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Dual-Channel Supply Chain Pricing Decisions under Discounted Advertising Value

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Abstract: Retailers advertise on different sales channels. When consumers buy online, the effectiveness of advertising is discounted because they cannot feel the product. Observing this phenomenon, this paper studies the advertising strategy of a dual-channel retailer. In this paper, we build stylized game models for the retailer's price and advertising levels on online and offline channels, respectively. Our contribution is to provide prescriptions for how dual-channel retailers make price and advertise decisions and determine which channel is more profitable for retailers. We find that advertising discounts are not always harming the retailer's profits. The level of advertising discounts causes retailers to increase the selling price when consumers engage in online shopping. Also, we derive that retailers can choose the sales channels based on the level of consumers' channel preference of consumers. Interestingly, an increase in the level of advertising discounts will contribute to the growth of the retailer's profit. Finally, by numerical analysis, we demonstrate the robustness of the results.

Keywords: dual-channel; pricing decisions; advertisement strategies; supply chain

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1. Introduction

1.1. Backgrounds and Motivations

Online sales have become increasingly important due to the new crown epidemic pandemic. According to the UN report, online retail sales are growing significantly in many countries, with South Korea reporting the highest share, reaching 25.9 percent in 2020 [1]. With the growth of e-commerce, it is becoming increasingly common for retailers to operate on a dual-channel model. Dual-channel operations are becoming increasingly popular for retailers. For example, the famous fast fashion brand Uniqlo has opened an online retail channel at JD. com, as well as a considerable number of offline brick-and-mortar shops. Consumers can buy products online as well as in traditional brick-and-mortar shops.

In addition, retailers implement advertising strategies to increase sales of their products [2,3]. The retailers' advertising strategies are different for different sales channels [4]. For example, in the online channel, retailers will advertise their products on retail platforms (e.g., JD App). Such an advertisement may appear on the opening page of the online sales app or the home page of the app. In the offline channel, retailers may place offline advertisements in physical shops or shopping centers to welcome more consumers into their shops. Figure 1 shows the two different forms of advertising for retailers online and offline.

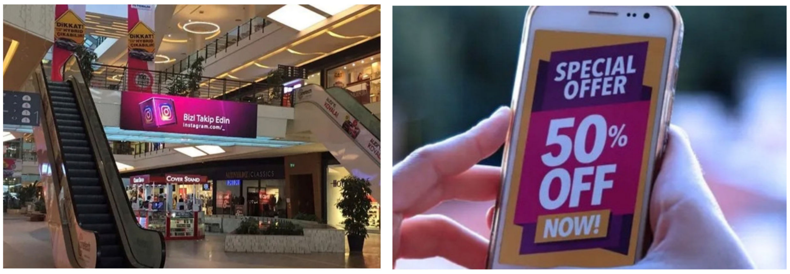

Figure 1. Illustrations of retailers' offline (**left**) and online (**right**) advertisements.

Although retailers use two different advertising strategies, they deliver different results. Online advertising is much less effective because consumers attracted to online advertising cannot know the actual quality of the product when they shop online. They cannot touch and try the product. Consumers attracted by offline advertising, on the other hand, can fully experience the product when shopping in a physical shop. Therefore, the effect of offline advertising is more pronounced.

Given the above observed issues in practice, we find that retailers' advertising has different impacts in different channels. Therefore, we believe this topic is important, and theoretical analysis of advertising can help dual-channel retailers become more competitive. In this paper, the main research question explores how the dual-channel retailer makes price and advertising decisions when considering the advertisement discount. We believe this paper contributes to the retailer's advertising operations in practice. We use game theory to address our core research question, which is widely used in operations management papers.

1.2. Research Questions and Main Findings

Observing the operational challenges of a retailer's advertising strategy in practice, this article explores a dual-channel retailer's optimal pricing and advertising strategy. The main research questions of this paper are as follows:

(i) What is the impact of advertising strategies in different sales channels on the profitability of a dual-channel retailer?
(ii) What should advertise levels and pricing decisions be for a dual-channel retailer across different sales channels?
(iii) Given optimal advertising levels and pricing decisions, which is more profitable for retailers, online or offline sales channels?

To address the above questions, in this paper, we develop a dual-channel retailer's pricing and advertising models to explore the performance of online and offline advertisements. We first develop the retailer's online pricing and advertising model, under which we consider online advertisement is discounted due to the uncertainty of online shopping. Next, we develop the retailer's offline pricing and advertising model. We derive the retailer's optimal strategies (prices, advertisement levels, and profits) under two sales channels from investigating two models. To understand the retailer's channel selection, we then compare the optimal profits under two models. We have several interesting findings in this paper, (1) advertising discounts are not always harming the retailer's profits. (2) An increase in the level of advertising discounts will contribute to the growth of the retailer's profit. (3) The level of advertising discounts causes the retailer to increase the selling price when consumers engage in online shopping.

1.3. Contributions and Paper Arrangements

We have several contributions to this paper. First, to the best of our knowledge, this is the first paper that compares the performance of online and offline advertisements. Second, we capture the features of online advertisement by employing an advertisement discount factor, which has hardly been considered in the current literature. Third, we give dual-channel retailers operational strategies on channel selection based on advertising strategies, which helps retailers be more profitable. All of our conclusions are derived from theoretical analysis and rigorous proof. The results have practical implications.

The remainder of this paper is as follows: Section 3 describes the models. We develop and analyze the models and compare the different models in Sections 4 and 5. In Section 6 we consider the extended models. To assess the robustness of the results, we perform a numerical analysis in Section 7. Section 8 summarises this paper's conclusions and management insights and provides recommendations for future research. All proofs are presented in Appendix A.

2. Literature Review

This article relates to two research streams: operational strategies for dual-channel supply chains and supply chain advertising strategies. We provide an overview of the relevant research as follows.

2.1. Dual-Channel Operation

It is increasingly common for manufacturers or retailers to adopt a dual-channel approach to selling their products [5–9]. Current studies have analyzed operational strategies, including pricing strategies, channel structure, etc. For example, Liu et al. [10] considered a supply chain with a manufacturer and two different retailers in different sales channels. They assumed that the two different have different service quality. They fund that the strategies of two retailers will be influenced by the other. Wang et al. [11] studied the effects of strategic inventory on the pricing and coordination of a dual-channel supply chain. They suggested that the retailers in different channels have the potential to reduce the double marginal effect, but they will harm the supply chain profits. Zhang et al. [12] investigated the quality decisions in a dual-channel supply chain. They considered a case where a manufacturer sells different quality products (high-quality and low-quality) in its own online and third-party sales channels. They implied that it is optimal for the manufacturer to sell high-quality products in its own online channel. Wang et al. [13] researched the retailer's buy online and pick up in-store strategies, which is a popular mode of dual-channel operations. Shao [14] uncovered how a retailer successfully moves omnichannel retailing to dual-channel retailing. Wang and He [15] thought that online customization is an importation sales mode. They claimed that the manufacturer should sell standard products on the online channel. Liu et al. [16] explored a retailer's channel selection issue. They fund that hybrid retail channel strategies are valuable, although price competition exists between different channels.

In addition, channel integration and differentiation are also vital research question that has received much attention [17–19]. Matsui [20] studied how a manufacturer recycles products using traditional and online channels. Shao [21] discovered how to sell mass customization products in different channels. They noted that dual-channel consistently outperforms single-channel. Yang et al. [22] analyzed the impacts of consumers' reviews when shopping online on a dual-channel supply chain. They claimed that providing consumers' reviews in the online store is not always beneficial. Mahar and Wright [23] revealed how to integrate channels by providing an In-Store Pickup and Return strategy. Tian et al. [24] investigated how to differentiate channels in a dual-channel supply chain if there are many free-riding consumers in the market. Gao et al. [25] considered the difference in bullwhip effects between different sales channels. They fund that the online retailer's price discount can enlarge the bullwhip effects, but such bullwhip effects can be less in the offline channel in certain conditions.

The above research solves the operational problem of dual-channel supply chains. Although this paper also studies a dual-channel supply chain, unlike the above studies, we take into account the advertising strategies in each channel. We analyzed the advertising strategies of different channels separately and compared the performance of different channels.

2.2. Advertising Strategies

Advertising strategies in supply chains are an essential research topic. Effective advertising strategies can help retailers or manufacturers increase sales [26–28]. Advertising level or effort is one of the critical advertising strategies [29]. Huang et al. [30] considered the impacts of the sequence of advertising on the manufacturers and retailers. Yan et al. [31] analyzed the joint promotion strategies for cross-market retailers. They suggested that when a retailer provides coupons to consumers, the retail price is relatively high. Karray et al. [32] uncovered the manufacturer's advertisement investment decisions. The fund that the cooperative advertising programs do not necessarily benefit the manufacturer. Liu et al. [33] investigated a fresh supply chain with the advertisement provided by the retailer. Choi and Liu [34] proposed an advertisement budget allocation program in fashion supply chains. They fund that the optimal advertisement investment decision is a polarized strategy, although the luxury fashion brands have different risk attitudes. Ma [35] presented the relationship between the advertising decision and the corporation's social responsibility. Liu et al. [36] highlighted that live-stream sales are a kind of advertisement. They uncovered a supply chain's price and quality decisions with online celebrity retailers.

In addition, advertising strategies are often made in conjunction with other decisions. For example, Du et al. [37] the green advertisement strategies of a platform. They illustrated the value of the green advertisement. Farshbaf-Geranmayeh and Zaccour [38] invested the joint price and advertising strategies with strategic consumers in a supply chain. Niu et al. [39] compared the relationships between advertisement and free trials. They revealed that free trial outperforms advertising if the effort of advertising is low. Khorshidvand et al. [40] studied a supply chain's jointly optimal strategies, including the price of sale channels, the advertisement level, and the green policy of the products. Karray and Martín-Herrán [41] uncovered the decision time of price and advertisement level. They fund that the manufacturer can change the decision time strategically to gain more profits.

The above research analyzed the advertising strategies of supply chains. This paper also develops an advertising strategy for a dual-channel supply chain. However, unlike the current studies, this paper considers the value discount of online advertising, which has not been studied in the current literature.

In this paper, we incorporate advertising decisions to a dual-channel retailer's operations, which is a novelty. Specifically, we consider the advertisement discount due to channel differences, which is first considered in the literature on advertising decisions. In addition, we highlight the retailer's channel decision when considering the advertisement discount, which is a realistic suggestion. Such a result contributes to not only the literature but also the retailer's operations in the real world. Finally, we have several counter-intuitive findings, which are different from the above research, such as advertising discounts are not always harming the retailer's profits. An increase in the level of advertising discounts will contribute to the growth of the retailer's profit to some extent. In addition, advertising discount has the opportunity to lead the retailer to increase the price.

3. The Problems and Assumptions

This paper considers a supply chain consisting of a retailer and n (quantities) consumers. The retailer (brands) sells its products in traditional online sales channels (shopping platforms such as JD.com and Taobao) and offline sales channels (physical stores) at costs c_i ($i = t, l$, where t denotes the online sales channel and l denotes the offline sales channel). The market demand for the product is D_i. The consumers' valuation for the product is x. This paper assumes that x is uniformly distributed between the signs 0 and 1, i.e., $x \sim U(0, 1)$. The probability density function is $f(x)$, and the cumulative distribution function is $F(x)$.

The retailer advertises the product to sell more of it. The level of advertising when selling products online and offline is e_i. The cost to the retailer to achieve this level of advertising is $C_i = ke_i{}^2/2$. This quadratic function is widely used in operations studies [37–39]. Advertising increases the consumer's utility. However, because consumers cannot touch and feel the product when they buy it online, the effect of advertising is discounted by a factor of β. Then we can derive consumers' utility when purchasing online and offline. Given the above reasons, this paper explores the retailer's pricing and advertisement strategies when selling the product online and offline. The notation used in this paper is shown in Table 1.

Table 1. The notations used in this paper.

Notations	Explanation
U_i	The consumers' utility when purchasing the product, where $i = t, l$.
D_i	The demand for the product, where $i = t, l$.
n	The number of consumers.
e_i	The advertisement levels, where $i = t, l$.
c_i	The cost for sale channels, where $i = t, l$.
C_i	The cost for the advertisement, where $i = t, l$.
k	The cost coefficient for the advertisements, where $0 < k < 1$.
β	The discount factor for the online advertisements, where $0 < \beta < 1$.
p_i	The retail prices, where $i = t, l$.
x	Consumers' valuation for the product, where $x \sim U(0,1)$.
π_i	The retailer's profits, where $i = t, l$.
$\Delta\pi$	The difference in profits between online and offline channels.

4. Models and Analysis

4.1. Online Sales Model

In the online sales model, the sales of a product increase due to the level of advertising by the retailer. As consumers cannot see the physical goods when they buy them online, there is a discount to the advertising effect for consumers. The proportion of consumers who choose the online channel is n. When purchasing online, a consumer's utility is $U_t = x - p_t + \beta e_t$, where β is the discount factor of the advertisement level. When $U_t > 0$, consumers will purchase products online. At this point, the consumer's demands are as follows:

$$D_t(p_t, e_t) = n \int_{p_t - \beta e_t}^{1} f(x)dx = n(1 + \beta e_t - p_t) \tag{1}$$

The retailer's profit function is as follows:

$$\pi_t(p_t, e_t) = (p_t - c_t)D_t(p_t, e_t) - C_e, \tag{2}$$

where $C_e = k{e_t}^2/2$ is the cost of advertisement.

Putting (1) into (2), we have:

$$\pi_t(p_t, e_t) = (p_t - c_t)n(1 + \beta e_t - p_t) - \frac{k{e_t}^2}{2} \tag{3}$$

The Hessian matrix of the profit function $\pi_t(p_t, e_t)$ with respect to p_t and e_t $H^t = \begin{vmatrix} \frac{\partial^2 \pi_t}{\partial {p_t}^2} & \frac{\partial^2 \pi_t}{\partial p_t \partial e_t} \\ \frac{\partial^2 \pi_t}{\partial e_t \partial p_t} & \frac{\partial^2 \pi_t}{\partial {e_t}^2} \end{vmatrix} = \begin{vmatrix} -2n & n\beta \\ n\beta & -k \end{vmatrix} = 2nk - n^2\beta^2$. When $2k > n\beta^2$, $H^t > 0$. Thus, the maximum profit for retailers exists. Considering the first-order conditions of $\pi_t(p_t, e_t)$ regarding p_t and e_t, respectively, i.e., $\frac{\partial \pi_t}{\partial p_t} = 0$ and $\frac{\partial \pi_t}{\partial e_t} = 0$, and solving these equations, we derive the optimal price and advertisement level as follows:

$$p_t^* = \frac{k + (k - n\beta^2)c_t}{2k - n\beta^2}, \tag{4}$$

$$e_t^* = \frac{n\beta(1-c_t)}{2k-n\beta^2} \tag{5}$$

To ensure the price p_t^* is positive, we consider the most interesting case where $k > n\beta^2$. Putting (4) and (5) into (3), we derive the retailer's optimal profit under the online sale model as follows:

$$\pi_t^* = \frac{nk(c_t-1)^2}{2(2k-n\beta^2)} \tag{6}$$

Up to here, we derive the retailer's optimal solutions under the online sale model. We find that the cost coefficient of advertisement, the number of consumers, the discount factor of advertisement, and the operations costs will impact the retailer's strategies. We will provide further analysis in Section 4.3.

4.2. Offline Sales Model

When consumers buy goods offline, the advertising effect is not discounted because they can experience (by touching, trying) the goods in person. At this point, the consumer's utility function for purchasing the product is $U_l = x - p_l + e_l$. When $U_l > 0$, consumers will buy products offline. The demand for offline sales is as follows:

$$D_l(p_l, e_l) = (1-n)\int_{p_l-e_l}^{1} f(x)dx = (1-n)(1+e_l-p_l) \tag{7}$$

Then, we derive the retailer's profit function as follows:

$$\pi_l(p_l, e_l) = (p_l - c_l)D_l(p_l, e_l) - C_e, \tag{8}$$

where $C_l = k{e_l}^2/2$ is the cost of advertisement.

Putting (8) into (7), we derive

$$\pi_l = (p_l - c_l)(1-n)(1+e_l-p_l) - \frac{k{e_l}^2}{2} \tag{9}$$

The Hessian matrix of the profit function $\pi_l(p_l, e_l)$ with respect to p_l and e_l $H^l = \begin{vmatrix} \frac{\partial^2\pi_l}{\partial {p_l}^2} & \frac{\partial^2\pi_l}{\partial p_l \partial e_l} \\ \frac{\partial^2\pi_l}{\partial r_l \partial p_l} & \frac{\partial^2\pi_l}{\partial {e_l}^2} \end{vmatrix} = \begin{vmatrix} -2+2n & 1-n \\ 1-n & -k \end{vmatrix} = (1-n)(2k+n-1)$. When $2k+n > 1$, $H^l > 0$. Thus, the maximum profit for retailers exists. Considering the first-order conditions of $\pi_l(p_l, e_l)$ regarding p_l and e_l, respectively, i.e., $\frac{\partial \pi_l}{\partial p_l} = 0$ and $\frac{\partial \pi_l}{\partial e_l} = 0$, and solving these equations, we derive the optimal price and advertisement level as follows:

$$p_l^* = \frac{k + c_l(k+n-1)}{2k+n-1} \tag{10}$$

$$e_l^* = \frac{(c_l-1)(n-1)}{2k+n-1} \tag{11}$$

To ensure the price p_l^* is positive, we consider the most interesting case where $k+n-1 > 0$. Putting (10) and (11) into (9), we derive the retailer's optimal profit under the offline sale model as follows:

$$\pi_l^* = \frac{(c_l-1)^2 k(1-n)}{2(2k+n-1)} \tag{12}$$

Up to here, similar to the online sales model, we derive the retailer's optimal solutions under the online sale model. We find that the cost coefficient of advertisement, the number of consumers, and the operations costs will impact the retailer's strategies. We will provide further analysis in Section 4.3.

4.3. Sensitivity Analysis

According to the optimal solutions under online and offline sales models, Sensitivity analysis was carried out on the key parameters of models, namely the advertising level cost factor, the advertising discount factor for consumers, the number of consumers, and the operating costs. The results are shown in Table 2.

Table 2. Sensitive analysis results under online and offline sales models *.

	k	β	n	c_t	c_l
$p_t^* \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$-$
$e_t^* \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$-$
$\pi_t{}^* \uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$-$
$p_l^* \uparrow$	$\downarrow$	$-$	$\downarrow$		$\uparrow$
$e_l^* \uparrow$	$\downarrow$	$-$	$\downarrow$		$\downarrow$
$\pi_l{}^* \uparrow$	$\downarrow$	$-$	$\downarrow$		$\downarrow$

* $\downarrow$ represents increase; $\uparrow$ represents decrease; $-$ represents no effect.

Then, to provide more insightful results, we have the following results.

Proposition 1. *The online retailer's decisions are as follows:*

(i) p_t^* *decreases with* k*, increases with* c_t*,* n*, and* β*.*
(ii) e_t^* *decreases with* k *and* c_t*, increases with* n *and* β*.*
(iii) π_t^* *decreases with* k *and* c_t*, increases with* n *and* β*.*

Proposition 1 shows that the level of advertising discounts causes the retailer to increase the selling price when consumers engage in online shopping. This could suggest that the retailer can adopt a high pricing strategy when consumer perceptions of advertising are weak in the market. In addition, the retailer's level of advertising increases consumer preference for the channel, when the retailer should pay a better level of advertising to attract more consumers. But the retailer's advertising level decision has to consider the cost of effort. Too high a cost of advertising level can hurt the retailer's profit even if it leads to more demand.

Furthermore, it follows from Proposition 1 that an increase in the level of advertising discounts will contribute to the growth of the retailer's profit. An increase in advertising level costs hurts retailers' profits. Higher advertising level costs lead to higher levels of advertising level degrees, indicating that marketing techniques such as advertising promotions by the retailer can attract more consumers and increase consumer loyalty to the channel.

Proposition 2. *The offline retailer's decisions are as follows:*

(i) p_l^* *decreases with* k *and* n*; increases with* c_l*.*
(ii) e_l^* *decreases with* k*,* n*, and* c_l*.*
(iii) π_l^* *decreases with* k*,* n*, and* c_l*.*

It follows from Proposition 2 that under the offline sales channel, the retailer's pricing decision is influenced by the monotonicity of advertising level costs. When the retailer's channel maintenance costs are high, a higher advertising level effort enables the retailer to execute a high pricing strategy. In actual dual-channel supply chain operations, high channel costs imply that retailers choose high-traffic, high-visibility online sales platforms, suggesting that consumers tend to be more receptive to higher-priced products on such online retail platforms. Proposition 2 also suggests that the retailer's high advertising level costs decrease its advertising level under the offline sales channel. At this point, the retailer should focus more on channel maintenance costs and compensate for the loss from its own lack of advertising level, such as advertising campaigns, by choosing a high-traffic, high-visibility platform.

From Proposition 2, it is clear that higher advertising level costs in the offline sales channel will hurt the retailer's profits, similar to the findings for the online sales channel. However, high advertising level costs can lead to higher advertising level levels, which can attract more consumers to shop on online retail platforms. Likewise, high channel maintenance costs hurt the retailer's profit. Furthermore, Proposition 2 illustrates that in the offline sales channel, retailers take advantage of the horse-trading effect of the internet platform to capture higher profits through the head platform. This is because the high-traffic, high-visibility platforms themselves carry their own advertising effects to attract more consumers.

5. The Retailer's Channel Decision

To further illustrate which sales channel makes the retailer more dynamic, we compared the sales margins of the two channels as follows:

$$\Delta\pi = \pi_t^* - \pi_l^* \tag{13}$$

Proposition 3. *When $\beta > \hat{\beta}$, $\Delta\pi > 0$, the online sales channel outperformed the offline sale channels; When $\beta < \hat{\beta}$, $\Delta\pi < 0$, the online sale channels perform worse than the offline sales channel.*

Proposition 3 yields a channel selection strategy for the retailer. The retailers can choose their sales channels based on the level of channel preference of consumers. If consumers in the market prefer online sales channels, the retailer should pay higher levels of advertising on online sales platforms such as Taobao and Jingdong to create more demand. If consumers prefer offline sales channels, retailers should put more advertising effort to boost their advertising levels with advertising effects.

6. Extended Models: Advertising Effect

With advertisement, the quantity of consumers in online channels will increase to $n + a_t e_t^E$, where consumers are sensitive to advertisements. In addition, to focus on the effect of advertisements on consumer quantity, we ignore the cost of advertising in this extended model. Thus, the demand function for the online retailer is

$$D_t^E\left(p_t^E, e_t^E\right) = \left(n + a_t e_t^E\right)\int_{p_t - \beta e_t^E}^{1} f(x)dx \tag{14}$$

The online retailer's profit function is as follows:

$$\pi_t^E\left(p_t^E, e_t^E\right) = \left(p_t^E - c_t\right)D_t^E\left(p_t^E, e_t^E\right) \tag{15}$$

Then we derive the optimal price and advertisement level for the online retailer as follows (the solution process in the extended model is similar to the basic models, thus we omit them here):

$$p_t^E = \frac{a + 2ac - n\beta}{3a}, \tag{16}$$

$$e_t^E = \frac{a + ac - 2n\beta}{3a\beta} \tag{17}$$

$$\pi_t^E = \frac{[a(-1+c) + n\beta]^3}{27a^2\beta} \tag{18}$$

For the offline retailer, the quantity of consumers will also increase to $1 - n + a_l e_l^E$. Thus, the demand function for the offline retailer is

$$D_l\left(p_l^E, e_l^E\right) = \left(1 - n + a_l e_l^E\right) \int_{p_l - e_l^E}^{1} f(x)dx \tag{19}$$

The offline retailer's profit function is as follows:

$$\pi_l^E = (p_l - c_l) D_l(p_l, e_l) \tag{20}$$

Then we derive the optimal price and advertisement level for the offline retailer as follows (the solution process in the extended model is similar with the basic models, thus we omit them here):

$$p_l^E = \frac{a + 2ac - \beta + n\beta}{3a}, \tag{21}$$

$$e_l^E = \frac{a + ac - 2\beta + 2n\beta}{3a\beta}, \tag{22}$$

$$\pi_l^E = \frac{[a(-1+c) + \beta - n\beta]^3}{27a^2\beta} \tag{23}$$

We derive the optimal solutions for the online and offline retailers above. Then, to understand the advertisement effects, we compare the optimal profits of the retailers as follows:

$$\Delta\pi^E = \pi_t^l - \pi_l^E \tag{24}$$

We have the following results.

Proposition 4. *When $\beta > \hat{\beta}_1$, $\Delta\pi^E > 0$, the online sales channel outperformed the offline sale channels; When $\beta < \hat{\beta}_1$, $\Delta\pi^E < 0$, the online sale channels perform worse than the offline sales channel.*

Proposition 4 provides similar results to Proposition 3, which shows the retailer's channel selection methods by comparing the optimal profits. Thus, Proposition 4 confirms the robustness of the results derived from the basic models. Even after considering the effects of the advertisement on consumer quantity, no channel has an absolute advantage. The retailer needs to tailor its channel strategies to different consumer preferences.

7. Numerical Analysis

To further illustrate the impact of factors such as the retailer's advertising level cost, cost, and advertising discount preference on the retailer's optimal profit, the propositions obtained in the previous section are analyzed numerically. The parameters are set as follows: $n = 0.1$, $k \in [0.1,\ 0.9]$, $\beta \in [0.1,\ 0.9]$, $c_i \in [10,\ 90]$.

Figure 2 shows how the optimal profit of the retailer under the online sales channel varies with the cost of advertising levels when considering different consumer channel preferences, and it can be seen that an increase in the cost of advertising levels reduces the profit of the retailer, in line with Proposition 1. Furthermore, Figure 2 illustrates that advertising level costs on the retailer's profit are higher if consumers' preference for the traditional channel is more severe. Therefore, the retailer can pay more for advertising levels if consumers are loyal to the sales channel.

Figure 3 shows the impact of the level of advertising discounts on the retailer's profit under the online sales channel. Consistent with Proposition 1, higher levels of advertising discounts lead to higher profits. In addition, Figure 3 illustrates the sensitivity of the retailer's profits to the cost of advertising levels. Lower advertising level costs ($k = 0.1$) have a more significant impact on the retailer's profit.

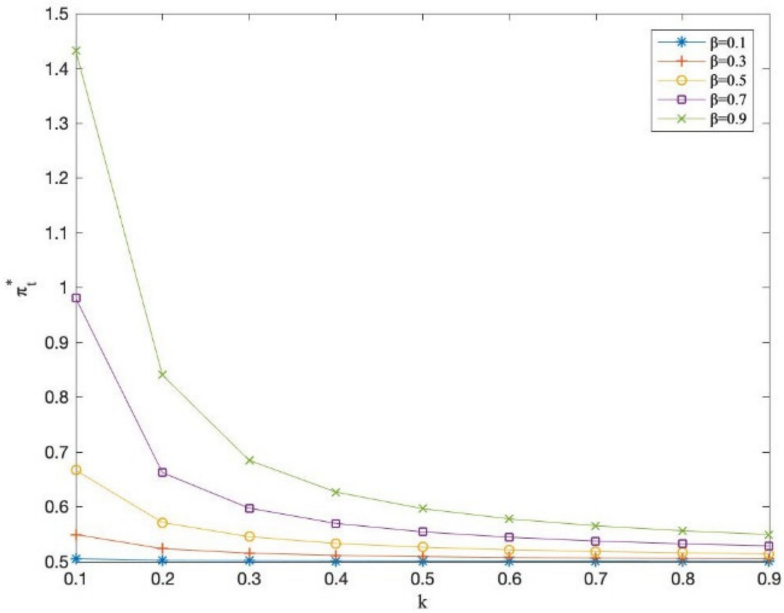

Figure 2. Variation of optimal profit with advertising level cost factor for the online sales channel.

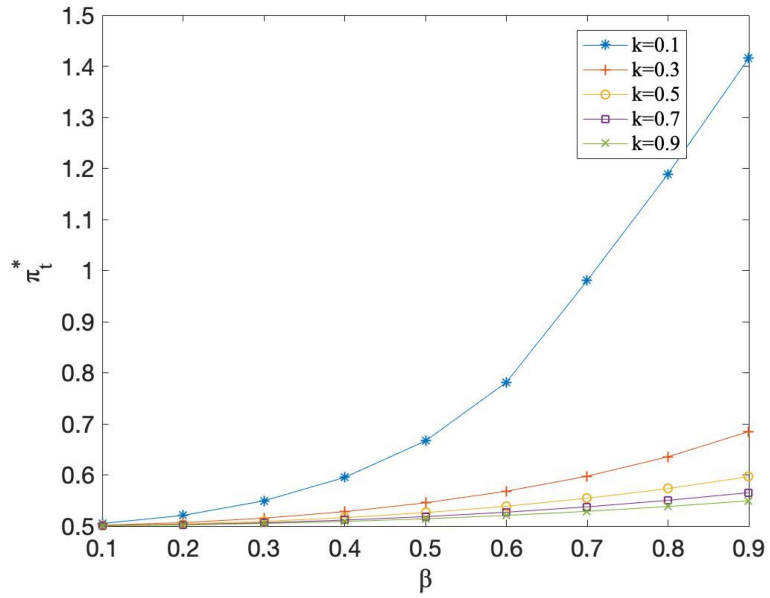

Figure 3. Variation in optimal profit with the level of advertising discounts for the online sales channel.

Figure 4 illustrates that the increase in advertising level costs in the offline channel makes retailers less profitable, in line with Proposition 2. Furthermore, Figure 4 shows that better channel maintenance costs (c = 90) give retailers better profits, suggesting that retailers can be more profitable with excellent levels of advertising. By comparing the impact of different channel maintenance costs on profits, it can be seen that retailers' profits are more sensitive to changes in high channel maintenance costs.

Figure 5 illustrates how the maintenance cost of the channel varies on profits under the offline sales channel. Consistent with Proposition 2, channel costs hurt the retailer's profit. In addition, the retailer's profit is more sensitive to changes in costs at low levels of advertising. At low channel maintenance costs, the retailer does not have to pay higher levels of advertising.

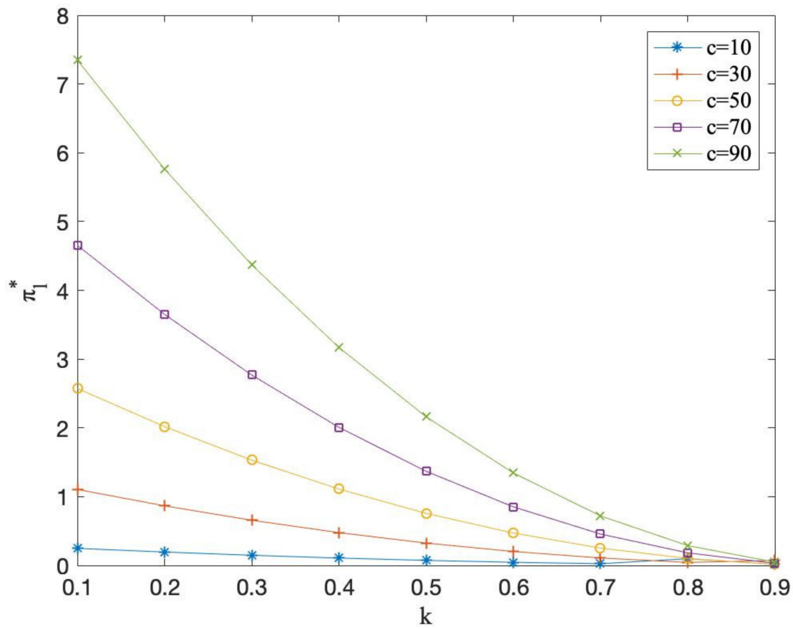

Figure 4. Variation of optimal profit with advertising level cost factor for the offline sales channels.

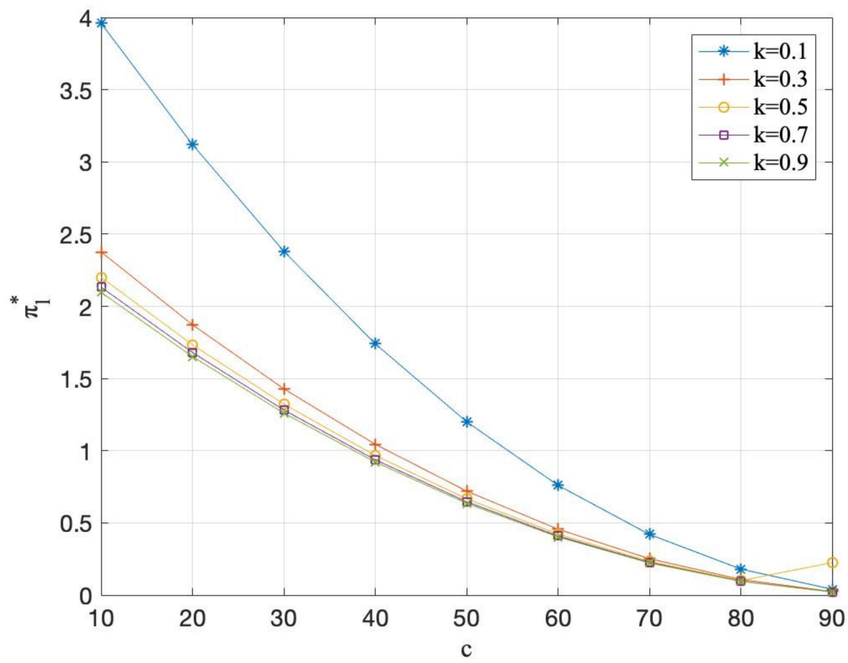

Figure 5. Optimal profit for an offline sales channel as a function of the cost of maintaining that channel.

8. Conclusions

8.1. Conclusion Remarks

Offline sales have attracted more consumers through their interactivity and immediate feedback, more finely tuned to their individual needs. The online sales approach has also brought new channel conflicts to the supply chain. Considering these problems in the actual operations and management of dual-channel supply chains, this paper explores the advertising conflicts between offline and online sales channels in the supply chain and the retailer's pricing and advertising level decisions.

This paper first develops a profit model for the online sales channel, considering the advertising discount factor. It derives the optimal pricing decision, advertising level decision, and optimal profit for the retailer under this model. The results show that advertising discounts can make the retailer increase the retail price and profit and stimulate the retailer to pay more for advertising levels, but paying more for advertising levels costs can also hurt the retailer's profit.

Subsequently, a profit model for the offline sales channel was developed, which considers the channel maintenance costs and obtained the optimal pricing decision, advertising level decision, and optimal profit for the retailer under this channel. The results show that advertising level costs in this channel have a monotonic effect on the retailer's pricing decisions and harm the retailer's profits and advertising levels. In addition, the study surprises by finding that channel maintenance costs can also reduce retailers' willingness to pay for advertising levels, which in turn hurts retailers' profits.

Finally, a retailer channel selection strategy that considers the level of advertising discounts is obtained. It is not always optimal to have a specific sales channel in practical supply chain operations management. The findings of this paper provide a reference for the retailer to make decisions when opening new sales channels.

8.2. Managerial Insights

This paper obtains optimal decisions for retailers under different channels. However, in practical operational management, it is more important to focus on the management insights of each decision. The sensitivity analysis of the optimal decisions led to the following management insights.

Regarding retailers' pricing decisions, the retailer in the online sales channel needs to find ways to increase customer loyalty and attract more consumers if they sell their products at higher prices. Paying for high levels of advertising is one effective way to achieve these goals. The retailer in the offline channel should focus on channel costs for that channel and choose high traffic, high popularity retail platforms to build their sales channels. But the retailer in this channel needs to be concerned about the impact of advertising level costs on pricing decisions, as lower advertising level costs can increase price competition in the market.

Regarding advertising level decisions, the retailer in the online channel needs to spend more on advertising levels to develop more loyal customers. The retailer in the offline channel does not need to spend more on advertising, as good offline channels carry their own advertising effects and can attract more consumers.

In terms of channel selection decisions, the retailer's decisions should focus on consumer loyalty to a channel and choose the sales channel that is most profitable for them to consider.

8.3. Future Research

Future research will take into account the risk appetite of consumers, as there is some risk in the perception of the product in operations management. Consumers may buy a product and then find that it does not meet their expectations. If consumers are risk-averse to this risk, how retailers in dual channels should make decisions is one of the issues that will need to be addressed. In addition, future research will consider the transfer of consumers across different sales channels. Some loyal customers in the offline channel may shift to the online channel to purchase products due to certain factors. Future research will consider how dual-channel retailers should respond in such situations.

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Appendix A

Proof of Proposition 1. With (4), finding the first-order derivatives of p_t^* with respect to the parameters k, n, c_t, and β, respectively. we have $\frac{\partial p_t^*}{\partial k} = \frac{n\beta^2(-1+c_t)}{(-2k+n\beta^2)^2} < 0$, $\frac{\partial p_t^*}{\partial c_t} = \frac{k-n\beta^2}{2k-n\beta^2} > 0$, $\frac{\partial p_t^*}{\partial n} = -\frac{k\beta^2(c_t-1)}{(-2k+n\beta^2)^2} > 0$, and $\frac{\partial p_t^*}{\partial \beta} = -\frac{2kn\beta(c_t-1)}{(-2k+n\beta^2)^2} > 0$.

With (5), finding the first-order derivatives of e_t^* with respect to the parameters k, n, c_t, and β, respectively, we have $\frac{\partial e_t^*}{\partial k} = \frac{2n\beta(c_t-1)}{(-2k+n\beta^2)^2} < 0$, $\frac{\partial e_t^*}{\partial c_t} = \frac{n\beta}{-2k+n\beta^2} < 0$, $\frac{\partial e_t^*}{\partial n} = -\frac{2k\beta(-1+c_t)}{(-2k+n\beta^2)^2} > 0$, and $\frac{\partial e_t^*}{\partial \beta} = -\frac{n(2k+n\beta^2)(-1+c_t)}{(-2k+n\beta^2)^2} > 0$.

With (6), finding the first-order derivatives of π_t^* with respect to the parameters k, n, c_t, and β, respectively, we have $\frac{\partial \pi_t^*}{\partial k} = -\frac{n^2\beta^2(-1+c_t)^2}{2(-2k+n\beta^2)^2} < 0$, $\frac{\partial \pi_t^*}{\partial c_t} = \frac{2kn(-1+c_t)}{4k-2n\beta^2} < 0$, $\frac{\partial \pi_t^*}{\partial n} = \frac{k^2(-1+c_t)^2}{(-2k+n\beta^2)^2} > 0$. and $\frac{\partial \pi_t^*}{\partial \beta} = \frac{4kn^2\beta(-1+c_t)^2}{(4k-2n\beta^2)^2} > 0$. □

Proof of Proposition 2. With (10), finding the first-order derivatives of p_l^* with respect to the parameters k, n, and c_l, respectively. we have $\frac{\partial p_l^*}{\partial k} = \frac{-1+c+n-cn}{(-1+2k+n)^2} < 0$ (recall that $c < 1$), $\frac{\partial p_l^*}{\partial c} = \frac{k+n-1}{2k+n-1} > 0$ (recall that $k + n - 1 > 0$), and $\frac{\partial p_l^*}{\partial n} = \frac{(-1+c)k}{(-1+2k+n)^2} < 0$.

With (11), finding the first-order derivatives of e_l^* with respect to the parameters k, n, and c_l, respectively. we have $\frac{\partial e_l^*}{\partial k} = -\frac{2(-1+c)(-1+n)}{(-1+2k+n)^2} < 0$, $\frac{\partial e_l^*}{\partial c} = \frac{n-1}{2k+n-1} < 0$, and $\frac{\partial e_l^*}{\partial n} = \frac{2(-1+c)k}{(-1+2k+n)^2} < 0$.

With (12), finding the first-order derivatives of π_l^* with respect to the parameters k, n, and c_l, respectively. we have $\frac{\partial \pi_l^*}{\partial k} = -\frac{(-1+c)^2(-1+n)^2}{2(-1+2k+n)^2} < 0$, $\frac{\partial \pi_l^*}{\partial c} = -\frac{(-1+c)k(-1+n)}{2k+n-1} < 0$, and $\frac{\partial \pi_l^*}{\partial n} = -\frac{(-1+c)^2k^2}{(-1+2k+n)^2} < 0$. □

Proof of Proposition 3. With (13), we have $\Delta\pi = \pi_t^* - \pi_l^* = \frac{(-1+n)n+2k\left(-1-2c(-1+n)+c^2(-1+n)+2n\right)-2n(-1+2k+n)c_t+n(-1+2k+n)c_t^2+\left(n-2cn+c^2n-n^2+2cn^2-c^2n^2\right)\beta^2}{2(-1+2k+n)(2k-n\beta^2)}$. Clearly when $\beta > \hat{\beta} = \sqrt{\frac{(-1+n)n+2k(-1-2c(-1+n)+c^2(-1+n)+2n)-2n(-1+2k+n)c_t+n(-1+2k+n)c_t^2}{(n-2cn+c^2n-n^2+2cn^2-c^2n^2)}}$, $\Delta\pi > 0$; when $\beta < \hat{\beta} = \sqrt{\frac{(-1+n)n+2k(-1-2c(-1+n)+c^2(-1+n)+2n)-2n(-1+2k+n)c_t+n(-1+2k+n)c_t^2}{(n-2cn+c^2n-n^2+2cn^2-c^2n^2)}}$, $\Delta\pi < 0$. □

Proof of Proposition 4. With (24), we have $\Delta\pi^E = \pi_t^l - \pi_l^E = \frac{(-1+2n)\left(3a^2(-1+c)^2+3a(-1+c)\beta+\left(1-n+n^2\right)\beta^2\right)}{27a^2}$. Clearly when $\beta > \hat{\beta}_1 = \sqrt{\frac{3a^2(-1+c)^2+3a(-1+c)\beta}{1-n+n^2}}$, $\Delta\pi^E > 0$; when $\beta < \hat{\beta}_1 = \sqrt{\frac{3a^2(-1+c)^2+3a(-1+c)\beta}{1-n+n^2}}$, $\Delta\pi^E < 0$. □

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