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Optimization Method for Allocating Peak-Period Parking Demand in Hub Parking Lot Clusters

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Abstract: With the expansion of urban scale and the popularization of multi-modal transportation, transportation hubs, as the link of multi-modal travel, are becoming increasingly important in urban development and residents' lives. In situations of high parking demand, the increase in road traffic volume and parking search delays exacerbates the service pressure on hub parking lots and the traffic congestion on surrounding roads. Therefore, reasonable parking demand allocation is one of the key solutions to this problem. Based on the analysis of the vehicle parking search process, this paper constructs a model for estimating parking search delay on roads outside hub parking lots and proposes an optimization model for parking demand allocation aimed at minimizing the total parking search delay of vehicles. Finally, taking a major transportation hub in Nanjing as a case study, data were obtained through field investigations and simulation experiments to identify peak parking demand periods and calibrate the model parameters. The results show that the average vehicle delay was reduced by 4.5%, with a total reduction of 13,860 s in vehicle delay for parking demands at the hub within one hour. Therefore, by optimizing the allocation of parking demand, the average delay for vehicles searching for parking can be reduced to a certain extent.

Keywords: hub parking cluster; peak parking demand periods; vehicle delay model; parking demand allocation; VISSIM simulation

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1. Introduction

Transportation hubs are major locations for transferring passengers and freight in cities. They are also critical nodes in urban road networks, playing an essential role in modern urban transportation systems. As gathering places for multimodal transport, these hubs face significant pressure for traffic conversion. The influx of a large number of private vehicles entering and exiting the hubs to drop off and pick up passengers poses severe challenges to the operational efficiency of the hubs and the traffic conditions both inside and around them.

Within large-scale transportation hubs, multiple parking facilities are often established, forming a hub parking cluster. These clusters, interconnected by urban roads, have a direct impact on the hubs' capability to accommodate private vehicles. However, the road resources around transportation hubs are limited. They need to accommodate both vehicles with hub parking demands and those with non-hub parking demands. This situation becomes particularly problematic during peak demand periods. The limited parking resources at the hub conflict with the high volume of parking demand. As a result, the service efficiency of the parking lots decreases, leading to congestion on the roads surrounding the hub.

Existing research on the parking demand of vehicles in a hub parking cluster mainly focuses on the search process within parking lots. There is a lack of studies on the search process for vehicles outside the parking lots on the urban roads. However, the off-street parking search process is an important component of the overall parking search process for vehicles in a hub parking cluster. During peak demand periods, the off-street parking search process for vehicles is characterized by higher traffic volumes and complex road structures compared to the on-street parking search process. Therefore, this paper studies the parking demand allocation method for hub parking facilities during the peak demand periods, aiming to effectively improve the parking efficiency and operational efficiency of transportation hubs. Reasonably allocating parking choices for vehicles at the terminals can, significantly, provide clear driving targets for vehicles, reduce delays caused by disordered parking searches, and alleviate traffic pressure on transportation hubs.

2. Related Research

This paper primarily encompasses research from the following three angles: investigation of driving characteristics under high traffic volume, analysis of parking demand allocation, and calibration of traffic simulation parameters.

In the theoretical research on high-volume traffic, Reference [1] introduced bounded rationality into the field of transportation, arguing that travelers cannot and do not always choose the option with the highest utility when making decisions. Reference [2] started with individual travelers and then expanded the model to heterogeneous travelers, thereby achieving a Rationally Inattentive User Equilibrium (RIUE). Reference [3] assumed that the inflow and outflow of road segments were equal, averaging the queueing delays across all demand flows, and posited that all demand flows experience delays when traversing the road network. After a detailed discussion of the characteristics of traffic flow in complex networks, Reference [4] analyzed network congestion. To describe the dynamic traffic flow characteristics of urban road networks, Reference [5] employed Morlet wavelet analysis to analyze the traffic flow characteristics. Reference [6] conducted an in-depth analysis of the traffic flow characteristics on King Fahad Road in the Al-Ahsa region of Saudi Arabia, utilizing survey data to assess the various factors affecting traffic flow. The study primarily focuses on key characteristics such as flow, density, and speed, aiming to establish a relationship between flow and density. Reference [7] clustered the traffic congestion patterns in Beijing under different flow conditions, revealing that these patterns vary significantly during the morning and evening peak hours. Notably, Monday and severely congested weekday morning peaks are particularly pronounced, while Friday and regular weekdays exhibit more severe characteristics during the evening peak. Reference [8] evaluated the traffic characteristics of the road network in Ramadi, Iraq, using sustainability indicators to analyze road efficiency. Reference [9] focused on the highways connecting urban agglomerations characterized by high traffic flow and density, establishing a dynamic collision risk assessment model that effectively evaluates traffic safety risks in high-flow vehicle environments.

In the study of parking demand allocation, Reference [10] considered the personalized needs of parking users and the avoidance of traffic conflicts, proposing an automatic parking allocation mechanism. They selected decision factors such as lane occupancy, travel distance, walking distance, and the occupancy of adjacent parking spaces to establish an optimal parking allocation model. Reference [11] investigated the problem of optimal allocation of vehicles for parking. Reference [12] proposed a novel intelligent parking system based on matching theory, which allocates parking spaces while simultaneously considering the preferences of both drivers and parking managers. Reference [13] proposed a Personalized Parking Guidance Service (PPGS) by constructing a bi-level programming model to describe the relationship between the personalized parking guidance information system and drivers. The simulation results show that under this model's guidance, peak congestion time is significantly reduced. Reference [14] introduced a distributed parking space allocation framework based on an adaptive pricing algorithm and virtual voting, providing users with a fair, fast, and cost-optimal parking allocation method. Reference [15] discussed parking demand, drivers' parking choice behavior, and parking-related policies, suggesting that factors such as accessibility, walking time, and parking fees should be given more attention in planning and policymaking to improve the efficiency and resource utilization of parking systems. Reference [16] proposed a parking space allocation model that considers the transition between dynamic and static traffic, optimizing parking space allocation by minimizing travelers' total travel time, thereby alleviating the resource waste and traffic congestion caused by the temporal and spatial concentration of parking demand. Reference [17] proposed a data-driven parking demand estimation framework, first dividing parking areas through statistical information grids and multi-density clustering algorithms and then estimating parking demand using support vector machines. Reference [18] proposed an integrated optimization strategy for dynamic parking space allocation, aimed at systematically optimizing the use of curbside parking spaces. Reference [19] proposed a machine learning- and game theory-based approach for dynamic pricing and the allocation of parking spaces in curbside parking scenarios. The problem is modeled as a Stackelberg game and solved by finding its Nash equilibrium. Reference [20] developed a polynomial logit model to study drivers' parking choice behavior, based on data obtained from a revealed preference survey on drivers' parking type choices and related factors in urban areas. Reference [21] established a rolling shared parking allocation model, optimizing the supply–demand matching in parking-dense areas by maximizing platform revenue and minimizing parking users' travel costs. Reference [22] proposed a multi-agent deep reinforcement learning framework to generate efficient online parking allocation strategies. Reference [23] viewed urban parking management as an online localized resource allocation problem and proposed a multi-agent system to address it. Reference [24] presented a framework for shared parking allocation and guidance optimization for autonomous vehicles and validated its effectiveness on the urban road network of Xi'an. Reference [25] designed a two-stage network-level parking space allocation method, first assigning parking lots to users using a polynomial logit model and then solving the constructed model using the NSGA-II algorithm by collecting indicators such as the number of rejections, occupancy rates, and profits of each parking lot to ensure the most favorable profits. Reference [26] proposed two linear integer programming models, the first for assigning a parking space to each driver and the second for assigning two parking spaces when no single space was suitable, solving the models using genetic algorithms and tabu search algorithms. Reference [27] designed a scalable dynamic parking allocation framework that effectively improves the quality of parking space allocation. Reference [28] formulated the parking resource allocation optimization problem as an integer linear programming (ILP) problem to minimize total costs and validated the feasibility and effectiveness of the proposed method based on real data. Reference [29] studied the parking allocation problem for multiple destinations and multiple parking lots, establishing and solving an equivalent mathematical programming model by analyzing the various factors influencing drivers' parking choices. Reference [30] addressed the issue of nighttime parking difficulties by proposing an integer linear programming model for the nighttime sharing of large shopping mall parking spaces. Reference [31] introduced an adaptive ant colony optimization algorithm for solving the parking allocation problem, which shows better performance compared to traditional algorithms. Reference [32] considered the impact of heterogeneity in temporary parking demand on allocation decisions and optimized parking space allocation through modeling to improve the utilization rate of parking resources. Reference [33] studied the following two allocation models for shared parking spaces in residential areas: "real-time allocation" and "fixed-time allocation". The results showed that the real-time allocation model exhibited user optimization advantages when supply exceeded demand, while the fixed-time allocation model demonstrated a more balanced performance in terms of resource utilization efficiency and system revenue. Reference [34] proposed a constrained optimization model for dynamic parking space allocation based on user priorities, parking lot organization structure, and shift scheduling to intelligently allocate parking spaces. Reference [35] investigated the allocation of shared parking spaces in hospitals and proposed an allocation model based on cumulative prospect theory to alleviate parking difficulties. Reference [36] proposed a nonlinear mixed-integer programming model to achieve optimal matching between parking supply and demand and solved the model, with the effectiveness of the model being validated through a case study in Beijing. Reference [37] proposed an optimization model based on the Nondominated Sorting Genetic Algorithm (NSGA-II) with an elite strategy, aiming to enhance the planning efficiency of autonomous vehicle parking facilities. Reference [38] uses the Technology Acceptance Model to analyze factors affecting users' adoption of shared autonomous vehicles (SAVs) and their parking choices, with the goal of improving urban traffic efficiency and sustainability. Reference [39] applies a Multinomial Logit Model (MNL) to examine users' behavior and influencing factors when choosing parking applications, providing practical guidance for the development of smart parking solutions, particularly in Ningbo's context.

In the area of traffic simulation parameter calibration, Reference [40] proposed the following nine-step process for calibrating the parameters in microscopic traffic simulation models: selecting evaluation indicators, collecting observational data, determining the parameters to be calibrated, designing experimental schemes, conducting simulation experiments, establishing intuitive functions, identifying alternative calibration parameters, evaluating simulation results, and verifying model validity. Reference [41] focused on the calibration of urban traffic microscopic simulation models, emphasizing the multivariate distribution of traffic characteristics. Reference [42] reviewed research on calibration methods for heterogeneous traffic conditions based on VISSIM and discussed the various methods for identifying and optimizing calibration parameters in VISSIM microscopic simulation software. Reference [43] conducted a detailed analysis of the advantages of VISSIM in microscopic traffic simulation and developed a collaborative simulation platform based on it. Reference [44] used VISSIM to calibrate and validate the car-following model based on peak-hour traffic flow and driving data from Medina, Saudi Arabia. Reference [45] calibrated the model using traffic flow and accident data from highways in Florida and simulated the conditions to obtain delay results. Reference [46] analyzed and compared 29 different traffic simulation software programs to evaluate their applicability to various real-world traffic scenarios. Reference [47] demonstrated the consistency between VISSIM simulation results and actual measured values through field data analysis of four signalized intersections in Miami, showing the effectiveness of this data-driven calibration method. Reference [48] proposed a dynamic calibration method based on detection data and validated the calibration of driving behavior parameters in a case model using VISSIM software.

According to the existing research, most studies focus on scenarios such as residential areas, commercial districts, and hospitals, while there is relatively little research on large transportation hubs. As critical nodes in urban transportation networks, transportation hubs are characterized by high parking demand and turnover rates. Additionally, the current studies often focus on parking demand allocation at the level of individual parking spaces or static parking optimization. However, parking lots at transportation hubs are typically organized in clusters, with multiple parking lots simultaneously serving a large number of vehicles and vehicles dynamically choosing between different lots. Therefore, optimizing parking demand allocation among parking lot clusters from a more macro-level perspective, rather than merely focusing on individual parking spaces, is essential for effectively guiding vehicle flow and reducing the delays caused by chaotic parking searches. This study aims to fill this research gap through the following methods:

1. Focusing on parking clusters at transportation hubs, our research analyzes the parking search process of vehicles on the surrounding roads and develops a corresponding delay calculation model.
2. With the goal of minimizing total vehicle delay, our research transforms the parking demand allocation problem among parking lot clusters into an optimization problem.

3. Materials and Methods

To address the problem of optimizing the distribution of parking demand in hub parking lots during peak periods, the methodology of this study can be divided into the following two parts: (1) analyzing the delays that occur from the moment a vehicle starts searching for a parking space until it enters the lot to establish a model for vehicle parking search delays on the external roads of hub parking lots; (2) transforming the parking demand distribution problem into an optimization problem based on the structure of the hub road network, with the goal of minimizing overall parking search delays.

3.1. Notation

The symbols involved in the text and their corresponding explanations are shown in Table 1.

Table 1. Symbols and Explanations in the Model.

Symbol	Detailed Definition
D_r	vehicle travel delay (s)
D_{sl}	vehicle delay at signalized intersections (s)
D_c	vehicle delay due to lane changing (s)
D_{qu}	vehicle queueing delay (s)
D_{it}	The delay experienced by vehicles entering the surrounding roads of the hub from intersection i to reach the target parking lot t (s)
D_{rit}	The travel delay experienced by vehicles entering the surrounding roads of the hub from intersection i to reach the target parking lot t (s)
D_{slij}	The delay experienced by vehicles at traffic signals when entering the surrounding roads of the hub from intersection i to reach the target parking lot t (s)
D_{ct}	The lane change delay experienced by vehicles at parking lot t (s)
D_{qut}	The queuing delay experienced by vehicles at parking lot t (s)
i	Intersection number
t	The number of parking lots accessible from intersection i
L	Length of the roadway (m)
L_{it}	The travel distance from intersection i to parking area t (m)
v	Vehicle travel speed (m/s)
v_s	Free-flow vehicle speed on the surrounding roads of the hub (m/s)
R	Duration of red light within a single signal cycle (s)
R_k	Duration of red light within a single signal cycle at traffic signal intersection k (s)
T	Total duration within a single signal cycle (s)
T_k	Total duration of a single signal cycle at traffic signal intersection k (s)
t_c	Acceptable minimum gap (s)
t_{ct}	Acceptable minimum gap at parking lot t (s)
t_f	Following time (s), which refers to the headway between vehicles when a lane-changing vehicle has the opportunity to merge into the target lane
t_{ft}	Following time at parking lot t (s)
q_p	Traffic flow rate of the target lane (veh/s)
q_{pt}	Traffic flow rate of the target lane at parking lot t (veh/s)
q_n	Initial lane traffic flow rate (veh/s)

Table 1. *Cont.*

Symbol	Detailed Definition
q_{nt}	Initial lane traffic flow rate at parking lot t (veh/s)
Q_i μ	Number of vehicles with parking demand at intersection i Service rate of the gate machine (veh/s)
μ_t	Service rate of the gate machine at parking lot t (veh/s)
λ	Vehicle arrival rate (veh/s)
λ_t	Vehicle arrival rate at parking lot t (veh/s)
p_{it}	The proportion of vehicles at parking lot t to the total arrival traffic flow at intersection i
c	Number of service counters

3.2. Optimization Methods for Parking Demand Allocation during Peak Demand Periods in Hub Parking Lot Clusters

To ensure that the research focuses on the influencing factors of road traffic flow, this study first makes the following assumptions regarding the calculation process of delays.

Assumption 1: *Vehicles can pass through the downstream intersection smoothly without affecting the operation of vehicles on the segment.* The assumption is intended to prevent scenarios where complete traffic congestion results in excessively large traffic delay values that are incomputable. Therefore, this assumption does not apply to situations of full congestion.

Assumption 2: *Vehicles can easily find parking spaces after passing through the parking lot barrier gate.* The assumption is made because this research focuses solely on the delays experienced by vehicles between the parking lots in the cluster, without considering scenarios inside the parking lots.

Assumption 3: *Pedestrians will only use the sidewalks at the intersections.* This assumption is intended to prevent vehicle delays caused by pedestrians crossing the road in violation of traffic rules, which are not considered in the model. Therefore, this assumption is applicable to areas with minimal or regulated pedestrian crossing behavior.

Assumption 4: *There will be no vehicle pick-up and drop-off activities within the segment, nor will there be any bus stops.* This assumption is intended to prevent delays on specific roads caused by the frequent starts and stops of public buses in practical applications. Therefore, this assumption applies to road segments without frequent pick-up/drop-off activities or bus stops.

Assumption 5: *Vehicles will arrive at the traffic signal uniformly, and there will be no scenario where a large volume of traffic arrives at the traffic signal simultaneously to wait.* This assumption is made because, when calculating signal-induced delays, the extent to which vehicles experience these delays is uneven. By assuming that vehicles arrive uniformly, we can focus on estimating the average delay. Therefore, this assumption is applicable in scenarios where traffic flow is relatively stable.

Assumption 6: *The arriving traffic flow at each intersection is homogeneous, with parking demands and driving characteristics that are identical to those of traffic at other intersections, ensuring that vehicles will not exhibit parking preferences for any specific parking lots.* This assumption eliminates the differences in the individual driver's parking selection behaviors, allowing the model to better analyze the overall distribution of parking demand.

Based on the above assumptions, the analysis of the vehicle searching for parking process on the roads outside the hub parking lot can be divided into the following two parts: searching for parking on the surrounding roads and queuing at the parking lot entrance. The delays for these two parts are shown in Figure 1.

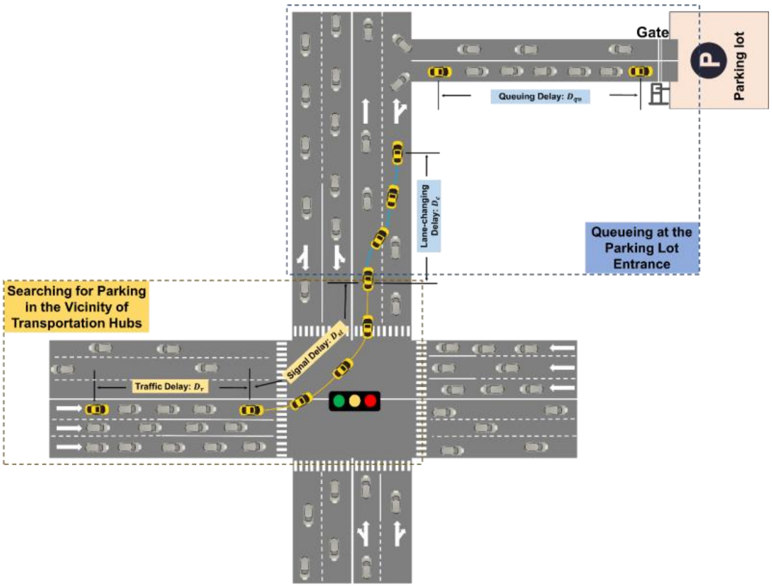

Figure 1. Schematic diagram of the vehicle searching for parking process on the roads outside the hub parking lot.

1. Searching for Parking in the Vicinity of Transportation Hubs

After vehicles enter the surrounding roads from non-hub roads, they need to drive on the surrounding roads. When encountering traffic signal intersections, they must wait or proceed according to the traffic signal. The process of searching for parking on the surrounding roads ends when they arrive at the target parking lot. The delays during this process can be divided into roadway travel delays and signal delays.

Travel delay refers to the delay experienced by vehicles while driving on the surrounding roads of the hub due to high traffic volume, which prevents them from achieving free-flow speeds. This portion of the delay is defined as the actual travel time of the vehicles minus the time required for the vehicles to travel the same road under free-flow conditions. The calculation formula is shown in Equation (1):

$$D_r = \frac{L}{v} - \frac{L}{v_s} \tag{1}$$

Traffic signal delay refers to the delay that vehicles experience while waiting at traffic lights when traveling to a target parking lot. This delay is not evenly distributed among vehicles, making it difficult to estimate the signal waiting delay for individual vehicles. Therefore, the average delay across the traffic flow is usually calculated for this part of the delay, as shown in Equation (2):

$$D_{sl} = \frac{R}{T} \times \frac{R}{2} \tag{2}$$

2. Searching for Parking in the Vicinity of Transportation Hubs

At the entrance to the parking lot, the process of vehicles entering the parking lot typically involves changing lanes to the far-right lane of the road and then slowing down to queue and pass through the entrance gate into the parking lot. During this process, the delays incurred by vehicles are concentrated during the lane-changing and the queuing at the gate. Therefore, the delay in this process can be divided into vehicle lane-changing delay and vehicle queuing delay.

Vehicle lane-changing delay refers to the delay caused by conflicts with other vehicles when a vehicle, after determining its target parking lot, changes lanes to the far-right lane of the road. Since the lane-changing vehicle must wait for an acceptable gap in the target lane that the driver deems safe before making the lane change, its driving logic is similar to the logic at major–minor road intersections. Therefore, this part of the delay can be considered as the delay incurred by minor road vehicles entering the main road, and it can be calculated using the acceptable gap theory, as shown in Equation (3):

$$D_c = \frac{1 - e^{-(q_p t_c + q_n t_f)}}{\frac{1}{t_f} e^{-q_p t_c} - q_n} + t_f \tag{3}$$

Vehicle queuing delay refers to the delay that occurs when vehicles, after completing a lane-changing maneuver, line up in the far-right lane of the road and wait to pass through the gate to enter the parking lot. This part of the delay can be calculated using queueing theory. Based on queueing theory, the parking lot gate should be categorized into single-server and multi-server scenarios, with the calculation formulas given as in Equation (4):

$$D_{qu} = \left\{ \begin{array}{l} \frac{1}{\mu - \lambda}(c = 1) \\ \frac{c\rho^c \rho}{c!(1-\rho)^c \lambda} \times \frac{1}{\sum_{k=0}^{c-1} \frac{1}{k!} \left(\frac{\lambda}{\mu}\right)^k + \frac{1}{c!(1-\rho)} \left(\frac{\lambda}{\mu}\right)^c} + \frac{1}{\mu}(\text{c>1}) \end{array} \right\} \tag{4}$$

Based on the above analysis, it is necessary to integrate the parking search delay for vehicles with parking demand based on the road network structure around the hub and to construct a model for the parking search delay of vehicles on the roads outside the parking lot. Assume that vehicles with parking demands enter the roads surrounding the hub from intersection i, pass through several traffic lights, and then through the last intersection j encountered before reaching the parking lot. They drive to the target parking lot t, complete lane-changing and queuing maneuvers, and enter the parking lot. Therefore, the total delay that the vehicle needs to bear is the sum of the aforementioned delays is as shown in Equation (5):

$$D_{it} = D_{rit} + D_{slij} + D_{ct} + D_{qut} = \frac{L_{it}}{v} - \frac{L_{it}}{v_s} + \sum_{k=i}^{j} \left(\frac{R_k}{T_k} * \frac{R_k}{2}\right) + \frac{1 - e^{-(q_{pt} t_{ct} + q_{nt} t_{ft})}}{\frac{1}{t_{ft}} e^{-q_{pt} t_{ct}} - q_{nt}} + t_{ft} + \frac{1}{\mu_t - \lambda_t} \tag{5}$$

In a given parking demand scenario, the traffic flow proportions arriving at each parking lot can be adjusted to minimize the total delay for vehicles with parking demand at the hub. Based on the road network structure around the hub, the expression for the parking demand traffic volume can be completed, and the objective function to be optimized as shown in Equation (6):

$$minD = \sum_i \sum_t Q_i p_{it} D_{it} \tag{6}$$

The optimization problem is subject to constraints as shown in Equations (7) and (8):

$$0 \le p_{it} \le 1 \tag{7}$$

$$\sum_t p_{it} = 1 \tag{8}$$

In the optimization model, the independent variable is p_{it}. By adjusting p_{it}, the distribution of parking demand can be adjusted to achieve the optimization goal of minimizing the total delay for vehicles with parking demand at the hub during peak demand periods. Since p_{it} is defined as the proportion of the number of vehicles for parking lot t relative to the total arriving traffic flow at intersection i, p_{it} must be within the range of 0 to 1. Moreover, all vehicles arriving at the intersections need to complete the search for parking, select the target parking lot, and finish the lane-changing and queuing before entering the parking lot. Therefore, for the arriving parking demand traffic at intersection i, the sum of p_{it} for all target parking lots must be 1, ensuring that all vehicles from the arriving parking demand traffic at intersection i complete the distribution of parking demand.

4. Case Study

Taking a large transportation hub A in Nanjing, Jiangsu Province, China, as the research object, this study obtains road traffic volumes and traffic flow direction ratios through field surveys and establishes a vehicle delay model under high demand scenarios. Combining the acquired video data and VISSIM simulations, the parameters within the model are calibrated. Finally, the particle swarm optimization algorithm (PSO) is used to solve the demand distribution optimization model, thereby obtaining the optimal parking demand distribution scheme under this research scenario.

4.1. Determining the Peak Parking Demand Period for the Hub

In this study, gate data from a single parking lot at Transportation Hub A were collected from 25 April 2022 to 15 May 2022, totaling 21 days, and comprising 107,042 gate records. To determine the peak parking demand period for the hub, an analysis of weekly and daily variations was conducted.

By calculating the average number of vehicles arriving and departing (where this number is the sum of vehicles arriving and leaving) each day over three consecutive weeks, a weekly variation graph of entry and exit volume can be shown in Figure 2. In the figure, the horizontal axis represents time, ranging from 0:00 to 24:00, while the vertical axis represents the volume of arrivals and departures. The three curves show the variation trends in arrival and departure volumes for different weeks.

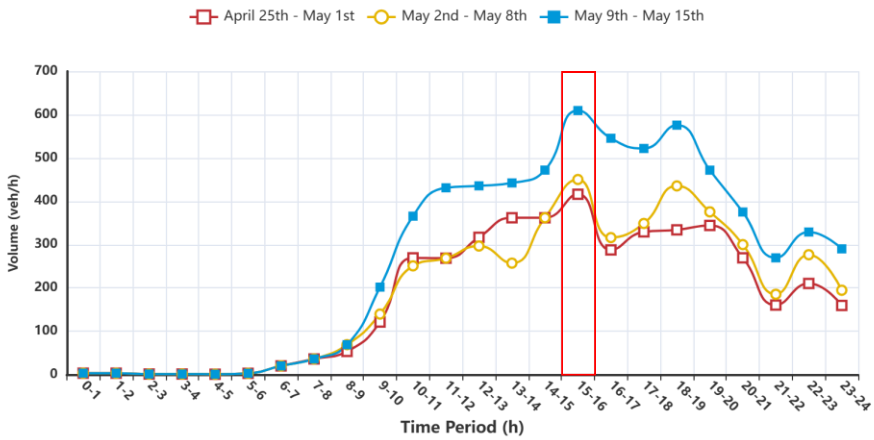

Figure 2. The weekly variation graph of the average number of vehicle arrivals and departures at the parking lot, with the red box indicating the peak period from 15:00 to 16:00.

It can be observed that the characteristics of vehicle entries and exits through the gate are highly similar from week to week, with consistent trends and peak periods. The peak volumes of vehicle entries and exits within a single day occur between 3 and 4 p.m.

After determining the weekly variation characteristics of the vehicle entries and exits through the gate at the hub, we proceeded to analyze the daily variation characteristics. A line graph depicting the hourly vehicle entries and exits at the hub from 0:00 to 24:00 within a single week is shown in Figure 3. In the figure, the horizontal axis represents time, ranging from 0:00 to 24:00, and the vertical axis represents the volume of arrivals and departures. The seven curves represent the trends in arrivals and departures on different weeks during day x.

In crosswise, variations in the average number of vehicle arrivals and departures exhibits a bimodal pattern, with the primary peak occurring from 15:00 to 16:00 and the secondary peak from 19:00 to 20:00. A vertical comparison of the daily variations within a single week shows that the average number of vehicle arrivals and departures on Friday is significantly higher than at other times during the week. By synthesizing the daily and weekly variation characteristics, the peak demand period within a single week is identified as Friday from 15:00 to 16:00.

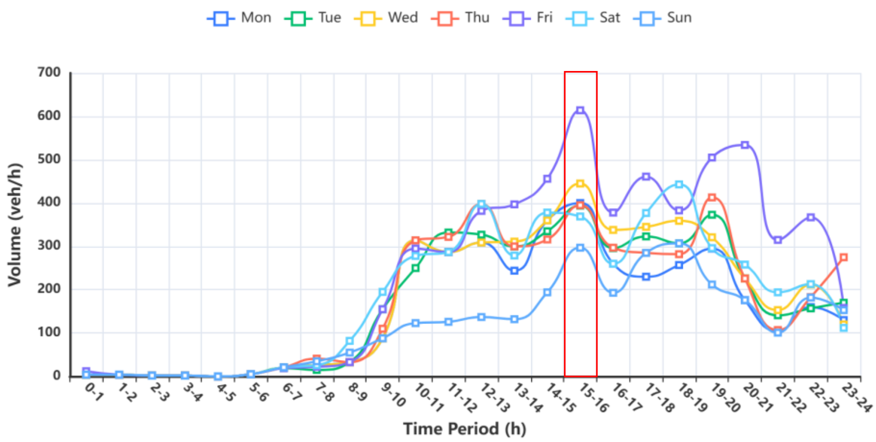

Figure 3. The daily variation graph of the average number of vehicle arrivals and departures at the parking lot, with the red box indicating the peak period from 15:00 to 16:00.

4.2. Calibration of Delay Model Parameters

In constructing the vehicle delay model, it is necessary to calibrate the parameters to facilitate subsequent optimization algorithm solutions for the delay model. The parameters in the model can be classified into those that can be obtained through field observations and those that cannot. For the former, this study conducted a traffic survey at the transportation hub A from 15:00 to 16:00 on Friday, 3 March 2023, to obtain parameters such as the service rate of the gates at various parking lots, the cycle duration of traffic signals, and the duration of red lights at various intersections. Detailed data can be found in Appendix A.

This large hub parking area has five parking lots open to the public, and there are seven major intersections on the surrounding roads of the hub. The directions of vehicles heading to the hub and surrounding intersections are shown in Figure 4. In the figure, the red lines represent the main roads around the hub, traffic lights indicate the major intersections, and the arrows along with the letter "A" denote different entry directions.

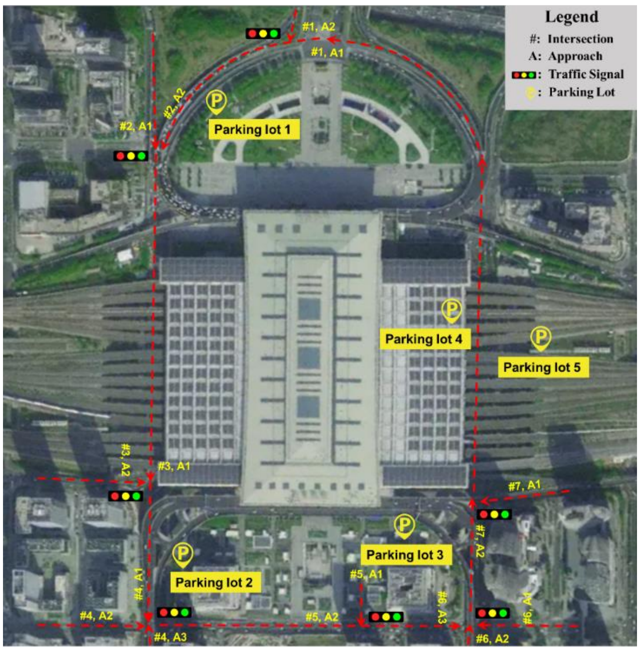

Figure 4. Diagram of Major Intersections, Parking Lots, and Vehicle Directions into the Surrounding Area of the Large Hub A.

For parameters that cannot be obtained through field observations, such as those related to delays caused by lane changes at the entrances of each parking lot, the minimum acceptable gap, and the follow-up time, this study uses a simulation experiment parameter calibration method to obtain them. Commonly used traffic simulation platforms include SimTraffic, CORSIM, and VISSIM. A comparison of the driving behavior models across these simulation platforms is shown in Table 2.

Table 2. Comparison of Driving Behavior Models Across Different Simulation Platforms.

Model Types	Simulation Platforms						
	VISSIM	**CORRSIM**	**SimTraffic**	**Paramics**	**AISSUN**	**MITSMLab**	**TransModeler**
Car-Following, Lane-Changing, and Gap Acceptance Models	✓ [1]	✓	✓	✓	✓	✓	✓
Turning Movement Model at Intersections	✓	✓	✓	✓	✓	✓	✓
Queue Formation and Dissipation Models	✓	✓	✓	✓	✓	✓	✓
Left-Turn Impact Model at Intersections	✓	✓	✓	✓	✓	×	—
Turning Speed Impact Model	✓	×	✓	×	×	✓	✓
Stopping Impact Model	✓	✓	×	✓	×	✓	✓
Vehicle Turn Signal Impact Model	×	×	×	×	×	×	—

[1] ✓ indicates the presence of the model, × indicates its absence, and—indicates that the information is unknown.

From the table, it can be observed that VISSIM, as a widely applicable simulation software, offers comprehensive driving behavior models, enabling the efficient setup and execution of simulations for real-world traffic scenarios. Additionally, VISSIM features a high degree of visualization, with both the user interface and the simulation environment being fully visualized. This allows for the observation of each vehicle's operational status and the retrieval of its operational parameters, facilitating the assessment of the simulation's accuracy and realism. Therefore, VISSIM is selected for the calibration of simulation experiment parameters.

Firstly, the experimental scenario is constructed in VISSIM based on the structure of the real road network. Next, it adjusts the road traffic flow and parking demand within a reasonable range to conduct simulation experiments and obtain vehicle delays at parking lot entrances. Based on the data obtained from the traffic survey, the target lane traffic flow in the simulation experiment is set to range from 200 vehicles per hour to 500 vehicles per hour, with a value taken every 50 vehicles per hour. The initial lane traffic flow is set to range from 100 vehicles per hour to 200 vehicles per hour, with a value taken every 20 vehicles per hour. A total of 42 sets of simulation experiment data were collected for each parking lot entrance. Finally, the parameters are calibrated using the least squares method. The results are shown in Table 3.

Table 3. Parameter Calibration Table for Simulation Experiments.

Parking Lot Number	P1	P2	P3	P4	P5
t_c	4.87	4.31	5.12	6.7	5.43
t_f	5.73	5.67	5.34	3.45	5.15
R^2	0.95	0.93	0.88	0.96	0.93

4.3. Optimization of Parking Demand during Peak Demand Periods

During the traffic survey of hub A, aside from obtaining the parameters that need calibration, it is also necessary to collect traffic flow data and parking lot arrival numbers during peak periods to determine the parking demand at the hub. The external traffic flow at major intersections around the hub and the number of vehicles arriving at each parking lot are presented in Table 4.

Table 4. Equivalent Hourly Arrival Traffic Flow at Each Intersection and Parking Lot.

Intersection Number	Equivalent Hourly Arrival Traffic Flow (veh/h)	Parking Lot Number	Equivalent Hourly Arrival Traffic Flow (veh/h)
Intersection 1	1260	P1	432
Intersection 2	444	P2	504
Intersection 3	144	P3	36
Intersection 4	144	P4	648
Intersection 5	636	P5	360
Intersection 6	960		
Intersection 7	336		
Total	3924	Total	2004

From the table, it can be seen that the total traffic flow of vehicles arriving at each intersection during the statistical period is approximately twice that of the parking lots. Moreover, under the assumptions of the delay calculation model, the incoming traffic at each intersection is considered to be homogeneous, so it can be assumed that about 50% of the vehicles arriving at each intersection have parking needs. After determining the parking demand, the optimization objective function of the model is shown in Equation (9):

$$minD = \sum_i \sum_t Q_i p_{it} D_{it} = \sum_i \sum_t Q_i p_{it} \left(\frac{L_{it}}{v} - \frac{L_{it}}{v_s} + \sum_{k=i}^{j} \left(\frac{R_k}{T_k} * \frac{R_k}{2} \right) + \frac{1 - e^{-(q_{pt} t_{ct} + q_{nt} t_{ft})}}{\frac{1}{t_{ft}} e^{-q_{pt} t_{ct}} - q_{nt}} + t_{ft} + \frac{1}{\mu_t - \lambda_t} \right) \quad (9)$$

Since this problem is a multivariable optimization problem, it is typically solved using metaheuristic algorithms. Therefore, this paper uses the Particle Swarm Optimization (PSO) algorithm for the solution. The initial values of the particle swarm are set as random arrays between 0 and 1, the velocity threshold is set to 0.5, and the maximum number of iterations is set to 1000. The optimized results are shown in Table 5.

Table 5. Results of Parking Demand Allocation Optimization.

Parking Lot Number	P1	P2	P3	P4	P5	Average Delay per Vehicle (s)
Number of Vehicles Served Before Optimization (veh/h)	432	504	36	648	360	162
Proportion of Passing Traffic Stopping Before Optimization	67%	100%	4%	64%	36%	
Number of Vehicles Served After Optimization (veh/h)	204	408	312	492	588	155
Proportion of Passing Traffic Stopping After Optimization	30%	53%	25%	44%	56%	

The results before and after the demand allocation optimization are shown in Figure 5. A comparison reveals that the parking demand allocation for Parking Lots 3 and 5 increased, while the parking demand allocation for Parking Lots 1, 2, and 4 decreased.

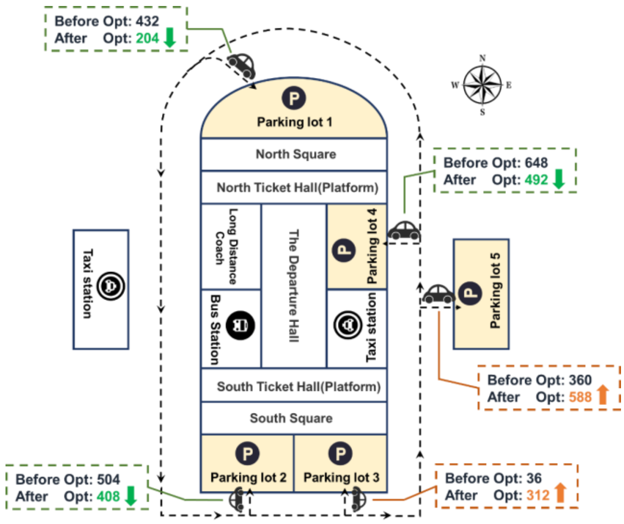

Figure 5. Comparison Chart of Results Before and After Parking Demand Allocation Optimization. A red upward arrow indicates that the demand for the parking lot has increased after optimization, while a green downward arrow indicates that the demand has decreased after optimization.

The corresponding explanations for the optimized allocation plan are as follows:

1. Due to the large driving distance between Parking Lot 1 and the other parking lots, many vehicles are unwilling to continue seeking parking after entering the surrounding roads of the hub. Instead, they choose to queue at the entrance of Parking Lot 1, resulting in higher queue delays. Guiding some of this traffic to other parking lots can significantly optimize the overall efficiency of parking at the hub.
2. The decrease in demand allocation for Parking Lot 2 is attributed to the low utilization rate of Parking Lot 3, which is relatively close to Parking Lot 2. Redirecting parked vehicles to Parking Lot 3 can improve the overall delays at the hub.
3. Parking Lots 4 and 5 are located close to each other, and neither has a spatial advantage over the other. However, the optimization plan reduces the allocation for Parking Lot 4 while increasing that for Parking Lot 5 because Parking Lot 5, with its dual service counters, has a higher service efficiency, resulting in significantly reduced queue delays.

Before optimizing the parking demand allocation, the average delay for vehicles was 162 s. After optimizing the parking demand allocation, the average delay was reduced to 155 s, an improvement of 4.5%, with the delay time for individual vehicles shortened by 7 s. During peak demand periods, the parking demand traffic volume is 1980 vehicles per hour. Therefore, after reallocating and guiding the parking demand vehicles during peak demand periods, the total delay for parking demand vehicles at the hub was reduced by 13,860 s within one hour.

5. Conclusions

Existing studies on the parking search behavior of vehicles with parking demands in a hub parking lot clusters mainly focus on the search process within the parking lots, lacking research on the search process on urban roads outside the hub parking lot clusters. However, the outside search process is an essential component of the overall search process for vehicles with parking demands in hub parking lot clusters. In contrast, this study analyzes the outside search process of vehicles in hub parking lot groups during peak demand periods, constructs mathematical expression models for various delays, and proposes an optimization model for parking demand allocation with the objective of minimizing delays.

Taking a large transportation hub in Nanjing as an example, this study demonstrates that through reasonable parking demand allocation, the average search delay time of vehicles during peak parking demand periods can be significantly reduced, effectively alleviating the congestion in hub parking lots and surrounding roads during peak demand periods. Additionally, it can provide data support for the management and control of outside parking search for vehicles with hub parking demands during peak hours.

In summary, this study effectively fills the research gap in the existing literature regarding the outside road search process of vehicles with parking demands in hub parking lot clusters. In practical applications, this method can be combined with intelligent transportation systems and navigation software to better serve users. It also provides strong support for improving the parking management level of transportation hubs and the connectivity efficiency of urban transportation.

However, this study still requires further research and improvement. Firstly, while the six assumptions proposed for the delay model help simplify the modeling process to some extent, it is acknowledged that they may have certain limitations in real-world applications. Therefore, future research should further explore the potential impact of these assumptions. Secondly, future research needs to further analyze the applicable scenarios and sensitivity. Thirdly, consider the impact of traffic and parking dynamics on the arrival rate and service rate. In addition, further optimization and improvement methods should be considered, such as constructing multi-objective optimization models [49] and employing Pareto optimization techniques [50]. Furthermore, factors such as traffic signal timing and green wave coordination of upstream and downstream traffic signals can be considered, which could significantly reduce the impact of traffic signals on the vehicle parking search process.

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Appendix A

This study conducted a traffic survey at Major Transport Hub A in Nanjing. The survey covered five parking lots and seven key intersections surrounding the hub. The data collected included traffic flow at the intersections, parking lot arrival data, vehicle queue delay data at each parking lot, gate service rates at the parking lots, and traffic signal parameters at the main intersections.

- Traffic Flow at the Intersections

By conducting traffic flow statistics at key intersections on roads surrounding the transport hub, the vehicle flow from various directions at these intersections was determined, as shown in Table A1. The definitions of the road intersections and directions in the table are provided in Figure 4 of the main text.

Table A1. Traffic Flow Statistics for Major Intersections on Roads Surrounding Hub A.

Intersection Number	Vehicle Approach	Traffic Flow (veh/h)	Total Intersection Flow (veh/h)
Intersection 1	Approach 1	900	1260
	Approach 2	360	
Intersection 2	Approach 1	444	828
	Approach 2	384	
Intersection 3	Approach 1	636	780
	Approach 2	144	
Intersection 4	Approach 1	24	1140
	Approach 2	120	
	Approach 3	996	
Intersection 5	Approach 1	1116	1476
	Approach 2	360	
Intersection 6	Approach 1	864	1828
	Approach 2	288	
	Approach 3	676	
Intersection 7	Approach 1	216	1244
	Approach 2	1028	

- Parking Lot Arrival Data

By recording the number of parked vehicles at the entrances and exits of the parking lots, the parking demand at each parking lot during peak demand periods was determined, as shown in Table A2.

Table A2. Parking Demand During Peak Demand Periods.

Parking Lot Number	Parking Lot 1	Parking Lot 2	Parking Lot 1	Parking Lot 2	Parking Lot 5
Parking Demand (veh/h)	432	528	36	848	648

- Vehicle Queue Delay Data

By observing each vehicle frame by frame in the videos recorded at the entrances and exits of the parking lots, the queue delay for each vehicle was determined. A 5 min delay analysis was conducted for each parking lot entrance and exit, and the average vehicle delay was calculated, as shown in Table A3.

Table A3. Vehicle Average Delay for Each Parking lot.

Parking Lot Number	Parking Lot 1	Parking Lot 2	Parking Lot 1	Parking Lot 2	Parking Lot 5
Average Delay (s)	14.97	15.92	13	12	43.43

- Gate Service Rates

By analyzing the time each vehicle passes through the gate in the video recorded at the entrances of the parking lots, the gate service rate for each parking lot could be determined. This rate was used to calculate the queue delay at the parking lot entrances. The statistical results are shown in Table A4.

Table A4. Gate Service Rates for Each Parking Lot at Hub A.

Parking Lot Number	Parking Lot 1	Parking Lot 2	Parking Lot 1	Parking Lot 2	Parking Lot 5
Average Time for Vehicles to Pass Through Gate (s)	5.8	6.8	5.2	10	5.6
Gate Service Rate (veh/h)	621	529	692	360	643

- Traffic Signal Parameters at the Main Intersections

By analyzing the red signal duration and cycle duration at signalized intersections on roads surrounding the hub, as recorded in the video footage, the parameters for the signal delay component in the delay model were determined. The statistical results are shown in Table A5.

Table A5. Signal Parameters for Major Signalized Intersections on Roads Surrounding Hub A.

<table>
<tr><th>Intersection Number</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th></tr>
<tr><td>Red Signal Duration (s)</td><td rowspan="4">\</td><td>61</td><td>40</td><td>38</td><td rowspan="4">\</td><td>37</td><td>25</td></tr>
<tr><td>Green Signal Duration (s)</td><td>40</td><td>44</td><td>57</td><td>40</td><td rowspan="3">\</td></tr>
<tr><td>Yellow Signal Duration (s)</td><td>3</td><td>3</td><td></td><td>3</td></tr>
<tr><td>Cycle Duration (s)</td><td>104</td><td>87</td><td>95</td><td>80</td></tr>
</table>

For Intersection 1, as it serves as the starting point within the study area, its traffic signals do not impact the subjects of this study, so no signal timing data were collected for this intersection. For Intersections 4 and 7, because the roads surrounding Hub A are counterclockwise one-way streets, the placement of traffic signals does not affect vehicles searching for parking around the hub; vehicles can proceed simply by staying on the left side of the road. For Intersection 5, as it is a merge point between the elevated road traffic and the traffic on the roads surrounding the hub, no traffic signals are installed.

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