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Low-Carbon Supply Chain Decision-Making and CSR Strategy Evolution Analysis Considering Heterogeneous Consumer Preferences

Jinghua Zhao, Ruishu Zhang, Zhuang Wang * and Shaoyun Cui

Business School, University of Shanghai for Science and Technology, Shanghai 200093, China; zhaojinghua@usst.edu.cn (J.Z.); 213501547@st.usst.edu.cn (R.Z.); 232481166@st.usst.edu.cn (S.C.)
* Correspondence: 232481203@st.usst.edu.cn

Abstract: Decision-making regarding the low-carbon supply chain, considering corporate social responsibility (CSR) and the heterogeneous preferences of consumers, has become an urgent topic to be explored. This paper explores the decision-making problem of a low-carbon supply chain considering the heterogeneous preferences of consumers under different CSR situations, analyzes the influence of important parameters on each equilibrium solution, compares the size relationship of each equilibrium solution under different CSR situations, and verifies the conclusions obtained through numerical simulation. Then, based on the obtained equilibrium solution, a CSR evolutionary game model of the low-carbon supply chain is constructed, and the evolutionary stability strategies of the two sides on the CSR game are solved. Finally, the evolutionary trajectory of the game system is intuitively presented using a simulation method, and the influence of the main parameters on the evolutionary trends of the two sides is analyzed. The findings are as follows: (1) When both manufacturers and retailers undertake CSR, the retail price and wholesale price are their lowest, while carbon emission reduction, total market demand, manufacturer utility, retailer utility, and supply chain total utility are the highest. (2) When a company undertakes CSR, carbon emission reduction, total market demand, manufacturer utility, retailer utility, and supply chain total utility all increase with the increase in the CSR degree of the company and the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers. (3) The evolutionary stability strategy for both manufacturers and retailers is to undertake CSR. In addition, the initial proportion of manufacturers and retailers that undertake CSR, the low-carbon preference of low-carbon consumers, and the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers can encourage both members of the supply chain to undertake CSR.

Keywords: CSR; low-carbon supply chain; consumer heterogeneity preference; Stackelberg game; evolutionary game

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1. Introduction

The booming industrial economy and deepening cooperation in the supply chain have improved people's living standards and created new demands for a better life. With such a social background, the concept of corporate social responsibility (CSR), which has gradually become popular with economic globalization, has gradually become a hot topic. CSR refers to the responsibility of enterprises to the natural environment and its stakeholders [1], that is, to consider their own profits and operating conditions, but also to consider the impact of their business operations on the natural environment and society. Changes in market demand make "Completely reasonable" no longer a commercial necessity pursued by all enterprises. More and more supply chain members have accelerated the transformation of their business philosophy; begun to respond to policy calls; paid attention to consumer demands; fulfilled their social responsibilities through consumer rights protection, employee care, community public welfare, and environmental protection; and strived to reduce the social cost of their business activities. For example, BYD reduced its CO_2 emissions by about 132,491 tons [1] in 2023 through energy-saving renovations; from 2011 to 2022, Walmart China donated more than CNY 17 million [2] to public welfare projects to improve the nutritional status of children in underdeveloped areas. These enterprises with a sense of social responsibility still achieve a win–win situation between their corporate and social benefits while undertaking CSR. In order to explore the law, many scholars have discussed it [1–5]. As shown in Table 1, we provide all the abbreviations used in this paper along with their meanings.

Table 1. Abbreviations and their meanings.

Abbreviation	Meaning
CSR	Corporate social responsibility
R&D	Research and development
BYD	Build Your Dream, a Chinese automobile brand
CS	Consumer surplus
PN	The model of no enterprise undertaking corporation social responsibility
PR	The model of retailer undertaking corporation social responsibility
PM	The model of manufacturer undertaking corporation social responsibility
PB	The model of manufacturers and retailers sharing corporation social responsibility
ESS	Evolutionarily stable strategy

At the same time, the negative effects of global warming are becoming increasingly prominent, and the frequency and intensity of natural disasters such as droughts, wildfires, and floods are increasing, making it imperative to limit greenhouse gas emissions. Countries around the world are gradually realizing the seriousness of the problem, and their cooperation on emission reduction is imperative. In order to alleviate the continuous deterioration of the environment, the United Nations General Assembly adopted the United Nations Framework Convention on Climate Change in 1992 to promote global cooperation on carbon emission reduction. Since then, the Kyoto Protocol and the Paris Agreement have been signed one after another, formulating the implementation plan of low-carbon targets based on international cooperation and also clarifying the responsibility of each party to reduce their energy consumption and emissions. Under the promotion of specific programs, governments have also begun to coordinate carbon emission reduction efforts and limit greenhouse gas emissions through legislation and other forms. For example, France passed the Green Growth Energy Transition Law in August 2015, establishing a timetable for green growth and energy transition in France. Germany's Climate Protection Act, adopted on 15 November 2019, requires that its total greenhouse gas emissions should be reduced by at least 55% compared to 1990 by 2030. Since joining the Paris Agreement in September 2016, the Chinese government has made a lot of efforts to invest in renewable energy, carbon trading mechanisms, and other areas. In recent years, it has strengthened its source management and systematic governance, accelerated the adjustment of its industrial structure and energy structure, and proposed in September 2020 that "carbon dioxide emissions strive to peak before 2030." Striving to achieve carbon neutrality by 2060 is the goal, thus opening the way for a coordinated reduction of carbon emission intensity and promoting the comprehensive green transformation of economic and social development. To promote the realization of the "dual carbon" goal as scheduled, it is still necessary to introduce a carbon tax mechanism and a carbon trading mechanism to complement each other and establish a sound carbon emission supervision system that covers other industries [6]. Under the influence of long-term policy publicity and education, the concept of low-carbon targets has gradually become known to the public. However, due to the existence of objective differences such as age, experience, income, gender and education level, consumers' preferences are characterized by heterogeneity [7–9]. On this basis, the scale of low-carbon consumer groups continues to grow, and the capacity of the low-carbon consumption market increases accordingly, injecting strong impetus into the carbon emission reduction activities of supply chain members [10].

In the face of a new market environment with heterogeneous consumer preferences, it has become an urgent issue to discuss how low-carbon supply chain members subject to carbon tax will make decisions and adopt CSR strategies when different enterprises undertake CSR. The main contributions of this paper are as follows:

(1) Considering CSR and consumers' heterogeneous preferences, this paper discusses the decisions made about low-carbon supply chain under different CSR situations and further uses an evolutionary game method to explore the evolutionary stability strategy of supply chain members.

(2) This paper innovatively incorporates different CSR scenarios and heterogeneous consumer preferences into its analysis of low-carbon supply chain decision-making problems, combining game theory and numerical simulation to analyze the CSR strategies of low-carbon supply chain members, and explores the influence of important parameters on decision variables and other equilibrium solutions. The trends identified provide a theoretical basis for the decision-making of low-carbon supply chain members and provides a certain reference for the government to formulate policies.

(3) Compared with previous research results, this paper obtains the optimal decision for the low-carbon supply chain considering the heterogeneity of consumer preferences and different CSR subjects and further uses evolutionary game theory and simulation analysis to obtain their evolutionarily stable strategy and change the trend of the low-carbon supply chain system, which provides a more comprehensive view for dynamic decision-making in the market, from the background of low carbon and CSR.

2. Related Work

In the research on supply chain decisions for carbon emission reduction, most scholars consider low-carbon supply chain decisions in terms of the emission reduction input of manufacturers. Among them, some scholars focus on the emission reduction and pricing decisions of the low-carbon supply chain, such as Xia et al. [11], who considered consignment contracts and resale contracts with fixed wholesale prices, developed a Stackelberg game model between manufacturers and platforms under the different sales contracts, and found that precision marketing increased the total carbon emissions under resale contracts while reducing the total carbon emissions under consignment contracts. Ma et al. [12] constructed centralized and decentralized decision-making models of a low-carbon garment supply chain under different game scenarios and found that centralized decision-making is beneficial to the entire supply chain when the low-carbon investment coefficient is moderate, while decentralized decision-making is more beneficial to the supply chain when the low-carbon investment coefficient is small or large. Wu Xinghua et al. [13] believe that in the centralized or decentralized decision-making mode of the low-carbon supply chain, with the enhancement of product network externalities, the carbon emission reduction level of unit products will increase, the retail price of products will increase, and the overall profits of the low-carbon supply chain will increase. In a decentralized decision-making mode, with the increase in product network externalities, the loss of the decision-making efficiency of the low-carbon supply chain increases. Other scholars have explored the investment and financing decisions of the low-carbon supply chain [14].

The literature considering consumer preferences mainly assumes consumers to be a group with certain or multiple preferences. For example, Zhou et al. [15] explored supply chain coordination under consumer preferences and retailers' fairness concerns. Sun et al. [16] found that only when the lag time of emission reduction technology is kept within a specific range will the increase in consumers' low-carbon preference have a positive impact on supply chain profits. In studies considering consumer heterogeneity, most scholars analyzed supply chain pricing strategies under the influence of consumer heterogeneity. For example, Sheng et al. [17] analyzed a supply chain's dynamic pricing strategies under consumer heterogeneity. Li et al. [18] discussed the optimal decision-making for recycling platforms under different trade-in models based on the heterogeneity of consumers' perception of quality. Long et al. [19] established a model that considers WTP heterogeneity or the multiple remanufacturing modes of different remanufactured products in one- and two-cycle closed-loop supply chain scenarios to determine the optimal recycling and remanufacturing decisions for manufacturers. Sarkar et al. [20] divided consumers into "traditional consumers" and "environmental consumers" and established a centralized decision-making model.

In the existing literature on the impact of CSR on the supply chain, one type of work focuses on the problems encountered by specific enterprises undertaking CSR: for example, in Tang Juan et al. [3], a manufacturer undertaking CSR and entrusting retailers to recycle used products in a closed-loop supply chain, the decision-making problem of product pricing, social responsibility effort level, and recycling rate. Lin Zhibing et al. [4] found that the CSR behavior of retailers exacerbated the impact of large demand disturbances on the profits of a green supply chain system. Xu Minli et al. [5] studied the impact of value co-creation and CSR on the decision-making and income of the members of a supply chain on an Internet recycling platform and found that the CSR behavior of the recycling platform is conducive to improving the recycling price and promoting the recycling of waste products, but that it will damage the economic profits of the recycling platform to a certain extent. In addition, a small number of scholars have discussed the influence of the CSR behavior of different entities in the supply chain on supply chain decision-making. For example, Modak et al. [1] built four closed-loop supply chain structures based on CSR and found that the model of the promotion of recycling by retailers to customers has the best profit level among the three decentralized decision-making models. Yao Fengmin et al. [2] studied the influence of CSR behavior on closed-loop supply chain pricing decisions under different power structures and found that the recovery rate was lowest when manufacturers undertook CSR.

Evolutionary games have become a research hotspot in recent years. A large number of scholars have studied problems in supply chain operation based on evolutionary games, and some scholars have built classic two-party evolutionary game models. For example, Kang and Tan [21] established an evolutionary game model to study the investment decisions of suppliers and manufacturers under the carbon cap-and-trade mechanism. Zhu et al. [22] introduced green sensitivity into the green supply chain system and established an evolutionary game model between enterprises and consumers. Yan Wenzhou et al. [23], combining this with evolutionary game theory, built an evolutionary game model based on a core party and subordinate party and explored the influencing factors of information sharing in a construction supply chain under a new organizational form from the perspective of benefit realization. With the deepening of relevant studies on evolutionary games, more and more scholars have studied the factors affecting the evolutionarily stable strategy (ESS) of various parties through tripartite evolutionary games. For example, Kang et al. [24] established a tripartite stochastic evolutionary game model of transnational corporations, foreign distributors, and governments. A dynamic strategy selection scheme was proposed by using a tripartite evolutionary path iterative algorithm, and it was found that consumers' preference for complex technology products significantly affects tripartite strategy. Wang Xianjia et al. [25] analyzed the equilibrium point and asymptotic stability of a tripartite evolutionary dynamic model of financial institutions, core enterprises, and micro, small and medium-sized enterprises through evolutionary game theory and the Lyapunov discriminant method. Zhang et al. [26] used evolutionary games to study a competitive closed-loop supply chain consisting of two leading Oems and two third-party remanufacturers. In addition, some scholars cleverly combine the static game method with the dynamic evolutionary game method to analyze the game players from both the micro and macro levels. Peng and Wang [27] embed evolutionary game theory into a pricing model to explore the dynamic interaction mechanism between shipping companies and freight forwarders in channel selection. The conditions in which shipping enterprises choose different channel strategies in the process of their long-term dynamic evolution are deduced. Li Chunfa et al. [28] obtained a payment matrix under different investment strategy combinations of manufacturers and retailers through a Stackelberg game model and explored the evolutionary path and stability of a targeted advertising investment strategy in different low-carbon product supply chains.

It can be seen from the above literature review that existing studies have conducted research on topics such as supply chain carbon emission reduction, consumer preference and heterogeneity, CSR, and supply chain evolutionary games and achieved some research results, but there is still room for further research and expansion, as follows:

(1) Most studies considering consumer preferences only consider a homogeneous consumer preference and assume consumers to be a group with certain or multiple preferences. However, this paper takes into account the heterogeneity of consumers and further describes the heterogeneous preferences of consumers directly through the demand function.

(2) Most of the literature on supply chain decision-making considering corporate social responsibility focuses on solving the problem of specific enterprises undertaking CSR. In contrast, this paper further studies the impact of the CSR behavior of different subjects in the supply chain on supply chain decision-making and discusses the changes in decision parameters and utility that occur under different conditions. (3) In the research on supply chains based on evolutionary games, many scholars study the supply chain problem by constructing a two-party or three-party evolutionary game model, but few studies combine a Stackelberg game and evolutionary game to study the problem. Therefore, this paper integrates the equilibrium solution of a supply chain under a Stackelberg game into the evolutionary game and uses the evolutionary game model to explore the CSR strategy of low-carbon supply chain members.

In summary, based on the background of the carbon tax imposed on manufacturers by the government, this study first designed a low-carbon supply chain decision model considering CSR and the heterogeneous preferences of consumers, attempted to explore the optimal decisions for and effectiveness of the low-carbon supply chain considering the heterogeneous preferences of consumers under different CSR scenarios, and the compared and analyzed the equilibrium solution under different CSR scenarios. Then, the utility equilibrium solution of the low-carbon supply chain was used to establish a two-party evolutionary game model, analyze the evolutionarily stable strategy of both parties, and explore the factors affecting their evolutionarily stable strategy (ESS).

3. Decision Model Construction and Solution

3.1. Model Description

As shown in Figure 1, the low-carbon supply chain consists of a manufacturer and a retailer, and the two sides play a Stackelberg game. The government imposes a carbon tax on the manufacturers in this supply chain based on the goal of carbon neutrality. In response to the carbon constraint policy, manufacturers, as leaders in the low-carbon supply chain, invest in carbon emission reduction research and development costs to reduce the carbon emission of products, and retailers, as followers, purchase and sell products from manufacturers.

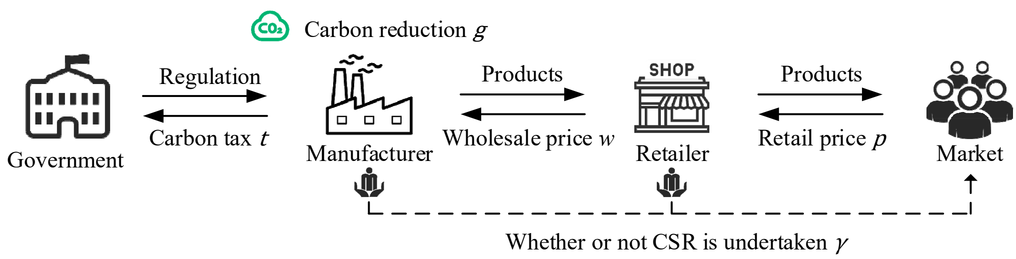

Figure 1. Decision-making model of low-carbon supply chain.

Enterprises in the low-carbon supply chain may undertake CSR, and enterprises that undertake CSR have a sense of social responsibility, will pay attention to consumers' welfare, and strive to enhance consumers' sense of gain. The four CSR models are the PN model, where neither manufacturer nor retailer undertakes CSR; the PR model, where only retailers bear CSR; and the PM model, where only the manufacturer undertakes CSR; while both manufacturers and retailers bear CSR in the PB model. Among them, the PN model can not only explore the optimal decision-making and utility level of enterprises without CSR but also serve as a benchmark model. By comparing this with the other three models, we can more intuitively understand the pricing, emission reduction, and utility of different CSR situations and help enterprises adjust their CSR strategies in practice.

The following hypotheses are proposed:

(1) The government monitors the carbon emissions generated by the production activities of the manufacturers and imposes a specific tax, the price of which is t.

(2) The cost function of the manufacturer's carbon emission reduction research and development is $I = \frac{1}{2}kg^2$, where g is the carbon emission reduction per unit product produced after R&D (Research and Development) investments and k is the cost factor of carbon emission reduction.

(3) The total market demand is composed of the low-carbon consumer demand and ordinary consumer demand. Both types of consumer demand are affected by price, but there is heterogeneity in their low-carbon preference. Ordinary consumer demand is not affected by products' carbon emission reduction, which is $\alpha - p$; low-carbon consumer demand is affected by products' carbon emission reduction, $\lambda\alpha - p + \theta g$, where α is the potential scale of ordinary consumers, λ is the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers, and θ is the low-carbon preference coefficient of low-carbon consumers [14,29,30]. Therefore, the total market demand is $Q = (1+\lambda)\alpha - 2p + \theta g$, and the enterprises in the supply chain make decisions according to the total market demand.

(4) In order to ensure the feasibility of this research, $\alpha > \frac{2(c+et)}{(1+\lambda)}$ is set to ensure that the total market demand and each utility are non-negative and consumers' low-carbon preference is set to a certain level, that is, $\theta > 2t$ [31,32]. In addition, the assumption that $k > (2t+\theta)^2$ indicates that manufacturers cannot improve the level of their carbon reduction technology indefinitely [33,34]. The research results are derived based on the above conditions.

(5) In view of the unavoidable limitations and deviations in the CSR practices of enterprises [35], with reference to the relevant literature [36–38], it is assumed that when enterprises undertake CSR, they will obtain a consumer surplus through the total market demand assessment and regard it as a part of their own utility; that is, the time-out increment of CSR undertaken by enterprises is γCS. γ is the CSR degree of the enterprise, $0 < \gamma \le 1$, and CS stands for consumer surplus:

$$CS = \int_{\frac{(1+\lambda)\alpha+\theta g-Q}{2}}^{\frac{(1+\lambda)\alpha+\theta g}{2}} Q dp = \frac{[(1+\lambda)\alpha - 2p + \theta g]^2}{4}.$$

The parameter settings are shown in Table 2.

Table 2. Basic symbol specification.

Parameter Symbol	Definition	Explanation
c	Cost	manufacturer's cost for producing unit product
w	Wholesale price	The price of the manufacturer's wholesale unit product (manufacturer's decision variable)
p	Retail price	The price of unit product sold by the retailer (retailer's decision variable)
t	Carbon tax price	The price of carbon tax levied by the government on manufacturers
e	Carbon emissions	The initial carbon emissions per unit product produced by the manufacturer
I	Research and development costs	Manufacturer's R&D costs for carbon emission reduction
g	Carbon reduction	Carbon emission reductions per unit of product produced by the manufacturer (manufacturer decision variable)
k	Research and development cost factor	Manufacturer's R&D cost coefficient for carbon emission reduction
α	The potential size of the average consumer	The potential size of consumers who do not have low-carbon preferences
λ	The ratio of the potential size of low-carbon consumers to that of ordinary consumers	The ratio of the potential size of consumers with a low-carbon preference to that of consumers without a low-carbon preference
θ	Preference coefficient	Low-carbon preference coefficient of low-carbon consumers
Q	Aggregate market demand	The total market demand of products in supply chain system
γ	Corporate CSR degree	The extent to which the manufacturer or retailer assumes corporate social responsibility
CS	Consumer surplus	When undertaking CSR, the consumer surplus is obtained through the total market demand assessment, which is used to characterize the investment of CSR
π_m^i	Manufacturer's profit	i = PN, PR, PM, PB, representing the manufacturer's profit in each of the four models
π_r^i	Retailer profit	i = PN, PR, PM, PB, representing retailer profit in the four models, respectively
U_m^i	Manufacturer's utility	i = PN, PR, PM, PB, representing the manufacturer's utility in each of the four models
U_r^i	Retailer utility	i = PN, PR, PM, PB, representing retailer utility in each of the four models
U_{sc}^i	Total supply chain utility	i = PN, PR, PM, PB, representing total supply chain utility in the four models, respectively

3.2. Method for Solving the Model

3.2.1. No CSR Model (Model PN)

In the low-carbon supply chain, manufacturers occupy a dominant position and bear all the low-carbon R&D input costs. Therefore, the solution sequence of the model is as follows: manufacturers decide the carbon emission reduction and wholesale price of low-carbon products, and retailers decide the retail price accordingly. The profit functions of manufacturers and retailers are as follows:

$$\pi_m^{PN} = [w - c - t(e - g)]Q - I \tag{1}$$

$$\pi_r^{PN} = (p - w)Q \tag{2}$$

In model PN, that is, in the absence of CSR, both manufacturers and retailers will make decisions to maximize their respective profits, regardless of the accumulation of consumer surplus. Therefore, the utility functions of both sides are as follows:

$$U_m^{PN} = \pi_m^{PN} \tag{3}$$

$$U_r^{PN} = \pi_r^{PN} \tag{4}$$

The decentralized decisions of the two sides constitute the sub-game's perfect Nash equilibrium, so a backward induction method is used to solve it. First, the second derivative of Equation (4) with respect to p can be obtained, $\frac{\partial^2 U_r^{PN}}{\partial p^2} = -4 < 0$; it is easy to know that

U_r^{PN} is a concave function with respect to p and there is an optimal p such that U_r^{PN} has a maximum value.

Then, with $\frac{\partial U_r^{PN}}{\partial p} = 0$, the retail price response function is

$$p^{PN} = \frac{[(1+\lambda)\alpha + \theta g + 2w]}{4} \tag{5}$$

By substituting Formula (5) into Formula (3), it can be obtained using the U_m^{PN} of w and g that the Hessian matrix $H = \begin{bmatrix} -2 & \frac{\theta-2t}{2} \\ \frac{\theta-2t}{2} & -k+t\theta \end{bmatrix}$. Because $k > (2t+\theta)^2 > \frac{(2t+\theta)^2}{8}$, $|H| = -2(-k+t\theta) - \left(\frac{\theta-2t}{2}\right)^2 > 0$, and we can see that the matrix is negative definite and U_m^{PN} is a joint concave function about w and g. Let $\frac{\partial U_m^{PN}}{\partial w} = 0$, $\frac{\partial U_m^{PN}}{\partial g} = 0$, and the optimal decision of the manufacturer is obtained as follows:

$$w^{PN^*} = \frac{(1+\lambda)\alpha[2k - t(2t+\theta)] - (c+et)[-4k + \theta(2t+\theta)]}{8k - (2t+\theta)^2} \tag{6}$$

$$g^{PN^*} = \frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{8k - (2t+\theta)^2} \tag{7}$$

By substituting Formulas (6) and (7) into Formula (5), the optimal decision for retailers can be obtained as follows:

$$p^{PN^*} = \frac{(1+\lambda)\alpha[3k - t(2t+\theta)] - (c+et)[-2k + \theta(2t+\theta)]}{8k - (2t+\theta)^2} \tag{8}$$

By further substituting each formula, the total market demand can be obtained as follows:

$$Q^{PN^*} = \frac{2k[(1+\lambda)\alpha - 2(c+et)]}{8k - (2t+\theta)^2} \tag{9}$$

The retailer utility is

$$U_r^{PN^*} = \frac{2k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[8k - (2t+\theta)^2\right]^2} \tag{10}$$

The manufacturer's utility is

$$U_m^{PN^*} = \frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{16k - 2(2t+\theta)^2} \tag{11}$$

The total utility of the supply chain is

$$U_{sc}^{PN^*} = \frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[12k - (2t+\theta)^2\right]}{2\left[8k - (2t+\theta)^2\right]^2} \tag{12}$$

3.2.2. Retailer CSR Model (Model PR)

In model PR, the order of model solving is unchanged. Retailers promise to seek a balance between their own interests and consumer surplus; manufacturers will make decisions based on their own profit maximization, regardless of the accumulation of consumer surplus. Therefore, the profit function and utility function of the manufacturer and retailer are, respectively:

$$\pi_m^{PR} = [w - c - t(e-g)]Q - I \tag{13}$$

$$\pi_r^{PR} = (p - w)Q \tag{14}$$

$$U_m^{PR} = \pi_m^{PR} \tag{15}$$

$$U_r^{PR} = \pi_r^{PR} + \gamma CS \tag{16}$$

Consistent with the PN model solving method, the other models are solved in Appendix A, and each equilibrium solution is obtained as follows:

$$w^{PR^*} = -\frac{(1+\lambda)\alpha[(\gamma-2)k + t(2t+\theta)] + (c+et)[2k(\gamma-2) + \theta(2t+\theta)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{17}$$

$$g^{PR^*} = \frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{4k(2-\gamma) - (2t+\theta)^2} \tag{18}$$

$$p^{PR^*} = -\frac{(1+\lambda)\alpha[(2\gamma-3)k + t(2t+\theta)] + (c+et)[-2k + \theta(2t+\theta)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{19}$$

$$Q^{PR^*} = \frac{2k[(1+\lambda)\alpha - 2(c+et)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{20}$$

$$U_r^{PR^*} = -\frac{(\gamma-2)k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[4k(\gamma-2) + (2t+\theta)^2\right]^2} \tag{21}$$

$$U_m^{PR^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{2\left[4k(\gamma-2) + (2t+\theta)^2\right]} \tag{22}$$

$$U_{sc}^{PR^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[6k(\gamma-2) + (2t+\theta)^2\right]}{2\left[4k(\gamma-2) + (2t+\theta)^2\right]^2} \tag{23}$$

3.2.3. Manufacturer CSR Model (Model PM)

In model PM, the order of model solving remains unchanged, and manufacturers will seek a balance between their own interests and consumer surplus. Retailers do not consider the accumulation of consumer surplus, and will make decisions according to their own profit maximization. Therefore, the profit function and utility function of the manufacturer and retailer are, respectively:

$$\pi_m^{PM} = [w - c - t(e-g)]Q - I \tag{24}$$

$$\pi_r^{PM} = (p - w)Q \tag{25}$$

$$U_m^{PM} = \pi_m^{PM} + \gamma CS \tag{26}$$

$$U_r^{PM} = \pi_r^{PM} \tag{27}$$

Each equilibrium solution is obtained as follows:

$$w^{PM^*} = \frac{(1+\lambda)\alpha[(\gamma-2)k + t(2t+\theta)] + (c+et)[-4k + \theta(2t+\theta)]}{2k(\gamma-4) + (2t+\theta)^2} \tag{28}$$

$$g^{PM^*} = -\frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{2k(\gamma-4) + (2t+\theta)^2} \tag{29}$$

$$p^{PM^*} = \frac{(1+\lambda)\alpha[(\gamma-3)k + t(2t+\theta)] + (c+et)[-2k + \theta(2t+\theta)]}{2k(\gamma-4) + (2t+\theta)^2} \tag{30}$$

$$Q^{PM^*} = -\frac{2k[(1+\lambda)\alpha - 2(c+et)]}{2k(\gamma-4) + (2t+\theta)^2} \tag{31}$$

$$U_r^{PM^*} = \frac{2k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[2k(\gamma-4) + (2t+\theta)^2\right]^2} \tag{32}$$

$$U_m^{PM^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{2\left[2k(\gamma-4) + (2t+\theta)^2\right]} \tag{33}$$

$$U_{sc}^{PM^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[2k(\gamma-6) + (2t+\theta)^2\right]}{2\left[2k(\gamma-4) + (2t+\theta)^2\right]^2} \tag{34}$$

3.2.4. Both Sides Having CSR Model (Model PB)

In model PB, the order of model solving remains unchanged, and manufacturers and retailers pay attention to consumer surplus at the same time and seek a balance between their own interests and the consumers' interests in their decision-making. Therefore, the profit function and utility function of the manufacturer and retailer are, respectively:

$$\pi_m^{PB} = [w - c - t(e-g)]Q - I \tag{35}$$

$$\pi_r^{PB} = (p-w)Q \tag{36}$$

$$U_m^{PB} = \pi_m^{PB} + \gamma CS \tag{37}$$

$$U_r^{PB} = \pi_r^{PB} + \gamma CS \tag{38}$$

Similarly, each equilibrium solution of the PB model is obtained as follows:

$$w^{PB^*} = \frac{(1+\lambda)\alpha[2k(\gamma-1) + t(2t+\theta)] + (c+et)[2k(\gamma-2) + \theta(2t+\theta)]}{k(6\gamma-8) + (2t+\theta)^2} \tag{39}$$

$$g^{PB^*} = -\frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{k(6\gamma-8) + (2t+\theta)^2} \tag{40}$$

$$p^{PB^*} = \frac{(1+\lambda)\alpha[3k(\gamma-1) + t(2t+\theta)] + (c+et)[2k + \theta(2t+\theta)]}{k(6\gamma-8) + (2t+\theta)^2} \tag{41}$$

$$Q^{PB^*} = \frac{-2k[(1+\lambda)\alpha - 2(c+et)]}{k(6\gamma-8) + (2t+\theta)^2} \tag{42}$$

$$U_r^{PB^*} = -\frac{(\gamma-2)k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[k(6\gamma-8) + (2t+\theta)^2\right]^2} \tag{43}$$

$$U_m^{PB^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{2\left[k(6\gamma-8) + (2t+\theta)^2\right]} \tag{44}$$

$$U_{sc}^{PB^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[4k(2\gamma-3) + (2t+\theta)^2\right]}{2\left[k(6\gamma-8) + (2t+\theta)^2\right]^2} \tag{45}$$

3.3. Sensitivity Analysis of Equilibrium Solutions in Different CSR Models

A sensitivity analysis of the equilibrium solutions of different CSR models is carried out. Specifically, the influence of the CSR degree of enterprises, the potential scale of low-carbon consumers, and the potential scale of ordinary consumers on the equilibrium results for each model is studied, and the reasons are analyzed. This section compares and analyzes the changes in equilibrium solutions in different models, including wholesale price, retail price, carbon emission reduction, total market demand, and utility (Propositions

1–9). Due to the complexity of the model, and considering the research's focus and space constraints, the impact of the carbon tax rate on carbon emission reduction per unit product will be directly analyzed in a numerical simulation.

Proposition 1. *Each equilibrium solution in the PN model, PR model, and PM model increases with the increase in the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers.*

Proof. because $k > (2t+\theta)^2$, $\frac{\partial p^{PN^*}}{\partial \lambda} = \frac{\alpha[3k - t(2t+\theta)]}{8k-(2t+\theta)^2} > 0$. Similarly verifiable is that $\frac{\partial w^{PN^*}}{\partial \lambda} > 0$, $\frac{\partial g^{PN^*}}{\partial \lambda} > 0$, $\frac{\partial Q^{PN^*}}{\partial \lambda} > 0$, $\frac{\partial U_r^{PN^*}}{\partial \lambda} > 0$, $\frac{\partial U_m^{PN^*}}{\partial \lambda} > 0$, and $\frac{\partial U_{sc}^{PN^*}}{\partial \lambda} > 0$.

Because $k > (2t+\theta)^2$, $\alpha > \frac{2(c+et)}{(1+\lambda)}$, so $\frac{\partial p^{PR^*}}{\partial \lambda} = \frac{\alpha[(2\gamma-3)k + t(2t+\theta)]}{4k(\gamma-2)+(2t+\theta)^2} > 0$. Similarly verifiable is that $\frac{\partial w^{PR^*}}{\partial \lambda} > 0$, $\frac{\partial g^{PR^*}}{\partial \lambda} > 0$, $\frac{\partial Q^{PR^*}}{\partial \lambda} > 0$, $\frac{\partial U_r^{PR^*}}{\partial \lambda} > 0$, $\frac{\partial U_m^{PR^*}}{\partial \lambda} > 0$, and $\frac{\partial U_{sc}^{PR^*}}{\partial \lambda} > 0$.

Because $k > (2t+\theta)^2$, $\alpha > \frac{2(c+et)}{(1+\lambda)}$, so $\frac{\partial p^{PM^*}}{\partial \lambda} = \frac{\alpha[(\gamma-3)k + t(2t+\theta)]}{2k(\gamma-4)+(2t+\theta)^2} > 0$. Similarly, it can be proven that $\frac{\partial w^{PM^*}}{\partial \lambda} > 0$, $\frac{\partial g^{PM^*}}{\partial \lambda} > 0$, $\frac{\partial Q^{PM^*}}{\partial \lambda} > 0$, $\frac{\partial U_r^{PM^*}}{\partial \lambda} > 0$, $\frac{\partial U_m^{PM^*}}{\partial \lambda} > 0$, and $\frac{\partial U_{sc}^{PM^*}}{\partial \lambda} > 0$. □

Proposition 1 has been proved.

Proposition 1 shows that when no enterprise undertakes CSR, retailers undertake CSR, or manufacturers undertake CSR, in a market with heterogeneous preferences, the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers will increase the total market demand and stimulate manufacturers to increase wholesale prices and reduce the carbon emission of products. Retailers also gradually increase retail prices under the premise of higher utility, and retailer utility, manufacturer utility, and total supply chain utility are all improved.

Proposition 2. *The change in each equilibrium solution in the PB model with the increase in the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers is as follows:*

(1) Carbon emission reduction, total market demand, retailer utility, manufacturer utility, and total supply chain utility increase with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers.

(2) When $0 < \gamma < 1 - \frac{t(2t+\theta)}{3k}$, the retail price increases with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers; when $1 - \frac{t(2t+\theta)}{3k} < \gamma \le 1$, the retail price decreases with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers.

(3) When $0 < \gamma < 1 - \frac{t(2t+\theta)}{2k}$, the wholesale price increases with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers; when $1 - \frac{t(2t+\theta)}{2k} < \gamma \le 1$, the wholesale price decreases with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers.

Proof because of $k > (2t+\theta)^2$, $\alpha > \frac{2(c+et)}{(1+\lambda)}$, therefore, when $0 < \gamma < 1 - \frac{t(2t+\theta)}{3k}$, $\frac{\partial p^{PB^*}}{\partial \lambda} = \frac{\alpha[3k(\gamma-1)+t(2t+\theta)]}{k(6\gamma-8)+(2t+\theta)^2} > 0$; when $1 - \frac{t(2t+\theta)}{3k} < \gamma \le 1$, $\frac{\partial p^{PB^*}}{\partial \lambda} = \frac{\alpha[3k(\gamma-1)+t(2t+\theta)]}{k(6\gamma-8)+(2t+\theta)^2} < 0$. **Similarly, when** $0 < \gamma < 1 - \frac{t(2t+\theta)}{2k}$, $\frac{\partial w^{PB^*}}{\partial \lambda} > 0$; when $1 - \frac{t(2t+\theta)}{2k} < \gamma \le 1$, $\frac{\partial w^{PB^*}}{\partial \lambda} < 0$. $\frac{\partial g^{PB^*}}{\partial \lambda} > 0$, $\frac{\partial Q^{PB^*}}{\partial \lambda} > 0$, $\frac{\partial U_r^{PB^*}}{\partial \lambda} > 0$, $\frac{\partial U_m^{PB^*}}{\partial \lambda} > 0$, and $\frac{\partial U_{sc}^{PB^*}}{\partial \lambda} > 0$. □

Proposition 2 has been proved.

Proposition 2 shows that when both retailers and manufacturers undertake CSR, if the CSR degree of the enterprises is low, the total market demand will increase with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers. Under the guarantee of an increase in the total market demand, manufacturers will appropriately increase the wholesale price of products in order to obtain greater benefits. In order to cater to the market, manufacturers make more efforts to increase the carbon emission reduction of products. Due to the growth of the total market demand, retailers also raise their retail price appropriately, and finally the manufacturer's utility, retailer's utility, and the total utility of the supply chain are all improved. If an enterprise has a high degree of CSR, supply chain members are more sensitive to consumer surplus. As the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers increases, both sides of the supply chain tend to improve their own utility by lowering prices. When manufacturers increase the carbon emission reduction of their products, wholesale prices will be reduced, and retailers will also reduce retail prices. In order to better improve consumer surplus, the total market demand increases in the process, so that the manufacturer utility, retailer utility, and supply chain total utility are improved. It is worth noting that with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers, the CSR degree of enterprises, which allows manufacturers or retailers to decide whether to raise or lower prices, is not consistent. According to the constraint condition $k > (2t+\theta)^2$, we can see that $1 - \frac{t(2t+\theta)}{2k} < 1 - \frac{t(2t+\theta)}{3k}$. The manufacturer starts to implement a price reduction strategy when the CSR level of both parties is lower, while the retailer will choose to reduce the price only when the CSR level of both parties is higher. In other words, when the CSR degree of the enterprise is in the interval $\left(1 - \frac{t(2t+\theta)}{2k}, 1 - \frac{t(2t+\theta)}{3k}\right)$, the manufacturer will lower the price as the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers increases, but the retailer will increase the retail price.

Proposition 3. *In the PR model, retail price decreases with the increase in CSR degree; wholesale price, carbon emission reduction, total market demand, retailer utility, manufacturer utility, and supply chain utility increase with the increase in CSR degree.*

Proof. Because $\mathrm{k} > (2\mathrm{t}+\theta)^2$, $\alpha > \frac{2(\mathrm{c}+\mathrm{et})}{(1+\lambda)}$ and $\theta > 2\mathrm{t}$ when $\frac{\partial \mathrm{p}^{\mathrm{PR}^*}}{\partial \gamma} = \frac{2\mathrm{k}[(1+\lambda)\alpha-2(\mathrm{c}+\mathrm{et})][-2\mathrm{k}+\theta(2\mathrm{t}+\theta)]}{\left[4\mathrm{k}(\gamma-2)+(2\mathrm{t}+\theta)^2\right]^2} < 0$. Similarly, it can be proven that $\frac{\partial w^{PR^*}}{\partial \gamma} > 0$, $\frac{\partial g^{PR^*}}{\partial \gamma} > 0$, $\frac{\partial Q^{PR^*}}{\partial \gamma} > 0$, $\frac{\partial U_r^{PR^*}}{\partial \gamma} > 0$, $\frac{\partial U_m^{PR^*}}{\partial \gamma} > 0$, and $\frac{\partial U_{sc}^{PR^*}}{\partial \gamma} > 0$. □

Proposition 3 has been proved.

Proposition 3 shows that when a retailer undertakes CSR, with the increase in its CSR degree, the manufacturer will consider that the total market demand will also increase and, in order to obtain higher benefits, it will appropriately increase the carbon emission reduction and wholesale price of its product. Retailers will try to boost consumer surplus in two ways: on the one hand, they will accept higher wholesale prices, which will help manufacturers to increase the carbon reduction of their products; on the other hand, they could keep retail prices as low as possible to expand market demand. In this process, retailers increase their own utility. At the same time, the benefits brought about by the increase in wholesale prices and total market demand are greater than the R&D costs invested by manufacturers, which increases their utility, and the total utility of the supply chain is thus improved.

Proposition 4. *In the PM model, the retail price decreases with the increase in CSR degree. Wholesale price, carbon emission reduction, total demand, retailer utility, manufacturer utility, and supply chain total utility increase with the increase in CSR degree.*

Proof. Because $k > (2t+\theta)^2$, $\alpha > \frac{2(c+et)}{(1+\lambda)}$, so

$\frac{\partial p^{PM^*}}{\partial \gamma} = \frac{k[(1+\lambda)\alpha-2(c+et)][-2k+\theta(2t+\theta)]}{\left[2k(\gamma-4)+(2t+\theta)^2\right]^2} < 0$. Similarly, it can be proven that $\frac{\partial w^{PM^*}}{\partial \gamma} > 0$, $\frac{\partial g^{PM^*}}{\partial \gamma} > 0$, $\frac{\partial Q^{PM^*}}{\partial \gamma} > 0$, $\frac{\partial U_r^{PM^*}}{\partial \gamma} > 0$, $\frac{\partial U_m^{PM^*}}{\partial \gamma} > 0$, and $\frac{\partial U_{sc}^{PM^*}}{\partial \gamma} > 0$. □

Proposition 4 has been proved.

Proposition 4 shows that when the manufacturer undertakes CSR, with the improvement of its CSR degree, the manufacturer will pay more attention to improving the consumer surplus. Since increasing the carbon emission reduction of the product can bring more surplus to the consumer than reducing the wholesale price, the manufacturer will invest more in research and development to increase the carbon emission reduction of the product. But the increase in research and development costs will also lead manufacturers to appropriately raise the wholesale price of their products. In view of the increase in the carbon reduction of products, retailers, in order to obtain greater profits, reduce the retail price appropriately to increase the total demand of the market. This increases retailer utility, manufacturer utility, and total supply chain utility.

Proposition 5. *In the PB model, retail price and wholesale price decrease with the increase in CSR degree. Carbon emission reduction, total demand, retailer utility, manufacturer utility, and supply chain total utility increase with the increase in CSR degree.*

Proof Because $k > (2t+\theta)^2$, $\alpha > \frac{2(c+et)}{(1+\lambda)}$, so

$\frac{\partial p^{PB^*}}{\partial \gamma} = \frac{3k[(1+\lambda)\alpha-2(c+et)][-2k+\theta(2t+\theta)]}{\left[k(6\gamma-8)+(2t+\theta)^2\right]^2} < 0$. Similarly, it can be proven that $\frac{\partial w^{PB^*}}{\partial \gamma} < 0$, $\frac{\partial g^{PB^*}}{\partial \gamma} > 0$, $\frac{\partial Q^{PB^*}}{\partial \gamma} > 0$, $\frac{\partial U_r^{PB^*}}{\partial \gamma} > 0$, $\frac{\partial U_m^{PB^*}}{\partial \gamma} > 0$, and $\frac{\partial U_{sc}^{PB^*}}{\partial \gamma} > 0$. □

Proposition 5 has been proved.

Proposition 5 shows that when both retailers and manufacturers have CSR behaviors, the manufacturers will increase the carbon emission reduction of their products with the improvement of the CSR degree of both ends of the chain, but at the same time reduce the wholesale price, mainly for two considerations: on the one hand, increasing the carbon emission reduction of products is conducive to the accumulation of consumer surplus; on the other hand, considering that retailers will also pay more attention to consumer welfare, reducing wholesale prices can give retailers more room to reduce prices, thereby increasing market aggregate demand and obtaining higher utility. The retailer will follow the manufacturer's strategy and reduce the retail price of the product so as to further stimulate the total market demand and increase the consumer surplus. In this process, the retailer's utility, the manufacturer's utility, and the total supply chain utility increase simultaneously.

3.4. Comparative Analysis of Equilibrium Solutions in Different CSR Models

We compare and analyze the equilibrium solutions in different CSR models, specifically to explore the size relationships of the equilibrium solutions in different CSR models and explain the reasons for this.

Proposition 6. *The size relationship between the wholesale prices of low-carbon products is as follows:*

(1) When $0 < \gamma < \frac{-4t^2+\theta^2}{2k}$*, the size of the relationship between low-carbon product wholesale prices is* $w^{PR^*} > w^{PN^*} > w^{PB^*} > w^{PM^*}$.

(2) When $\frac{-4t^2+\theta^2}{2k} < \gamma \le 1$*, the size of the relationship between low-carbon product wholesale prices is* $w^{PR^*} > w^{PN^*} > w^{PM^*} > w^{PB^*}$.

Proof. By subtracting the wholesale price of low-carbon products in model PM from the wholesale price of low-carbon products in model PB, we obtain $w^{PB^*} - w^{PM^*} =$

$-\frac{\gamma k[(1+\lambda)\alpha-2(c+et)](2\gamma k+4t^2-\theta^2)}{[2k(\gamma-4)+(2t+\theta)^2][k(6\gamma-8)+(2t+\theta)^2]}$, As $k > (2t+\theta)^2$, $0 < \gamma \le 1$, $\theta > 2t$, and $\alpha > \frac{2(c+et)}{(1+\lambda)}$, it can be inferred that $2k(\gamma-4)+(2t+\theta)^2 < 0$ and $k(6\gamma-8)+(2t+\theta)^2 < 0$; when $0 < \gamma < \frac{-4t^2+\theta^2}{2k}$, $w^{PB^*} - w^{PM^*} > 0$, and when $\frac{-4t^2+\theta^2}{2k} < \gamma \le 1$, $w^{PB^*} - w^{PM^*} < 0$. Similarly, it can be proven that $w^{PN^*} > w^{PR^*}$,

$w^{PR^*} > w^{PM^*}$, and $w^{PR^*} > w^{PB^*}$. □

Proposition 6 has been proved.

Proposition 6 shows that in the PR model in which only retailers bear CSR, retailers will allow manufacturers to transfer the R&D cost caused by the increase in the product's carbon reduction through higher pricing, so the wholesale price is higher than the PN model, in which no enterprises bear CSR, making it the highest of the four models. Regardless of whether the retailer undertakes CSR, when the manufacturer undertakes CSR, it will reduce the wholesale price of the product in order to increase the consumer surplus. If the enterprise has a low degree of CSR, the wholesale price in the PB model is higher than that in PM model, because retailers who bear CSR in the PB model can accept a higher wholesale price. In order to improve their own utility, manufacturers set a higher wholesale price than that in the PM model, but in order to ensure higher utility, the wholesale price is still lower than that in the PN model. If an enterprise has a high CSR level, manufacturers and retailers pay more attention to consumer surplus due to the high CSR level of both sides. In this case, reducing wholesale price can better improve the total market demand and promote an increase in their own utilities. Therefore, the wholesale price set by manufacturers in the PB model is the lowest among the four models.

Proposition 7. *The size relationship between the retail prices of low-carbon products is as follows:* $p^{PN^*} > p^{PM^*} > p^{PR^*} > p^{PB^*}$.

Proof. By subtracting the retail price of low-carbon products in model PN from the retail price of low-carbon products in model PM, we can obtain

$p^{PN^*} - p^{PM^*} = \frac{\gamma k[(1+\lambda)\alpha-2(c+et)][-2k+\theta(2t+\theta)]}{[8k-(2t+\theta)^2][2k(\gamma-4)+(2t+\theta)^2]}$; from $k > (2t+\theta)^2$, $0 < \gamma \le 1$, and $\alpha > \frac{2(c+et)}{(1+\lambda)}$, it can be inferred that $8k-(2t+\theta)^2 > 0$, $2k(\gamma-4)+(2t+\theta)^2 < 0$, and $-2k+\theta(2t+\theta) < 0$, so $p^{PN^*} - p^{PM^*} > 0$. Similarly, it can be proven that $p^{PM^*} > p^{PR^*}$, $p^{PR^*} > p^{PB^*}$. □

Proposition 7 has been proved.

Proposition 7 shows that in the PN model without CSR, both manufacturers and retailers aim to maximize their own interests, and retail prices are the highest in this case. In the PM model in which manufacturers undertake CSR, retailers appropriately reduce retail prices in accordance with the manufacturers' decisions, which can further stimulate the total market demand and increase their own profits, so the retail price is lower than in the PN model. In the PR model in which retailers bear CSR, because only retailers pay attention to consumer surplus, retailers will directly increase consumer surplus by reducing retail prices, so the retail price is lower than the retail price in the PM model. In the PB model, due to the CSR consensus between the manufacturer and the retailer, the manufacturer provides the retailer with more pricing space through a lower wholesale price, so the retailer can set a retail price lower than in the other three scenarios.

Proposition 8. *The size relationship between unit the carbon emission reduction of products is as follows:* $g^{PB^*} > g^{PR^*} > g^{PM^*} > g^{PN^*}$.

Proof. By subtracting the unit carbon reduction of low-carbon products in model PB from the unit carbon reduction of low-carbon products in model PR, we can obtain $g^{PB^*} - g^{PR^*} =$

$\frac{2\gamma k[(1+\lambda)\alpha-2(c+et)](2t+\theta)}{\left[4k(\gamma-2)+(2t+\theta)^2\right]\left[k(6\gamma-8)+(2t+\theta)^2\right]}$; from $k > (2t+\theta)^2$, $0 < \gamma \leq 1$ and $\alpha > \frac{2(c+et)}{(1+\lambda)}$, it can be inferred that $4k(\gamma-2)+(2t+\theta)^2 < 0$ and $k(6\gamma-8)+(2t+\theta)^2 < 0$, so $g^{PB^*} - g^{PR^*} > 0$. Similarly, it can be proven that

$g^{PR^*} > g^{PM^*}$, $g^{PM^*} > g^{PN^*}$. □

Proposition 8 has been proved.

Proposition 8 shows that in the PN model without CSR, manufacturers and retailers only focus on improving their own profits, and the carbon emission reduction of products is the lowest in this case. In the PM model where manufacturers bear CSR, in order to increase consumer surplus, manufacturers will invest in more research and development to improve the carbon emission reduction of their products. In the PR model where retailers undertake CSR, retailers accept the higher wholesale price brought about by the increase in carbon emission reduction, and the carbon emission reduction is even higher than that of the PM model. In the PB model, since both manufacturers and retailers have CSR behaviors, in order to further increase the total market demand and consumer surplus, the consensus of the supply chain members contributes to the optimal product carbon emission reduction of the four cases.

Proposition 9. *The size relationship between the market demand for low-carbon products is as follows:* $Q^{PB^*} > Q^{PR^*} > Q^{PM^*} > Q^{PN^*}$. *The size relationship between retailer utility is* $U_r^{PB^*} > U_r^{PR^*} > U_r^{PM^*} > U_r^{PN^*}$. *The size relationship between the manufacturers' utilities is* $U_m^{PB^*} > U_m^{PR^*} > U_m^{PM^*} > U_m^{PN^*}$. *The size relationship between the total utility of the supply chain is as follows:*

$$U_{sc}^{PB^*} > U_{sc}^{PR^*} > U_{sc}^{PM^*} > U_{sc}^{PN^*}.$$

Proof. By subtracting the total market demand in model PR from the total market demand in model PB, we can obtain $Q^{PB^*} - Q^{PR^*} = \frac{4\gamma k^2[(1+\lambda)\alpha-2(c+et)]}{\left[4k(\gamma-2)+(2t+\theta)^2\right]\left[k(6\gamma-8)+(2t+\theta)^2\right]}$. From $k > (2t+\theta)^2$, $0 < \gamma \leq 1$, and $\alpha > \frac{2(c+et)}{(1+\lambda)}$, it can be inferred that $4k(\gamma-2)+(2t+\theta)^2 < 0$ and $k(6\gamma-8)+(2t+\theta)^2 < 0$, so $Q^{PB^*} - Q^{PR^*} > 0$. Similarly, it can be proven that $Q^{PR^*} > Q^{PM^*}$ and $Q^{PM^*} > Q^{PN^*}$.

Subtracting the retailer utility in model PR from the retailer utility in model PB yields $U_r^{PB^*} - U_r^{PR^*} = \frac{4\gamma k^3(\gamma-2)[(1+\lambda)\alpha-2(c+et)]^2\left[k(5\gamma-8)+(2t+\theta)^2\right]}{\left[4k(\gamma-2)+(2t+\theta)^2\right]^2\left[k(6\gamma-8)+(2t+\theta)^2\right]^2}$. From

$k > (2t+\theta)^2$ and $0 < \gamma \leq 1$, it can be inferred that $k(5\gamma-8)+(2t+\theta)^2 < 0$ and $\gamma - 2 < 0$, so $U_r^{PB^*} - U_r^{PR^*} > 0$. Similarly, it can be proven that $U_r^{PR^*} > U_r^{PM^*}$ and $U_r^{PM^*} > U_r^{PN^*}$.

Subtracting the manufacturer utility in model PR from the manufacturer utility in model PB yields $U_m^{PB^*} - U_m^{PR^*} = \frac{\gamma k^2[(1+\lambda)\alpha-2(c+et)]^2}{\left[4k(\gamma-2)+(2t+\theta)^2\right]\left[k(6\gamma-8)+(2t+\theta)^2\right]}$; from

$k > (2t+\theta)^2$ and $0 < \gamma \leq 1$, we can obtain $4k(\gamma-2)+(2t+\theta)^2 < 0$ and $k(6\gamma-8) + (2t+\theta)^2 < 0$, so $U_r^{PB^*} - U_r^{PR^*} > 0$. Similarly, it can be proven that $U_m^{PR^*} > U_m^{PM^*}$ and $U_m^{PM^*} > U_m^{PN^*}$.

By subtracting the total supply chain utility in model PR from the total supply chain utility in model PB, we can obtain that

$U_{sc}^{PB^*} - U_{sc}^{PR^*} = \frac{\gamma k^2[(1+\lambda)\alpha-2(c+et)]^2\left[4(\gamma-2)(11\gamma-16)k^2+2k(7\gamma-12)(2t+\theta)^2+(2t+\theta)^4\right]}{\left[4k(\gamma-2)+(2t+\theta)^2\right]^2\left[k(6\gamma-8)+(2t+\theta)^2\right]^2}$. From $k > (2t+\theta)^2$ and $0 < \gamma \leq 1$, we can obtain that

$4(\gamma-2)(11\gamma-16)k^2 + 2k(7\gamma-12)(2t+\theta)^2 + (2t+\theta)^4 > 0$, so

$U_{sc}^{PB^*} - U_{sc}^{PR^*} > 0$. Similarly, it can be proven that $U_{sc}^{PR^*} > U_{sc}^{PM^*}$ *and*$U_{sc}^{PM^*} > U_{sc}^{PN^*}$. □

Proposition 9 has been proved.

Proposition 9 shows that in the PN model without CSR, manufacturers and retailers only pay attention to their own profits. In this case, the total market demand is the lowest, and the manufacturer's utility, retailer's utility, and supply chain's total utility are also at their lowest level within the four scenarios. In the PM model in which manufacturers bear CSR, the carbon emission reduction of products is higher than that of the PN model and the retail price is lower, which makes the total market demand of the PM model greater than that of the PN model. The utility brought about by the increase in total market demand is higher than the loss caused by the R&D cost of emission reduction and the price reduction. Both retailer utility and supply chain total utility are greater than in the PN model. In the PR model in which retailers bear CSR, the retail price is lower than in the PM model but the product carbon emission reduction is higher than in the PM model, so the total market demand is higher than in the PM model, and the gains brought about by the increase in total market demand are higher than the loss of the retailers, so the retailer's utility is higher than in the PM model, and the increase in total market demand and wholesale price makes the manufacturer's utility higher than in the PM model. Therefore, the total profit of the supply chain is higher than that of the PM model. In the PB model, the product's carbon emission reduction is higher than in the other three scenarios and the retail price is lower than in the other three scenarios, so the total market demand is the best. In addition, the gain brought about by the total market demand is higher than the loss of the emission reduction research and development cost and the wholesale price reduction, and the utility of the manufacturer is higher than in the other three scenarios. Similarly, the increase in the total market demand not only makes up for the loss of the retailer's price reduction, but it also makes the retailer utility reach its optimal level across the four cases, and the total profit of the supply chain is therefore at its optimal level across the four models.

4. Construction and Solving of Evolutionary Game Model

In reality, the choice of CSR strategy by manufacturers and retailers is a complex dynamic evolution process, and there are many external factors that affect the realization of an evolutionarily stable strategy (ESS). Therefore, a game matrix [27,28] is established based on the equilibrium solution obtained above, and an evolutionary game analysis is carried out on the CSR strategies of manufacturers and retailers in the low-carbon supply chain to discuss whether manufacturers and retailers will ultimately undertake CSR and to explore how external factors affect the CSR strategies of both sides. Specifically, the manufacturer's utility and retailer's utility in the four CSR models are used to construct the payment matrix of both sides in terms of their CSR strategies, and then the replicator dynamic equation and local equilibrium point of both sides are solved, and the evolutionary equilibrium strategy of both sides is obtained by combining this with the Lyapunov method. Finally, the ESS of both parties is verified by a numerical simulation method, and the influence of important parameters on the strategy selection process of both parties is analyzed.

4.1. Construction of Evolutionary Game Model

Considering that enterprises are often bounded rational agents when carrying out CSR activities, the upstream and downstream enterprises of the low-carbon supply chain play games with each other based on the complex market environment and choose whether to undertake CSR in order to achieve the maximum benefits, which is a process of continuous trial and error and continuous optimization, and thus we can build an evolutionary game model of the CSR strategy of a low-carbon supply chain [22,23].

The following assumptions and explanations are presented:

(1) Both the manufacturer group and the retailer group are bounded rational agents, and both groups continue to learn and imitate the most effective strategy in the repeated-game process.

(2) The probability that the manufacturer chooses to undertake a strategy is x, $x \in [0, 1]$; The probability that the manufacturer chooses not to undertake that strategy is $1 - x$.

(3) The probability that the retailer chooses to undertake a strategy is y, $y \in [0,1]$; The probability that the retailer chooses not to undertake that strategy is $1-y$.

According to the above assumptions, both manufacturers and retailers have two strategies, to undertake CSR or not to undertake CSR, and the payment matrix of the game played between the two sides can be established, as shown in Table 3.

Table 3. Payment matrix of low-carbon supply chain members.

Both Sides of the Game	The Probability that the Retailer Chooses to Undertake CSR (y)	The Probability that the Retailer Chooses not to Undertake CSR ($1-y$)
The probability that the manufacturer chooses to undertake CSR (x)	$(U_m^{PB^*}, U_r^{PB^*})$	$(U_m^{PM^*}, U_r^{PM^*})$
The probability that the manufacturer chooses not to undertake CSR ($1-x$)	$(U_m^{PR^*}, U_r^{PR^*})$	$(U_m^{PN^*}, U_r^{PN^*})$

Corresponding to the utility equilibrium solution of the four CSR models, when both manufacturers and retailers undertake CSR, the benefits for both sides are $U_m^{PB^*}$ and $U_r^{PB^*}$. When the manufacturer has CSR, but the retailer does not undertake CSR, the benefits for both sides are $U_m^{PM^*}$ and $U_r^{PM^*}$. When the manufacturer does not have CSR, but the retailer undertakes CSR, the benefits for both sides are $U_m^{PR^*}$ and $U_r^{PR^*}$. When neither manufacturer nor retailer undertakes CSR, the benefits for both sides are $U_m^{PN^*}$ and $U_r^{PN^*}$.

4.2. Solving the Evolutionary Game Model

For manufacturers, the expected benefit when choosing to undertake the CSR strategy is

$$U_{x1} = yU_m^{PB^*} + (1-y)U_m^{PM^*} \tag{46}$$

The expected benefit of choosing not to undertake the CSR strategy is

$$U_{x2} = yU_m^{PR^*} + (1-y)U_m^{PN^*} \tag{47}$$

The average expected benefit for manufacturers is:

$$\overline{U_x} = xU_{x1} + (1-x)U_{x2} \tag{48}$$

The expected benefit for retailers when they choose to undertake CSR strategies is

$$U_{y1} = xU_r^{PB^*} + (1-x)U_r^{PR^*} \tag{49}$$

The expected benefit for retailers who choose not to undertake CSR strategies is as follows:

$$U_{y2} = xU_r^{PM^*} + (1-x)U_r^{PN^*} \tag{50}$$

The average expected benefit for retailers is:

$$\overline{U_y} = yU_{y1} + (1-y)U_{y2} \tag{51}$$

From this, the replicator dynamic equation for the manufacturer can be obtained as follows:

$$F(x) = \frac{dx}{dt} = x\left(U_{x1} - \overline{U_x}\right) \tag{52}$$

The replicator dynamic equation for the retailer is:

$$F(y) = \frac{dy}{dt} = y(U_{y1} - \overline{U_y}) \tag{53}$$

Combining (52) and (53), the evolution process of the CSR game between manufacturers and retailers can be described as a two-dimensional dynamic system:

$$\begin{cases} F(x) = x(1-x)\left[yU_m^{PB^*} + (1-y)U_m^{PM^*} - yU_m^{PR^*} - (1-y)U_m^{PN^*}\right] \\ F(y) = y(1-y)\left[xU_r^{PB^*} + (1-x)U_r^{PR^*} - xU_r^{PM^*} - (1-x)U_r^{PN^*}\right] \end{cases} \tag{54}$$

Let $F(x) = \frac{dx}{dt} = 0$ and $F(y) = \frac{dy}{dt} = 0$, and set each equilibrium point of the system: (0,0), (0,1), (1,0), (1,1). Another (x_5, y_5) is not discussed, where $x_5 = \frac{U_r^{PN^*} - U_r^{PR^*}}{U_r^{PB^*} + U_r^{PN^*} - U_r^{PM^*} - U_r^{PR^*}}$ and $y_5 = \frac{U_m^{PN^*} - U_m^{PM^*}}{U_m^{PB^*} + U_m^{PN^*} - U_m^{PM^*} - U_m^{PR^*}}$. According to Proposition 9, we know that $U_r^{PN^*} - U_r^{PR^*} < 0$; if $x_5 \in (0,1)$,

$U_r^{PB^*} + U_r^{PN^*} - U_r^{PM^*} - U_r^{PR^*} < 0$, and $\frac{U_r^{PN^*} - U_r^{PR^*}}{U_r^{PB^*} + U_r^{PN^*} - U_r^{PM^*} - U_r^{PR^*}} < 1$, we find that $U_r^{PN^*} - U_r^{PR^*} > U_r^{PB^*} + U_r^{PN^*} - U_r^{PM^*} - U_r^{PR^*}$, namely $U_r^{PB^*} - U_r^{PM^*} < 0$. This is contradictory to Proposition 9. It can be seen that x_5 is not in the manufacturer's strategy range, and likewise y_5 is not in the retailer's strategy range. Therefore (x_5, y_5) is not a combination of two strategies, and its stability is not discussed.

Then, it is further analyzed whether the four local equilibrium points are evolutionarily stable strategies (ESSs), and the first partial derivatives of x, y are obtained, respectively, for the two-dimensional replication dynamic equation of Equation (54). The Jacobian matrix can be obtained as follows:

$$J = \begin{bmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{55}$$

wherein

$$a_{11} = (2x-1)\left[U_m^{PN^*} + (y-1)U_m^{PM^*} - y\left(U_m^{PB^*} + U_m^{PN^*} - U_m^{PR^*}\right)\right] \tag{56}$$

$$a_{12} = x(x-1)\left(U_m^{PR^*} + U_m^{PM^*} - U_m^{PB^*} - U_m^{PN^*}\right) \tag{57}$$

$$a_{21} = y(y-1)\left(U_r^{PR^*} + U_r^{PM^*} - U_r^{PB^*} - U_r^{PN^*}\right) \tag{58}$$

$$a_{22} = (2y-1)\left[(1-x)U_r^{PN^*} - U_r^{PR^*} + x\left(U_r^{PM^*} + U_r^{PR^*} - U_r^{PB^*}\right)\right] \tag{59}$$

In order to obtain the evolutionarily stable strategy (ESS) of both sides, the Lyapunov decision rule was used to judge the stability of the four equilibrium points [24–26]. Firstly, each pure strategy equilibrium point was substituted into the Jacobian matrix, and the diagonal matrix of each equilibrium point was obtained:

(1) The diagonal matrix of the equilibrium point (0,0) is

$$J = \begin{bmatrix} U_m^{PM^*} - U_m^{PN^*} & 0 \\ 0 & U_r^{PR^*} - U_r^{PN^*} \end{bmatrix} \tag{60}$$

(2) The diagonal matrix of equilibrium (1,0) is

$$J = \begin{bmatrix} U_m^{PN^*} - U_m^{PM^*} & 0 \\ 0 & U_r^{PB^*} - U_r^{PM^*} \end{bmatrix} \tag{61}$$

(3) The diagonal matrix of the equilibrium point (0,1) is

$$J = \begin{bmatrix} U_m^{PB^*} - U_m^{PR^*} & 0 \\ 0 & U_r^{PN^*} - U_r^{PR^*} \end{bmatrix} \tag{62}$$

(4) The diagonal matrix of equilibrium point (1,1) is

$$J = \begin{bmatrix} U_m^{PR^*} - U_m^{PB^*} & 0 \\ 0 & U_r^{PM^*} - U_r^{PB^*} \end{bmatrix} \tag{63}$$

When all the eigenvalues of the diagonal matrix are less than 0, the equilibrium point has asymptotic stability, that is, the point is an ESS. When at least one eigenvalue of the diagonal matrix is greater than 0, the equilibrium point is not asymptotically stable, that is, the point is unstable. In summary, in combination with Proposition 9, the asymptotic stability of each equilibrium point is determined by the eigenvalues of that point, as shown in Table 4.

Table 4. Stability analysis of system's local equilibrium points.

Equilibrium Point	Eigenvalue		Stability
(0,0)	$U_m^{PM^*} - U_m^{PN^*}$	$U_r^{PR^*} - U_r^{PN^*}$	Unstable point
(1,0)	$U_m^{PN^*} - U_m^{PM^*}$	$U_r^{PB^*} - U_r^{PM^*}$	Unstable point
(0,1)	$U_m^{PB^*} - U_m^{PR^*}$	$U_r^{PN^*} - U_r^{PR^*}$	Unstable point
(1,1)	$U_m^{PR^*} - U_m^{PB^*}$	$U_r^{PM^*} - U_r^{PB^*}$	ESS

According to Table 4, the only ESS in this two-dimensional system is (1,1), that is, in the low-carbon supply chain, both manufacturers and retailers choosing to undertake CSR. Even though manufacturers and retailers can receive "free rider" benefits when they do not undertake CSR and the other side undertakes CSR, both sides can still resolve to undertake CSR strategies. The reason is that no matter what strategy the other side chooses, enterprises in the low-carbon supply chain can always receive higher returns when they undertake CSR than when they do not undertake CSR.

5. Numerical Modeling

In order to verify and more intuitively display the propositions and conclusions obtained, in this section, PYTHON software (3.8.10) will be used for a numerical simulation analysis, which mainly includes:

(1) Exploring the influence of the CSR degree of enterprises in different CSR models (PN model, PR model, PM model, PB model) in the low-carbon supply chain and the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers on the optimal decision in different CSR models and comparing the optimal decisions in different CSR models. We can further analyze the impact of the carbon tax rate on the carbon emission reduction per unit product.

(2) Comparing the total market demand, manufacturer utility, retailer utility, and supply chain total utility in different CSR models and exploring the relationship between the CSR degree of enterprises, the potential scale of low-carbon consumers, the potential scale of ordinary consumers, the impact of total market demand, the utility of each enterprise, and the total supply chain utility in different CSR models.

(3) Verifying the ESS of the CSR game in the supply chain, and explore how different initial CSR commitment ratios, ratios of the potential scale of low-carbon consumers to the potential scale of ordinary consumers, and low-carbon preferences of the low-carbon consumers affect the evolution trend of both sides.

5.1. The Influence of Important Parameters on Decision Variables

Referring to the previous literature [11,13], first set $\alpha = 100, c = 5, \theta = 3$, k = 200, e = 10, $t = 0.5$. Then, according to Proposition 2, take $\gamma = 0.5$ under the condition $0 < \gamma < 1 - \frac{t(2t+\theta)}{2k}$; when $1 - \frac{t(2t+\theta)}{2k} < \gamma \leq 1$, take $\gamma = 0.997$. By substituting in the specific values of each parameter, the impact of the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers on wholesale prices under the different CSR degrees of enterprises is obtained, as shown in Table 5.

Table 5. Influence of the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers λ on wholesale prices.

Condition	λ Value	w^{PN^*}	w^{PR^*}	w^{PM^*}	w^{PB^*}
$0 < \gamma < 1 - \frac{t(2t+\theta)}{2k}$	0.2	35.1263	35.1689	31.5318	30.1220
	1.2	60.2525	60.3378	53.0636	50.2439
	2.2	85.3788	85.5068	74.5954	70.3659
	3.2	110.5051	110.6757	96.1272	90.4878
	4.2	135.6313	135.8446	117.6590	110.6098
$1 - \frac{t(2t+\theta)}{2k} < \gamma \leq 1$	0.2	35.1263	35.2543	26.7567	9.7936
	1.2	60.2525	60.5086	43.5133	9.5872
	2.2	85.3788	85.7630	60.2700	9.3808
	3.2	110.5051	111.0173	77.0267	9.1744
	4.2	135.6313	136.2716	93.7833	8.9680

It can be seen from Table 5 that the wholesale price increases with the increase in λ in the PN model, PR model, and PM model, which is consistent with Proposition 1. When $0 < \gamma < 1 - \frac{t(2t+\theta)}{2k}$, the wholesale price in the PB model w^{PB^*} increases with the increase in λ; when $1 - \frac{t(2t+\theta)}{2k} < \gamma \leq 1$, the wholesale price in the PB model w^{PB^*} decreases with the increase in λ, consistent with Proposition 2. It can be seen that when both manufacturers and retailers have CSR behaviors and the CSR degree of enterprises is high, the manufacturers will adopt bolder product pricing strategies, and the wholesale price will decrease as the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers increases.

Similarly let $\alpha = 100, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$. Then, according to Proposition 2, take $\gamma = 0.5$ under the condition $0 < \gamma < 1 - \frac{t(2t+\theta)}{3k}$; under the condition $1 - \frac{t(2t+\theta)}{3k} < \gamma \leq 1$, $\gamma = 0.997$ is taken. The specific values of each parameter are substituted to obtain the influence of the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers on retail prices under the different CSR degrees of enterprises, as shown in Table 6.

Table 6. Influence of the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers λ on retail prices.

Condition	λ Value	p^{PN^*}	p^{PR^*}	p^{PM^*}	p^{PB^*}
$0 < \gamma < 1 - \frac{t(2t+\theta)}{3k}$	0.2	47.7525	43.6149	45.9827	40.2846
	1.2	85.5050	77.2297	81.9653	70.5691
	2.2	123.2576	110.8446	117.9480	100.8537
	3.2	161.0101	144.4595	153.9306	131.1382
	4.2	198.7626	178.0743	189.9133	161.4228
$1 - \frac{t(2t+\theta)}{3k} < \gamma \leq 1$	0.2	47.7525	35.3306	43.6315	9.9484
	1.2	85.5050	60.6612	77.2629	9.8968
	2.2	123.2576	85.9919	110.8944	9.8452
	3.2	161.0101	111.3225	144.5258	9.7936
	4.2	198.7626	136.6531	178.1573	9.7420

As can be seen from Table 6, the retail price increases with the increase in λ in the PN model, PR model, and PM model, which is consistent with Proposition 1. In the PB model, the retail price p^{PB^*} increases with the increase in λ when γ is relatively small. When γ is large, the retail price p^{PB^*} in the PB model decreases with the increase in λ, which is consistent with Proposition 2. Combined with Table 5, it can be found that when γ is high, retailers in the PR model and PB model are trying to reduce their own profit space, trying to stimulate the market through the market strategy of "small profits and high sales", so as to achieve increased consumer surplus and self-utility. In addition, it is found that even if the wholesale price set by the manufacturer in the PM model is lower than that in the PR model, the retail price set by the retailer in the PR model is still higher than that in the PM model because the retailer does not pay attention to the consumer surplus.

Taking $\alpha = 100, \gamma = 0.5, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$, the influence of the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers on carbon emission reduction is obtained, as shown in Table 7.

Table 7. Influence of the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers λ on carbon emission reductions.

λ **Value**	g^{PN^*}	g^{PR^*}	g^{PM^*}	g^{PB^*}
0.2	0.2525	0.3378	0.2890	0.4065
1.2	0.5050	0.6757	0.5780	0.8130
2.2	0.7576	1.0135	0.8671	1.2195
3.2	1.0101	1.3514	1.1561	1.6260
4.2	1.2626	1.6892	1.4451	2.0325

As can be seen from Table 7, carbon emission reductions increase with the increase in λ in the PN model, PR model, PM model, and PB model, which is consistent with Proposition 1 and Proposition 2. It can also be found from the table that both manufacturers and retailers can increase the carbon emission reduction of products when they undertake CSR. When both parties undertake CSR, the carbon emission reduction is at its highest.

Taking $\alpha = 100, \lambda = 1, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$, the impact of corporate CSR level on wholesale price is obtained, as shown in Table 8.

Table 8. Influence of CSR degree γ on wholesale price.

γ **Value**	w^{PN^*}	w^{PR^*}	w^{PM^*}	w^{PB^*}
0.01	55.2273	55.2284	55.1139	55.1145
0.21	55.2273	55.2542	52.7200	52.4324
0.41	55.2273	55.2866	50.0563	48.5714
0.61	55.2273	55.3285	47.0746	42.5352
0.81	55.2273	55.3846	43.7143	31.7647

As can be seen from Table 8, in the PR model, the wholesale price increases with the increase in γ; the wholesale price in the PM model and PB model decreases with the increase in γ, which is consistent with Proposition 3, Proposition 4, and Proposition 5. Combined with Proposition 6, when $0 < \gamma < \frac{-4t^2+\theta^2}{2k}$, that is, $0 < \gamma < 0.02$, the size of the relationship between low-carbon product's wholesale prices as follows: $w^{PR^*} > w^{PN^*} > w^{PB^*} > w^{PM^*}$. When $\frac{-4t^2+\theta^2}{2k} < \gamma \leq 1$, that is, $0.02 < \gamma \leq 1$, the size of the relationship between low-carbon product's wholesale prices is:

$w^{PR^*} > w^{PN^*} > w^{PM^*} > w^{PB^*}$. Proposition 6 is verified. This indicates that when both parties bear CSR and the CSR degree of both parties is low, the manufacturer will set a higher wholesale price than in the PM model. The reason for this is that manufacturers are far away from the consumer side, and their weak attention to CSR struggles to have a real impact on consumer surplus. Compared with uncertain retailer decisions and market reactions, the benefits of higher wholesale prices are more obvious, so there is "free rider" behavior. Of course, as corporate CSR levels increase, this phenomenon will soon disappear.

Taking $\alpha = 100, \lambda = 1, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$, the impact of corporate CSR level on retail price is obtained, as shown in Table 9.

Table 9. Influence of CSR degree γ on retail price.

γ **Value**	p^{PN^*}	p^{PR^*}	p^{PM^*}	p^{PB^*}
0.01	77.9545	77.8426	77.8987	77.7863
0.21	77.9545	75.3390	76.7200	73.7838
0.41	77.9545	72.1975	75.4085	68.0220
0.61	77.9545	68.1387	73.9403	59.0141
0.81	77.9545	62.6923	72.2857	42.9412

As can be seen from Table 9, retail prices in the PR model, PM model, and PB model all decrease with the increase in γ, which is consistent with Proposition 3, Proposition 4, and Proposition 5. At the same time, combining this with Table 5, we can obtain the relative size of the retail price in four kinds of circumstances:

$p^{PN^*} > p^{PM^*} > p^{PR^*} > p^{PB^*}$, which proves Proposition 7. This shows that whether the manufacturer, the retailer, or both parties undertake CSR, the retailer will reduce the retail price. As the CSR degree of the enterprise increases, the retail price will drop more and more when both parties undertake CSR, a drop which is much larger than in the other three cases.

Taking $\alpha = 100, \lambda = 1, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$, the influence of CSR degree on carbon emission reduction is obtained, as shown in Table 10.

Table 10. Influence of CSR degree γ on carbon emission reduction.

γ **Value**	g^{PN^*}	g^{PR^*}	g^{PM^*}	g^{PB^*}
0.01	0.4545	0.4569	0.4557	0.4580
0.21	0.4545	0.5085	0.4800	0.5405
0.41	0.4545	0.5732	0.5070	0.6593
0.61	0.4545	0.6569	0.5373	0.8451
0.81	0.4545	0.7692	0.5714	1.1765

As can be seen from Table 10, the product's carbon emission reduction in the PR model, PM model, and PB model increases with the increase in γ, which is consistent with Proposition 3, Proposition 4, and Proposition 5. And combined with Table 7, we can obtain the relationship between the models' carbon emissions:

$g^{PB^*} > g^{PR^*} > g^{PM^*} > g^{PN^*}$, which proves Proposition 8. This indicates that the CSR behavior of any company has a positive effect on the improvement of a product's carbon emission reduction in the production activities of a low-carbon supply chain. Meanwhile, a counter-intuitive phenomenon can be observed: although the product's carbon emission reduction is a decision variable for the manufacturer, at the same CSR level, the retailer's CSR awareness has a greater impact on the product's carbon emission reduction than that of the manufacturer.

We now explore the impact of carbon tax rates on a unit product's carbon emission reduction through numerical simulation, taking $\alpha = 100, \lambda = 1, c = 5, \theta = 3$, k = 200, e = 10, $\gamma = 0.5$, as shown in Table 11.

Table 11. Influence of carbon tax rate t on carbon emission reduction.

t **Value**	g^{PN^*}	g^{PR^*}	g^{PM^*}	g^{PB^*}
0.1	0.3784	0.5056	0.4329	0.6078
0.3	0.4174	0.5580	0.4776	0.6711
0.5	0.4545	0.6081	0.5202	0.7317
0.7	0.4899	0.6559	0.5609	0.7897
0.9	0.5235	0.7015	0.5996	0.8451

It can be seen from Table 11 that the carbon emission reduction per unit product in the PN model, PR model, PM model, and PB model increases with the increase in t, but its marginal utility decreases. This indicates that the increase in the carbon tax rate is conducive to the increase in the product's carbon reduction during production activities, but the existence of diminishing marginal utility means that the increase in the carbon tax rate will not increase the product's carbon reduction without limitation. In addition, the relationship between the size of the carbon emission reductions in the four models is consistent with Proposition 8, which once again proves that proposition.

5.2. The Influence of Important Parameters on Total Market Demand and Utility

Firstly, $\alpha = 100, \gamma = 0.5, c = 5, \theta = 3, k = 200, e = 10, t = 0.5, \lambda \in [0.2, 4]$ were used for numerical simulation to obtain the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers. Under different CSR scenarios, the total market demand (Figure 2a), manufacturer's utility (Figure 2b) and retailer's utility were obtained (Figure 2c), as well as the total supply chain utility (Figure 2d). It can be seen from Figure 2 that in the PN model, PR model, PM model, and PB model, the total market demand, manufacturer utility, retailer utility, and supply chain total utility all increase with the increase in the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers, which is consistent with Proposition 1 and Proposition 2. In other words, regardless of whether the manufacturer or retailer undertakes CSR, the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers can bring about higher total market demand, thus prompting an increase in the utility of each member of the supply chain and the total utility of the supply chain.

$\alpha = 100, \lambda = 1, c = 5, \theta = 3, k = 200, e = 10, t = 0.5, \gamma \in [0, 1]$ were numerically simulated to obtain the effects of corporate CSR on total market demand (Figure 3a), manufacturer utility (Figure 3b), retailer utility (Figure 3c), and supply chain total utility (Figure 3d) under different CSR scenarios. From Figure 3, it can be seen that in the PR model and the PM and PB models the total market demand; manufacturer utility; retailer utility; and total supply chain utility increase with the increase in the enterprise's CSR degree, consistent with Proposition 3; Proposition 4; and Proposition 5. That is, when manufacturers or retailers undertake CSR in low-carbon supply chains, the increase in the enterprise's CSR degree stimulates the total market demand; as a result, the manufacturer utility, retailer utility, and total supply chain utility increase.

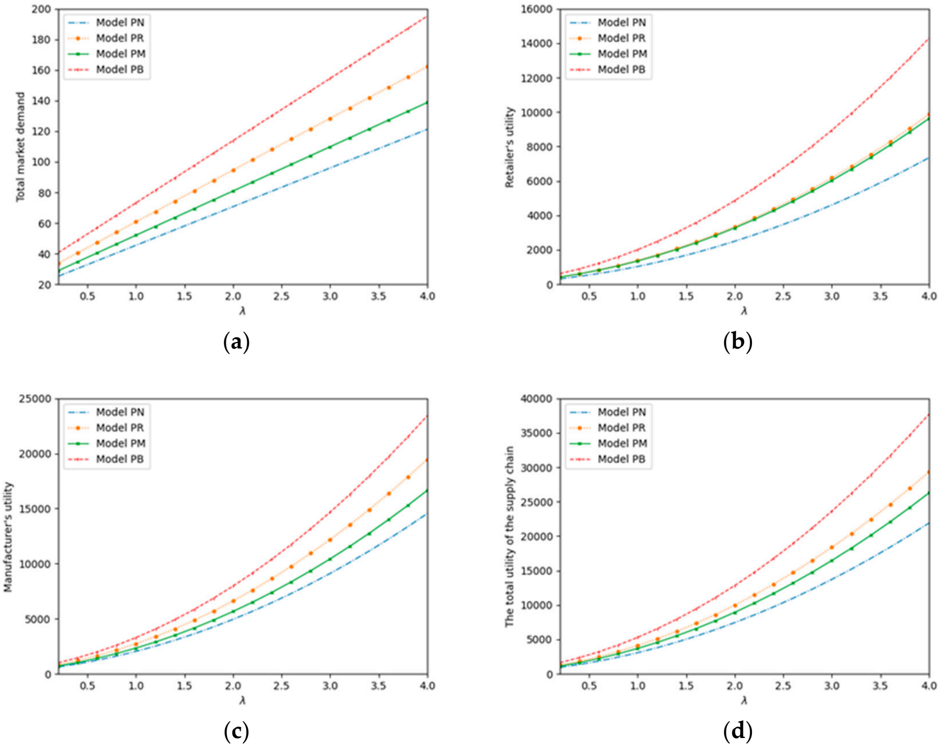

Figure 2. Influence of the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers λ on total market demand and utility. (**a**) The effect of λ on total market demand; (**b**) the effect of λ on retailer utility; (**c**) the effect of λ on manufacturer utility; and (**d**) the effect of λ on retailer utility.

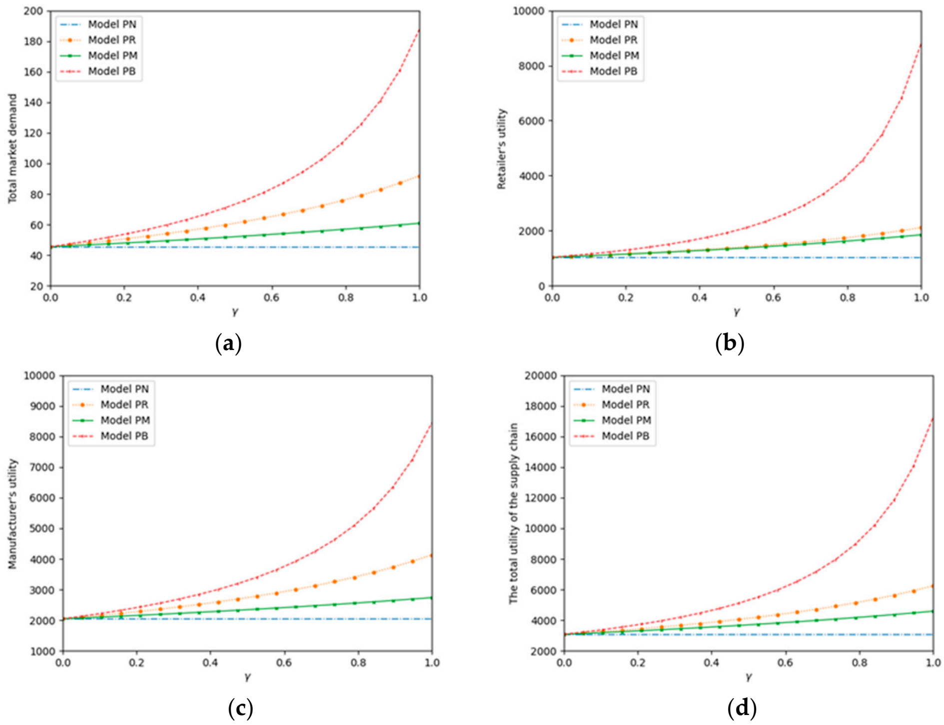

Figure 3. Influence of CSR degree γ on total market demand and utility. (**a**) The effect of γ on total market demand; (**b**) the effect of γ on retailer utility; (**c**) the effect of γ on the manufacturer's utility; and (**d**) the effect of γ on the total utility of the supply chain.

According to Figures 2 and 3, in the PN model, total market demand, manufacturer utility, retailer utility, and supply chain total utility are their lowest. In the PM model, in order to increase the consumer surplus, the manufacturer increases the unit carbon emission reduction of the product and reduces the wholesale price, prompting the retailer to reduce the retail price. Therefore, the total market demand, manufacturer utility, retailer utility, and supply chain total utility are all greater than in the PN model. In the PR model, on the one hand, retailers reduce the retail price of products; on the other hand, they accept higher wholesale prices to encourage manufacturers to increase the carbon emission reduction of products. This leads to a higher total market demand, manufacturer utility, retailer utility, and supply chain total utility than in the PM model. In the PB model, both manufacturers and retailers undertake CSR, which makes the total market demand, manufacturer utility, retailer utility, and supply chain total utility reach their optimal level among the four models and verifies Proposition 9. The simulation results directly reflect the impact of the four CSR models on total market demand, retailer utility, manufacturer utility, and supply chain total utility. From the perspective of the two cases in which a single enterprise undertakes CSR, the CSR behavior of a retailer is more effective and altruistic than that of a manufacturer, indicating that low-carbon supply chain is more effective in the context of no enterprise undertaking CSR. The awakening of the CSR consciousness of retailers in the supply chain plays a greater role in the accumulation of total market demand, the utility of each member of the low-carbon supply chain, and the total utility of the supply chain. However, it should be noted that, on the basis of retailers' CSR commitment, manufacturers' awareness of CSR greatly improves the total market demand, retailers' utility, manufacturers' utility, and the supply chain's total utility. The higher the CSR level of enterprises, the more obvious this synergistic effect will be, especially in the improvement of retailers' utility.

5.3. The Influence of Important Parameters on the CSR Strategies of Low-Carbon Supply Chain Members

5.3.1. Different Initial CSR Ratios

The basic parameter values are unchanged: $\alpha = 100, \lambda = 1, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$. When the initial proportion of CSR undertaken by retailers is $y = 0.1$ and the initial proportion x of CSR undertaken by manufacturers is 0.1, 0.3, 0.5, 0.7, and 0.9, respectively, the impact of the initial proportion of CSR undertaken by the manufacturers on the system's evolution trajectory can be obtained, as shown in Figure 4.

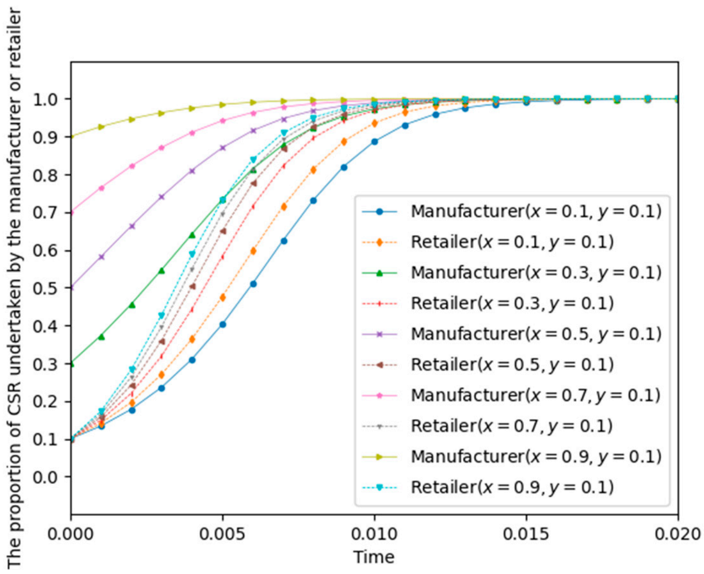

Figure 4. Evolution trajectory of low-carbon supply chain members under different initial CSR proportions of manufacturers.

In the case that $x = 0.1$ is the initial proportion of CSR undertaken by the manufacturer and the initial proportion y of CSR undertaken by the manufacturer is 0.1, 0.3, 0.5, 0.7 and 0.9, respectively, the impact of the initial proportion of CSR undertaken by the retailer on the system's evolution trajectory can be obtained, as shown in Figure 5.

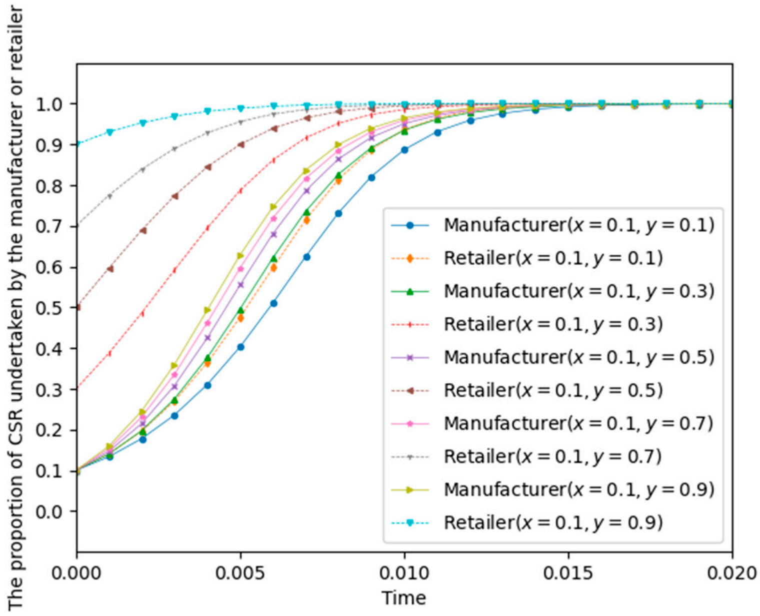

Figure 5. Evolution trajectory of low-carbon supply chain members under the initial CSR proportions of different retailers.

According to Figures 4 and 5, no matter whether the initial proportion of CSR undertaken by the manufacturer or the initial proportion of CSR undertaken by the retailer is changed, the strategy of each member in the low-carbon supply chain will eventually evolve into a CSR commitment, but the time taken for both parties to reach a stable strategy is different. If the initial proportion of CSR commitment is the same, the retailer will reach a stable state before the manufacturer. Moreover, an increase in the initial proportion of CSR undertaken by manufacturers or the increase in the initial proportion of CSR undertaken by retailers accelerates the evolution of CSR strategies for these low-carbon supply chain members.

Further, we move to explore how important parameters such as the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers and low-carbon preferences affect the CSR strategies of supply chain members. Since different initial CSR ratios will affect the strategy evolution of both parties, it is assumed that the initial CSR ratio of both parties is 0.5.

5.3.2. The Ratio of the Potential Scale of Low-Carbon Consumers to the Potential Scale of Ordinary Consumers

Setting $\alpha = 100, c = 5, \theta = 3, k = 200, e = 10, t = 0.5$, and λ as 1, 2, and 3, respectively, the influence of the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers on the system's evolution trajectory can be obtained, as shown in Figure 6.

As can be seen in Figure 6, the speed at which manufacturers and retailers evolve towards a CSR commitment is significantly accelerated as the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers increases, indicating that an increase in the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers improves the utility of manufacturers and retailers, thus speeding up the speed at which both sides undertake CSR.

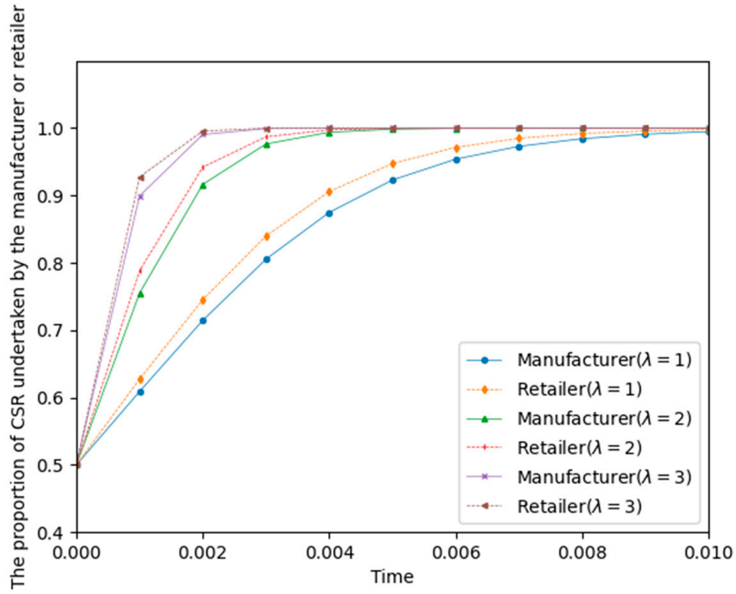

Figure 6. Influence of the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers on the system's evolution trajectory.

5.3.3. Low-Carbon Preferences of Low-Carbon Consumers

Setting $\alpha = 100, \lambda = 1, c = 5, k = 200, e = 10, t = 0.5$, and θ as 3, 5, and 7, respectively, the influence of low-carbon consumers' preferences on the system's evolution trajectory can be obtained, as shown in Figure 7.

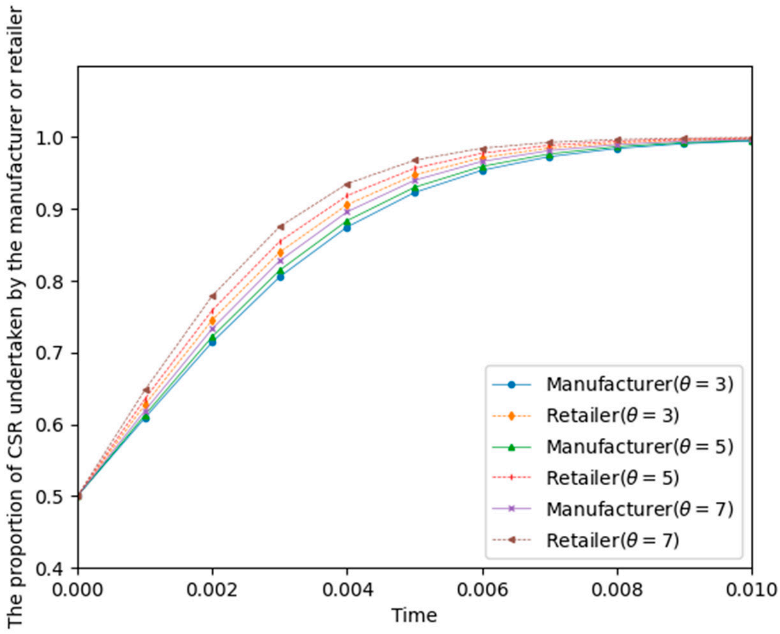

Figure 7. Influence of low-carbon preference on the evolution trajectory of the system.

As can be seen from Figure 7, with the increase in low-carbon consumers' preferences, the speed at which manufacturers and retailers evolve to undertaking CSR strategies becomes faster, indicating that an increase in low-carbon consumers' preference promotes the growth of market demand, thus bringing higher benefits to manufacturers and retailers and accelerating the speed at which both sides undertake CS; however, this effect is limited.

6. Conclusions

6.1. Research Results

By establishing the PN model, where no enterprise undertakes CSR; the PR model, where retailers undertake CSR; the PM model, where manufacturers undertake CSR; and the PB model, where manufacturers and retailers undertake CSR at the same time, this paper studies the optimal decision-making of low-carbon supply chain members, considering the heterogeneous preferences of consumers, and solves the market in four cases. We established the total market demand, manufacturer's utility, retailer's utility, and supply chain total utility and analyzed the influence of the CSR degree of the enterprises, the potential scale of low-carbon consumers, and potential scale of ordinary consumers on decision-making, utility, market total demand, and the supply chain total utility of each member of the supply chain, respectively, and compared the size of each member's optimal decision-making, utility, and supply chain total utility in different CSR relationship models. Then, using the optimal utility of the four CSR models, an evolutionary game model of CSR strategy was constructed to obtain an ESS. Finally, our propositions were verified by numerical simulation. From the above research, the following conclusions can be drawn:

(1) In the PN model, the retail price of low-carbon products is the highest, while carbon emission reduction, total market demand, manufacturer utility, retailer utility, and supply chain total utility are the lowest. In PB model, retail price and wholesale price are the lowest, while carbon emission reduction, total market demand, manufacturer utility, retailer utility, and supply chain total utility are the highest. When the CSR level is low, the wholesale price in the PN model is the highest. When the corporate CSR level is high, the wholesale price in the PR model is the highest.

(2) In PN model, PR model, PM model, and PB model, carbon emission reduction, total market demand, manufacturer utility, retailer utility and total supply chain utility increase with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers. In the PN model, PR model, and PM model, the retail price and wholesale price increase with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers. In the PB model, when the CSR degree of enterprises is low, the retail price and wholesale price increase with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers. When the CSR degree of enterprises is high, the retail price and wholesale price decrease with the increase in the ratio of the potential scale of low-carbon consumers to the potential scale of ordinary consumers, but manufacturers are more sensitive to the CSR degree of enterprises than retailers. In addition, in the four CSR models, carbon emission reductions increase with the increase in carbon tax rate, with a marginal diminishing effect.

(3) In the PR model, PM model, and PB model, carbon emission reduction, total market demand, manufacturer utility, retailer utility, and supply chain total utility increase with the increase in the CSR degree. In the PR, PM, and PB models, retail price decreases with the increase in the CSR degree. In the PM model and PB model, the wholesale price decreases with the increase in the CSR degree. In the PR model, the wholesale price increases with the increase in the CSR degree.

(4) In the CSR game system composed of manufacturers and retailers, there is only one evolutionarily stable strategy, which is that both manufacturers and retailers will eventually bear CSR. The initial proportion of CSR undertaken by manufacturers, the initial proportion of CSR undertaken by retailers, and the increase in the ratio between the potential scale of low-carbon consumers and the potential scale of ordinary consumers can effectively promote the CSR commitment of supply chain members. Although the increase in low-carbon consumers' preference can also promote the CSR commitment of supply chain members, its effect is limited.

In summary, first of all, the retail price of low-carbon products in the PN model is the highest but the total market demand and various utilities are the lowest, while those in the PB model are the opposite, and as the proportion of low-carbon consumers increases, the carbon emission reduction and utility of all models increase, while retail prices and wholesale prices are affected by CSR levels. Secondly, manufacturers and retailers will eventually assume social responsibility in the CSR game system, and an increase in the proportion of low-carbon consumers can promote CSR commitments, although the effect is limited.

6.2. Theoretical Implications

Considering consumers' heterogeneous preferences, this study classifies and discusses low-carbon supply chain decisions under four different CSR scenarios and discusses the evolutionary equilibrium strategies of supply chain members. This study broadens the scope of research in the field of corporate social responsibility and provides a new research perspective for the evolutionary game of the corporate social responsibility strategy of low-carbon supply chain members. These research results will enrich consumer theory and low-carbon supply chain management theory.

6.3. Managerial Implications

By analyzing the influence trends of important parameters on decision variables and other equilibrium solutions, this research provides an optimization scheme for the decision-making of low-carbon supply chain members under heterogeneous consumer preferences. In addition, the utility of low-carbon supply chain members in different CSR situations and the factors affecting their corporate social responsibility strategies are further analyzed to provide practical guidance for enterprises to formulate CSR strategies and government departments to formulate relevant policies. The specific management implications are as follows:

Firstly, four supply chain decision-making models under different CSR scenarios were established, and the influence of each model on the decision parameters and utility was analyzed. This shows that different CSR strategies and their undertakers have a significant impact on the performance indicators of supply chain members, so managers need to consider the potential results of adopting different CSR strategies to maximize their own interests.

Secondly, this study found that the proportion of potential low-carbon consumers to ordinary consumers significantly affects many factors such as market demand, utility, and pricing, which reflects the importance of understanding and catering to diverse consumer preferences. Therefore, managers need to consider targeted marketing and product development strategies to attract low-carbon consumers while also taking into account the preferences of ordinary consumers.

Third, this study shows that carbon emission reductions increase with the increase in the carbon tax rate, but with a marginal diminishing effect. This shows the importance of government policies and regulations in promoting environmental protection and reducing carbon emissions. Therefore, government departments can appropriately raise the price of carbon tax to promote carbon emission reductions, while corporate managers should be aware of external policy adjustments and adjust their strategies accordingly to align environmental regulations with corporate goals.

Finally, in a CSR game system containing manufacturers and retailers, both sides will eventually assume that CSR is an evolutionarily stable strategy. This shows that the common commitment of manufacturers and retailers to corporate social responsibility is critical to environmental sustainability and corporate development. Managers should give priority to the collaboration and coordination of corporate social responsibility initiatives among supply chain members. Government departments should also take relevant reward and punishment measures to encourage enterprises to actively assume social responsibility in order to achieve the maximization of their overall impact and utility.

6.4. Limitations and Future Research Directions

Due to the limitations of our objective resources and our subjectivity, although this paper explores the decision-making of low-carbon supply chain members, considering the heterogeneous preferences of consumers and CSR, and the evolution of CSR strategies of low-carbon supply chain, there are still some shortcomings. Firstly, the connotations of CSR are abstract and extensive. In reality, it covers many aspects such as social care, environmental protection, employee welfare, etc. Establishing quantitative CSR is a complex project involving multiple disciplines, and subsequent research could construct a more intuitive and comprehensive quantitative method for evaluating CSR. Moreover, the CSR levels of manufacturers and retailers may not be consistent, and the CSR levels of manufacturers and retailers could be distinguished in future studies.

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Appendix A. Solving Process of Optimal Solution

(Model PR)

Proof. First, the second derivative of U_r^{PR} with respect to p can be obtained $\frac{\partial^2 U_r^{PR}}{\partial p^2} = -4 + 2\gamma < 0$, it is easy to know that U_r^{PR} is a concave function with respect to p, there is an optimal p such that U_r^{PR} has a maximum value.

Then, with $\frac{\partial U_r^{PR}}{\partial p} = 0$, the retail price response function is:

$$p^{PR} = \frac{(\gamma - 1)[(1+\lambda)\alpha + \theta g] - 2w}{-4 + 2\gamma} \tag{A1}$$

Substitute p^{PR} into U_m^{PR}, can be obtained by U_m^{PR} of w and g Hessian matrix $H = \begin{bmatrix} \frac{4}{\gamma-2} & \frac{\theta-2t}{2-\gamma} \\ \frac{\theta-2t}{2-\gamma} & -k + \frac{2t\theta}{2-\gamma} \end{bmatrix}$. Because $0 < \gamma \leq 1, k > (2t+\theta)^2 > \frac{(2t+\theta)^2}{4(2-\gamma)}$, so $|H| = \frac{4}{\gamma-2}\left(-k + \frac{2t\theta}{2-\gamma}\right) - \left(\frac{\theta-2t}{2-\gamma}\right)^2 > 0$, the Hessian matrix is negative definite, U_m^{PR} is a joint concave function about w and g. let $\frac{\partial U_m^{PR}}{\partial w} = 0$, $\frac{\partial U_m^{PR}}{\partial g} = 0$, the optimal decision of the manufacturer is obtained as follows:

$$w^{PR^*} = -\frac{(1+\lambda)\alpha[(\gamma-2)k + t(2t+\theta)] + (c+et)[2k(\gamma-2) + \theta(2t+\theta)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{A2}$$

$$g^{PR^*} = \frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{4k(2-\gamma) - (2t+\theta)^2} \tag{A3}$$

By substituting Formula (A2) and Formula (A3) into Formula (A1), the optimal decision of retailers can be obtained as follows:

$$p^{PR^*} = -\frac{(1+\lambda)\alpha[(2\gamma-3)k + t(2t+\theta)] + (c+et)[-2k+\theta(2t+\theta)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{A4}$$

By further substituting each formula, the total market demand can be obtained as follows:

$$Q^{PR^*} = \frac{2k[(1+\lambda)\alpha - 2(c+et)]}{4k(2-\gamma) - (2t+\theta)^2} \tag{A5}$$

Retailer utility is:

$$U_r^{PR^*} = -\frac{(\gamma-2)k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[4k(\gamma-2) + (2t+\theta)^2\right]^2} \tag{A6}$$

Manufacturer's utility is:

$$U_m^{PR^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{2\left[4k(\gamma-2) + (2t+\theta)^2\right]} \tag{A7}$$

The total utility of supply chain is:

$$U_{sc}^{PR^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[6k(\gamma-2) + (2t+\theta)^2\right]}{2\left[4k(\gamma-2) + (2t+\theta)^2\right]^2} \tag{A8}$$

□

(Model PM)

Proof. First, the second derivative of U_r^{PM} with respect to p can be obtained $\frac{\partial^2 U_r^{PM}}{\partial p^2} = -4 < 0$, it is easy to know that U_r^{PM} is a concave function with respect to p, there is an optimal p such that U_r^{PM} has a maximum value.

Then, with $\frac{\partial U_r^{PM}}{\partial p} = 0$, the retail price response function is:

$$p^{PM} = \frac{(1+\lambda)\alpha + \theta g + 2w}{4} \tag{A9}$$

Substitute p^{PM} into U_m^{PM}, can be obtained by U_m^{PM} of w and g Hessian matrix $H = \begin{bmatrix} \frac{\gamma-4}{2} & \frac{\theta(2-\gamma)}{4} - t \\ \frac{\theta(2-\gamma)}{4} - t & -k + t\theta + \frac{\gamma\theta^2}{8} \end{bmatrix}$. Because $0 < \gamma \leq 1$, $k > (2t+\theta)^2 > \frac{(2t+\theta)^2}{2(4-\gamma)}$, so $|H| = \frac{\gamma-4}{2}\left(-k + t\theta + \frac{\gamma\theta^2}{8}\right) - \left(\frac{\theta(2-\gamma)}{4} - t\right)^2 > 0$, the Hessian matrix is negative definite, U_m^{PM} is a joint concave function about w and g. let $\frac{\partial U_m^{PM}}{\partial w} = 0$, $\frac{\partial U_m^{PM}}{\partial g} = 0$, the optimal decision of the manufacturer is obtained as follows:

$$w^{PM^*} = \frac{(1+\lambda)\alpha[(\gamma-2)k + t(2t+\theta)] + (c+et)[-4k+\theta(2t+\theta)]}{2k(\gamma-4) + (2t+\theta)^2} \tag{A10}$$

$$g^{PM^*} = -\frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{2k(\gamma-4) + (2t+\theta)^2} \tag{A11}$$

By substituting Formula (A10) and Formula (A11) into Formula (A9), the optimal decision of retailers can be obtained as follows:

$$p^{PM^*} = \frac{(1+\lambda)\alpha[(\gamma-3)k + t(2t+\theta)] + (c+et)[-2k+\theta(2t+\theta)]}{2k(\gamma-4)+(2t+\theta)^2} \tag{A12}$$

By further substituting each formula, the total market demand can be obtained as follows:

$$Q^{PM^*} = -\frac{2k[(1+\lambda)\alpha - 2(c+et)]}{2k(\gamma-4)+(2t+\theta)^2} \tag{A13}$$

Retailer utility is:

$$U_r^{PM^*} = \frac{2k^2[(1+\lambda)\alpha - 2(c+et)]^2}{\left[2k(\gamma-4)+(2t+\theta)^2\right]^2} \tag{A14}$$

Manufacturer's utility is:

$$U_m^{PM^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2}{2\left[2k(\gamma-4)+(2t+\theta)^2\right]} \tag{A15}$$

The total utility of supply chain is:

$$U_{sc}^{PM^*} = -\frac{k[(1+\lambda)\alpha - 2(c+et)]^2\left[2k(\gamma-6)+(2t+\theta)^2\right]}{2\left[2k(\gamma-4)+(2t+\theta)^2\right]^2} \tag{A16}$$

□

(Model PB)

Proof. First, the second derivative of U_r^{PB} with respect to p can be obtained $\frac{\partial^2 U_r^{PB}}{\partial p^2} = -4+2\gamma < 0$, it is easy to know that U_r^{PB} is a concave function with respect to p, there is an optimal p such that U_r^{PB} has a maximum value.

Then, with $\frac{\partial U_r^{PB}}{\partial p} = 0$, the retail price response function is:

$$p^{PB} = \frac{(\gamma-1)[(1+\lambda)\alpha + \theta g] - 2w}{-4+2\gamma} \tag{A17}$$

Substitute p^{PB} into U_m^{PB}, can be obtained by U_m^{PB} of w and g Hessian matrix $H = \begin{bmatrix} \frac{6\gamma-8}{(\gamma-2)^2} & \frac{2[t(\gamma-2)+\theta-\gamma\theta]}{(\gamma-2)^2} \\ \frac{2[t(\gamma-2)+\theta-\gamma\theta]}{(\gamma-2)^2} & -k+\frac{\theta[-4t(\gamma-2)+\gamma\theta]}{2(\gamma-2)^2} \end{bmatrix}$. Because $0 < \gamma \le 1$, $k > (2t+\theta)^2 > \frac{(2t+\theta)^2}{2(4-3\gamma)}$, so $|H| = \frac{6\gamma-8}{(\gamma-2)^2}\left(-k+\frac{\theta[-4t(\gamma-2)+\gamma\theta]}{2(\gamma-2)^2}\right) - \left(\frac{2[t(\gamma-2)+\theta-\gamma\theta]}{(\gamma-2)^2}\right)^2 > 0$, the Hessian matrix is negative definite, U_m^{PB} is a joint concave function about w and g. let $\frac{\partial U_m^{PB}}{\partial w} = 0$, $\frac{\partial U_m^{PB}}{\partial g} = 0$, the optimal decision of the manufacturer is obtained as follows:

$$w^{PB^*} = \frac{(1+\lambda)\alpha[2k(\gamma-1)+t(2t+\theta)] + (c+et)[2k(\gamma-2)+\theta(2t+\theta)]}{k(6\gamma-8)+(2t+\theta)^2} \tag{A18}$$

$$g^{PB^*} = -\frac{[(1+\lambda)\alpha - 2(c+et)](2t+\theta)}{k(6\gamma-8)+(2t+\theta)^2} \tag{A19}$$

By substituting Formula (A18) and Formula (A19) into Formula (A17), the optimal decision of retailers can be obtained as follows:

$$p^{PB^*} = \frac{(1+\lambda)\alpha[3k(\gamma-1)+t(2t+\theta)]+(c+et)[2k+\theta(2t+\theta)]}{k(6\gamma-8)+(2t+\theta)^2} \tag{A20}$$

By further substituting each formula, the total market demand can be obtained as follows:

$$Q^{PB^*} = \frac{-2k[(1+\lambda)\alpha-2(c+et)]}{k(6\gamma-8)+(2t+\theta)^2} \tag{A21}$$

Retailer utility is:

$$U_r^{PB^*} = -\frac{(\gamma-2)k^2[(1+\lambda)\alpha-2(c+et)]^2}{\left[k(6\gamma-8)+(2t+\theta)^2\right]^2} \tag{A22}$$

Manufacturer's utility is:

$$U_m^{PB^*} = -\frac{k[(1+\lambda)\alpha-2(c+et)]^2}{2\left[k(6\gamma-8)+(2t+\theta)^2\right]} \tag{A23}$$

The total utility of supply chain is:

$$U_{sc}^{PB^*} = -\frac{k[(1+\lambda)\alpha-2(c+et)]^2\left[4k(2\gamma-3)+(2t+\theta)^2\right]}{2\left[k(6\gamma-8)+(2t+\theta)^2\right]^2} \tag{A24}$$

□

Notes

1 Data Source: BYD Social Responsibility Report 2023 (Simplified)

2 Data Source: https://www.walmart.cn/sustainability/ (accessed on 18 May 2023)

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