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Evolution Analysis of Green Innovation in Small and Medium-Sized Manufacturing Enterprises

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Abstract: In recent years, green innovation has gained substantial attention and popularity from the manufacturing industry around the world. As an essential part of the manufacturing industry, small and medium-sized manufacturing enterprises (SMMEs) are vital participants that promote green innovation to realize sustainable development. However, how green innovation evolves in SMMEs is unclear, which hinders SMMEs from implementing or even adopting green innovation. This study attempted to essentially reveal the evolution of green innovation in SMMEs based on complex systems theory. First, this study divided green innovation into green product innovation and green process innovation, defined the state variables of the two components, and dissected the symbiotic interactions between them. This study then designed a nonlinear dynamic model followed by extensive simulations to theoretically and visually describe how green innovation evolves. This study found that green innovation with non-zero levels in both dimensions is desired and determines the evolutionary paths with corresponding measures that can guide SMMEs to realize green innovation at desired stable states. Besides, symmetric mutualism is discovered to be the optimal symbiotic interaction. Based on these findings, regulatory subjects and SMMEs can duly adjust the inputs on green innovation and the symbiotic interactions within green innovation to better manage green innovation practices.

Keywords: green innovation; SMMEs; complex systems theory; evolutionary stability; evolutionary paths

1. Introduction

Rapidly growing industrialization and urbanization consume natural resources in quantity and generate large amounts of environmentally unfriendly emissions such as greenhouse gas and suspended particulates [1,2], causing mounting problems that plague human existence, such as resource scarcity, natural disasters, detrimental water, rising air temperature, global diseases, and flabby economy, just to name a few [3,4]. Compelled by the deteriorating environment and unsustainable economy, governments, scholars, industry practitioners, and environmentalists highlight the importance of green development [5,6] and call for relevant cooperation, practices, studies, policies, action plans, etc., to propel green development. At the same time, customers and consumers are gradually aware of environmental sustainability and display relevant behaviors such as using green products [7]. In this context, green innovation, a strategy to balance economic growth and environmental protection, enjoys more and more attention and popularity at the micro, meso, and macro levels. Enterprises, industries, and countries are the subjects at the three levels, respectively [8–10].

The manufacturing industry is a representative subject at the meso level because it has a broad scope and directly manifests the productivity of a country. Besides, the manufacturing industry is resource-intensive or energy-intensive and can produce massive emissions [11]. Thus, the manufacturing industry is responsible for current environmental

and economic states and urgently needs to transform traditional practices into green innovation practices. Small and medium-sized manufacturing enterprises (SMMEs) are vital subjects at the micro-level in the manufacturing industry, as they provide plenty of employment opportunities and boost economic development [12]. Accompanied by positive externalities, SMMEs emit the most carbon emissions and pollutants (up to 60% and 70% of the corresponding total emissions, respectively) [13], which is often ignored due to the scale of SMMEs [14]. Nevertheless, SMMEs still face environmental pressures such as carbon neutrality [15] and sustainable development [16]. Therefore, considering green innovation in SMMEs not only conforms to the situation but also acts as a necessary means to enhance enterprise competitiveness.

Compared with large manufacturing enterprises, SMMEs face many distinct barriers, involving economic and financial barriers, organizational and managerial barriers, legal and regulatory barriers, technological barriers, knowledge and competency-related barriers, and market barriers. Among these barriers, the internal barriers are more manageable than the external barriers and thus should be handled as a matter of priority, especially the evolution of green innovation in SMMEs. However, existing literature places emphasis on the macro and meso evolution of green innovation [17–19], and the micro evolution driven by the interactive links between the multiple dimensions of green innovation stays to be fully explained [20]. Therefore, the research objective is to uncover how green innovation evolves at the micro level in order to provide detailed guidance for enterprises and regulatory subjects to advance green innovation. This paper attempted to realize the above objective by answering the following research questions.

1. *What are the end states of green innovation in SMMEs? Which states are the ones SMMEs expect, and which states are the ones SMMEs want to avoid? Under what conditions can SMMEs encounter green innovation at a given end state?*
2. *How green innovation in SMMEs evolves from an unexpected state to an expected state?*
3. *What SMMEs and regulatory subjects should do to better propel green innovation?*

Most current methods handling the barriers of green innovation are proposed based on reductionism and mechanism, do not dispose of green innovation from multiple aspects, and can hardly be applied to analyze the complex, dynamic, nonlinear, and multi-dimensional properties of green innovation as a complex system. Thus, more appropriate methods are needed to tackle the evolution of green innovation in SMMEs. Complex systems theory is eminently suitable to dynamically model the structures and explain the bottom-up behaviors of complex systems. Therefore, this paper analyzed the evolution of green innovation in SMMEs based on complex systems theory. This paper categorized green innovation into green product innovation and green process innovation and dissected the state variables of these two kinds of green innovation as well as the nonlinear interactions between them, which underlies the evolution model of green innovation. The paper dynamically reveals the rich evolutionary processes of green innovation and discovers the conditions and evolutionary paths that enable green innovation to stabilize at ideal states.

The remainder of the paper is organized as follows. Section 2 reviewed related literature. Section 3 established and analyzed the evolution model of green innovation in SMMEs. Section 4 conducted numerical simulations. Section 5 displayed findings and discussion, and Section 6 presented contributions and managerial implications. Finally, Section 7 concluded the paper and proposed future research directions.

2. Literature Review

2.1. Green Innovation

Green innovation, as an advanced version of traditional innovation, combines innovation-driven and green ideas to balance economic growth and environmental protection. Since green innovation is still in its growth stage [21], the concepts of green innovation are constantly supplemented and updated by researchers from their own perspectives [14], and an agreed-upon and standardized definition does not emerge [22]. Besides, green innovation is usually used interchangeably with the other three terms, that is, eco-innovation,

environmental innovation, and sustainable innovation [12,23]. Sustainable innovation covers economic, environmental, and social dimensions, while the remaining three terms address a narrower set of content and ignore the social dimension [24,25]. This difference does not affect micro-level (i.e., enterprise-level) studies. Thus, this paper viewed these terms as similar terminologies.

Green innovation can be divided into several types according to separate classification criteria, such as innovative subjects, manners, meanings, manifestations, and fields. This paper classifies green innovation based on innovative manifestations. As the definitions of green innovation are not unified, the classes of green innovation in terms of innovative manifestations are diverse, and the main categories are summarized as follows: (1) green product innovation and green process innovation [5,20,26–28]; (2) green product innovation, green process innovation, and green management innovation [29–31]; (3) green product innovation, green process innovation, green management innovation and green business innovation [1,28,32]. This paper adopts the first classification considering the following two situations. On the one hand, although green innovation has various manifestations, green product innovation, and green process innovation are the primary and fundamental under each division. These two sorts of green innovation account for the vast majority of all innovation manifestations [33]. On the other hand, SMMEs are constrained by financial resources, knowledge resources, management competency, etc. As a result, SMMEs cannot touch all kinds of innovations and treat them equally, and the dominating activities in SMMEs revolve around products and processes [34]. It should be noted that innovative green technology can trigger green product innovation and green process innovation [35,36]. In other words, green technology innovation permeates green product innovation and green process innovation in this paper.

2.2. SMME-Specific Challenges for Green Innovation

SMMEs often perform worse than large manufacturing enterprises when it comes to green innovation. The reason behind the reality is that SMMEs face disparate barriers in the adopting, implementing, or expanding phases of green innovation. Such barriers can be summarized into six interwoven categories, including economic and financial barriers, organizational and managerial barriers, legal and regulatory barriers, technological barriers, knowledge and competency-related barriers, and market barriers [14,37,38].

Cash crunches usually prevent SMMEs from adopting and conducting green innovations. These cash crunches mainly root from economic and financial barriers, among which less return of high investment and costs involved in green innovation [39], destitute government subsidies and financial support [40], and unavailable bank loads [41] are the major constituents. Favorable organizational and managerial states endow enterprises with a powerful endogenous impetus to experience and maintain green innovation practices. However, SMMEs are continually puzzled on this aspect. The frequently-mentioned barriers under this category cover passive attitudes and unstable belief towards green innovation [42], inadequate training and consultative activities aiming at green practices [43], deficient human resources in green innovation [44], and poor interactions with other stakeholders participating in green innovation [45]. Legal and regulatory conditions are vital factors throughout the development of green innovation, especially in the early stage of green innovation. Current regulations and policies are complex and unsound, making themselves drag on the floor, followed by debased green innovation initiatives and practices [46,47]. The other main legal and regulatory barrier is that non-policy aids (e.g., training and consultancy programs) are provided by governments [48]. Technology is the hard-nucleus power that SMMEs need to rely on to develop green products and processes. Technological barriers mainly include a lack of new or competent technologies/infrastructure to innovate [39,49], complex designing processes to achieve green targets [50], severe technology dependence on external providers (e.g., benchmarking manufacturing enterprises) [51], and technological uncertainty regarding green innovations [52]. Although external support for green innovation exists, SMMEs are still emphasized to possess certain knowledge and

competency which require much time and energy to obtain. Thus, the major barriers under this category encountered by enterprises can be a lack of knowledge and information (e.g., legal knowledge and technological information) on green innovation [53,54] and a lack of competency to accomplish innovation practices (e.g., the ability to carry out R&D and seek innovation chances) [48]. Generally, market demands are the basis for enterprises to implement green innovation, and huge demands can drive even small enterprises to actually perform green innovation in their daily operations. For SMMEs, lack of access to the market and weak market responsiveness are the major market barriers [44,55].

The various barriers imply that green innovation is indeed tough for SMMEs. To pursue successful green innovation, SMMEs must conquer these barriers and the derived ones spawned by them, among which the internal barriers are more controllable than the external barriers and should be tackled preferentially, especially the unclear evolution of green innovation. However, most of the studies focusing on the evolution of green innovation stay at the macro and meso levels (e.g., green innovation networks). The micro-level evolution of green innovation remains a research gap. This paper attempted to fill the gap.

2.3. Current Methods to Propel Green Innovation in SMMEs

Multifarious methods of handling the above barriers appear in existing studies, and they are mainly put forward from the perspectives of manufacturing enterprises and governments. Manufacturing enterprises are the practitioners that really execute green innovation at the frontline. Governments, the crucial non-industrial actors, regulate the behaviors of manufacturing enterprises in green innovation.

In terms of manufacturing enterprises, many measures proposed in the literature can be considered in order to get out of the internal dilemmas encountered in green innovation. Such measures include devising eco-innovation maturity models [56], developing strategies and business modes related to green innovation [23,57], improving external interaction patterns directed at green innovation practices [29,58], implementing information systems concerning green innovation [22,59], ranking green innovation solutions [38], cultivating green innovation-oriented enterprise culture [4], and using green procurement [60], just to name a few. From the angle of governments, introducing new or improved regulations and policies, which can be in the forms of notices, guidebooks, plans, opinions, action programs, etc., is the primary approach used to reward, punish or coordinate the behaviors of manufacturing enterprises and their stakeholders [19,61,62].

Existing methods are instructive to propel green innovation in SMMEs, yet three aspects need to be noticed if comprehensiveness, complexity, and dynamics are taken into account. First, green innovation is a multi-dimensional concept. Disposing of green innovation in more dimensions is more all-sided, but most of the works do not intend to explore green innovation on multiple levels. Second, green innovation in SMMEs is complex, while most of the studies ignore the fact and discuss green innovation utilizing reductionism. Third, green innovation is dynamic, but many countermeasures are presented based on mechanism. Hence, more applicable ways of researching green innovation are needed.

2.4. Complex Systems Theory

Complex systems theory, one theory transcending reductionism and mechanism, is able to model the structures of complex systems and dynamically delineate the operating mechanism and bottom-up behaviors of the systems [63]. The core idea of complex systems theory is that complex systems consist of many components linked by nonlinear interactions, making the systems exert global behaviors and phenomena that are different from those of system components and cannot be inferred by using the superposition theorem [64,65].

Complex systems theory has been widely used to explore the evolution driven by the complex interactions within various complex systems, yielding a lot of robust applications and relevant fruits. For example, Niu et al. [66] developed a dynamic model to describe

the evolution of information development driven by different actors in manufacturing enterprises. Zhu et al. [67] established a nonlinear evolution model to investigate how manufacturing technology and information technology interacted and together determined the evolutionary trajectories of technology fusion. Pal et al. [68] studied the stability of two ecological systems jointed by migration based on two classical dynamic models. Çalış et al. [69] devised a delayed model to analyze the dynamic response of a financial system dominated by four interacted state variables. Xu et al. [70] discussed the chaos control of a supply chain system made up of customers, distributors, and manufacturers by using a dynamic model with effective control parts. Therefore, through building a nonlinear dynamic model about green product innovation and green process innovation, which are the primary components of green innovation, this paper explored the evolution of green innovation in SMMEs.

3. Evolution Model of Green Innovation in SMMEs

On the grounds of the literature review, green innovation in this paper was viewed as the nonlinear superposition of green product innovation and green process innovation, and we utilized complex systems theory and the Logistic equation to conditionally construct a dynamic model for green innovation with the purpose to quantify the bottom-up evolution of green innovation at the micro level. Through model analysis, we obtained detailed evolutionary stability and paths of green innovation. By setting data, many numerical simulations were performed to verify our model. The methodology proposed in this paper is graphically displayed in Figure 1.
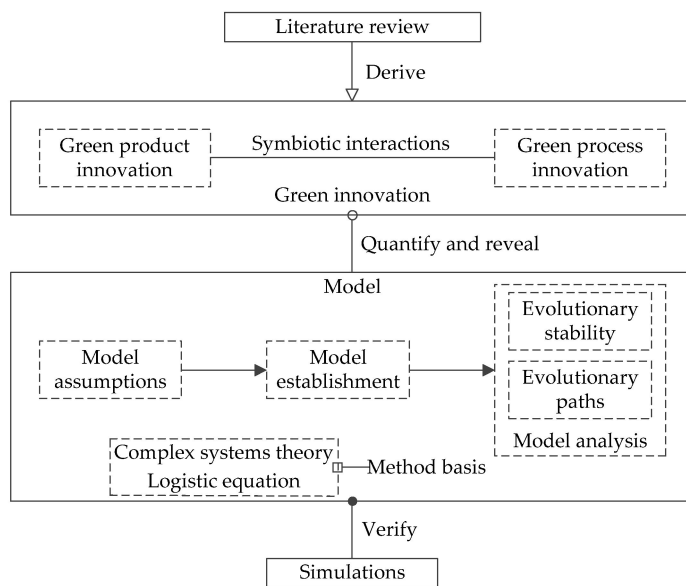

Figure 1. The schematic diagram of our methodology.

3.1. Model Assumptions

Green innovation mainly consists of green product innovation and green process innovation and is a dynamic process in nature. This paper investigated the evolution process based on the Logistic equation, which is perceived as an excellent tool to depict how objects grow and develop [71]. This paper made the following three assumptions to facilitate the subsequent model establishment and model analysis.

Assumption 1. *Both the development of green product innovation and green process innovation is a function of time denoted as t. t is abstracted into the changes of all the factors affecting the development of green product innovation and green process innovation, respectively. Under all the influencing factors, the states of green product innovation and green process innovation continuously fluctuate, which together portrays the evolutionary process of green innovation.*

Assumption 2. *If green product innovation develops independently, its development process obeys the growth law revealed by the Logistic equation. The same is true for green process innovation.*

Assumption 3. *During the symbiotic development process of green product innovation and green process innovation, various interactions are possible to emerge between the two categories of green innovation. Such an interaction can restrain or promote the growth of green product innovation and green process innovation.*

3.2. Model Establishment

Green innovation in SMMEs can be viewed as a complex system where green product innovation and green process innovation are two main components (i.e., subsystems). Green product innovation means an innovation that introduces new or improved products to reduce negative environmental externalities and create economic benefits [26,72]. Green process innovation refers to an innovation bringing enhanced or new processes, which facilitates the generation of green products [28,45]. In complex systems theory, the states of a system are represented by state variables, which can be the properties or features of the system. The state variables of green product innovation and green process innovation are defined as green product innovation level and green process innovation level, respectively.

When green product innovation and green process innovation all grow in isolation, we utilized the Logistic equation to denote the changes in green product innovation level and green process innovation level over time. Consequently, the evolution of green innovation is denoted as:

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= \alpha_1 x_1 - \beta_1 {x_1}^2 \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= \alpha_2 x_2 - \beta_2 {x_2}^2 \end{aligned} \tag{1}$$

Let x_1 and x_2 represent the green product innovation level and green process innovation level, respectively. The parameter α_1 indicates the growth rate of green product innovation level when green product innovation develops independently with constant inputs, and the same is true for the parameter α_2. The greater the value of α_i ($i = 1, 2$) is, the more the inputs are. The parameters β_1 and β_2 denote the self-inhibiting effects of green product innovation level and green process innovation level, respectively. Generally, $\beta_i > 0$ ($i = 1, 2$). The reason behind this fact is that green product innovation and green process innovation can consume limited resources bit by bit, eventually leading to self-inhibiting effects if consumption reaches a certain threshold. Green product innovation and green process innovation are connected by nonlinear interactions which are not embodied in Equation (1). Thus, Equation (1) cannot stand for the actual development of green innovation, and the evolution model of green innovation was modified as follows:

$$\begin{aligned} \frac{\mathrm{d}x_1}{\mathrm{d}t} &= \alpha_1 x_1 - \beta_1 {x_1}^2 + \gamma_1 x_1 x_2 \\ \frac{\mathrm{d}x_2}{\mathrm{d}t} &= \alpha_2 x_2 - \beta_2 {x_2}^2 + \gamma_2 x_1 x_2 \end{aligned} \tag{2}$$

where the product $x_1 x_2$ represents the nonlinear interactions between green product innovation and green process innovation. The parameters γ_1 and γ_1 are the symbiotic coefficients which measure the influences that the nonlinear interactions exert on the growth of green product innovation level and green process innovation level, respectively. The values of γ_1 and γ_2 determine which categories the symbiotic interactions between green product innovation and green process innovation belong to. On the basis of community ecology theory [73], symbiotic interactions between green product innovation and green process innovation are divided into six categories, that is, neutralism, competition, amensalism, parasitism, commensalism, and mutualism, as shown in Table 1.

Table 1. The relationships of symbiotic coefficients and symbiotic interactions.

Interactions	γ_i ($i = 1, 2$)	Description
Neutralism	$\gamma_1 = 0$, $\gamma_2 = 0$	Green product innovation and green process innovation do not affect each other
Competition	$\gamma_1 < 0$, $\gamma_2 < 0$	Green product innovation and green process innovation inhibit each other
Amensalism	$\gamma_1 < 0$, $\gamma_2 = 0$	Green product innovation is inhibited, and green process innovation is unaffected
	$\gamma_1 = 0$, $\gamma_2 < 0$	Green product innovation is unaffected, and green process innovation is inhibited
Parasitism	$\gamma_1 > 0$, $\gamma_2 < 0$	Green product innovation is benefited, and green process innovation is inhibited
	$\gamma_1 < 0$, $\gamma_2 > 0$	Green product innovation is inhibited, and green process innovation is benefited
Commensalism	$\gamma_1 > 0$, $\gamma_2 = 0$	Green product innovation is benefited, and green process innovation is unaffected
	$\gamma_1 = 0$, $\gamma_2 > 0$	Green product innovation is unaffected, and green process innovation is benefited
Mutualism	$\gamma_1 > 0$, $\gamma_2 > 0$	$\gamma_1 \neq \gamma_2$, Green product innovation and green process innovation are not equally benefited $\gamma_1 = \gamma_2$, Green product innovation and green process innovation are equally benefited

3.3. Model Analysis

3.3.1. Analysis of Evolutionary Stability

Green innovation in SMMEs is a dynamic process and finally evolves towards a steady state which can be one of the equilibrium points of our evolution model. For Equation (2), let $\frac{dx_1}{dt} = 0$ and $\frac{dx_2}{dt} = 0$, then we obtained four equilibrium points, denoted as $E_1(0, 0)$, $E_2(\frac{\alpha_1}{\beta_1}, 0)$, $E_3(0, \frac{\alpha_2}{\beta_2})$ and $E_4(\frac{\alpha_1\beta_2 + \alpha_2\gamma_1}{\beta_1\beta_2 - \gamma_1\gamma_2}, \frac{\alpha_1\gamma_2 + \alpha_2\beta_1}{\beta_1\beta_2 - \gamma_1\gamma_2})$. The equilibrium point E_1 shows that the green product innovation level and green process innovation level become zero when green product innovation and green process innovation synergistically evolve for a period of time, indicating that green innovation disappears in SMMEs. E_2 denotes that the green process innovation level turns zero when the two categories of green innovation interact for a period of time, implying that green innovation degrades into green product innovation. The same is true for E_3. E_4 represents that both green product innovation level and green process innovation level are non-zero when the two categories of green innovation together develop for some time, meaning that green innovation in both dimensions healthily grows.

Driven by different symbiotic interactions, green innovation is likely to stabilize at the above four equilibrium points. It is necessary to conduct a stability analysis on each equilibrium point to determine the stable points in the evolutionary process of green innovation and the corresponding stable conditions. The Jacobian matrix of Equation (2) is:

$$J = \begin{bmatrix} \alpha_1 - 2\beta_1 x_1 + \gamma_1 x_2 & \gamma_1 x_1 \\ \gamma_2 x_2 & \alpha_2 - 2\beta_2 x_2 + \gamma_2 x_1 \end{bmatrix} \tag{3}$$

According to Lyapunov stability theory, when an equilibrium point of Equation (2) satisfies $det J = |J| = (\alpha_1 - 2\beta_1 x_1 + \gamma_1 x_2)(\alpha_2 - 2\beta_2 x_2 + \gamma_2 x_1) - \gamma_1 x_1 \gamma_2 x_2 > 0$ and tr $J = \alpha_1 - 2\beta_1 x_1 + \gamma_1 x_2 + \alpha_2 - 2\beta_2 x_2 + \gamma_2 x_1 < 0$, the equilibrium point is a stable point. The stability analyses of all the equilibrium points of Equation (2) are shown in Table 2.

As shown in Table 2, the stability of each equilibrium point was analyzed as follows. If $\alpha_1 < 0$ and $\alpha_2 < 0$, the growth rates of green product innovation level and green process innovation level are negative, and the green innovation could only stabilize at E_1; under this condition, the green innovation is destined to vanish in SMMEs no matter what the internal symbiotic interactions between green product innovation and green process

innovation are. If $\alpha_1 > 0$ and $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$, the equilibrium point E_2 is stable; the result represents the fact that the green product innovation level and green process innovation level severally stabilize at $\frac{\alpha_1}{\beta_1}$ and 0, which is immune to the categories of the internal symbiotic interactions. If $\alpha_2 > 0$ and $\gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$, the stable point is E_3; under this condition, the green product innovation level and green process innovation level severally stabilize at 0 and $\frac{\alpha_2}{\beta_2}$, regardless of the internal interactions. Remarkably, when E_2 and E_3 are satisfied at the same time, the stable points can be E_2 or E_3, and which one is the final stable point depends on the initial states, such as the initial green innovation level. If $\gamma_1\gamma_2 < \beta_1\beta_2$, $\alpha_1 > 0$, $\alpha_2 < 0$, $\gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$ and $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$, or if $\gamma_1\gamma_2 < \beta_1\beta_2$, $\alpha_1 < 0$, $\alpha_2 > 0$, $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$, $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$, E_4 is the unique stable point; under these conditions, the green product innovation level and green process innovation level stabilize at $\frac{\alpha_1\beta_2+\alpha_2\gamma_1}{\beta_1\beta_2-\gamma_1\gamma_2}$ and $\frac{\alpha_1\gamma_2+\alpha_2\beta_1}{\beta_1\beta_2-\gamma_1\gamma_2}$, respectively, and the symbiotic interactions between the green product innovation and green process innovation can only be mutualism, commensalism, and parasitism. If $\gamma_1\gamma_2 < \beta_1\beta_2$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$ and $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$, E_4 is also the only stable point, but the symbiotic interactions are unlimited. The above analysis proves that the growth rates of green product innovation level and green process innovation level and the categories of the internal interactions within green innovation can influence the evolutionary homeostasis of green innovation.

Table 2. Stability analysis of each equilibrium point.

| E_i (i = 1, 2, 3, 4)	det J/tr J	Stable Conditions	Symbiotic Interactions
E_1	$\alpha_1\alpha_2 / \alpha_1 + \alpha_2$	$\alpha_1 < 0, \alpha_2 < 0$	All
E_2	$-\alpha_1\left(\alpha_2 + \frac{\alpha_1\gamma_2}{\beta_1}\right) / \alpha_2 - \alpha_1\left(1 - \frac{\gamma_2}{\beta_1}\right)$	$\alpha_1 > 0, \gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$	All
E_3	$-\alpha_2\left(\alpha_1 + \frac{\alpha_2\gamma_1}{\beta_2}\right) / \alpha_1 - \alpha_2\left(1 - \frac{\gamma_1}{\beta_2}\right)$	$\alpha_2 > 0, \gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$	All
E_4	$\frac{(\alpha_1\beta_2+\alpha_2\gamma_1)(\alpha_1\gamma_2+\alpha_2\beta_1)}{\beta_1\beta_2-\gamma_1\gamma_2} / \frac{-\beta_1(\alpha_1\beta_2+\alpha_2\gamma_1)-\beta_2(\alpha_1\gamma_2+\alpha_2\beta_1)}{\beta_1\beta_2-\gamma_1\gamma_2}$	$\gamma_1\gamma_2 < \beta_1\beta_2$	$\begin{array}{c}\alpha_1 > 0, \alpha_2 < 0,\\ \gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}, \gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}\end{array}$ → Mutualism Commensalism Parasitism; $\begin{array}{c}\alpha_1 < 0, \alpha_2 > 0,\\ \gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}, \gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}\end{array}$ → Mutualism Commensalism Parasitism; $\begin{array}{c}\alpha_1 > 0, \alpha_2 > 0,\\ \gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}, \gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}\end{array}$ → All

3.3.2. Analysis of Evolutionary Paths

Green innovation is a whole, and green product innovation and green process innovation are two major parts of the whole. In terms of complex systems theory, only when all parts of a system keep harmonious and balanced development can the system realize whole good emergence. The stable states represented by E_1, E_2, and E_3 mean that at least one dimension of green innovation finally disappears in SMMEs, while the stable state represented by E_4 denotes that both dimensions of green innovation develop to non-zero levels in the end. Therefore, the states represented by E_1, E_2, and E_3 are non-ideal and not expected to occur during the evolutionary process of green innovation, and the state represented by E_4 is the ideal stable state.

Green innovation is in the growing stage, and non-ideal stable states may appear. SMMEs can take relevant measures to create routes guiding them to better states or ideal stable states before encountering undesired states. This section analyzes the evolutionary paths and the corresponding measures from non-ideal stable states to the ideal stable state.

If green innovation stabilizes at E_1, three paths evolving towards E_4 are available: the direct path from E_1 to E_4 and the indirect paths from E_1 to E_4 over E_2 or E_3. SMMEs need to take relevant measures to follow each upgrading path, as shown in Figure 2. Specially, to follow the direct path from E_1 to E_4, SMMEs can take one of the three kinds of measures

below: increase the inputs on green product innovation (i.e., increase α_1) and keep the influences that the internal symbiotic interactions of green innovation exert on the growth of green product innovation level and green process innovation level below and above $-\frac{\alpha_1\beta_2}{\alpha_2}$ and $-\frac{\alpha_2\beta_1}{\alpha_1}$, respectively (i.e., keep $\gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$, $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$); increase the inputs on green process innovation (i.e., increase α_2) and keep $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$ and $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$; increase α_1 and α_2 and keep $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$ and $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$. To follow the indirect path from E_1 to E_4 over E_2, SMMEs need to take measures to realize two transformations in turn: the transformation from E_1 to E_2 and the transformation from E_2 to E_4. For the first transformation, SMMEs can increase α_1 and keep $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$. For the second transformation, SMMEs can select one of the below measures: increase γ_2, and keep $\alpha_2 < 0$ and $\gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$; decrease α_1, and keep $\alpha_2 > 0$ and $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$; increase γ_2, and keep $\alpha_2 > 0$ and $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$. It is worth nothing that the measures under the second transformation are just what SMMEs should do when their green innovation stagnates at E_2. Thus, this paper did not specify the path and the corresponding measures if green innovation in SMMEs stabilizes at E_2. Similarly, to follow the indirect path from E_1 to E_4 over E_3, SMMEs can take related measures, as shown in Figure 2. If green innovation in SMMEs is stuck at E_3, there exists one evolutionary path to E_4, as Figure 2 shows. To follow the path from E_3 to E_4, SMMEs can implement one of the below measures: decrease α_2 and keep $\alpha_1 > 0$ and $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$; increase γ_1 and keep $\alpha_1 < 0$ and $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$; increase γ_1 and keep $\alpha_1 > 0$ and $\gamma_2 > -\frac{\alpha_2\beta_1}{\alpha_1}$.
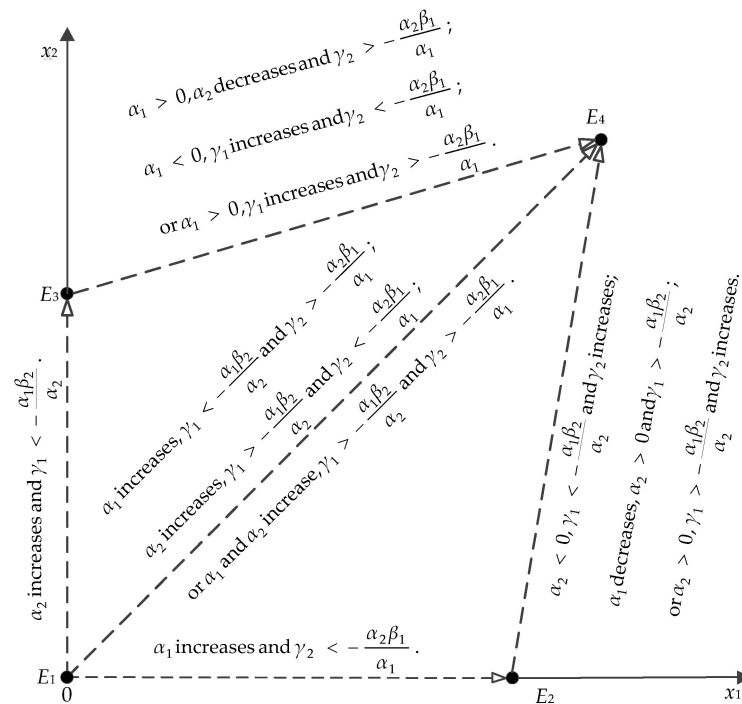
Figure 2. Evolutionary paths and the corresponding conditions of green innovation from non-ideal stable states to the ideal stable state.

4. Simulations

In order to further support our theoretical results, this section conducted a series of numerical simulations to visually show the evolutionary features of green innovation represented by the proposed evolution model.

4.1. Evolutionary Stability of Green Innovation

Based on the proposed evolution model, this section used MATLAB (version 7.0; MathWorks: Nedick, MA, USA, 2004)to perform numerical simulations to analyze the evolutionary stability of green innovation. The iteration length t is set as 1000, and the simulation results are shown in the following figures.

First, we analyzed the stability of green innovation represented by Equation (2) with $\alpha_1 = -0.02$ and $\alpha_2 = -0.01$. Under this condition, the unique evolutionary stable point of green innovation is E_1 no matter what kind of symbiotic interactions exist between green product innovation and green process innovation, as shown in Figure 3. In other words, green innovation with α_1 and α_2 satisfying the given condition is hard to implement for a long time and eventually disappears in SMMEs.
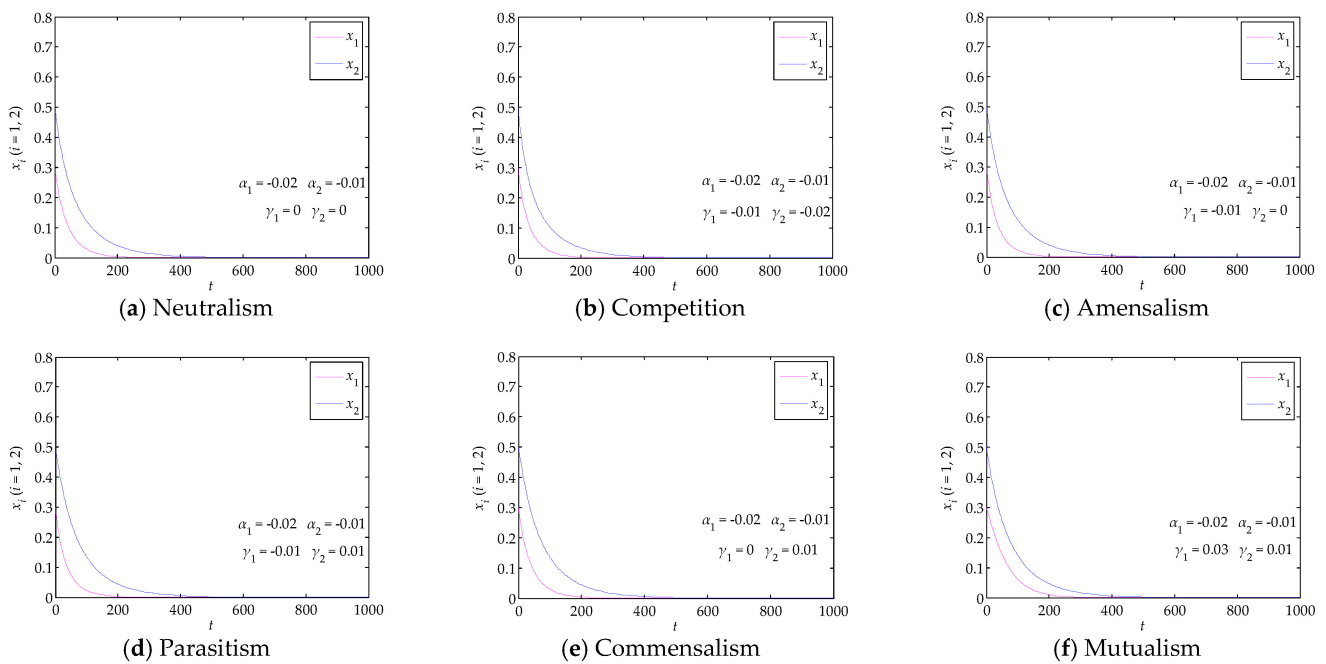

Figure 3. Evolutionary stability of green innovation at E_1.

When Equation (2) is set with $\alpha_1 = -0.05$, $\alpha_2 = 0.01$, $\beta_2 = 0.015$, and $\gamma_1 < 0.075$, the condition that E_3 is the unique stable point is satisfied, and green innovation finally stabilizes at E_3 (0, 0.67) under arbitrary symbiotic interactions, as shown in Figure 4. Under this condition, the green product innovation level and green process innovation level eventually converge to 0 and 0.67, respectively, indicating that SMMEs do not conduct any green product innovation activities and only maintain green process innovation at a certain level. When the condition that E_2 is stable holds, green innovation in SMMEs would exhibit similar evolutionary processes. Therefore, numerical simulations about green innovation evolving towards the equilibrium point E_2 are omitted.

When Equation (2) satisfies $\alpha_1 = -0.014$, $\alpha_2 = 0.014$, $\beta_1 = 0.05$, $\beta_2 = 0.02$, $\gamma_1 > 0.02$, $\gamma_2 < 0.05$, and $\gamma_1\gamma_2 < 0.001$, the condition that E_4 is the only stable point holds, and the green innovation stabilizes at $\left(\frac{-0.028+1.4\gamma_1}{0.1-100\gamma_1\gamma_2}, \frac{0.07-1.4\gamma_2}{0.1-100\gamma_1\gamma_2} \right)$ under the symbiotic interactions of parasitism, commensalism, and mutualism, as shown in Figure 5. Under such conditions, the two dimensions of green innovation grow reversely and always hold a certain level, implying that SMMEs carry out green product innovation and green process innovation simultaneously with obvious differences. When the green innovation stabilizes at E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$, similar evolutions are presented under the three kinds of symbiotic interactions. As a result, this paper did not present the corresponding numerical simulations. In

terms of integrality and comprehensiveness, green innovation stabilizing at E_4 is superior to green innovation stabilizing at other equilibrium points (i.e., E_1, E_2, and E_3).
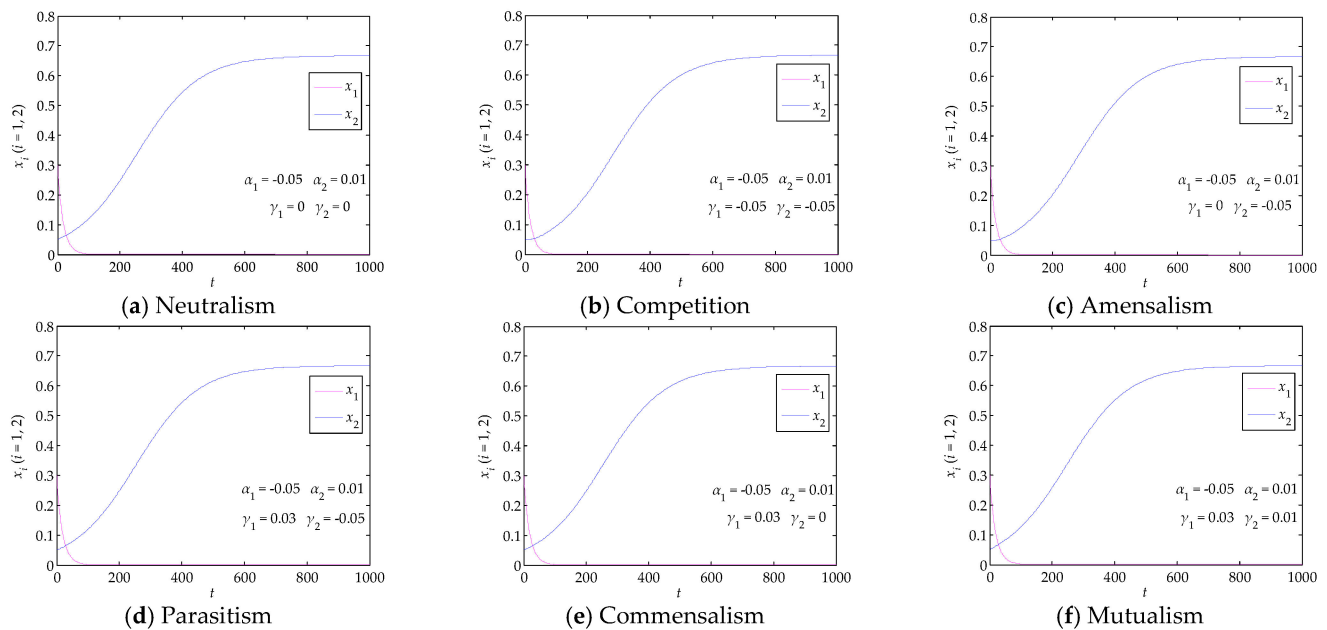

Figure 4. Evolutionary stability of green innovation at E_3.
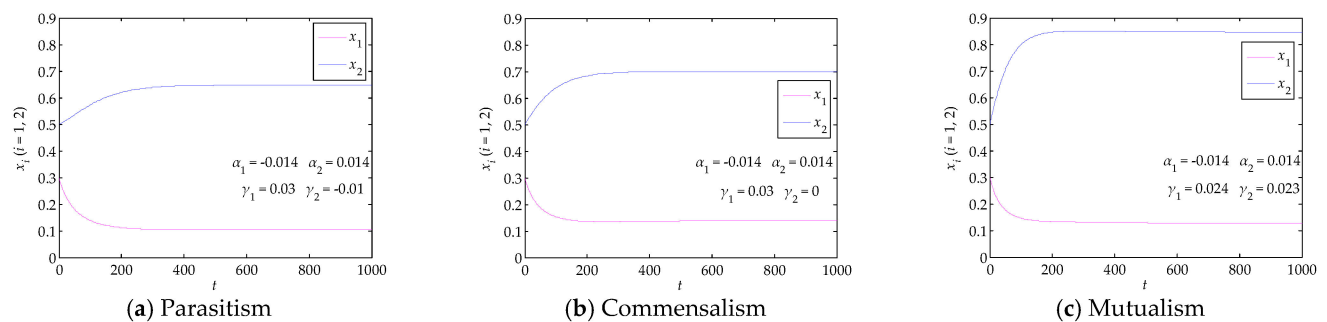

Figure 5. Evolutionary stability of green innovation at E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$.

When Equation (2) is set with $\alpha_1 = 0.02$, $\alpha_2 = 0.01$, $\beta_1 = 0.025$, $\beta_2 = 0.015$, $\gamma_1 > -0.03$, $\gamma_2 > -0.0125$, and $\gamma_1\gamma_2 < 0.000375$, the condition that E_4 is the only stable point satisfies, and the green innovation would converge to $\left(\frac{0.3+10\gamma_1}{0.375-1000\gamma_1\gamma_2}, \frac{0.25+20\gamma_2}{0.375-1000\gamma_1\gamma_2} \right)$ under all symbiotic interactions. Under such a condition, SMMEs conduct green innovation practices in both dimensions, and the green product innovation level and green process innovation level continually rise until their upper limits are reached. Compared with the case that green innovation stabilizes at E_4 with $\alpha_1\alpha_2 < 0$, green innovation stabilizing at E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$ enables healthier and superior innovation levels and thus gains more popularity and expectation.

When green innovation stabilizes at E_4, six symbiotic interactions play different roles, and mutualism is the optimal symbiotic interaction. Specifically, compared to other symbiotic interactions, mutualism enables SMMEs to realize stable green innovation at higher levels when inputs in green innovation are fixed, as shown in Figures 5 and 6. It should be pointed out that symmetric mutualism (i.e., $\gamma_1 = \gamma_2 > 0$) outperforms asymmetric mutualism (i.e., $\gamma_1 \neq \gamma_2$, $\gamma_1 > 0$ and $\gamma_2 > 0$) in green innovation. As shown in Figure 7c,d, green innovation in both dimensions under symmetric mutualism stabilizes at (0.132, 0.858) and (1, 1), respectively; however, as shown in Figures 5f and 6f, green innovation in both

dimensions under asymmetric mutualism can only stabilize at (0.125, 0.844) and (0.946, 0.730), respectively; obviously, the former green innovation levels are higher than the latter green innovation levels, for example, (0.132, 0.858) exceeds (0.125, 0.844). We observed similar phenomena and drew consistent conclusions when green innovation stabilizes at other equilibrium points, as shown in Figures 3, 4 and 7a,b.
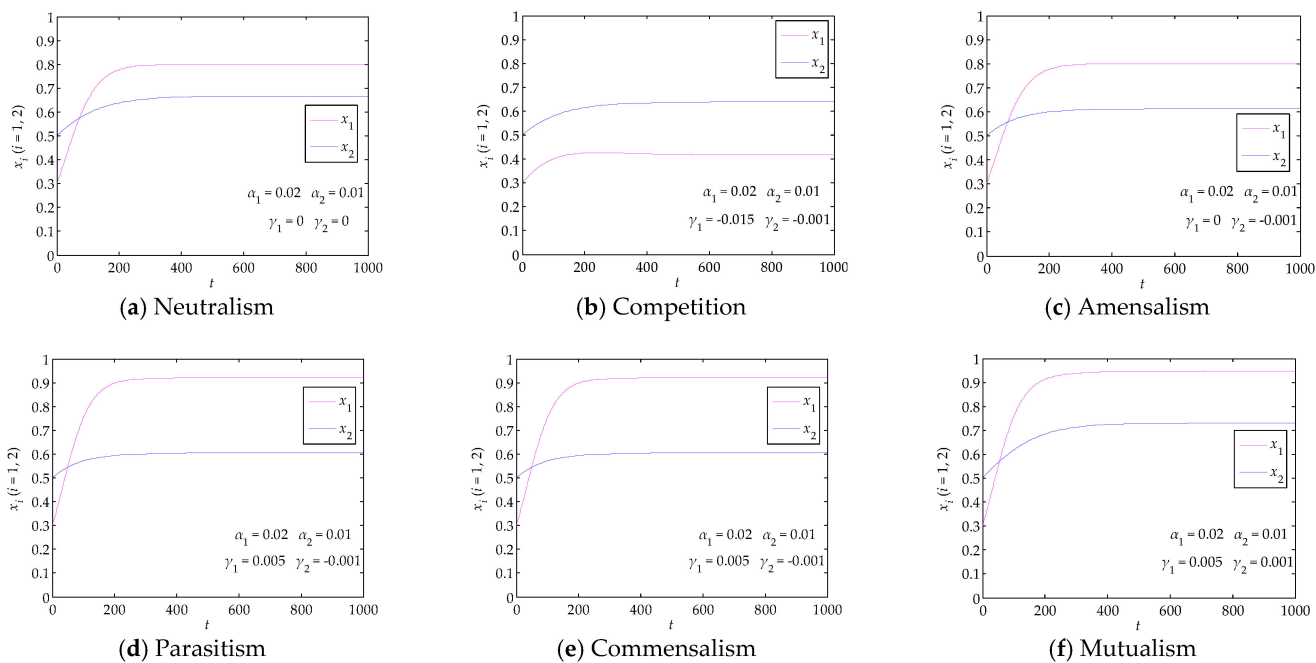

Figure 6. Evolutionary stability of green innovation at E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$.
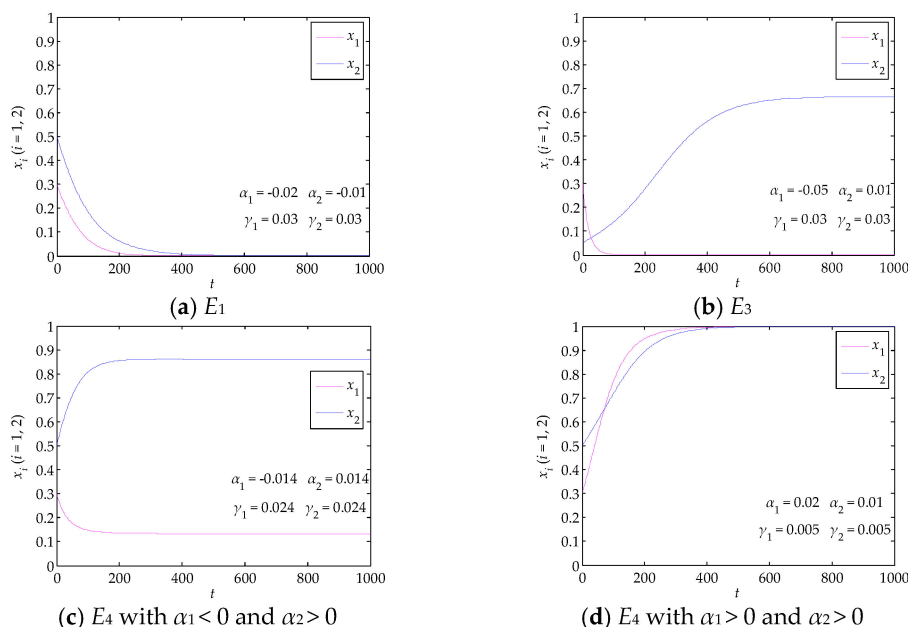

Figure 7. Evolutionary stability of green innovation at equilibrium point when $\gamma_1 = \gamma_2 > 0$.

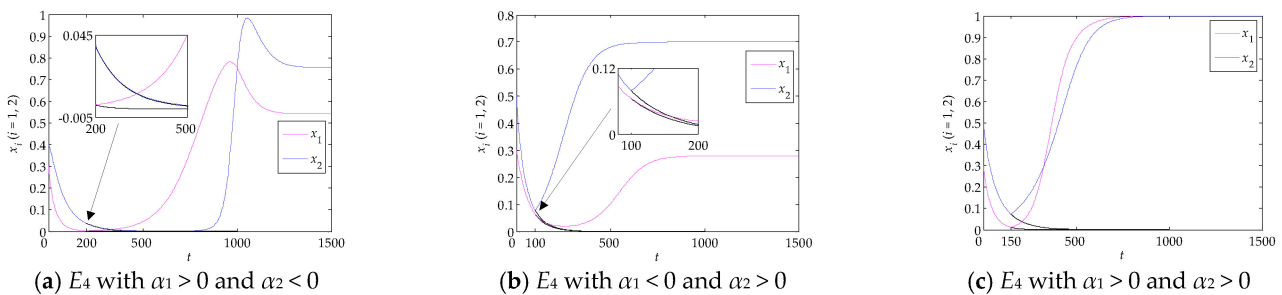### 4.2. Evolutionary Paths of Green Innovation

This section also used MATLAB to conduct numerical simulations to anatomize how green innovation evolves from a non-ideal stable point to the ideal point E_4. The iteration

length t is set as 1500, and the parameter values under different transitions are shown in Table 3. Then, we obtained the evolutionary paths from E_1, E_2, and E_3 to E_4.

Table 3. Parameter values before and after the transitions between two stable nodes.

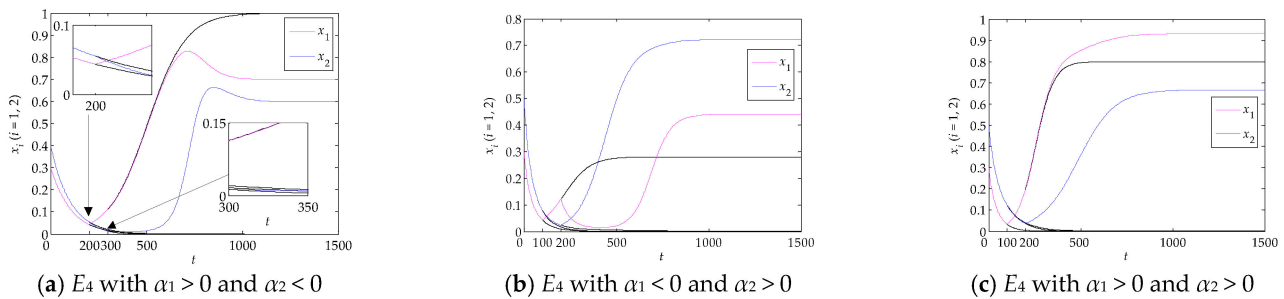| Original Points	Middle Points	Final Points	Parameters Values before and after Transitions (10^{-2})					
			α_1	α_2	β_1	β_2	γ_1	γ_2
E_1	None	E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$	$-2/1$	-1	1	3	$-2/-0.6$	6
		E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$	-1.4	$-1.4/1.4$	5	2	$3/4$	0
		E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$	$-2/2$	$-1/1$	2.5	1.5	0.5	0.5
E_1	E_2	E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$	$-1/1/1$	-1	1	3	$1/1/-0.5$	$4/-5/4$
		E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$	$-1.4/1.4/-1.4$	$-1.4/-1.4/1.4$	5	2	$0.5/0.5/5$	$0/4/0.1$
		E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$	$-2/2/2$	$-1/-1/1$	2.5	1.5	0.5	$0/-1/0$
E_3	None	E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$	1	$1/-1$	1	3	$-6/-1$	4
		E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$	-1.4	1.4	5	2	$1/3$	-1
		E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$	2	1	2.5	1.5	$-3.5/0.5$	0.1

Based on the parameter values in Table 3, we observed how green innovation evolves towards E_4 without transitional stable points before it stabilizes at E_1. Specifically, to make green innovation convert from E_1 to E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$, SMMEs can keep $\gamma_1 < -\frac{\alpha_1 \beta_2}{\alpha_2} = 0.03$ and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1} = 0.01$, and increase α_1 from -0.02 to 0.01, and these measures can be taken at the right time; as shown in Figure 8a, green innovation evolves towards E_1 before taking these measures at $t = 200$, but it gradually deviates from the path to E_1 (i.e., the path denoted by the dotted lines in Figure 8a) and follows the path to E_4 (i.e., the path denoted by the solid lines in Figure 8a) instead of taking these measures at $t = 200$. To make green innovation convert from E_1 to E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$, SMMEs can increase α_2 from -0.014 to 0.014 and keep $\gamma_1 > -\frac{\alpha_1 \beta_2}{\alpha_2} = 0.02$ and $\gamma_2 < -\frac{\alpha_2 \beta_1}{\alpha_1} = 0.05$; these measures can be taken at $t = 100$, as shown in Figure 8b. To make green innovation convert from E_1 to E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$, SMMEs can maintain $\gamma_1 > -\frac{\alpha_1 \beta_2}{\alpha_2} = -0.03$ and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1} = -0.0125$ and raise α_1 and α_2 from -0.02 and -0.01 to 0.02 and 0.01, respectively; these measures can be taken at $t = 150$, as shown in Figure 8c. The dotted lines in Figure 8 represent the virtual evolutionary paths of green innovation after taking measures, and the solid lines stand for the actual evolutionary paths of green innovation. Such expressions of evolutionary paths also exist in the following figures. The results presented in Figure 8 verify the theoretical analysis of the direct evolutionary paths from E_1 to E_4.

(a) E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$　　　**(b)** E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$　　　**(c)** E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$

Figure 8. Evolutionary paths from E_1 to E_4: **(a)** α_1 increases, $\gamma_1 < -\frac{\alpha_1 \beta_2}{\alpha_2}$ and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1}$, **(b)** α_2 increases, $\gamma_1 > -\frac{\alpha_1 \beta_2}{\alpha_2}$ and $\gamma_2 < -\frac{\alpha_2 \beta_1}{\alpha_1}$, and **(c)** α_1 and α_2 increase, $\gamma_1 > -\frac{\alpha_1 \beta_2}{\alpha_2}$ and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1}$; the corresponding measures are taken at **(a)** $t = 200$, **(b)** $t = 100$ and **(c)** $t = 150$.

　　　Similarly, using the parameter values in Table 3, we obtained the evolutionary paths of green innovation from E_1 to E_4 via E_2, as shown in Figure 9. To be specific, to drive green innovation from E_1 to E_2 and E_2 to E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$, SMMEs can first increase α_1 from -0.01 to 0.01 and make $\gamma_2 < 0.01$ by decreasing γ_2 from 0.04 to -0.05 at $t = 200$, which can enable green innovation to switch from E_1 to E_2; then, SMMEs can raise γ_2 from -0.05 to 0.04, and keep $\alpha_2 < 0$ and $\gamma_1 < 0.03$ at $t = 300$, and these measures are able to drive green innovation from E_2 to E_4, as shown in Figure 9a. To drive green innovation from E_1 to E_2 and E_2 to E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$, increasing α_1 from -0.014 to 0.014 and maintaining $\gamma_2 < 0.05$ at $t = 100$ are the primary measures that SMMEs can select to realize the transformation from E_1 to E_2; based on existing actions, decreasing α_1 from 0.014 to -0.014, and making $\alpha_2 > 0$ and $\gamma_1 > 0.02$ at $t = 200$ are the following actions that SMMEs can take to realize the transition from E_2 to E_4, as shown in Figure 9b. To drive green innovation from E_1 to E_2 and E_2 to E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$, SMMEs should first make green innovation convert from E_1 to E_2 and raise α_1 from -0.02 to 0.02 and keep $\gamma_2 < 0.0125$ at $t = 100$ are feasible measures; then, SMMEs can take actions to realize the evolution from E_2 to E_4, and increasing γ_2 from -0.01 to 0, making $\alpha_2 > 0$ by increasing it from -0.01 to 0.01 and keeping $\gamma_1 > -0.03$ at $t = 200$ are doable, as shown in Figure 9c. The corresponding evolutionary paths under the three cases in that green innovation evolves from E_1 to E_4 via E_2 are depicted in the three subfigures of Figure 9, respectively. When green innovation converges towards E_2, SMMEs can refer to what they do in the second phase of the evolution from E_1 to E_4 via E_2 and make relevant parameters satisfy specified conditions, and then green innovation could present desired evolutionary paths to E_4. Similarly, by taking the right measures (i.e., making related parameters meet stated conditions) at the right time, we observed the evolutionary paths that green innovation follows to evolve from E_1 to E_4 over E_3. Thus, this section omits the numerical simulations about green innovation evolving from E_1 to E_4 over E_3 and evolving from E_2 to E_4.

(**a**) E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$　　　(**b**) E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$　　　(**c**) E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$

Figure 9. Evolutionary paths from E_1 to E_4 over E_2: α_1 increases, $\gamma_2 < -\frac{\alpha_2\beta_1}{\alpha_1}$ and then (**a**) γ_2 increases, $\alpha_2 < 0$ and $\gamma_1 < -\frac{\alpha_1\beta_2}{\alpha_2}$, (**b**) α_1 decreases, $\alpha_2 > 0$, and $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$, and (**c**) γ_2 increases, $\alpha_2 > 0$ and $\gamma_1 > -\frac{\alpha_1\beta_2}{\alpha_2}$; the corresponding measures are successively taken at (**a**) $t = 200$ and $t = 300$, (**b**) $t = 100$ and $t = 200$, and (**c**) $t = 100$ and $t = 200$.

　　　Finally, based on the parameter values in Table 3, we surveyed how green innovation converges to E_4 without stable middle points before it is caught in the state represented by E_3. Concretely, to enable green innovation to switch from E_3 to E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$, SMMEs can decrease α_2 from 0.01 to -0.01, and keep $\alpha_1 > 0$ and $\gamma_2 > 0.01$ at $t = 100$, then the desired transition is possible, at shown in Figure 10a. To enable green innovation to switch from E_3 to E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$, SMMEs can increase γ_1 from 0.01 to 0.03 and maintain $\alpha_1 < 0$ and $\gamma_2 < 0.05$ at $t = 300$; by doing so, green innovation following a satisfactory evolutionary path is available, as shown in Figure 10b. To enable green innovation to switch from E_3 to E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$, increasing γ_1 from -0.035 to 0.005 and keeping $\alpha_1 > 0$ and $\gamma_2 > -0.0125$ at $t = 500$ are viable measures for SMMEs to create a path that guides their green innovation to E_4, as shown in Figure 10c.

(**a**) E_4 with $\alpha_1 > 0$ and $\alpha_2 < 0$ (**b**) E_4 with $\alpha_1 < 0$ and $\alpha_2 > 0$ (**c**) E_4 with $\alpha_1 > 0$ and $\alpha_2 > 0$
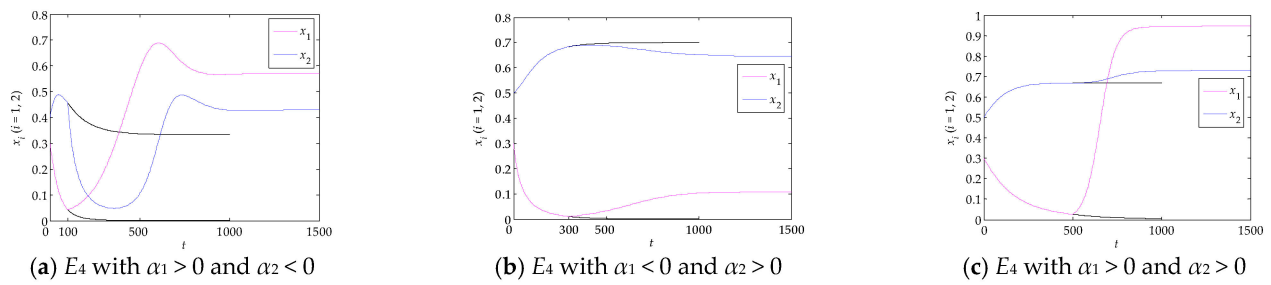
Figure 10. Evolutionary paths from E_3 to E_4: (**a**) α_2 decreases, $\alpha_1 > 0$ and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1}$, (**b**) γ_1 increases, $\alpha_1 < 0$, and $\gamma_2 < -\frac{\alpha_2 \beta_1}{\alpha_1}$, and (**c**) γ_1 increases, $\alpha_1 > 0$, and $\gamma_2 > -\frac{\alpha_2 \beta_1}{\alpha_1}$; the corresponding measures are taken at (**a**) $t = 100$, (**b**) $t = 300$, and (**c**) $t = 500$.

5. Findings and Discussion

5.1. Findings

The research objective is to explore how green innovation evolves in SMMEs. Theoretical analysis and numerical simulations revealed evolutionary stability and paths, and the following findings are extracted. First, green innovation in SMMEs has four end-states represented by E_i ($i = 1, 2, 3, 4$) during its evolutionary processes; E_1, E_2, and E_3 are unexpected end-states for SMMEs, and E_4 is the unique end state that SMMEs expect. Second, when green innovation in SMMEs stabilizes at E_1, there exists one direct evolutionary path and two indirect paths leading to E_4; when green innovation in SMMEs stabilizes at E_2 or E_3, one upgrading path evolving towards E_4 is available. Third, the inputs on green product innovation and green process innovation (i.e., α_1 or α_2) and the types or strength of the symbiotic interactions between green product innovation and green process innovation (i.e., γ_1 and γ_2) are key factors that determine the end states and the evolutionary paths of green innovation. Besides, symmetric mutualism is the optimum for symbiotic interactions. The above findings can yield beneficial management implications for SMMEs and regulatory subjects to drive green innovation.

5.2. Discussion

Green innovation becomes increasingly important for enterprises to relieve environmental pressures and enhance competitiveness. Currently, more and more SMMEs carry out green innovation in their products and production processes. However, SMMEs often get discouraged and want to give up when they develop green innovation practices. The issue behind this phenomenon is that how green innovation evolves in SMMEs remains unclear. Identifying all the end states of green innovation and determining the trajectories evolving towards the desired end states and the corresponding conditions will facilitate SMMEs to conduct green innovation.

In this study, we explored the evolution of green innovation in SMMEs based on complex systems theory and the Logistic equation. Differing from earlier studies that highlighted one dimension of green innovation [25,52,58], this study found that green innovation with non-zero levels in each dimension was expected. In terms of complex systems thinking, an explanation for this difference might be that green innovation missing any one dimension is crippled and cannot reach whole good emergence due to lacking nonlinear interactions between green product innovation and green process innovation. In addition, many previous studies demonstrated that exogenous measures (e.g., environmental policies and gross domestic product) [29,74,75] and improving external interactions [29,58] could promote green innovation. The results are different from our finding that changing internal interactions of green innovation was quantitatively proved to be an effective endogenous measure for SMMEs to steer green innovation towards expected states. It is surprising that mutualism could lead green innovation in arbitrary states to a stable state. The account for result assumes that mutualism benefits green innovation in all dimensions. Besides, symmetric mutualism is the most conducive to green innovation, for symmetric mutualism

is a kind of equilibrium relationship formed by the game between green product innovation and green process innovation. Previous studies usually qualitatively showed that increasing inputs could benefit green innovation [44,49], while this study quantitatively revealed that both increasing inputs and decreasing inputs could play a positive role in green innovation. It is intuitive that increasing inputs can improve green innovation if inputs in green innovation are indeed deficient. However, if inputs in green innovation reach or exceed the upper limit, adding inputs unceasingly might bring diminishing marginal returns or even huge losses [39]; under this condition, decreasing inputs is helpful. The reason for this phenomenon is that excessive inputs can induce chaotic green innovation by unbearable complexity, and decreasing inputs at this time can pull green innovation from chaos to healthy evolutionary paths by reducing complexity. Several previous studies provided guidance to advance green innovation beyond the micro level [1,17–19,61], while our findings could guide regulatory subjects and SMMEs to take measures to drive green innovation during its evolutionary process at the micro level. This research enriches the evolutionary theories related to green innovation and expands the application fields of complex systems theory.

This research mainly has two limitations. First, green innovation is a complex system with many components, and the proposed model cannot be directly applied if the division of green innovation changes. Second, the nonlinear interactions between green product innovation and green process innovation are quantized as $x_1 x_2$ in this paper, but there remain other quantified expressions such as $x_1{}^2 x_2$. Hence, the quantification of the nonlinear interactions within green innovation in this study is not all-sided.

6. Contributions and Managerial Implications

This research mainly has two contributions. First, unlike previous literature that concentrated on the macro and meso evolution of green innovation, this paper explored the evolutionary stability and paths of green innovation from a micro angle because enterprises are the real subjects that implement green innovation. The research results not only facilitate SMMEs to judge whether their green innovation could be stable but also provides the evolutionary paths and as well as supporting measures for SMMEs to convert green innovation into desired stable states if their green innovation cannot reach targeted states within a period of time. Besides, our research results are instructive to large manufacturing enterprises to manage and control their green innovation and also help regulatory organizations enact relevant policies. Second, given the complexity and dynamics of green innovation in SMMEs, this paper utilized complex systems theory to construct nonlinear dynamic models to essentially depict the evolution of green innovation in a quantitative manner, which avoids the defects originating from the mechanism and reductionism of existing methods. The proposed modeling approach provides a brand-new idea for us to discuss other complex issues fundamentally.

To make green innovation in SMMEs turn to or stay at desired states, this paper puts forward some managerial implications. On the one hand, regulatory subjects should establish regulatory mechanisms and introduce relevant policies and measures for SMMEs with green innovation in different evolutionary stages. In the infancy stage of green innovation, regulatory subjects could inject initial impetus to promote green innovation from top to bottom by giving administrative orders, directly inputting resources, issuing subsidy coupons, etc., for the inputs on green innovation in the infancy stage are deficient and adding the inputs on green innovation facilitates green innovation in SMMEs when the inputs on green innovation are insufficient. In the growing stage of green innovation, regulatory subjects could set up leading groups and expert groups to normalize the orders for SMMEs to conduct green innovation and to guide SMMEs to optimize resource allocation in green innovation, for only reasonable inputs allocated on green innovation can yield healthy green innovation in SMMEs. In the maturation stage of green innovation, regulatory subjects could establish evaluation systems and governance systems to assess and monitor the real states of green innovation in SMMEs, respectively, in order to help

SMMEs correct deviations in time when they are short of relevant technology systems and human resources. On the other hand, SMMEs should get rid of old ideas and actively enhance their own innovation levels to form the endogenous power of green innovation. Besides, SMMEs need to follow the evolutionary rules of green innovation and encourage internal components related to green innovation to collaborate zealously in order to yield healthy symbiotic interactions. Once SMMEs begin green innovation practices, they need to pay dynamic attention to the inputs of green product innovation and green process innovation as well as the symbiotic interactions between them, for our research results reveal that the inputs on green innovation and the symbiotic interactions within green innovation are key factors affecting the end states and the evolutionary paths of green innovation; by recording the changes of relevant parameters, SMMEs can evaluate the current states of their own green innovation and forecast future movements of green innovation; if current states or future movements of green innovation are not satisfactory, SMMEs can take the right measures at the right time to create desired evolutionary paths that green innovation can follow to reach ideal states. When SMMEs and regulatory subjects adopt the above actions, both enterprise competitiveness and environmental sustainability have the potential to improve.

7. Conclusions and Future Research

This research developed a nonlinear dynamic model to study the evolution of green innovation in SMMEs. Viewing green innovation as a complex system and defining its components and state variables enable the quantitative formulation of the model by providing the necessary elements. Based on the proposed model, we obtained the evolutionary stability, evolutionary paths, and corresponding conditions of green innovation in SMMEs. This research discovered a non-zero stable state of green innovation which is an ideal evolutionary end that SMMEs should take measures to reach. Among the internal measures available, controlling inputs on green innovation and the interactions between green product innovation and green process innovation are encouraged to be effective measures in this paper. By taking the proposed measures at the right time in SMMEs, green innovation is able to stabilize at the ideal state or turn to the ideal state by following the designed, evolutionary paths that avoid unexpected stable states. This paper demonstrated the validity of the proposed model and the theoretical results through a series of numerical simulations. Simulation results also show that symmetric mutualism is the optimal symbiotic interaction for SMMEs to advance green innovation because keeping the symbiotic interactions within green innovation as symmetric mutualism could enable green innovation to realize the greatest growth in each dimension without influencing the realization of the ideal stable state. This research can enrich the evolutionary theories of green innovation and provide practical guidance for SMMEs and regulatory subjects to judge current healthy states of green innovation, forecast the future trends of green innovation, and formulate controlling measures.

Future work mainly involves two aspects. On the one hand, green innovation has other categories in theory and in practice, and thus discussing the evolution of green innovation under new divisions is a research direction for the future. On the other hand, nonlinear interactions within green innovation can manifest in other forms. Therefore, future work can study the evolution of green innovation with different quantified interactions.

Data Availability Statement: Data sharing not applicable—no new data generated data sharing does not apply to this article as no new data were created or analyzed in this study.

Conflicts of Interest: The authors declare no conflict of interest.

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