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Systematic Review

Exploring the Benefits and Drawbacks of AR and VR Technologies for Learners of Mathematics: Recent Developments

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Abstract: Despite the growing interest in the field, the overall impact of augmented reality (AR) and virtual reality (VR) on mathematics learning remains unclear, with previous studies reporting mixed results. Moreover, to date, no systematic review has evaluated the potential of AR/VR in mathematics education, including its benefits and drawbacks for learners. To address this gap, the present systematic literature review aims to identify research trends, determine characteristics and methodologies, and explore the potential benefits and drawbacks of AR/VR technologies in mathematics learning based on existing empirical studies. In accordance with the PRISMA guidelines, we analyzed 59 peer-reviewed journal articles published in English that focused on AR/VR implementation in mathematics education. The review determined that geometry was the most widely studied topic of mathematics, with several studies focusing on the use of AR/VR to assist students with learning disabilities. The present review offers evidence for the potential of AR/VR potential in consolidating learners' socio-emotional, cognitive/meta-cognitive, and pedagogical development in mathematics learning. Nevertheless, a few issues, including technological glitches, cost, start-up effort, health issues, and unfamiliarity with AR/VR, pose challenges to the successful application of AR/VR in the classroom. This systematic review contributes to the existing body of knowledge in the field and recommends avenues for future research.

Keywords: augmented reality; digital technology; immersion; learning outcomes; mathematics education; mixed reality; recent developments; systematic review; virtual reality

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1. Introduction

The use of new technologies in education has become increasingly popular in recent years with the rise of augmented reality (AR) and virtual reality (VR) technologies, which promise to enhance students' learning process [1]. These nascent technologies can produce highly realistic representations as well as allowing subjects to interact with virtual objects. These effects are amplified when AR and VR are combined, offering multiple perspectives to learners [2]. VR technologies allow learners to access a virtual world, actively immerse themselves in it, and interact with objects, thereby enhancing their mathematical thinking skills and spatial abilities [3]. Moreover, AR technologies allow learners to work with objects in the real world, with the opportunity to explore their features and manipulate them without requiring them to become disconnected from reality [4,5].

AR/VR technologies have witnessed significant advancements in recent years and have the potential to revolutionize various fields, including engineering [6], medicine [7], tourism [8], industry [9], entertainment and gaming [10], and education [11]. Moreover, the use of AR and VR technologies in the educational landscape is expected to become more widespread in the near future [12–15]. In particular, the use of AR/VR in the field of mathematics education has attracted considerable attention in recent years as a useful tool to enhance learning motivation and student outcomes [16]. However, previous studies have yielded mixed results regarding the effects of AR/VR technologies on students' mathematical learning processes [16,17]. On the one hand, the integration of AR and VR applications into mathematics education has the potential to consolidate students' understanding of mathematical concepts and boost their learning motivation and spatial abilities [18–22]. On the other hand, negative attitudes toward AR and VR may generate unfavorable learning outcomes [23], and the mathematics learning process may be negatively impacted by the occurrence of negative side effects (e.g., headaches and eye strain) that may manifest after working in virtual environments [3]. Learning modes, curriculum design [24], culture, and learner characteristics [25] may influence the effectiveness of AR and VR technologies for students' mathematics learning. To summarize, these findings indicate that the extent to which AR/VR technologies can enhance mathematics learning has yet to be conclusively determined, and, thus, a more thorough and systematic examination of the existing literature is warranted to help ensure that the recent developments and research trends, the role of AR/VR in mathematics education, and the successful implementation of these technologies are guided by empirical evidence rather than hype and speculation [26].

In light of the above-mentioned research gap, the present systematic literature review will focus on existing empirical studies—in particular, those pertaining to the overall role that AR/VR technology plays in students' mathematics learning, including its benefits and drawbacks for learners. Moreover, research is included which explores the extent to which achievement outcomes are examined and which has identified ways to optimize the use of AR/VR, considering its benefits and drawbacks for users to support effective mathematics learning processes. The present literature review also reports research trends (e.g., publication years, geographical distributions, study contexts and domains, methodological bases of the studies, and distribution of the AR and VR studies) and the most popular digital tools used in the examined studies. Overall, this comprehensive systematic review will inform successful and sustainable design and practices in AR/VR-based mathematics education and help guide future research in this field by identifying key knowledge gaps.

The study is guided by the following research questions:

1. What are the research trends and overall characteristics of studies concerning the use of AR/VR technologies in mathematics learning?
    - How have the reviewed studies developed over time?
    - How are the authors of studies distributed geographically?
    - What are the study domains of the reviewed studies?
    - What are the methodological bases of the reviewed studies?
    - What research trends emerge in relation to the use of AR/VR in mathematics learning?
2. Which digital tools (software and hardware) are used in the reviewed studies on AR and VR research in mathematics education?
3. What potential benefits do AR and VR technologies offer for mathematics learning?
    - What are the potential benefits of AR and VR technologies for mathematics learners?
    - What are the potential drawbacks of AR and VR technologies for mathematics learners?

2. Background

2.1. Conceptualization of AR and VR Technology

While AR and VR technologies are closely related, they offer different approaches to interacting with reality and virtuality. While AR technologies overlay virtual information onto the real world and allow users to interact with both real and virtual content, VR technologies transport the user to a virtual environment with the aim of fully immersing them in that world [27,28]. Mixed reality (MR) blends both types of interaction within a single object. Milgram and Kishino [28] first proposed the reality–virtuality continuum—a central model for understanding AR/VR that introduced the MR construct. Their model proposed an approach to categorizing different MR display systems based on a taxonomy of key factors (see Figure 1). AR and VR may be viewed as lying at opposite ends of the reality–virtuality continuum [2]. For the user, the difference lies in the level of immersion [8]: in AR, the perspective of the user remains primarily that of the real world, while in VR, the user is wholly immersed in a virtual environment.

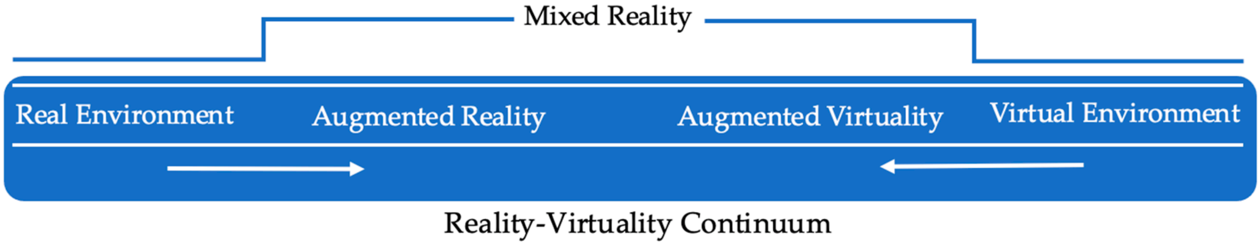

Figure 1. Simplified representation of a reality–virtuality continuum [28].

Researchers have highlighted several key features that characterize AR and VR technologies. According to Azuma [27], AR enhances the real-world view by overlaying or blending virtual objects with it, rather than replacing it entirely, with the aim of creating a seamless coexistence of real and virtual objects within the same space, giving the user the impression that the virtual objects are supplementing reality. Azuma et al. [29] describe the three main defining characteristics of AR systems as follows: (a) they generally involve the seamless integration of virtual and real-world elements; (b) they provide real-time interactivity; and (c) they accurately align and register 3D objects. Yung and Khoo-Lattimore [8] emphasized the three crucial constituents of VR as follows: first, visualization enables the user to explore the virtual environment by looking around, often facilitated by a head-mounted display; second, immersion involves the creation of a convincing virtual world that suspends the disbelief of users and provides realistic representations of objects; and third, interactivity measures the level of user engagement with the virtual environment and is often achieved through the use of sensors and input devices, such as joysticks or keyboards.

2.2. Previous Reviews of AR/VR in Mathematics Education

Previous literature reviews pertaining to AR/VR in education have primarily focused on either the use of AR in STEM education (e.g., [30]) or in general educational sciences (e.g., [12,31]), with few systematic reviews assessing the combined effectiveness of AR and VR for mathematics education. While several reviews have focused solely on the use of AR in mathematics education, to the best of our knowledge, no systematic review to date has evaluated the combined impact of AR and VR technologies in mathematics learning processes. Our search of the literature revealed several systematic review studies that were exclusively on the use of AR in mathematics education [16,17]. Ahmad and Junaini [16] used SCOPUS as a database and recruited 19 journal articles published between 2015 and 2019. Palanci and Turan [17] focused on the methodological trends of studies using AR in mathematics education, reviewing 86 studies (i.e., conference proceedings and journal articles) sourced from the Web of Science (WoS) database. These reviews focused exclusively on AR technology (omitting VR) and offered no comprehensive analysis or discussion of the effectiveness of both AR and VR technologies on mathematics learning from various perspectives (e.g., cognitive, affective, socio-emotional, psychological, and attitudinal outcomes). However, they reported several common benefits and difficulties associated with the use of AR in mathematics education. Accordingly, the most frequently mentioned opportunities offered by AR for students include improvements in learning motivation and confidence as well as enhanced learning through visualization, spatial abilities, and interactive engagement. The reviews also revealed that the use of AR technology is associated with problems in mathematics learning, such as difficulties in creating visualizations and understanding mathematics concepts visually, as well as technical problems and cost.

A recent scoping review [32] referred to AR and VR collectively as "extended reality" (XR). The study aimed to explore the existing research on XR, with a particular focus on the pedagogical implications of immersive extended realities in the context of teaching and learning engineering mathematics. Although Lai and Cheong [32] presented a framework for implementation of the XR technology, the available evidence in support of the impact of AR/VR in mathematics learning is limited.

Consequently, the discernible lack of research in the realm of mathematics education regarding the collective influence of AR/VR accentuates the burgeoning demand for comprehensive systematic reviews on this topic. The present study is well-equipped to address this requirement and provide a timely exploration of the potential benefits and drawbacks of both AR and VR technologies in mathematics education.

3. Materials and Methods

The present systematic review aims to understand the value of AR/VR technology, focusing on its benefits and drawbacks in the context of mathematics learning. The review promises to generate broad conclusions regarding the merit of focused conceptualizations, approaches, and applications in the field of AR/VR by identifying research trends and presenting interpretable patterns [33,34]. We adhere to the "referred reporting items for systematic reviews and meta-analyses" (PRISMA) guidelines [26] to enhance the trustworthiness and transparency of the review with respect to the selection of the studies and report synthesis. In light of the rapid development of technology, we have aimed to explore the most recent research on learning mathematics with AR/VR with a focus on empirical research articles. The literature search was conducted in August 2022 using two well-known electronic databases (i.e., WoS and SCOPUS), and we applied the following search strings with Boolean logic: "Title (augmented reality OR virtual reality) AND Abstract (math*)."

The electronic search yielded 740 records at the identification stage. MS Excel and EndNote X9 software were employed to organize and manage the identified records, and 60 duplicated records were discarded through EndNote X9. At the screening stage, we followed five manuscript selection criteria: (a) document type: the study was published in a peer-reviewed journal and presented empirical data; (b) language: the study was written in English; (c) publication year: the study is quite recent and incorporates most recent technological developments—we therefore restricted the publication interval to the last five years (2018–2022); (d) domain: the study was carried out in the field of mathematics education; and (e) research focus: the study provided empirical results on the benefits or drawbacks of AR/VR technology for mathematics learners. Based on these criteria, we screened the titles, abstracts, and keywords of 680 studies, and then examined the full texts of 151 papers. In the final step, the eligibility check ensured the inclusion of 59 papers in the present systematic review (see Figure 2).

Analyses of the 59 papers (see Table 6 in Section 5) were conducted separately by the first two authors, and the main focus was particularly on the role of AR/VR technologies and answering the developed research questions. We screened the eligible papers several times to ensure familiarity with the empirical data and evidence relating to the research questions. Two scholars conducted an independent coding of the papers in accordance with the principles of qualitative content analysis [35]. To establish intercoder reliability, all codes were compared, and a high reliability rate (0.91) above the recommended threshold (0.80) was obtained [35,36].

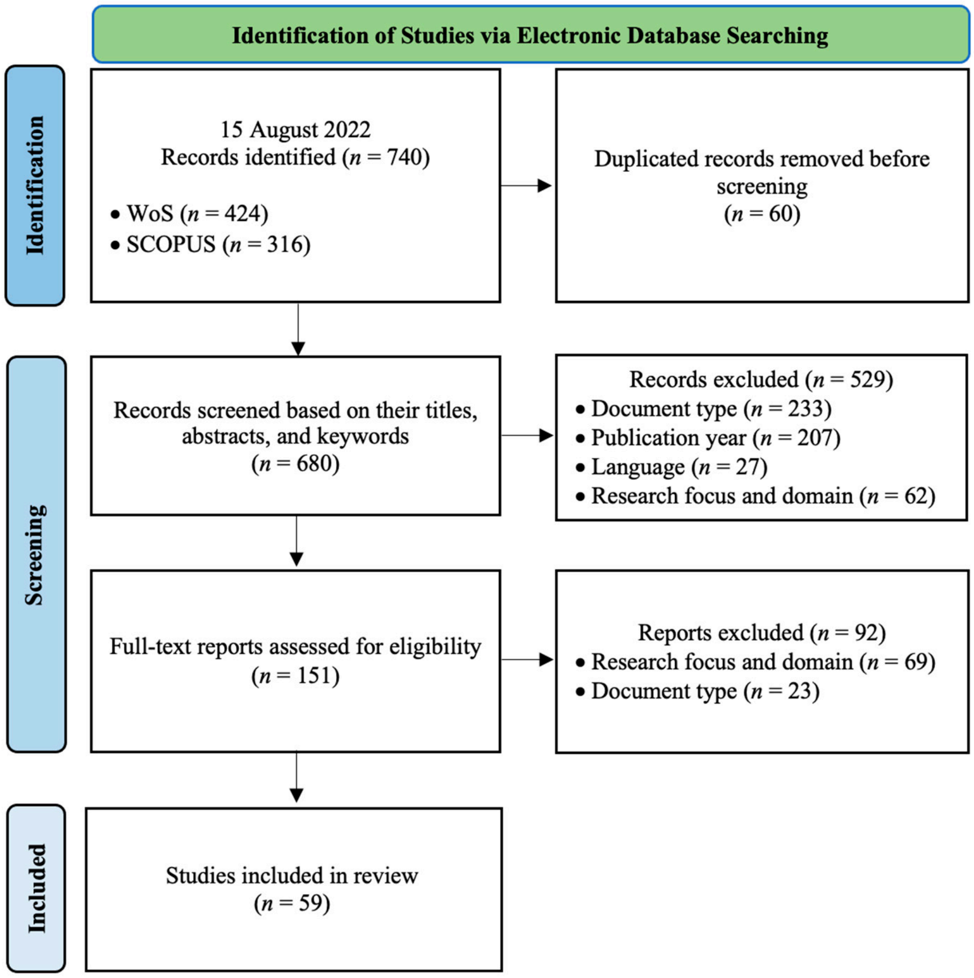

Figure 2. Flow chart of the article selection process.

4. Results and Discussion

In this literature review, we have organized the synthesis of the results and discussion into three main categories, which are oriented by the research questions: (a) overall research trends, (b) digital tools used in the studies (software and hardware), and (c) the potential of AR/VR technologies for mathematics learning.

4.1. Research Trends on Learning Mathematics with AR/VR Technology

4.1.1. Publication Years

The analyses revealed a notable increase in the number of empirical studies on the use of AR/VR in mathematics education published in peer-reviewed journals after 2018 (see Figure 3). This finding highlights the considerable interest of mathematics education researchers in this topic and the fact that applications of AR/VR technologies in the educational landscape have proliferated in recent years [12–15,22]. The fact that the tools/devices required for AR/VR implication are becoming more affordable and accessible [10,15,37] may have contributed to this result. Forecasts regarding the increased adoption of AR/VR technologies in education, reported in the 2019 and 2020 Horizon Reports by the New Media Consortium [38] and Educause [39], are in line with our results.

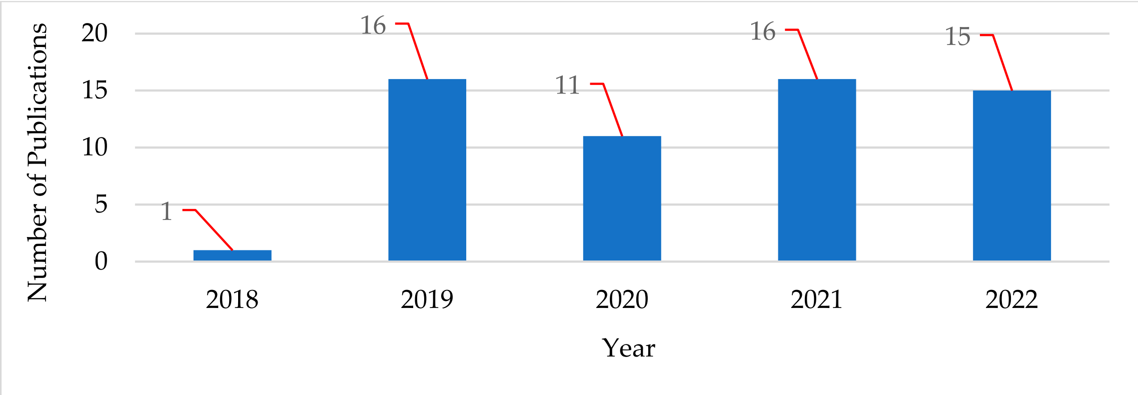

Figure 3. The publication years of the studies.

4.1.2. Geographical Distribution

An analysis was conducted on the country affiliations of all authors (n = 202) to determine the global outlook on the researchers' contributions to the implementation of AR/VR in mathematics education. Our analysis observed a diverse set of contributors to this research area, with 25 different countries being represented (see Figure 4). The numbers/figures displayed on the world map in Figure 4 represent the quantity of researchers from each country who are making contributions to the field. Researchers from Asia (45%, n = 90) and Europe (33%, n = 66) are the dominant contributors to the field. North America is the third largest contributor (20%, n = 40), followed by South America (1%, n = 3) and Australia (1%, n = 3).

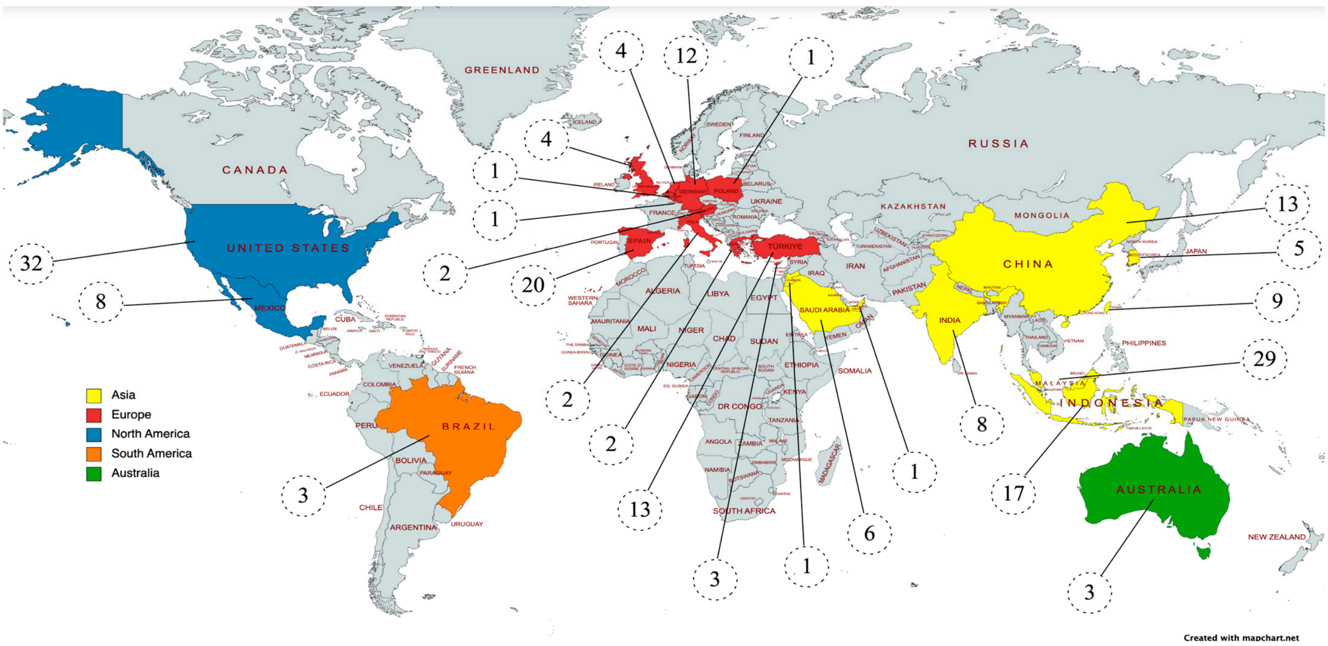

Figure 4. Geographical distribution of the authors of the reviewed studies.

The heterogeneous geographical distribution of researchers may be attributable to the varying levels of funding, research priorities, and educational policies across different regions worldwide. Furthermore, the prevalence of technology and digital devices in education may be higher in certain regions, leading to greater interest in the use of AR/VR in mathematics education. It is also worth noting that these results may reflect the biases of the research community or the authors themselves, such as language barriers, access to information and resources, and cultural factors that may influence the decision to pursue research in this area [40]. Overall, the reported results attest to considerable scholarly interest and investment in researching the use of AR/VR in mathematics education and demonstrate that researchers from a diverse set of countries are contributing to this important area of research.

4.1.3. Study Domains

The analysis demonstrated that research on the use of AR/VR in mathematics learning is heavily focused on geometry. Algebra is the second most studied area, followed by a mixture of a few domains (e.g., geometry, algebra, and calculus), then calculus, probability and other branches, such as financial mathematics and school mathematics (see Table 1).

Table 1. Study domains.

Category	*n*	%
Geometry	31	53
Algebra	13	22
Mixture of geometry, algebra, and calculus	6	10
Calculus	5	8
Probability	1	2
Other (financial mathematics and school mathematics)	3	5

It is unsurprising that geometry emerged as the most popular subject domain, given that it lends itself well to visualization and AR/VR technology can visualize geometric objects in the real sense [19]. According to our analysis, however, the effectiveness of AR/VR technologies for learning subjects in various foundational areas of mathematics, including calculus, analysis, arithmetic, logic, probability, and statistics, has not been sufficiently researched.

Lai and Cheong [32] argued that XR, including AR and VR, cannot be universally applied across all areas of mathematics given that the benefits derived from visualization are variable. However, we adopt a different perspective, asserting that visualization plays a significant role in the learning of different mathematics subjects and has the potential to enhance students' understanding across multiple mathematical fields [41,42]. While we recognize that the potential utilization of AR and VR in mathematics education may involve different levels of difficulty depending on the constraints of the subjects, we contend that the primary constraint may be closely tied to the ingenuity and proficiency of the developer as well as the technological infrastructure [37].

To summarize, AR/VR technologies can be useful for users in learning various fields of mathematics. For instance, AR/VR can be used to visualize geometrical objects and structures in a more immersive, realistic, and interactive way, allowing individuals to explore, render, and manipulate shapes and figures in three dimensions. This opportunity may especially help learners to improve their spatial reasoning skills and deepen their understanding of geometrical concepts. Concerning algebra, AR/VR can be used to create visual representations of abstract algebraic concepts (e.g., equations and functions). This can make these concepts more tangible and accessible for learners, particularly for those who struggle with abstract reasoning. Moreover, AR/VR can be used to visualize complex concepts of calculus, for example complex mathematical functions and important constructs and methods, such as derivatives and integrals. This may help learners to develop a more intuitive understanding of concepts from calculus and may facilitate mathematical problem-solving skills. In probability, as another important field of mathematics, AR/VR can be used to simulate probabilistic scenarios, such as coin tosses or dice rolls, and visually represent the outcomes. This technological support may help learners to develop a conceptual understanding of probability and to figure out how to calculate probabilities in different contexts.

In each of these domains, AR and VR can function as an important tool to provide learners with more immersive and interactive learning experiences. The significance of

AR/VR in mathematics education lies in its potential to engage and motivate learners, enhance their conceptual understanding, and facilitate problem-solving and critical thinking skills [3,16–22,43,44]. However, further empirical data are needed to substantiate the potential benefits of AR/VR technologies.

In conclusion, while there is a significant focus on using AR/VR for teaching geometry and partly algebra, the potential benefits and drawbacks of these technologies in other areas of mathematics are yet to be fully explored.

4.1.4. Methodological Bases of the Studies

Concerning the research methodologies, almost half of the reviewed studies applied quantitative research methods, followed by qualitative research methods, then mixed/multiple method and design-based research. Researchers most frequently focused on K-12 students' mathematics learning using AR/VR technologies (secondary school students and primary school students), which aligns with earlier reviews [16,17]. K-12 students were followed by undergraduates (other than pre-service teachers), a mixture of teachers and students, pre-service teachers (PSTs), adults, in-service teachers (ISTs), and preschoolers. The vast majority of the studies recruited a relatively small number of participants (fewer than 100, see Table 2).

Table 2. Methodological bases of the studies.

Category	Sub-Category	*n*	%
Research method	Quantitative research methods	28	47
	Qualitative research methods	13	22
	Mixed/multiple methods	13	22
	Design-based research method	5	8
Sample	Secondary school students	22	37
	Primary school students	14	24
	Undergraduates other than PSTs	9	15
	Mixture of teachers and students	7	12
	PSTs	3	5
	Adults	2	3
	ISTs	1	2
	Preschoolers	1	2
Sample Size	1–100	49	83
	101–500	7	12
	501–1000	2	3
	Not mentioned	1	2

Owing to the prevalence of quantitative research methods among the reviewed studies (contrary to the findings of Palancı and Turan [17] but consistent with the findings of Ahmad and Junaini [16]), one might have expected the studies to have relatively large sample sizes; contrary to this expectation, however, most of the reviewed studies were found to have small sample sizes. On the one hand, this underscores the need for both large-scale studies that can help visualize the big picture in terms of the impact of AR/VR technologies on mathematics learning and in-depth qualitative research studies that facilitate comprehensive examination of this impact. On the other hand, the recruitment of larger samples may be challenging due to the costs and logistical challenges associated with the implementation of AR/VR technologies in educational settings. Moreover, the distribution of the study participants highlights the shortage of studies on AR/VR that concentrate on pre- and in-service mathematics teachers. This indicates the need for research on the role of AR/VR technologies in mathematics teacher education in particular.

4.1.5. Research Trends in the Use of AR and VR in Mathematics Learning

Our analysis revealed that the vast majority of the studies (80%, n = 47) focused exclusively on the use of AR technologies in mathematics education. By comparison, only a limited number of studies (15%, n = 9) exclusively evaluated the role of VR technologies in mathematics education, and even fewer (5%, n = 3) concentrated on AR and VR technologies in combination [2,4,45].

The reasons that AR technologies were utilized more commonly than VR technologies in educational settings—mathematics education in this case—are likely multiple. One key factor is likely the relatively high cost and complexity of VR systems, which may be prohibitive for many educational institutions. Another factor is the requirement for specialized equipment and tools (e.g., head-mounted displays), which can also be tiresome to use for long periods of time and may cause fatigue. Moreover, the development of VR applications requires considerable technical expertise [2,46]. AR technologies tend to be more accessible than VR technologies, as users can easily access AR applications on their own mobile devices. Finally, the relatively poor adoption of VR technology in education may also be due to a lack of awareness of its potential benefits and applications. However, as VR technologies continue to evolve and become more affordable, their use for educational purposes will likely increase.

Another significant result concerns the lack of attention afforded to the combined use of AR and VR technologies in mathematics education. One study considered the use of AR, VR, and MR together, and two compared the effectiveness of AR and VR in learning outcomes. This result highlights the need for more comparative research to identify the benefits and limitations of both VR and AR technologies in mathematics education. By examining the potential opportunities and drawbacks associated with using these technologies in conjunction with one another, researchers and educators can obtain a more comprehensive understanding of how these technologies may best be used in the classroom to optimize students' mathematics learning.

4.2. Digital Tools Used in the Reviewed Studies

Our analysis revealed that various hardware and software tools were used to support mathematics learning with AR and VR technologies (see Table 3). Mobile devices were the most popular hardware tools reported in the examined studies, which may be attributed to their cost-effective advantages and suitability for use in classroom settings, as well as their accessibility, affordability, and portability compared to many stationary gadgets [47]. In particular, mobile devices can provide ideal platforms for AR applications [48]. The results revealed that the most frequently employed software programs in the reviewed studies were 3D modeling programs (e.g., Unity, Vuforia, and HP Reveal/Aurasma), which was not unexpected, given that these professional software packages are required to create 3D objects and images in virtual environments. Another significant finding was that GeoGebra was only included in a few studies (8%, n = 5), contrary to our expectation that GeoGebra would be mentioned more frequently in the studies, given that it is a context-specific, free-to-use open-source software program that is widely used in mathematics education [49]. This software is useful for developing 3D models and visualizations for AR applications and supports scripting and programming languages [50]. However, GeoGebra is not specifically optimized for AR/VR platforms, and its compatibility with such platforms is limited. The small number of studies using GeoGebra may be related to this limitation. This result should stimulate researchers and developers to develop free and open-source mathematics/geometry software tailored specifically to AR/VR platforms that can support students' mathematics learning.

Table 3. Digital tools (hardware and software) used in the reviewed studies.

Category	Sub-Category	*n*	%
Hardware	Tablet PCs	27	46
	Smartphones	24	41
	AR/VR Glasses-Headsets-Controllers	13	22
	QR code/Marker-based systems	11	19
	Desktops	10	17
	Calculator	4	7
	Camera	2	3
	Checklists, guidelines booklets, MagicBook	2	3
	3D Printers	1	2
	Sandbox	1	2
	Projector	1	2
	MP3 player	1	2
Software	Unity	16	27
	Vuforia	11	19
	HP Reveal/Aurasma	8	14
	GeoGebra	5	8
	Adobe Illustrator, Adobe Photoshop, and Adobe Audition	5	8
	Game-based applications (LetsGo Hiking, Beijing Travel Plan, Kesfet Kurtul)	6	10
	C#	3	5
	3ds Max	2	3
	Zappar	2	3
	NeoTrie	1	2
	Krpano	1	2
	ENTITI Creator	1	2
	Maya	1	2
	Mixamo	1	2
	Zoom	1	2
	Blender	1	2
	PhET	1	2
	VisualMath	1	2

4.3. Potential Benefits and Drawbacks of AR and VR Technologies for Mathematics Learning

In this section, we outline and discuss in detail the key findings regarding the reported benefits and drawbacks of AR and VR technologies that have been identified in mathematics learning processes. As noted above, of the reviewed studies, the majority of the reported results were related to AR technologies, and evidence regarding the role of VR in mathematics learning was relatively limited. This shows that the adoption of VR technologies in the context of mathematics education is relatively infrequent in comparison to AR technologies, which calls for future research that focuses especially on VR technologies.

4.3.1. Benefits of Using AR/VR Technologies in Mathematics Learning

The analysis revealed that both AR and VR technologies positively impacted students' mathematics learning. All reviewed studies (n = 59), which were scrutinized under three distinct categories (see Table 4), provided empirical evidence for favorable outcomes associated with using AR or VR in mathematics education. The most prevalent favorable outcomes observed were socio-emotional outcomes, with cognitive and metacognitive outcomes following closely behind and pedagogical outcomes being less prominent. We devised these categories based on the classifications obtained in our previous studies [40,51–54].

Table 4. Benefits of using AR/VR in mathematics learning.

Category	Sub-Category	AR		VR	
		n	%	*n*	%
Socio-emotional outcomes	Learning interest, curiosity	20	34	6	10
	Learning motivation	20	34	6	10
	Enthusiasm, enjoyment, entertaining	19	32	6	10
	Social interaction, interactivity/dynamism	18	31	6	10
	Satisfaction	10	17	3	5
	Attitude, perception	10	17	2	3
	Collaboration, teamwork	6	10	1	2
	Sense of confidence	4	7	1	2
	Anxiety, stress	3	5	-	-
Cognitive and meta-cognitive outcomes	Achievement/performance, active learning, understanding	31	53	9	15
	Visual thinking/visualization	19	32	3	5
	Problem-solving	14	24	-	-
	Spatial thinking/ability	7	12	4	7
	Autonomy, independency	7	12	2	3
	Memory retention	4	7	-	-
	Mathematical/computational/critical thinking	4	7	-	-
	Noticing/awareness, attention/concentration	3	5	5	8
	Proof and reasoning	2	3	2	3
	Creativity	1	2	1	2
	Cognitive load	1	2	-	-
	Inquiry	1	2	-	-
Pedagogical outcomes	Usefulness	15	25	2	3
	Engagement	13	22	5	8
	Competence development	4	7	1	2

Benefits of Using AR/VR from Socio-Emotional Perspective

An important observation is that each study documented evidence concerning the favorable impacts of AR/VR technology on mathematics learning, with socio-emotional benefits being the most prevalent.

In particular, a significant number of studies (AR: 34%, n = 20; VR: 10%, n = 6) noted that AR/VR boosted students' learning interest and motivation as well as their curiosity with respect to mathematics learning. According to a significant portion of the studies (AR: 32%, n = 19; VR: 10%, n = 6), users demonstrated enthusiasm and derived enjoyment in learning mathematics through the use of AR/VR. An additional salient characteristic of AR/VR, as revealed by the outcomes, is its capacity to provide users with interactive and dynamic learning experiences, which, in turn, foster social interaction among them (AR: 31%, n = 18, VR: 10%, n = 6). Several studies established that numerous users expressed satisfaction with their use of AR/VR in mathematics learning (AR: 17%, n = 20, VR: 5%, n = 3), leading to the development of a positive attitude toward the use of AR/VR in mathematics education (AR: 17%, n = 20, VR: 3%, n = 2). According to several studies, the use of AR/VR in mathematics education has been associated with enhanced opportunities for peer collaboration and the cultivation of teamwork skills (AR: 10%, n = 6; VR: 2%, n = 1), as well as heightened levels of confidence among users (AR: 7%, n = 4; VR: 2%, n = 1). Furthermore, several investigations have revealed that AR can help reduce anxiety and stress that students experience in relation to mathematics (AR: 5%, n = 3).

The reported results suggest that AR and VR positively impact students' socio-emotional development with respect to mathematics education and that the role of socio-emotional factors in mathematics cannot be understated with respect to the learning process and the achievement of proficiency [55]. This positive outcome may be associated with the change in students' perception of reality that results from AR/VR encounters, which offers them educational prospects that are individually linked to pertinent information [18].

Benefits of Using AR/VR from the Cognitive/Meta-Cognitive Perspective

The benefits of using AR/VR technology for mathematics education extend beyond the realm of socio-emotional outcomes. Notably, research has revealed that AR/VR can enhance students' mathematics learning experiences by promoting cognitive and meta-cognitive development. Based on the analyses, it has been found that AR/VR technology has a positive impact on students' academic performance in mathematics, as it facilitates active learning and enhances conceptual understanding (AR: 53%, $n = 31$; VR: 15%, $n = 9$).

According to empirical research, the efficacy of AR and VR in promoting students' mathematical success and enhancing their levels of learning surpasses that of traditional methods [4]. However, in studies comparing the effects of AR and VR on mathematics learning and achievement, there was no significant difference found between these two technologies. Demitriadou, Stavroulia, and Lanitis [4] revealed that AR and VR technologies were equally effective for mathematics learning.

Several studies, e.g., [43,44,56,57], have indicated that AR/VR represents a significant opportunity for students to enhance their visual thinking skills through the provision of rich visualizations in both physical and virtual environments (AR: 32%, $n = 19$; VR: 5%, $n = 3$). Moreover, AR and VR technologies were found to be supportive of students' development of spatial abilities (AR: 12%, $n = 7$; VR: 7%, $n = 4$). While research has reported that AR contributes to students' problem-solving skills (24%, $n = 14$), there is no evidence concerning the effects of VR on the problem-solving skills of learners. Several studies have indicated that, through the use of AR/VR, students become more independent and autonomous in their mathematics learning processes (AR: 12%, $n = 7$, VR: 3%, $n = 2$).

Relatively few studies have reported on students' mathematical learning processes from the cognitive and meta-cognitive perspectives. These outcomes include the enhancement of memory retention (AR: 7%, $n = 4$); mathematical and critical thinking (AR: 7%, $n = 4$); awareness, attention, and noticing (AR: 5%, $n = 3$; VR: 8%, $n = 5$); reasoning and proof (AR: 3%, $n = 2$; VR: 3%, $n = 2$); creativity (AR: 2%, $n = 1$; VR: 2%, $n = 1$); and inquiry (AR: 2%, $n = 1$). These are all essential skills for success in mathematics and various other aspects of life. By enhancing these skills, AR/VR technologies have the potential to make a significant contribution to students' cognitive processes and academic development [51,58]. A single study reported that AR could reduce mathematics learners' cognitive load, particularly with the help of visual aids. Mathematics is often a challenging subject that requires considerable mental effort, and anything that can reduce the cognitive load can positively impact students' learning experiences.

An additional noteworthy outcome indicated that several studies (17%, $n = 10$) examined the efficiency of AR/VR technologies in facilitating mathematics learning among students with learning disabilities (e.g., autism and dyscalculia). The findings demonstrated that the implementation of visually-based AR/VR technologies can effectively support these students in learning mathematics.

Benefits of Using AR/VR Technologies from Pedagogical Perspective

Regarding the pedagogical outcomes, we identified three main benefits for mathematics students. The analysis revealed that AR/VR technologies were useful for learning mathematics; in addition, learners found AR/VR applications and tools to be user-friendly (AR: 25%, $n = 15$, VR: 3%, $n = 2$). Several studies noted that AR/VR had a positive effect on student engagement in mathematics education (AR: 22%, $n = 13$; VR: 8%, $n = 5$) and that this emerging technology contributed to students' mathematics competence development, including the mathematical modelling competence (AR: 7%, $n = 4$, VR: 2%, $n = 1$). The perceived ease of use of AR/VR technologies by students underscores the pedagogical significance of these innovative approaches, despite being novel learning instruments for many [59].

Overall, these results suggest that the emerging technology of AR/VR holds promise as an effective tool to support students' mathematical learning, enhancing their engagement and the development of their mathematical competence, which are among the main pedagogical goals of mathematics education [40,60].

4.3.2. Drawbacks to Using AR/VR Technology in Mathematics Learning

In addition to the numerous reported favorable outcomes of AR/VR technologies, it has also been observed that AR/VR technologies may entail certain drawbacks for students engaged in mathematics learning (see Table 5). Only a limited number of studies reported potential drawbacks to AR (29%, n = 17) and VR technologies (7%, n = 4) in mathematics education, basically including pedagogical, socio-emotional, and cognitive issues. This invites future studies exploring further possible drawbacks of AR/VR technologies, which will be crucial in developing robust AR/VR designs.

Table 5. Drawbacks of AR/VR technology in mathematics education.

Category	Sub-Category	AR		VR	
		n	%	*n*	%
Pedagogical outcomes	Technological glitches, technical deficiencies	9	15	4	7
	Cost	4	7	3	5
	Time-consuming	4	7	-	-
	Lack of user knowledge/experience in using AR tools	3	5	-	-
	Health problems	1	2	-	-
Socio-emotional outcomes	Being bored	2	3	-	-
	Lack of interaction and communication	1	2	1	2
Cognitive outcomes	Cognitive load	1	2	-	-

The most frequently cited drawback to AR/VR technologies concerned technical deficiencies (e.g., poor infrastructure, lack of devices and software) and technological glitches (e.g., internet connection problems, audio-visual problems) (AR: 15%, n = 9; VR: 7%, n = 4). The accessibility of AR/VR applications for all students may be limited due to cost-related factors (AR: 7%, n = 4; VR: 5%, n = 3). Several studies (7%, n = 4) have highlighted that AR applications may be time-consuming, with lengthy waiting times for users due to the lack of adequate devices as a contributing factor. Consequently, it has been reported that even a small number of students who had to wait without being able to participate in learning activities soon became bored (3%, n = 2). One of the notable negativities pertained to the absence of prior AR experience among students and their need for professional support to effectively use AR apps (5%, n = 3). Several studies have reported that AR apps may have the potential to cause health issues due to the high screen time involved and the small screen sizes of the mobile devices (2%, n = 1), and that they may increase the cognitive load of the users in obliging them to absorb information from both real and virtual settings simultaneously (2%, n = 1). Users of AR/VR applications may be restricted in terms of social interaction and communication, particularly in single-user modes (2%, n = 1).

These reported drawbacks may negatively impact the effectiveness of AR/VR technologies for mathematics learning. However, it should be noted that, while the reviewed studies reported the benefits of AR/VR that were applicable to the majority of students in their samples, the drawbacks of AR/VR have been reported for only a small number of students in the study samples. It is noteworthy that the advancement of digital technologies has emerged as a promising avenue by which the reported obstacles may be overcome in the foreseeable future [14]. Overall, to enhance mathematics learning with the successful application of AR/VR, it is important to address the potential drawbacks of AR/VR technologies and to ensure that these technologies are accessible to all students.

5. Summary of the Review Results and the Limitations of the Study

Figure 5 summaries the key findings of the present systematic review, which correspond to the three major categories: (a) socio-emotional outcomes, (b) cognitive/metacognitive outcomes, and (c) pedagogical outcomes. In our model, A, B, C, D, E, and F represent the positive effects of AR/VR technologies on learning outcomes (benefits of AR/VR for learners) and A^I, B^I, C^I, and F^I represent the negative effects of AR/VR technologies on learning outcomes (drawbacks of AR/VR for learners) reported in the previous section. See Table 6 for a list of the included studies, the assigned study numbers (1–59), and the main characteristics and methodologies of the studies.

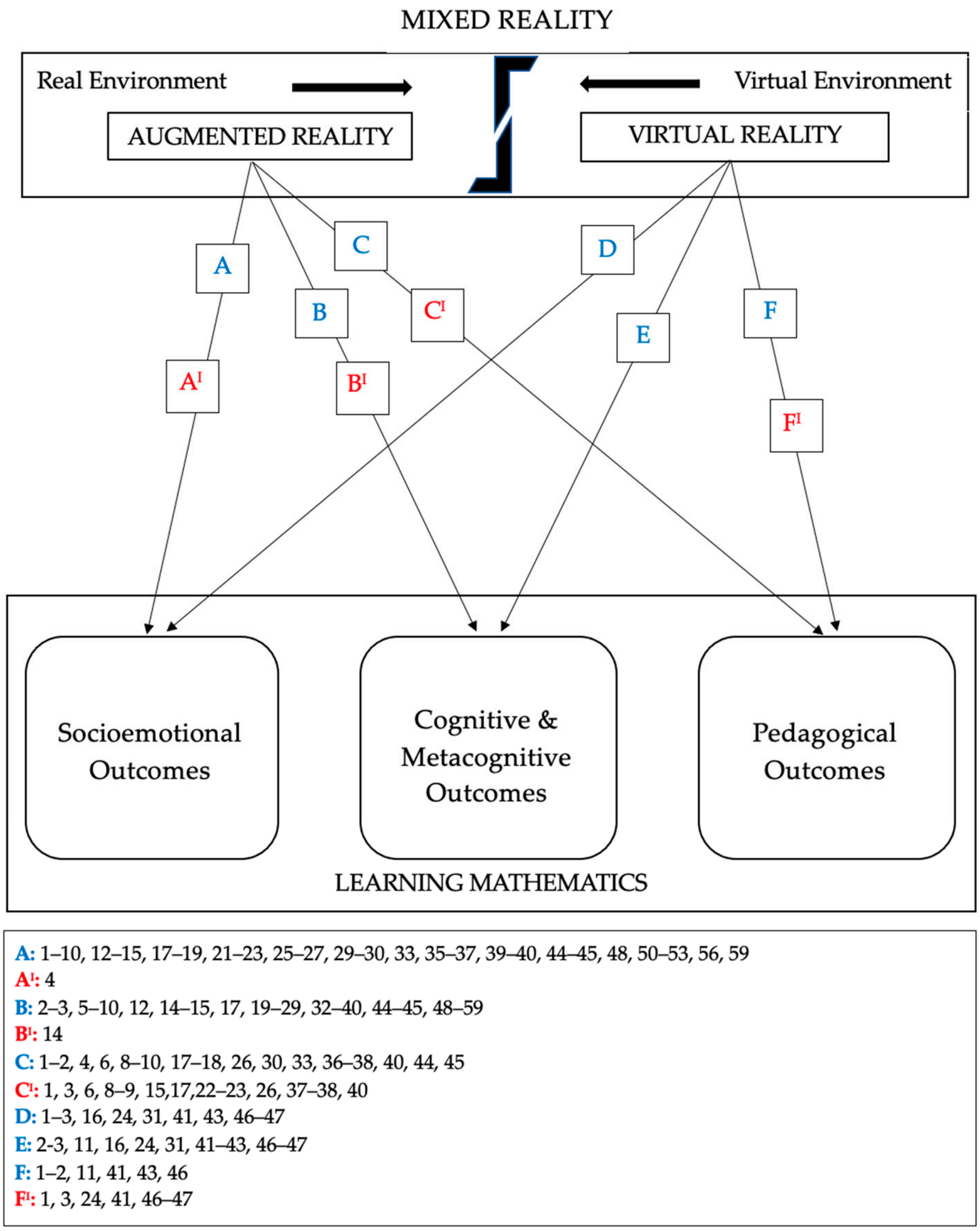

Figure 5. Summary of the key results integrated into the reality–virtuality continuum model.

Table 6. The list of the reviewed studies.

Study No	Author(s)	Country	Sample	Research Method	Technology	Domain
1	Cabero-Almenara, et al. [2]	Spain	Undergraduates	Quantitative	AR, VR, MR	Geometry
2	Demitriadou, et al. [4]	Cyprus	Primary school students	Quantitative	AR, VR	Geometry
3	Medina Herrera, et al. [45]	Mexico	Undergraduates	Mixed	AR, VR	Calculus, Geometry
4	Rebollo, et al. [61]	Spain, Italy	Primary school students	Quantitative	AR	Algebra
5	Monteiro Paulo, et al. [5]	Brazil	Undergraduates	Qualitative	AR	Calculus
6	Kounlaxay, et al. [62]	South Korea	Undergraduates, ISTs	Quantitative	AR	Geometry
7	Bos, et al. [63]	Netherlands	Undergraduates	Design-based research	AR	Calculus, Algebra
8	Jesionkowska, et al. [43]	England, Belgium	Secondary school students, ISTs	Qualitative	AR	Geometry
9	Hsieh and Chen [64]	Taiwan, China	Secondary school students, ISTs	Mixed	AR	Algebra, Geometry
10	Alqarni and Alzahrani [65]	Saudi Arabia	Secondary school students	Quantitative	AR	Geometry
11	Cangas, et al. [66]	Spain, Poland	Secondary school students	Qualitative	VR	Geometry
12	Ozcakir and Cakiroglu [67]	Turkey	Secondary school students	Quantitative	AR	Geometry
13	Ozcakir and Ozdemir [68]	Turkey	Secondary school students	Mixed	AR	Algebra, Geometry
14	Li, et al. [69]	China, USA	Secondary school students	Qualitative	AR	Calculus
15	Gargrish, et al. [70]	India	Secondary school students	Design-based research	AR	Geometry
16	Su, et al. [71]	Taiwan	Secondary school students	Quantitative	VR	Geometry
17	Schutera, et al. [72]	Germany	Secondary school students, ISTs	Qualitative	AR	Geometry
18	Mailizar and Johar [73]	Indonesia	Secondary school students	Quantitative	AR	Geometry
19	Cai, et al. [19]	China, USA	Secondary school students	Mixed	AR	Probability
20	Kellems, et al. [74]	USA	Secondary school students	Qualitative	AR	Algebra
21	Morris, et al. [75]	USA	Secondary school students	Quantitative	AR	Algebra
22	Miundy, et al. [76]	Malaysia	Primary school students, ISTs	Mixed	AR	Algebra

Table 6. *Cont.*

Study No	Author(s)	Country	Sample	Research Method	Technology	Domain
23	Moreno, et al. [77]	Mexico	Undergraduates	Mixed	AR	Other
24	Xie, et al. [78]	China	Primary school students	Mixed	VR	Geometry
25	Arican and Ozcakir [79]	Turkey	PSTs	Qualitative	AR	Geometry
26	Chen [44]	Taiwan	Secondary school students	Quantitative	AR	Algebra, Geometry
27	Cheng, et al. [80]	Taiwan, Australia	Secondary school students	Mixed	AR	Algebra
28	Aldalalah, et al. [81]	Saudi Arabia, Arab Emirates	Secondary school students	Quantitative	AR	Geometry
29	Root, et al. [82]	USA	Adults	Quantitative	AR	Other
30	Ibili, et al. [83]	Turkey, Australia	ISTs	Quantitative	AR	Geometry
31	Akman and Cakir [84]	Turkey	Primary school students	Qualitative	VR	Algebra
32	Wu [85]	Taiwan	Primary school students	Quantitative	AR	Algebra
33	Pozo-Sánchez, et al. [86]	Spain	Secondary school students	Quantitative	AR	Geometry
34	Ahmad [87]	Jordan	Secondary school students	Quantitative	AR	Geometry
35	Saundarajan, et al. [88]	Malaysia	Secondary school students	Quantitative	AR	Algebra
36	Kellems, et al. [89]	USA	Secondary school students	Quantitative	AR	Algebra
37	Kazanidis and Pellas [90]	Greece	Undergraduates	Mixed	AR	Other
38	Kellems, et al. [91]	USA	Adults	Quantitative	AR	Algebra
39	Flores-Bascuñana, et al. [92]	Spain	Secondary school students	Quantitative	AR	Geometry
40	Gargrish, et al. [93]	India	Undergraduates	Quantitative	AR	Geometry
41	Rodríguez, et al. [56]	Spain	Primary and secondary school students, ISTs	Qualitative	VR	Geometry
42	Jones, et al. [94]	USA	Undergraduates	Qualitative	VR	Calculus
43	Akman and Cakir [95]	Turkey	Primary school students	Mixed	VR	Algebra
44	Haas, et al. [96]	Luxembourg, Austria	PSTs	Mixed	AR	Geometry
45	del Cerro Velázquez and Morales Méndez [57]	Spain	Undergraduates	Quantitative	AR	Calculus
46	Yiannoutsou, et al. [97]	Spain, UK	Primary school students	Design-based research	VR	Geometry

Table 6. *Cont.*

Study No	Author(s)	Country	Sample	Research Method	Technology	Domain
47	Shi, et al. [98]	China	Secondary school students	Quantitative	VR	Calculus
48	Cahyono, et al. [99]	Indonesia, Germany	Secondary school students, ISTs	Design-based research	AR	Geometry
49	Amir, et al. [100]	Indonesia	PSTs	Qualitative	AR	Geometry
50	Andrea, et al. [101]	Indonesia	Primary school students	Quantitative	AR	Geometry
51	Nabila and Junaini [102]	Malaysia	Primary school students	Quantitative	AR	Geometry
52	Awang, et al. [103]	Malaysia	Primary school students	Quantitative	AR	Geometry
53	Elsayed and Al-Najrani [104]	Saudi Arabia	Primary school students	Quantitative	AR	Geometry
54	Hanafi, et al. [105]	Malaysia	Primary school students	Quantitative	AR	Algebra, Geometry
55	Hanid, et al. [106]	Malaysia	Primary school students	Qualitative	AR	Geometry
56	Miundy, et al. [107]	Malaysia	Primary school students	Mixed	AR	Algebra
57	Ozcakir and Cakiroglu [108]	Turkey	Secondary school students	Design-based research	AR	Geometry
58	Rohendi and Wihardi [109]	Indonesia	Secondary school students	Qualitative	AR	Geometry
59	Stotz and Columba [110]	USA	Preschoolers	Mixed	AR	Algebra

The results of our systematic literature review show very clearly that the impact of AR on learning outcomes can be both positive and negative. However, according to the empirical results of the examined studies, the benefits of AR for learners outweigh its drawbacks. Moreover, similarly to AR technology, VR has proven to be beneficial for users in their acquisition of mathematical skills, as evidenced by the various aspects highlighted in Section 4. Based on the presented findings, VR technology was observed to have limitations solely in terms of its impact on pedagogical outcomes. This appears to be a crucial aspect in which the impacts of VR and AR partly differ from each other.

Despite implementing the most recent PRISMA guidelines to enhance the transparency, accuracy, and quality of the study, and conducting a thorough search strategy, the study had certain limitations concerning the manuscript selection criteria. Our emphasis was on peer-reviewed journal articles that were published in English and indexed in selected prestigious databases (i.e., WoS and SCOPUS). Furthermore, our sampling methodology entailed the exclusion of literary works such as books, book chapters, and papers in conference proceedings, as well as studies not penned in the English language. It can be assumed that there exist studies that were indexed in electronic sources other than those which we have selected and, therefore, may have been omitted. Overall, the methodological approach we adopted for the selection process may have excluded studies that hold relevance to the investigation at hand.

6. Conclusions

The present study underscores the escalating scholarly attention devoted to exploring the efficacy of integrating AR/VR technology into mathematics education to optimize learning outcomes and pedagogical practices. The review provides evidence that indicates the noteworthy potential for the use of AR/VR in advancing students' socio-emotional, cognitive/meta-cognitive, and pedagogical development in mathematics learning processes. The main results of this systematic review are promising in that they substantiate the notion that the use of AR/VR technology constitutes an efficacious approach to enhancing students' mathematics learning outcomes. Notwithstanding, this review highlights certain apprehensions surrounding the incorporation of AR/VR in mathematics education, largely associated with potential technical and technological inadequacies that may hinder students' mathematics learning.

7. Recommendations and Implications

The results of this systematic literature review study have significant implications for future research and development efforts which are aimed at improving mathematics education through the integration of AR/VR technology.

Concerning the integration of AR/VR technologies into mathematics learning, the study has shown their potential to enhance students' engagement and mathematics achievement. To achieve these goals, mathematics educators can use AR/VR to create interactive and immersive mathematical environments that allow learners to explore mathematical concepts in a more engaging and intuitive way. They can design AR/VR applications that provide real-world scenarios that require mathematical problem-solving skills, and utilize AR/VR simulations to enable students to visualize complex mathematical concepts. Additionally, AR/VR technology can be employed to personalize the learning experience for learners, making mathematics more accessible and attractive for those with learning difficulties or disabilities. To effectively integrate AR/VR technology into their teaching practice, mathematics teachers should be provided with training and support, and encouraged to collaborate and share best practices within their mathematics education community. By incorporating these suggestions and pedagogical implications, mathematics educators may harness the full potential of AR/VR technology to enhance mathematics learning and promote students' learning processes.

However, there are still many areas where more research is needed to fully understand the potential benefits and limitations of AR/VR technology in mathematics education. The present investigation, through its rigorous and systematic analysis, has offered compelling evidence in favor of integrating AR/VR technology into mathematics education. Nonetheless, to further strengthen the validity of these findings, it is recommended that future research endeavors consider the possibility of a novelty effect of AR/VR on learning outcomes.

The studies reviewed in the present analysis are limited in number and scope, indicating the need for further research on the effectiveness of AR/VR in different educational settings and with diverse learner populations. To optimize the effectiveness of AR/VR technology in mathematics education, future research should explore its application in foundational areas of mathematics, such as calculus, logic, probability, and statistics, using both longitudinal large-scale quantitative and in-depth qualitative studies with diverse samples. Moreover, research on the amalgamation of AR/VR technology with other pedagogical approaches, such as flipped classroom and blended learning models, in the field of mathematics teacher education is urgently needed. This line of inquiry may hold significant promise for improving mathematics learning outcomes and is thus of critical importance.

Efforts should also be made to develop affordable, accessible, and user-friendly AR/VR software systems that are specifically designed to facilitate mathematics learning. Such development necessitates close collaboration among expert software developers and mathematics educators. The lack of empirical evidence regarding the effectiveness of VR in enhancing mathematics learning highlights the need for further research endeavors to elucidate the impact of VR technology on students' mathematics learning and their mathematical competence and problem-solving skills. Comparative research examining the differential effects of AR and VR technologies on mathematics learning will also inform educators and policymakers regarding the most appropriate technology for their desired learning outcomes.

Overall, the findings of this systematic literature review have the potential to furnish mathematics educators with critical insights that can facilitate the improvement of the course design, delivery, and effectiveness and overall quality of instruction with AR/VR. This potential is based on consideration of both the opportunities and limitations of AR and VR technology in relation to its effects on mathematics learning, thereby contributing significantly to the discourse on pedagogical innovation. As such, future studies should aim to address the reported gaps to fully understand the potential of AR/VR technologies in enhancing and transforming mathematics education, in line with the requirements of the digital age.

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