of the local electric field of particles and an understanding of the interactions between particles and materials and the behavior of the electric fields of particles are vital to the study of such problems.

In environmental treatments, electrolysis is often used to treat sewage [7]. The voltage type used in electrolysis evolved from DC voltage to pulse voltage. Nanowire-assisted disinfection water treatment technology [8] was introduced when nanotechnology was combined with a field enhancement treatment. In the application of field-enhanced nano-particles in groundwater treatment [9], when a DC voltage is applied to wastewater that is rich in special particles, the degree of aggregation of nano-particles is lower, which increases the interactions between nano-particles and silicon dioxide and improves the reactivity and persistence of nano-particles, depending on the characteristics of the distribution of the local electric field between nano-particles and the interactions between nano-particles under an electric field.

In biomedicine, cell–cell fluids and cell–cell interactions are the key components of the pathophysiological state of micro-vessels, and cells are composed of elementary charged particles. On the one hand, artificially using the chemotaxis of cells enables one to control their migration along the gradient of an electric field under the action of a DC electric field [10]. On the other hand, therapeutic drugs can be accurately targeted to inflamed tissues [11] or the nervous system [12]. Nano-particles (NPs) interact well with drug protein synapses and, thus, can be transported to the central nervous system through the blood barrier. When a migrating cell receives a direction-changing signal from an electric field, the interactions between the cells decrease, and there is more freedom in the individual behaviors of the cells. The inhibition of mitosis in cancer cells can also be accomplished with electrical activity in the cells. An extremely low-intensity (<2 V/cm) medium-frequency electric field (100–300 kHz) can successfully inhibit the proliferation of cancer cells [13]. Simultaneously, electrical stimulation can be used in cells to trigger the release of drug molecules [14]. In addition, the adsorption strength of polymers in medical imaging depends on the surface charges of nano-particles, which are closely related to the surrounding electric fields and the interactions between the nano-particles [15]. Therefore, analysis of the interactions between cells or nano-particles and the behavior of the electric field is very important in the medical field. The key to this research is to obtain the distribution of the local electric field of micro-cells or nano-particles and to understand the interactions and behaviors of particles under the action of an electric field.

In summary, the particles' charging behaviors, the interactions between particles, and the characteristics of the behavior of the electric field play an important role in the fields of insulation, nano-materials, medicine, biology, environmental governance, etc. The key of this research is to obtain the distribution of the local electric field under particle interactions. Therefore, it is of great importance to propose a local electric field calculation method that can consider particle interactions in all fields. In fact, particles are not only polarized by themselves, but are also affected by other particle polarization processes [16], that is, there are interactions between particles; in particular, when the particle spacing is small, these interactions will be more significant. At present, the classical dipole approximation model or finite element method is mainly used in the calculation of the particle electric field. Although the finite element method is accurate, the calculation process for a micro-multi-particle system is too complex and depends on calculation resources, and is thus difficult to apply in the calculation of the local electric fields of large-scale micro-particles. The dipole approximation model is often used to calculate the local electric fields in mixtures. This model considers the medium in the electric field as a point dipole [17] by considering that the particle is infinitely small and that the electric field around the particle is uniform, and it further obtains the effective dipole moment of the particle. The accuracy of calculating the electric field distribution using the dipole approximation model in a uniform field is affected by the ratio of the permittivity of the particles to that of the environment and the distance between particles. When the difference between the particles' and environment's permittivity is small and the distance between the particles is large, the approximation accuracy of the dipole approximation model is higher [18,19]. The dipole approximation model was also used to calculate the charging of particles [20–22]. When calculating the maximum field strength of a mixture, an approximation algorithm [23] that considered the effect of the surrounding particles was introduced, and Ye Qizheng et al. proposed an enhanced-dipole model [24]. This model could be used to calculate the effective dielectric constant [25] of a mixture and the charge of an electron avalanche particle [26]. However, this model could only guarantee the accuracy of the calculation at the maximum field strength of the particle surface.

In addition to the dipole approximation model, the higher-order multipole model [27] or continuous mirror dipole model is commonly used when the permittivity is large or the distance between two medium particles is small. The method of image is generally used to conduct electromagnetic calculations, and its basis is the uniqueness theorem. Because the boundary conditions are complex, the calculation of spherical dielectric particles becomes more difficult; therefore, it can only be applied in the case of fewer particles. The computational processes of the higher-order multipole model and the method of image are complex, and convergence is difficult when the permittivity of the dielectric particles is large [23,28]. Therefore, Ye Qizheng et al. proposed a displaced dipole model [29] by assuming that the interactions between particles cause a center offset of the dipole moment. The displaced dipole model simplifies the calculation of the distribution of a charged sphere near an infinitely flat conductor to a large extent. This was applied to the calculation of the electric field distribution in a two-sphere system, and a good result was obtained. However, compared to the finite element method, the displaced dipole model produces a larger error when the distance between two particles is less than the radii of the particles.

The above-mentioned methods do not consider the influence of the interactions between particles on the local electric fields in various fields. Actually, the calculation of the effective dipole moment is the focus of local electric field calculation in a two-particle system. In other words, the particles in a two-particle system interact with each other. The polarization process of one particle is not only affected by the external electric field, but also by the polarized electric field of the other particle. At present, when calculating the effective dipole moment of a particle, only the dipole moment of the particle itself in the external electric field is considered, whereas the dipole moment produced by the polarized electric field of the other particle is ignored. In this paper, we define the concept of the mutual-coupling dipole moment based on the physical essence of the interactions between two particles and propose a mutual-coupling dipole model that considers the interactions between particles to accurately calculate the distribution of the electric field around the particles, thus realizing an effective solution of the local electric field in a two-particle system and compensating for the large error in the calculation of the electric field caused by the classical dipole approximation model when the distance between particles is small. The results will have important applications in the analysis of particle interactions and the control of particles in electric fields in many fields, such as in the calculation of the insulation of mixed dielectrics, the microscopic transport of medicines, the control of bio-cells and micro-fluids in electric fields, and environmental governance.

2. A Mutual-Coupling Dipole Model for Calculating the Electric Field Distribution in a Two-Particle System

Two particles of radius R and permittivity ε_i are placed in environmental media with the permittivity ε_e and are polarized by an external uniform electric field E_0. If the ratio of permittivity is relatively small and the distance between two particles is larger than the radius of the two particles, it is appropriate to use the dipole approximation model to calculate the distribution of the local electric field around the particle, which is equal to calculating the distribution of the electric field of the particle in an infinitely large external uniform electric field E_0. The dipole moment can be expressed as:

$$M_0 = \frac{\varepsilon_\mathrm{i} - \varepsilon_\mathrm{e}}{\varepsilon_\mathrm{i} + 2\varepsilon_\mathrm{e}} 4\pi\varepsilon_\mathrm{e} R^3 E_0 = 4\pi k E_0 \varepsilon_\mathrm{e} R^3, \tag{1}$$

where M_0 is the dipole moment, with $k = (\varepsilon_\mathrm{i} - \varepsilon_\mathrm{e})/(\varepsilon_\mathrm{i} + 2\varepsilon_\mathrm{e})$.

According to the dipole approximation model, the electric field outside a particle can be expressed in spherical coordinates as follows:

$$\boldsymbol{E} = \frac{\boldsymbol{M}_0}{4\pi\varepsilon_\mathrm{e} r^3}(2\boldsymbol{r}_0 \cos\theta + \boldsymbol{\theta}_0 \sin\theta), \tag{2}$$

where $\boldsymbol{E}$ is the local electric field generated by the dipole near the sphere of the particle, M_0 is the dipole moment, r is the radius outside the sphere, $\boldsymbol{r}_0$ is the unit radial vector, θ is the horizontal angle, and $\boldsymbol{\theta}_0$ is the unit angle vector.

According to the dipole approximation model, for a particle in a uniform electric field, the electric field outside a particle is as follows:

$$\boldsymbol{E}_\mathrm{out} = \frac{\boldsymbol{M}_0}{4\pi\varepsilon_\mathrm{e} r^3}(2\boldsymbol{r}_0 \cos\theta + \boldsymbol{\theta}_0 \sin\theta) + \boldsymbol{E}_0, \tag{3}$$

where $\boldsymbol{E}_\mathrm{out}$ is the electric field outside the particle.

At the left pole of the sphere and along the direction of the electric field, according to Equation (3), the dipole approximation model for calculating the actual electric field is:

$$E'_\mathrm{T} = E_0 + \frac{2M_0}{4\pi\varepsilon_\mathrm{e} R^3} = (2k+1)E_0, \tag{4}$$

At the upper pole of the sphere and in the direction of the vertical outfield, according to Equation (3), the dipole approximation model for calculating the actual electric field is:

$$E'_\mathrm{L} = E_0 - \frac{M_0}{4\pi\varepsilon_\mathrm{e} R^3} = (1-k)E_0. \tag{5}$$

When the distance between the two particles is large, the above describes the process of calculating the electric field near the particle sphere using the dipole approximation model, which ignores the influence of the interactions between particle when calculating the particle electric field. However, when the distance is small, there are interactions between the particles. In other words, during the polarization process of the applied electric field, one particle is affected by the polarization of the electric field produced by the other particles. This influence is mutual for two particles when the distance between them is small; their own polarization processes will be affected by the combined action from the applied electric field and the electric field of polarization from another particle. The interaction between particles is the key to accurately calculating the electric field in a two-particle system with a small distance. For this reason, the following ideas are proposed to solve this problem: This paper considers that, when the distance between two particles is small, the electric field that causes the polarization process of the particles includes the external field and the polarized electric field from another particle. The particle polarization process caused by the external electric field is characterized by the self-dipole moment, and that caused by another particle is characterized by the mutual-coupling dipole moment; this process is similar to that of another particle. Thus, for two-particle systems with a small distance, the actual moment of the particles is the mutual-coupling moment caused by the external electric field and the polarized electric field from the other particle. The specific mathematical definitions and descriptions of this concept are given in the following.

When the distance between two particles is small, the effect of the interactions between particles on the particles' polarization should be considered. It is assumed that, in a two-particle system, the actual moment of particle 1 is the self-coupling moment caused by the external electric field M_{11} plus the mutual-coupling moment M_{21}, which is caused by particle 2. The actual moment of particle 2 is the self-coupling moment caused by the external electric field M_{22} plus the mutual-coupling moment M_{12}, which is caused by particle 1. Therefore, considering this interaction, the new moments belonging to the two particles are $M_{1'} = M_{11} + M_{21}$ and $M_{2'} = M_{22} + M_{12}$.

From the above analysis, a mutual-coupling dipole model of two-particle systems considering the interactions between particles can be obtained, as shown in Figure 1. The radius of both particles is R, the distance between the two particles is D, and the horizontal uniform electric field is E_0. E_1 is defined as the sum of the field strength, which is caused by the electric field of the dipole moment generated by particle 2 and the external field at position 1. E_2 is the sum of the field strength caused by the electric field of the dipole moment generated by particle 1 and the external field at position 2. $E_{1'}$ and $E_{2'}$ are the actual electric fields at positions 1 and 2, respectively.

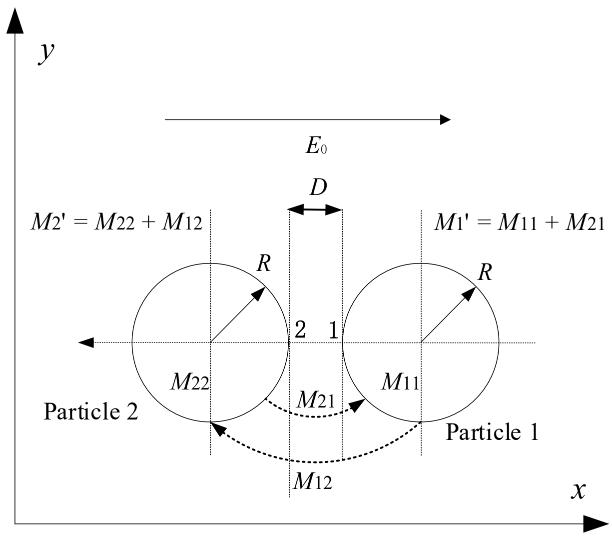

Figure 1. A mutual-coupling dipole model in a two-particle system.

2.1. Transverse External Electric Field

When the line connecting the centers of the particles is parallel to the applied electric field, it is called the transverse external electric field. In this situation, at positions 1 and 2, the extra-sphere electric fields are $E_{1\mathrm{T}}$ and $E_{2\mathrm{T}}$. The actual electric fields $E_{1\mathrm{T}}{}'$ and $E_{2\mathrm{T}}{}'$ are:

$$\begin{cases} E_{1\mathrm{T}} = E_{0\mathrm{T}} + \frac{2M'_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}}(R+D)^3} \\ E'_{1\mathrm{T}} = E_{1\mathrm{T}} + \frac{2M'_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}}R^3} \end{cases}, \tag{6}$$

$$\begin{cases} E_{2\mathrm{T}} = E_{0\mathrm{T}} + \frac{2M'_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}}(R+D)^3} \\ E'_{2\mathrm{T}} = E_{2\mathrm{T}} + \frac{2M'_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}}R^3} \end{cases}, \tag{7}$$

where $E_{0\mathrm{T}}$ denotes the transverse external electric field. In the transverse external electric field, considering the interaction, the new dipole moments belonging to particles 1 and 2 are $M_{1\mathrm{T}}{}'$and $M_{2\mathrm{T}}{}'$, respectively, which satisfy:

$$\begin{cases} M'_{1\mathrm{T}} = M_{11\mathrm{T}} + M_{21\mathrm{T}} \\ M'_{2\mathrm{T}} = M_{22\mathrm{T}} + M_{12\mathrm{T}} \end{cases}. \tag{8}$$

In particular, if the materials and radii of the two particles are identical, the same self-coupling dipole moments belong to these particles, and the mutual-coupling dipole moment caused by the interaction is also the same. Namely, $M_{1\mathrm{T}}{}' = M_{2\mathrm{T}}{}'$.

According to Equation (4), by solving Equations (6) and (7) and considering them as a linear approximation, the actual electric field can be obtained as follows:

$$\begin{cases} E'_{1\mathrm{T}} = (2k+1)E_{1\mathrm{T}} \\ E'_{2\mathrm{T}} = (2k+1)E_{2\mathrm{T}} \end{cases}. \tag{9}$$

At the same time, in Equations (6)–(9), when the external electric field is transverse, the new dipole moment of the particles considering the influence of their interaction is:

$$M'_{1\mathrm{T}} = M'_{2\mathrm{T}} = 4\pi k\varepsilon_{\mathrm{e}}E_{0\mathrm{T}}R^3\left[\frac{(R+D)^3}{(R+D)^3 - 2kR^3}\right]. \tag{10}$$

The self-coupling dipole moments belonging to particles 1 and 2, $M_{11\mathrm{T}}$ and $M_{22\mathrm{T}}$, are:

$$M_{11\mathrm{T}} = M_{22\mathrm{T}} = \frac{\varepsilon_{\mathrm{i}} - \varepsilon_{\mathrm{e}}}{\varepsilon_{\mathrm{i}} + 2\varepsilon_{\mathrm{e}}}4\pi\varepsilon_{\mathrm{e}}R^3E_{0\mathrm{T}} = 4k\pi\varepsilon_{\mathrm{e}}E_{0\mathrm{T}}R^3. \tag{11}$$

In this case, the mutual-coupling dipole moment is expressed as:

$$M'_{12\mathrm{T}} = M'_{21\mathrm{T}} = 4\pi k\varepsilon_{\mathrm{e}}E_{0\mathrm{T}}R^3\left[\frac{2kR^3}{(R+D)^3 - 2kR^3}\right]. \tag{12}$$

From the above results, in the transverse external electric field, the distribution of the local electric field in the vicinity of the sphere can be obtained; the calculation model is shown in Figure 2. Considering the symmetry of the electric field's distribution, the position at one-half of the arc from a to b on particle 1 is selected for the calculation. The origin is at the center of particle 1, the x-axis is in the direction of the external electric field E_0, the y-axis is perpendicular to the direction of the external electric field E_0, and θ is the angle between the x-axis and the connecting line between point m and the center of the circle (point m is an arbitrary point on the upper half-circle of particle 1), which rotates counterclockwise.

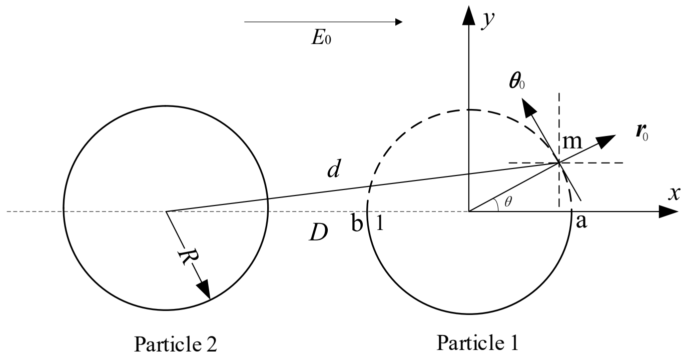

Figure 2. The calculation model for the two-particle system in the transverse external electric field.

Referring to Equation (2), which describes the electric field created by the point dipole surface of a particle sphere in spherical coordinates, the surface electric field of particle 1 is calculated as follows:

$$\boldsymbol{E}_{21\mathrm{T}} = \frac{\boldsymbol{M'}_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} d_{\mathrm{T}}{}^{3}}(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{13}$$

$$\boldsymbol{E}_{10\mathrm{T}} = \frac{\boldsymbol{M'}_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} R^{3}}(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{14}$$

$$\boldsymbol{E'}_{1\mathrm{T}} = \boldsymbol{E}_{10\mathrm{T}} + \boldsymbol{E}_{21\mathrm{T}} = \left(\frac{\boldsymbol{M'}_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} d_{\mathrm{T}}{}^{3}} + \frac{\boldsymbol{M'}_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} R^{3}}\right)(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{15}$$

where $\boldsymbol{E}_{10\mathrm{T}}$ and $\boldsymbol{E}_{21\mathrm{T}}$ are the electric fields of particles 1 and 2, respectively. $\boldsymbol{E}_{1\mathrm{T}}{}'$ is the actual electric field in this case, d_{T} is the distance from the center of particle 2 to a point on the surface of particle 1, and $d_{\mathrm{T}} = \sqrt{(2R + D + R\cos\theta)^2 + (R\sin\theta)^2}$.

Under the condition of a transverse electric field, by solving Equations (13)–(15), the analytical expressions of the electric field components E_{xT} and E_{yT}, which are along the *x*-axis and *y*-axis of the electric field on the upper surface of particle 1, can result in:

$$\begin{cases} E_{\mathrm{xT}} = \left(\frac{M'_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} d_{\mathrm{T}}{}^{3}} + \frac{M'_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} R^{3}}\right)\left(2\cos^2\theta - \sin^2\theta\right) + E_{0\mathrm{T}} \\ E_{\mathrm{yT}} = \left(\frac{M'_{2\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} d_{\mathrm{T}}{}^{3}} + \frac{M'_{1\mathrm{T}}}{4\pi\varepsilon_{\mathrm{e}} R^{3}}\right)(3\sin\theta\cos\theta) \end{cases}. \tag{16}$$

According to the mutual-coupling dipole model, the analytical expressions of the electric field at position a to b on the surface of particle 1 are calculated using the following equation:

$$E'_{1\mathrm{T}} = \sqrt{E_{\mathrm{xT}}{}^2 + E_{\mathrm{yT}}{}^2}. \tag{17}$$

2.2. Longitudinal External Electric Field

When the line connecting the centers of the particles is perpendicular to the applied electric field, it is called a longitudinal external electric field. In this situation, at positions 1 and 2, the extra-sphere electric fields $E_{1\mathrm{L}}$ and $E_{2\mathrm{L}}$ and the actual electric fields $E_{1\mathrm{L}}{}'$and $E_{2\mathrm{L}}{}'$ are:

$$\begin{cases} E_{1\mathrm{L}} = E_{0\mathrm{L}} - \frac{M'_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}(R+D)^3} \\ E'_{1\mathrm{L}} = E_{1\mathrm{L}} - \frac{M'_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}} R^3} \end{cases}, \tag{18}$$

$$\begin{cases} E_{2\mathrm{L}} = E_{0\mathrm{L}} - \frac{M'_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}(R+D)^3} \\ E'_{2\mathrm{L}} = E_{2\mathrm{L}} - \frac{M'_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}} R^3} \end{cases} \tag{19}$$

where E_{0L} denotes the longitudinal external electric field. In the longitudinal external electric field, considering the interaction, the new dipole moments belonging to particles 1 and 2 are M_{1L}'and M_{2L}', respectively, which satisfy:

$$\begin{cases} M'_{1L} = M_{11L} + M_{21L} \\ M'_{2L} = M_{22L} + M_{12L} \end{cases} \tag{20}$$

In particular, if the materials and radii of the two particles are identical, the same self-coupling dipole moments belong to these particles, and the mutual-coupling dipole moment caused by the interaction is also the same. Namely, $M_{1L}' = M_{2L}'$.

According to Equation (5), solving the pairs (18) and (19) is considered a linear approximation, and the actual electric field can be obtained:

$$\begin{cases} E'_{1L} = (1-k)E_{1L} \\ E'_{2L} = (1-k)E_{2L} \end{cases} \tag{21}$$

At the same time, in Equations (18)–(21), when the external electric field is longitudinal, the new dipole moment of the particles considering the influence of the interaction of the particles is:

$$M'_{1L} = M'_{2L} = 4k\pi\varepsilon_e R^3 E_{0L}\left[\frac{(R+D)^3}{(R+D)^3 + kR^3}\right] \tag{22}$$

The self-coupling dipole moments belonging to particles 1 and 2, M_{11L} and M_{22L}, are:

$$M_{11L} = M_{22L} = \frac{\varepsilon_i - \varepsilon_e}{\varepsilon_i + 2\varepsilon_e} 4\pi\varepsilon_e R^3 E_{0L} = 4k\pi\varepsilon_e E_{0L} R^3 \tag{23}$$

In this case, the mutual-coupling dipole moment is expressed as:

$$M'_{12L} = M'_{21L} = 4\pi k\varepsilon_e E_{0L} R^3\left[\frac{-kR^3}{(R+D)^3 + kR^3}\right] \tag{24}$$

The calculation method is the same as that when using the mutual dipole model in the horizontal direction when the electric field direction becomes perpendicular to the connecting line between the centers of the two particles; the calculation model is shown in Figure 3. The position at one-half of the arc from p to q on particle 1 is selected for the calculation. The origin is the center of particle 1, the y-axis is in the direction of the external electric field E_0, the x-axis is perpendicular to the direction of the external electric field E_0, and θ is the angle between the y-axis and the connecting line between point m and the center of the circle, which rotates counterclockwise.

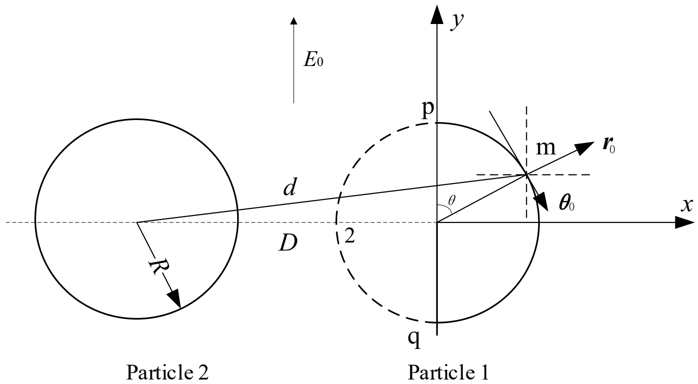

Figure 3. The calculation model for the two-particle system in the longitudinal external electric field.

Referring to Equation (2), which describes the electric field created by the point dipole surface of a particle sphere in spherical coordinates, the distribution of the surface electric field of particle 1 is calculated as follows:

$$\boldsymbol{E}_{21\mathrm{L}} = \frac{\boldsymbol{M'}_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}{d_{\mathrm{L}}}^3}(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{25}$$

$$\boldsymbol{E}_{10\mathrm{L}} = \frac{\boldsymbol{M'}_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}R^3}(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{26}$$

$$\boldsymbol{E'}_{1\mathrm{L}} = \boldsymbol{E}_{10\mathrm{L}} + \boldsymbol{E}_{21\mathrm{L}} = \left(\frac{\boldsymbol{M'}_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}{d_{\mathrm{L}}}^3} + \frac{\boldsymbol{M'}_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}R^3}\right)(2\boldsymbol{r}_0 cos\theta + \boldsymbol{\theta}_0 sin\theta), \tag{27}$$

where $\boldsymbol{E}_{10\mathrm{L}}$ and $\boldsymbol{E}_{21\mathrm{L}}$ are the electric fields of particles 1 and 2, respectively. $\boldsymbol{E}_{1\mathrm{L}}'$ is the actual electric field in this case, d_{L} is the distance from the center of particle 2 to a point on the surface of particle 1, and $d_{\mathrm{L}} = \sqrt{(2R + D + R\sin\theta)^2 + (R\cos\theta)^2}$.

Under the condition of a longitudinal electric field, by solving Equations (25)–(27), the analytical expressions of the electric field components E_{xL} and E_{yL}, which are along the x-axis and y-axis of the electric field on the upper surface of particle 1, can result in:

$$\begin{cases} E_{\mathrm{xL}} = \left(\frac{M'_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}d^3} + \frac{M'_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}R^3}\right)(3\sin\theta\cos\theta) \\ E_{\mathrm{yL}} = \left(\frac{M'_{2\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}d^3} + \frac{M'_{1\mathrm{L}}}{4\pi\varepsilon_{\mathrm{e}}R^3}\right)\left(2\cos^2\theta - \sin^2\theta\right) + E_{0\mathrm{L}} \end{cases}. \tag{28}$$

According to the mutual-coupling dipole model, the analytical expressions of the electric field at positions a to b on the surface of particle 1 are calculated using the following equation:

$$E'_{1\mathrm{L}} = \sqrt{{E_{\mathrm{xL}}}^2 + {E_{\mathrm{yL}}}^2}. \tag{29}$$

The basic theoretical equations of the mutual-coupling dipole model in the transverse and longitudinal fields are obtained in this section, and the theoretical solution of the electric field's distribution in the two-particle system is complete in the transverse and longitudinal fields. To facilitate the application of this method in different fields, such as those of high-voltage insulation, medicine, biology, nano-fluids, and micro-fluidic control, the theoretical equations of the mutual-coupling dipole model used in different situations are summarized and described by a flowchart, as shown in Figure 4.

Because many parameters are used in this study, in order to make it easier for readers to understand and apply the results, all of the parameters used in this study are summarized and shown in Table 1.

Table 1. The parameters used in this paper.

Parameters	Meaning
R	the radius of the two particles
D	the distance between the two particles
ε_{i}	the permittivity of the two particles
ε_{e}	the permittivity of the environment
E_0	the external uniform electric field in the dipole approximation
M_0	the dipole moment
k	$k = (\varepsilon_i - \varepsilon_e)/(\varepsilon_i + 2\varepsilon_e)$
$\boldsymbol{E}$	The local electric field generated by the dipole model's vicinity the sphere of the particle
r	the radius outside the sphere
$\boldsymbol{r}_0$	unit radial vector
θ	horizontal angle
$\boldsymbol{\theta}_0$	the unit angle vector
$\boldsymbol{E}_{\mathrm{out}}$	the electric field outside the particle

Table 1. *Cont.*

Parameters	Meaning
$E_T{}'$	at the left pole of the sphere and along the direction of the electric field, the dipole approximation model for calculating the actual electric field
$E_L{}'$	at the upper pole of the sphere and in the direction of the vertical outfield, the dipole approximation model for calculating the actual electric field
E_{0T}	the transverse external electric field
E_{1T}	the extra-sphere electric field at position 1
E_{2T}	the extra-sphere electric field at position 2
$M_{1T}{}'$	new dipole moments belonging to particle 1 (in the transverse external electric field)
$M_{2T}{}'$	new dipole moments belonging to particle 2 (in the transverse external electric field)
M_{11T}	the self-coupling dipole moment belonging to particle 1 (when the external electric field is transverse)
M_{22T}	the self-coupling dipole moment belonging to particle 2 (when the external electric field is transverse)
$M_{12T}{}'$	the mutual-coupling dipole moment belonging to particle 1 (when the external electric field is transverse)
$M_{21T}{}'$	the mutual-coupling dipole moment belonging to particle 2 (when the external electric field is transverse)
d_T	$d_T = \sqrt{(2R + D + R\cos\theta)^2 + (R\sin\theta)^2}$
E_{xT}	along the *x*-axis, the analytical expressions of the electric field components on the upper surface of particle 1 (when the external electric field is transverse)
E_{yT}	along the *x*-axis, the analytical expressions of the electric field components on the upper surface of particle 1 (when the external electric field is transverse)
E_{1T}	the analytical expressions of the electric field at position a to b on the surface of particle 1
E_{21T}	the electric field of particle 2
E_{10T}	the electric field of particle 1
$E_{1T}{}'$	actual electric field at position 1
$E_{2T}{}'$	actual electric field at position 2
E_{0L}	the longitudinal external electric field
E_{1L}	the extra-sphere electric field at position 1
E_{2L}	the extra-sphere electric field at position 2
$M_{1L}{}'$	new dipole moments belonging to particle 1 (in the longitudinal external electric field)
$M_{2L}{}'$	new dipole moments belonging to particle 2 (in the longitudinal external electric field)
M_{11L}	the self-coupling dipole moment belonging to particle 1 (when the external electric field is longitudinal)
M_{22L}	the self-coupling dipole moment belonging to particle 2 (when the external electric field is longitudinal)
$M_{12L}{}'$	the mutual-coupling dipole moment belonging to particle 1 (when the external electric field is longitudinal)
$M_{21L}{}'$	the mutual-coupling dipole moment belonging to particle 2 (when the external electric field is longitudinal)
d_L	$d_L = \sqrt{(2R + D + R\sin\theta)^2 + (R\cos\theta)^2}$
E_{xL}	along the *x*-axis, the analytical expressions of the electric field components on the upper surface of particle 1 (when the external electric field is longitudinal)
E_{yL}	Along the *x*-axis, the analytical expressions of the electric field components on the upper surface of particle 1 (when the external electric field is longitudinal)
E_{21L}	the electric field of particle 2
E_{10L}	the electric field of particle 1
$E_{1L}{}'$	the actual electric field
δ_1	the relative errors of the results calculated with the mutual-coupling dipole model
δ_2	the relative errors of the results calculated with the dipole model
$E_{\text{Mutual-coupling}}$	the result calculated with the mutual-coupling dipole model
E_{FEM}	the result calculated with the finite element method
E_{Dipole}	the result calculated with the dipole approximation model

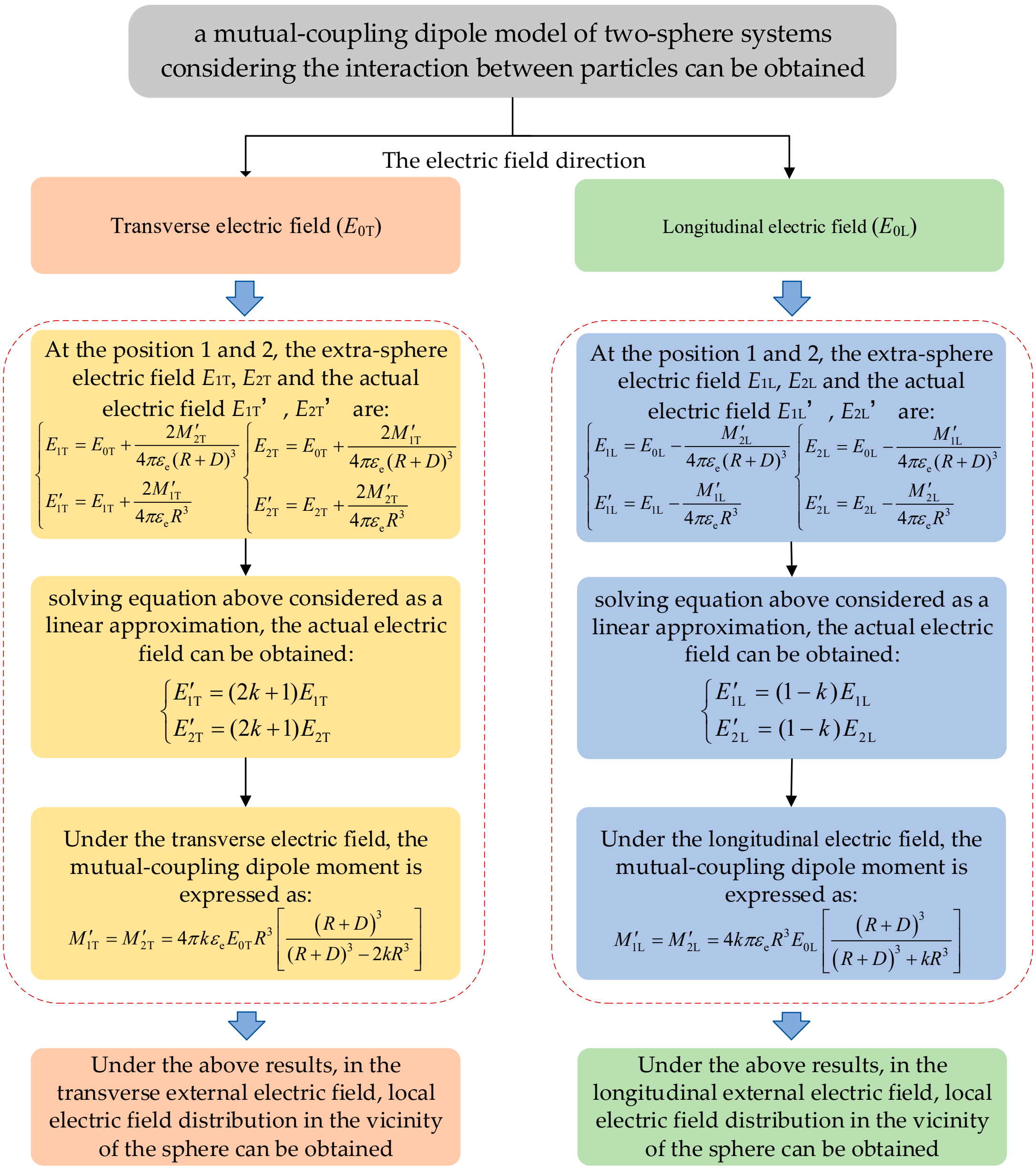

Figure 4. The mutual-coupling dipole model for calculating the equations under different scenarios.

3. Analysis and Comparison of Results

The strength of the local electric field in any mixture can be accurately calculated using the finite element method. The results of the mutual-coupling dipole model and the dipole approximation model were compared with those of the finite element method to verify the validity of the mutual-coupling dipole model. The finite element method was carried out by using the electrostatic field module of the COMSOL 5.6 finite element software. The solution of the electric field of the two-particle system was simplified into a two-dimensional model because of the symmetry of the structure. A computer with an 11th Gen Intel Core (TM) i7, a hard disk of 500 G, and a computer memory of 16 GB was used in the calculation of the electric field of the two-particle system. The size of the calculation domain was 1500 times the radius of the particle. The model used a free triangular mesh with a minimum element of three microns and a relative tolerance of 0.001. The external electric field E_0 was set to 1×10^5 V/m, the particle radius R was 1×10^{-4} m, ε_i was 5, 10, 25, and 50, and the distance between the two particles D was 0.2 R, 0.6 R, R, and 1.5 R. In some dielectric mixtures, the finite element method could accurately obtain the local field strength. The results were calculated using the mutual-coupling dipole model and dipole approximation model, which were, respectively, compared with the results of the calculation based on the finite element method to verify the effectiveness of the results of the calculation of the mutual-coupling dipole model.

The results calculated with the finite element method were taken as the standard for analyzing the errors of the results obtained with the mutual-coupling dipole model and dipole approximation model. The relative error was calculated using the following formula:

$$\delta_1 = \frac{\left|E_{\text{Mutual-Coupling}} - E_{\text{FEM}}\right|}{E_{\text{FEM}}} \times 100\%, \tag{30}$$

$$\delta_2 = \frac{\left|E_{\text{Dipole}} - E_{\text{FEM}}\right|}{E_{\text{FEM}}} \times 100\%, \tag{31}$$

where δ_1 and δ_2 are the relative errors of the results calculated using the mutual-coupling dipole model and dipole approximation model, respectively. $E_{\text{Mutual-coupling}}$ is the result calculated using the mutual-coupling dipole model, E_{FEM} is the result calculated using the finite element method, and E_{Dipole} is the result calculated using the dipole approximation model.

At the same time, the overall error between the results calculated with the mutual-coupling dipole model, the dipole approximation model, and the finite element method was calculated. In this study, the Euclidean distance was used to obtain the overall similarity of the two results. The Euclidean distance was calculated as:

$$dist(X, Y) = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}, \tag{32}$$

where X represents the resulting curve calculated with the mutual-coupling dipole model or the dipole approximation model, and Y represents the resulting curve calculated with the finite element method. x_{i} is the electric field value of the resulting curve calculated with the mutual-coupling dipole model or the dipole approximation model at the *i*th point, and y_{i} is the electric field value of the resulting curve calculated with the finite element method at the *i*th point. i = 1,2,3 ... n ... , n is the number of curve sampling points.

The Euclidean distance was used to measure the characteristics of the overall similarity of the two calculation results with the following formula:

$$\text{sim}(X, Y) = \frac{1}{1 + \text{dist}(X, Y)}. \tag{33}$$

According to the above formula, the Euclidean distance calculated between the two curves can be converted into (0,1]. Generally, the similarity is $0 \leq$ sim $(x,y) \leq 1$. The closer the value is to 1, the higher the similarity.

3.1. Transverse External Electric Field

Under the transverse external electric field, to verify the mutual-coupling dipole model, the electric field distributions were calculated with the mutual-coupling dipole model, the dipole approximation model, and the finite element method in certain situations; when the ratio of the distance (between two particles) to the particle radius D/R was 0.2, 0.6, 1, or 1.5, the ratio of the permittivity of the particle to that of the environment $\varepsilon_{\text{i}}/\varepsilon_{\text{e}}$ was 5, 10, 25, or 50, respectively. The calculation results for the three models are shown in Figures 5–8.

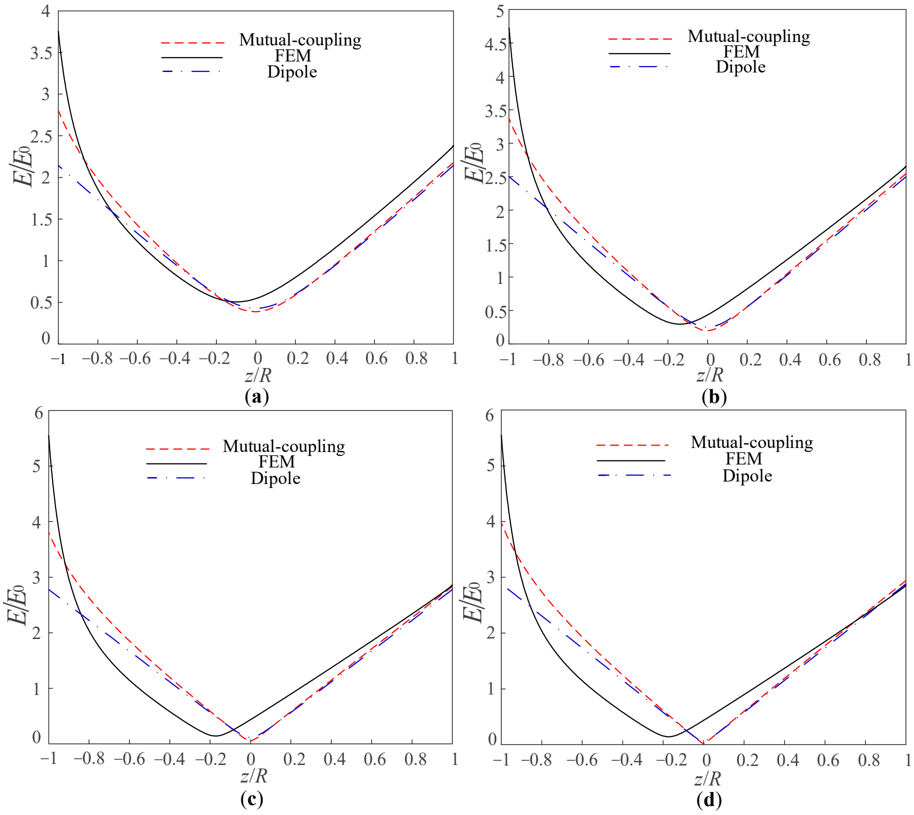

Figure 5. Calculation results of the electric field distribution on particle 1 when D/R = 0.2: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 5; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 10; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 25; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 50.

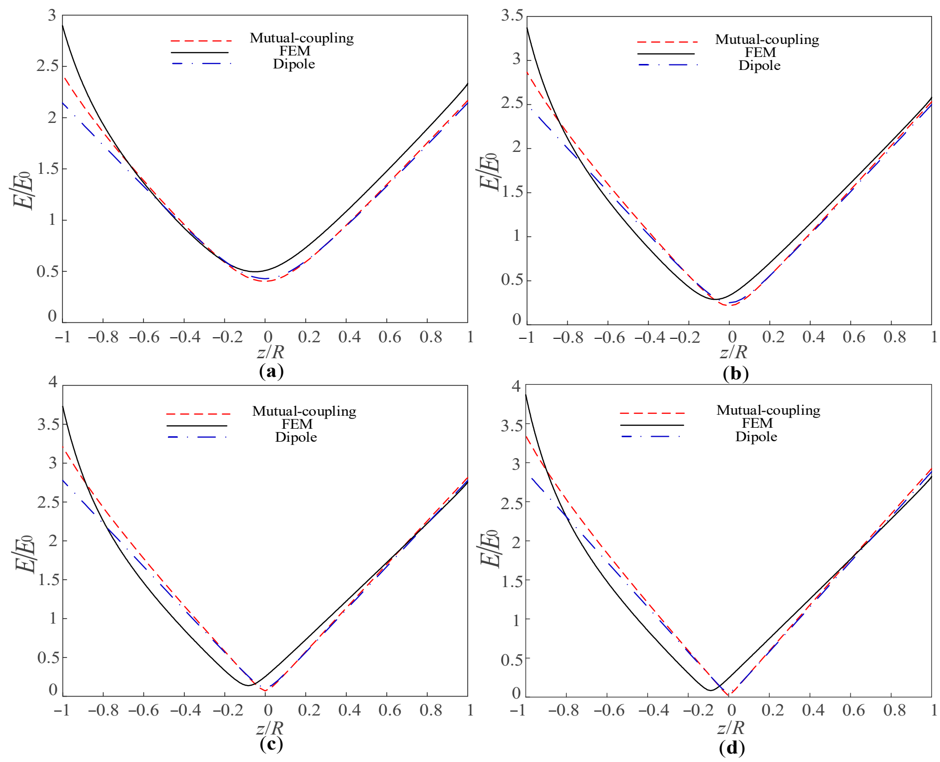

Figure 6. Calculation results of the electric field distribution on particle 1 when D/R = 0.6: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 5; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 10; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 25; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 50.

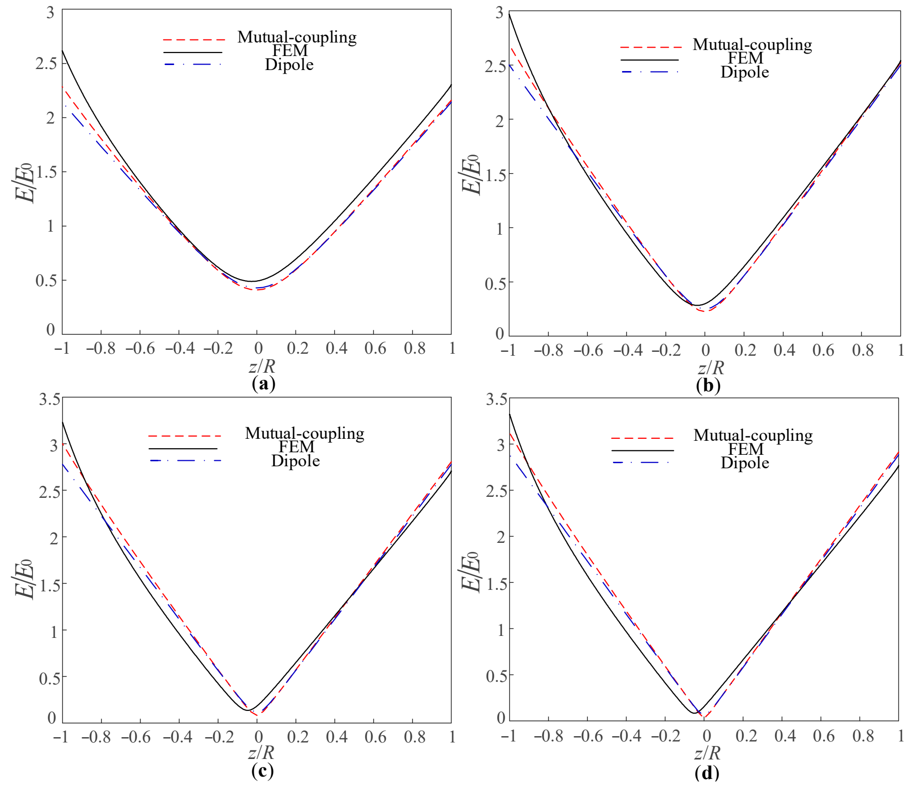

Figure 7. Calculation results of the electric field distribution on particle 1 when D/R = 1: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 5; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 10; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 25; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 50.

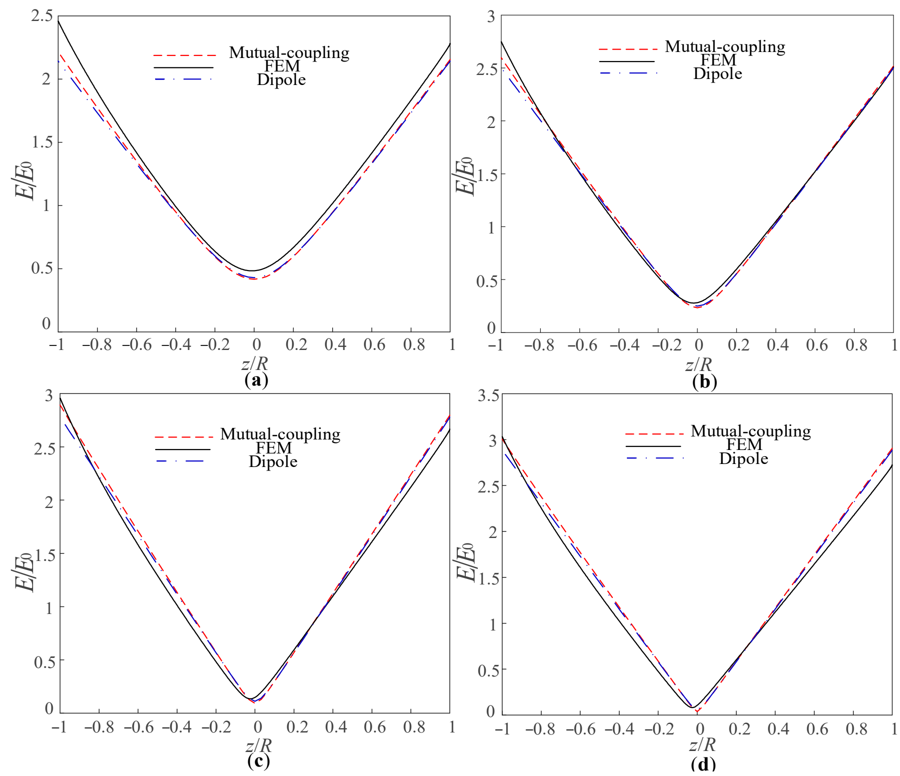

Figure 8. Calculation results of the electric field distribution on particle 1 when D/R = 1.5: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 5; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 10; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 25; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 50.

Figure 5 shows that the electric field distribution was calculated with the mutual-coupling dipole model, the dipole approximation model, and the finite element method from a to b on the surface of particle 1 in certain situations, which were when the ratio of the distance (between the two particles) to the particle radius D/R was 0.2 and the ratio of permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. It is clear from Figure 4 that, in the right part of the arc from a to b (from $z/R = 0$ to $z/R = 1$ in Figure 4), the calculation results of the mutual-coupling dipole model are consistent with the results of the dipole approximation model, and the difference in the results is mainly reflected in the left part of the a–b arc, which is the region that is significantly affected by the interaction between the two particles. The position $z/R = -1$ is the position with the most significant effect between the two particles, that is, the interaction near this position is the strongest and will have a great impact on the calculation of the electric field. Based on the results of the finite element method, the calculation accuracy of the mutual-coupling dipole model is much higher than that of the dipole approximation model under different permittivity ratios. The results show that, when the distance is small, in comparison with the dipole approximation model used to calculate the electric field in two- or multi-particle systems, the mutual-coupling dipole model proposed in this paper can more effectively reflect the influence of particle interactions on the local electric field, especially near the position where the particle interactions are most significant. The mutual-coupling dipole model proposed in this paper can take into account the interactions of particles and has a higher accuracy when calculating electric fields. It will contribute to the understanding of the interactions of particles and to achieving particle control in many fields, such as those of electrical insulation, medicine, and nano-fluids.

Figure 6 shows that the electric field distribution was calculated with the three models from a to b on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 0.6 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. Comparing Figures 5 and 6, it is clear that, with the increase in the distance (between the two particles), the error between the results calculated with the mutual-coupling dipole model and the finite element method gradually decreased. When ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ is 5, for the region where the interaction is significant (from $z/R = -1$ to $z/R = 0$ in Figure 5), the results calculated with the mutual-coupling dipole model are close to those of the finite element method, and its accuracy is better than that of the dipole approximation model; especially at the position ($z/R = -1$), which has the strongest interaction, the mutual-coupling dipole model has a higher accuracy. The calculation error between the mutual-coupling dipole model and the finite element method is 16.41% (at $z/R = -1$), while this figure for the dipole approximation is 26.04%. This indicates that particle interactions have a significant effect on the electric field distribution. The mutual-coupling dipole model proposed in this paper can take into account the influence of the interactions caused by particles on the electric field very well. Compared with the dipole approximation, the mutual-coupling dipole model improves the accuracy of the field calculation by about 10% by considering the interactions caused by particles.

With an increase in the permittivity ratio, there is a gradual decrease in the accuracy of the mutual-coupling dipole model, but its overall calculation result is better than that of the dipole approximation model.

Figure 7 shows that the electric field distribution was calculated with the three models from a to b on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 1 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. It can be seen in Figure 7 that, with a further increase in distance (between the two particles), the calculation error of the mutual-coupling dipole model shows a marked decline. At the same time, the results calculated with the mutual-coupling dipole model and the dipole approximation model gradually become closer with the increase in distance (between the two particles). This indicates that the interactions caused by particles are weakened with the increase in the distance, which is because the influence of a particle's moment on the polarization process of another particle decreases with the increase in distance, thus causing the calculation results of the mutual-coupling dipole model and the dipole approximation model to converge with the increase in distance. However, in the region where the interactions between the two particles are most significant (near the $z/R = -1$), the mutual-coupling dipole model still has obvious advantages over the dipole approximation model.

Figure 8 shows that the electric field distribution was calculated with the three models from a to b on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 1.5 and ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. In Figures 7 and 8, it is clear that, with a further increase in D/R, the calculation results of the mutual-coupling dipole model and the dipole approximation model tend to be more consistent. Additionally, the overall calculation error between the two models and the finite element model decreases further; in particular, when the ratio of permittivity $\varepsilon_i/\varepsilon_e$=10, the results calculated with the three models are very close. The results illustrate that, under the transverse field, when $D/R > 1.5$, the interactions between the two particles show a marked weakening. In other words, when $D/R > 1.5$, the polarized electric field of a particle will not have much of an influence on the polarization process of the other particle, and the results calculated with the mutual-coupling dipole model and the dipole approximation model tend to be more consistent. In a certain application case, $D/R = 1.5$ can be used as an important criterion for whether or not to consider the interactions of particles in the transverse electric field. That is to say, when the ratio of the distance and the particle radius D/R is less than 1.5, the interactions between particles need to be considered, while when $D/R > 1.5$, the interactions between particles can be ignored.

Compared with other methods, the innovation and contribution of the mutual-coupling dipole model proposed in this paper are mainly that it considers the influence of interactions on the particle polarization process and the local electric field. The consideration of the effect of interactions between particles in a two-particle system on the distribution of the local electric field is an innovation of the mutual-coupling dipole model. The most obvious position at which the influence between the two particles is the strongest is the point of the pole adjacent to the two particles, where the electric field distribution shows a significant change caused by the interactions between the two particles. Therefore, three models were used to calculate the value of the electric field, analyze the error at this point, and analyze the similarity of the corresponding electric field calculation curves for the half arc in which the two particles are close to each other.

Under a transverse external electric field, the relative errors of the results of the calculation of the electric field with the mutual-coupling dipole model, the dipole approximation model, and the finite element method at that position are shown in Table 2. The similarities of the curves among the calculation results of the three models are presented in Table 3.

Table 2. The relative errors of the three models at position 1 (mutual-coupling/FEM and dipole/FEM).

D/R	**Position**	**Mutual-Coupling/FEM and Dipole/FEM**	$\varepsilon_i/\varepsilon_e$			
			5	**10**	**25**	**50**
0.2	1	Mutual-coupling/FEM	25.34%	28.69%	31.40%	32.43%
		Dipole/FEM	42.95%	47.07%	49.94%	50.97%
0.6	1	Mutual-coupling/FEM	16.41%	14.98%	13.94%	13.55%
		Dipole/FEM	26.04%	25.84%	25.57%	25.44%
1	1	Mutual-coupling/FEM	12.66%	9.48%	7.05%	6.13%
		Dipole/FEM	18.12%	15.79%	13.94%	13.22%
1.5	1	Mutual-coupling/FEM	9.86%	5.51%	2.17%	0.89%
		Dipole/FEM	12.83%	9.01%	6.02%	4.87%

Table 3. The similarity of the curves under a transverse external electric field (mutual-coupling/FEM and dipole/FEM).

D/R	Mutual-Coupling/FEM and Dipole/FEM	$\varepsilon_i/\varepsilon_e$			
		5	10	25	50
0.2	Mutual-coupling/FEM	0.3849	0.2533	0.1889	0.1712
	Dipole/FEM	0.2736	0.2121	0.1716	0.1586
0.6	Mutual-coupling/FEM	0.5214	0.4478	0.3515	0.3205
	Dipole/FEM	0.3888	0.3649	0.3244	0.3073
1	Mutual-coupling/FEM	0.5591	0.5958	0.4835	0.4388
	Dipole/FEM	0.4623	0.4988	0.6930	0.4492
1.5	Mutual-coupling/FEM	0.5834	0.7346	0.5933	0.5297
	Dipole/FEM	0.5227	0.6443	0.6254	0.5808

According to Table 2, it can be observed that, under a transverse external electric field and when D/R and $\varepsilon_i/\varepsilon_e$ are different, at position 1, where the interaction between the two particles is the strongest, the calculation results of the mutual-coupling dipole model show a better accuracy than those of the dipole approximation model. This means that, compared with the dipole approximation model, the mutual-coupling dipole model can effectively reflect the impact of the interaction on the electric field. In particular, when the distance is small and the interaction between particles is the strongest, the mutual-coupling dipole model has more advantages than the commonly used dipole approximation model. The accuracy of the mutual-coupling dipole model is much higher than that of the dipole approximation model. By analyzing the interactions caused by the particles over small distances, the mutual-coupling dipole model can provide an important reference for the micro-fields of medicine, biology, micro-fluidic control, and nano-particles. With an increase in the distance (between the two particles), the interaction between the two particles is gradually weakened, the errors of the mutual-coupling dipole model, dipole approximation model, and finite element method are reduced, and the results of the mutual-coupling dipole model and dipole approximation model are similar. In addition, it can be observed in Table 2 that, except for D/R = 0.2, the calculation error of the mutual-coupling dipole model gradually decreases with the increase in $\varepsilon_i/\varepsilon_e$ when D/R is 0.6, 1, and 1.5. In particular, when D/R = 1.5 and $\varepsilon_i/\varepsilon_e$ = 50, the error between the mutual-coupling dipole model and the finite element method is only 0.89%.

According to Table 3, when the D/R is less than 0.6 and the ratio of permittivity $\varepsilon_i/\varepsilon_e$ is different, the similarity of the curves between the calculation results of the mutual-coupling dipole model and the finite element model is higher than that obtained with the dipole approximation model and the finite element model. This shows that the mutual-coupling dipole model can calculate more accurately than the dipole approximation model when the distance (between the two particles) is small. It is clear that, compared with the dipole approximation model, the mutual-coupling dipole model is suitable for use in calculating the electric field when the distance is small. At the same time, it can be observed that, with an increase in the distance (between the two particles), the interaction between the two particles gradually weakens, and the similarity of the curves between the calculation results of the mutual-coupling dipole model and the finite element model gradually approaches that of the dipole approximation model and the finite element model. Additionally, with an increase in the distance between the two particles, the interaction between the two particles is reduced, and the similarity between the two curves of the two models (the mutual-coupling dipole model and finite element model) gradually increases. It is clear that the results calculated with the mutual-coupling dipole model and the dipole approximation model tend to be consistent under a larger particle distance (between the two particles), and the interaction between particles is substantially weakened.

3.2. Longitudinal External Electric Field

To verify the mutual-coupling dipole model under a longitudinal external electric field, the electric field distribution was calculated with the mutual-coupling dipole model, the dipole approximation model, and the finite element method in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 0.2, 0.6, 1, or 1.5 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. The calculation results of the three models are shown in Figures 9–12.

Figure 9 shows that the electric field distribution was calculated with the mutual-coupling dipole model, the dipole approximation model, and the finite element method from p to q on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 0.2 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. It is clear from Figure 8 that the mutual-coupling dipole model shows a better calculation accuracy than that of the dipole approximation model. Especially at the three pole points ($z/R = 0$, $z/R = -1$ and $z/R = 1$), the mutual-coupling dipole model shows an excellent calculation accuracy. When $D/R = 0.2$ and $\varepsilon_i/\varepsilon_e = 5$, the error in the calculation results between the dipole approximation model and the finite element method reaches 44.57% at the point that has the strongest interaction between the two particles, while this figure for the mutual-coupling dipole model is only 8.65%. This means that the interaction of particles has a significant effect on the electric field. The mutual-coupling dipole model proposed in this paper can take into account the effect of interactions on the electric field. Compared with the dipole approximation model, the mutual-coupling dipole model improves the accuracy of the electric field calculation by about 36% by considering interactions, and the accuracy of the mutual-coupling dipole model is better than that of the dipole approximation model at this point.

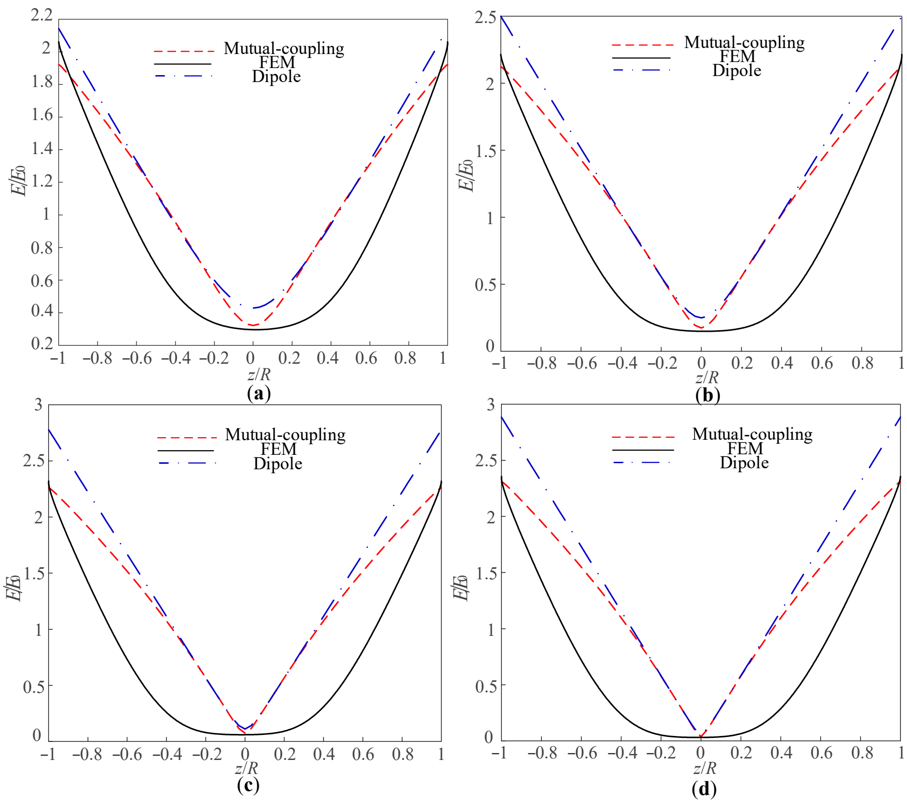

Figure 9. Calculation results of the electric field distribution on particle 1 when $D/R = 0.2$: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e = 5$; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e = 10$; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e = 25$; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e = 50$.

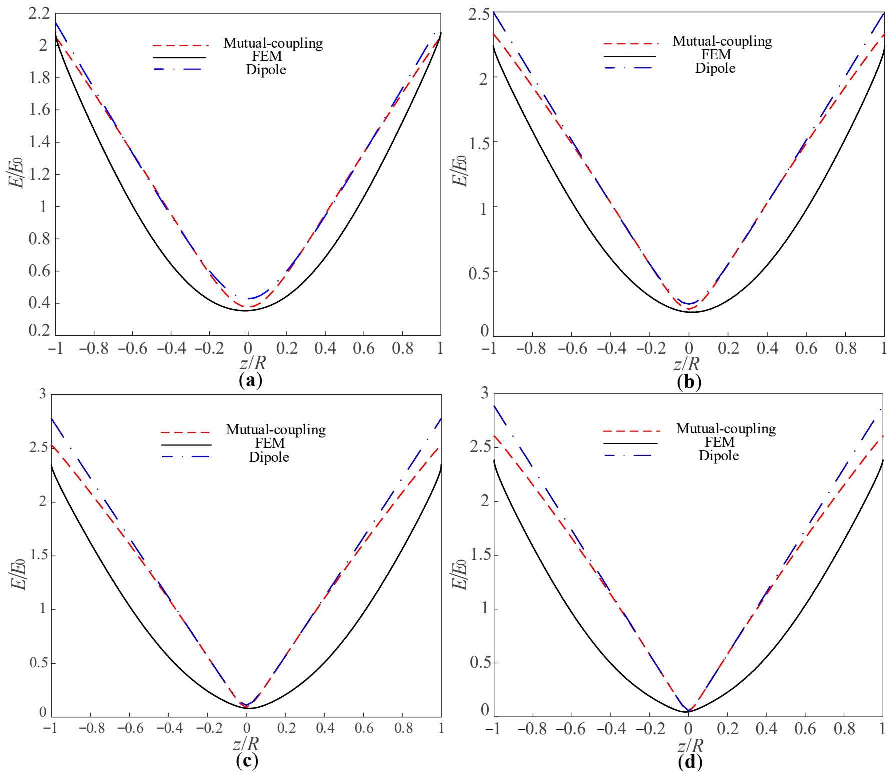

Figure 10. Calculation results of the electric field distribution on particle 1 when $D/R = 0.6$: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e = 5$; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e = 10$; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e = 25$; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e = 50$.

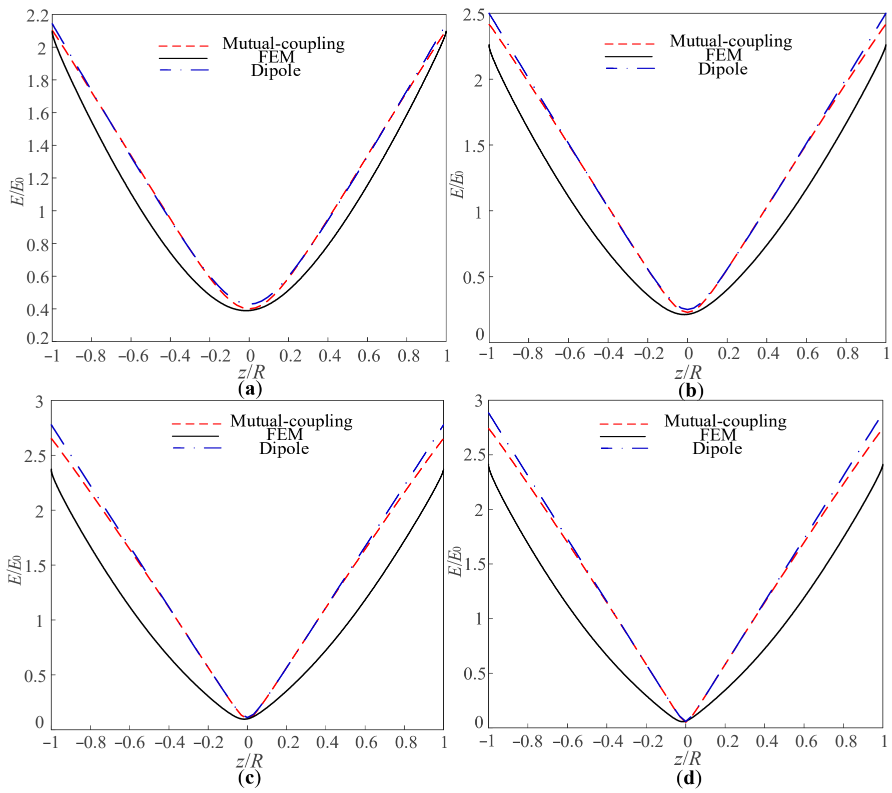

Figure 11. Calculation results of the electric field distribution on particle 1 when $D/R = 1$: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e = 5$; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e = 10$; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e = 25$; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e = 50$.

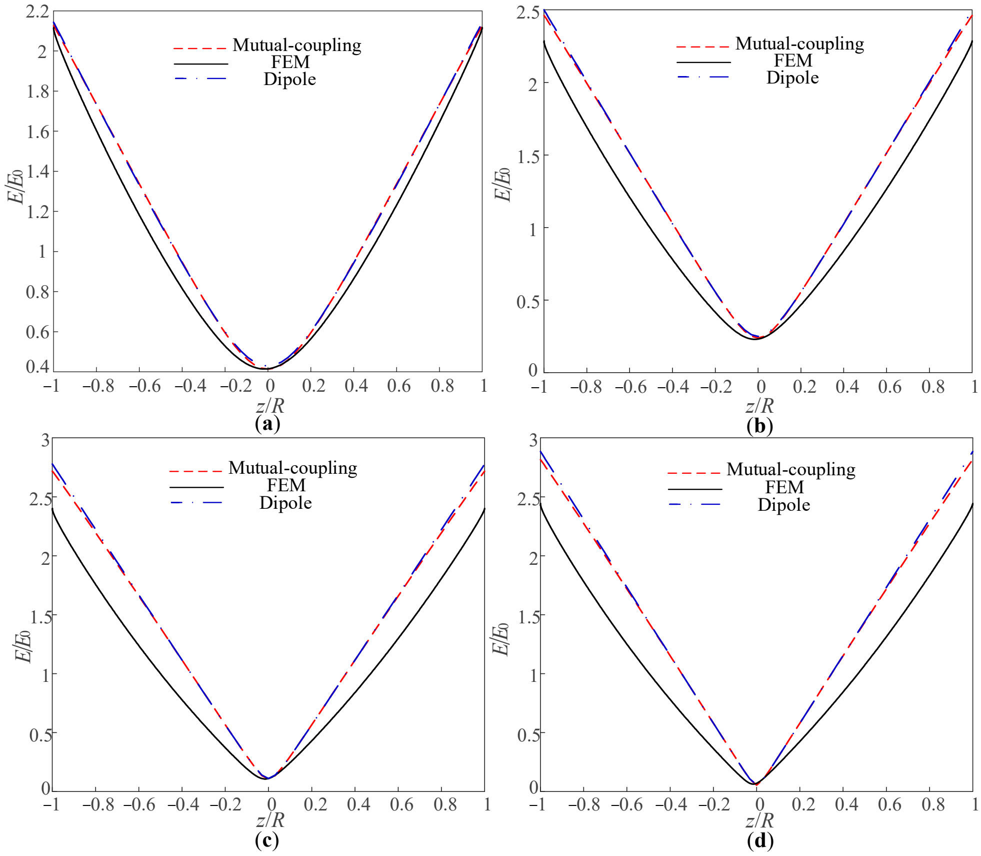

Figure 12. Calculation results of the electric field distribution on particle 1 when D/R = 1.5: (**a**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 5; (**b**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 10; (**c**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 25; (**d**) Calculation results when $\varepsilon_i/\varepsilon_e$ = 50.

The results show that, when the distance (between the two particles) is small, the interaction between the two particles affects the calculation of the electric field distribution, which can be clearly reflected by the mutual-coupling dipole model. Furthermore, with the increase in $\varepsilon_i/\varepsilon_e$, the results calculated with the mutual-coupling dipole model are close to the figure calculated with the dipole approximation near this position, whereas near the position of the maximum value of the electric field, the calculation accuracy of the mutual-coupling dipole model is much higher than that of the dipole approximation model.

Figure 10 shows that the electric field distribution was calculated with the three models from p to q on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 0.6 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. When the ratio of particle to environment permittivity $\varepsilon_i/\varepsilon_e$ is 5 at $z/R = 0$, compared with the finite element method, the calculation error of the mutual-coupling dipole model is 6.61%, which is much smaller than the figure for the dipole approximation model (20.97%). However, compared with Figure 9, the calculation errors of the mutual-coupling dipole model and the dipole approximation model decrease with the increase in the distance (between the two particles). As $\varepsilon_i/\varepsilon_e$ increases and when $\varepsilon_i/\varepsilon_e$=10, the deviation between the results calculated with the mutual-coupling dipole model and the finite element method gradually increases, but the overall error is still less than that of the results calculated with the dipole approximation model. With the further increase in $\varepsilon_i/\varepsilon_e$ until 25 or 50, the curve obtained by using the mutual-coupling dipole model is close to the curve calculated with the dipole approximation model (especially near the position $z/R = 0$). However, the calculation accuracy is still higher than that of the dipole approximation model (especially near $z/R = -1$ and $z/R = 1$).

Figure 11 shows that the electric field distribution was calculated with the three models from p to q on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 1 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. It is clear that, when the distance (between the two particles) increases to 1, the error between the results calculated with the mutual-coupling dipole model and the finite element method gradually decreases. When $\varepsilon_i/\varepsilon_e = 5$, the results calculated with the mutual-coupling dipole model are close to those of the dipole approximation model. However, with the increase in $\varepsilon_i/\varepsilon_e$, the overall calculation accuracy of the mutual-coupling dipole model is higher than that of the dipole approximation model under different levels of permittivity.

Figure 12 shows that the electric field distribution was calculated with the three models from p to q on the surface of particle 1 in certain situations in which the ratio of the distance (between the two particles) to the particle radius D/R was 1.5 and the ratio of the permittivity of the particle to that of the environment $\varepsilon_i/\varepsilon_e$ was 5, 10, 25, or 50. From Figures 11 and 12, it can be observed that, with a further increase in D/R, the results calculated with the mutual-coupling dipole model are closer to those of the dipole approximation model, and the calculation error between the above two methods and the finite element method further decreases. It is clear that, under a longitudinal external electric field, the interaction between the two particles shows a marked weakening after $D/R > 1.5$, that is, after $D/R > 1.5$, the polarized electric field of a particle has little effect on the polarization process of the other particle, and the results calculated with the mutual-coupling dipole model and the dipole approximation model are basically consistent. In a certain case, $D/R = 1.5$ can be used as an important criterion for considering the interactions of particles in a longitudinal external electric field. In other words, when the ratio of the distance and the particle radius D/R is less than 1.5, the interactions between particles need to be considered, and when D/R is greater than 1.5, the interactions between particles can be ignored.

Considering a longitudinal external electric field, the relative errors of the results of the calculation of an electric field with the mutual-coupling dipole model, the dipole approximation model, and the finite element method at a position are listed in Table 4. The similarities of the curves between the calculation results of the three models are presented in Table 5.

According to Table 4, under a longitudinal external electric field, it can be observed that, with a smaller distance between the two particles, there is a greater difference in the calculation of the electric field at position 2 (where the particles interact most strongly) between the mutual-coupling dipole model and the dipole approximation model. This indicates that the smaller the distance between two particles is, the more obvious the interaction between the two particles will be. In other words, when the distance is small, the polarized electric field has a significant influence on the polarization of the other particle. At this time, the electric field shows changes at position 2 because of the strong interaction between the particles, whereas the dipole approximation model produces a large calculation error. In Table 4, it can be observed that, with different values of $\varepsilon_i/\varepsilon_e$ and when the distance (between the two particles) is small (such as $D/R < 0.6$), the mutual-coupling dipole model, which considers the interaction of particles, has a much higher accuracy than that of the dipole approximation model. It is clear that the mutual-coupling dipole model can effectively reflect the interaction between two particles, which affects the calculation of the electric field distribution when the distance is small. This model not only has higher accuracy when calculating the electric field distribution, but can also compensate for the large error caused by the dipole approximation model when the distance (between the two particles) is small. It will have significance in the analysis of particle interactions over small distances in fields such as medicine, biology, micro-fluidic control, and nano-particles.

Table 4. The relative errors of the three models at position 2 (mutual-coupling/FEM and dipole/FEM).

D/R	Position	Mutual-Coupling/FEM and Dipole/FEM	$\varepsilon_i/\varepsilon_e$ 5	10	25	50
0.2	2	Mutual-coupling/FEM	8.65%	16.55%	21.41%	22.00%
		Dipole/FEM	44.57%	67.14%	83.87%	88.53%
0.6	2	Mutual-coupling/FEM	6.16%	12.47%	10.12%	1.55%
		Dipole/FEM	20.97%	33.07%	25.38%	21.10%
1	2	Mutual-coupling/FEM	3.57%	1.01%	8.15%	15.56%
		Dipole/FEM	9.88%	17.02%	11.78%	5.61%
1.5	2	Mutual-coupling/FEM	0.41%	2.94%	5.49%	23.07%
		Dipole/FEM	3.23%	7.88%	0.11%	18.43%

Table 5. The similarity of the curves under a longitudinal external electric field (mutual-coupling/FEM and dipole/FEM).

D/R	Mutual-Coupling/FEM and Dipole/FEM	$\varepsilon_i/\varepsilon_e$ 5	10	25	50
0.2	Mutual-coupling/FEM	0.2969	0.2245	0.2059	0.1959
	Dipole/FEM	0.2791	0.1996	0.1729	0.1609
0.6	Mutual-coupling/FEM	0.4147	0.2682	0.2227	0.2274
	Dipole/FEM	0.4043	0.2521	0.2022	0.2018
1	Mutual-coupling/FEM	0.5337	0.3717	0.3026	0.2605
	Dipole/FEM	0.5181	0.3394	0.2638	0.2423
1.5	Mutual-coupling/FEM	0.6550	0.4200	0.3247	0.2980
	Dipole/FEM	0.6546	0.4126	0.3138	0.2862

According to Table 5, for different distances (between the two particles) and different values of $\varepsilon_i/\varepsilon_e$, the curve similarity between the mutual-coupling dipole model and the finite element method is higher than that between the dipole approximation model and the finite element method, which shows that the mutual-coupling dipole model has a better accuracy when calculating the electric field distribution than the dipole approximation model does. At the same time, it can be seen that, with an increase in the distance (between the two particles), the curve similarity between the mutual-coupling dipole model and the finite element method and the curve similarity between the dipole approximation model and the finite element method gradually increase. In addition, as the distance (between the two particles) further increases, the curve similarity between the mutual-coupling dipole model and the dipole approximation model gradually increases, which means that as the distance (between the two particles) increases (such as $D/R > 1.5$), the interaction between the two particles is weakened, and the results calculated with the mutual-coupling dipole model and the dipole approximation model gradually become more consistent. In addition, with the increase in $\varepsilon_i/\varepsilon_e$, the curve similarity of the mutual-coupling dipole model gradually decreases, which indicates that the calculation error of the mutual-coupling dipole model would increase when $\varepsilon_i/\varepsilon_e$ is large, but the result calculated with the mutual-coupling dipole model is still better than that calculated with the dipole approximation model.

4. Conclusions

In this study, the influence of interactions between particles on an electric field was considered, the mutual-coupling dipole moment caused by interactions between particles was defined for the first time, and a new analytical expression of the dipole moment of a particle considering the interactions between particles was derived. On this basis, a mutual-coupling dipole model considering interactions between particles was proposed. Compared with the dipole approximation model and the finite element method, it was confirmed that, when the distance between two particles is small ($D/R < 0.6$), the mutual-coupling dipole model proposed in this paper has a higher accuracy in the calculation of the local electric field in a two-particle system than that of the dipole approximation model, and the curve similarity between the results of the mutual-coupling dipole model and finite element method is also higher than that between the results of the dipole approximation model and finite element method. When the distance between two particles is smaller ($D/R = 0.2$), the interaction between them will be very strong because of the influence of their polarized electric fields. In this case, the dipole approximation model has a large error in the calculation of the local electric field, whereas the mutual-coupling dipole model can effectively reflect the influence of the interaction between the particles on the local electric field and can be used to accurately calculate the electric field distribution when the distance between the two particles is smaller. With an increase in the distance between particles, especially when $D/R > 1.5$, the polarized electric field of one particle will not generate a significant effect on the polarization process of the other particle, and the interaction between them will be weakened; as a result, the results of the calculation of the mutual-coupling dipole model and the dipole approximation model tend to be consistent. Therefore, $D/R = 1.5$ can be used as a key criterion for considering the interactions between particles in practical applications. Interactions between particles are strong when $D/R < 0.6$. In this case, the mutual-coupling dipole model proposed in this paper can effectively reflect the interactions between particles, and it can be used to effectively solve for the local electric fields of particles and analyze the interactions between particles. These results can be widely and effectively applied in many fields, such as in the calculation of the insulation of mixed dielectrics, the microscopic transport of medicines, the control of bio-cells and micro-fluids in an electric field, and environmental governance.

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