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Settlement Selection Strategic Analysis for Self-Operated E-Commerce Platforms under Market Competition

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Abstract: This paper focuses on the settlement selection strategic analysis for self-operated e-commerce platforms on hybrid e-commerce platforms under market competition. Taking factors such as the market share, price competition, commission, and customer loyalty into account, a multi-leader–follower game model with the platforms as leaders and the manufacturers as followers is established. Then, we solve the model with the help of some mathematical techniques and describe some numerical experiments to analyze settlement strategies for the self-operated platforms and their impact on other members in the network. The numerical results reveal the following revelations: a lower commission rate is more suitable for the self-operated platforms; once the commission rates are determined, the self-operated platforms prefer to settle in the hybrid platforms under lower medium price competition; when the price competition is fierce, as customer loyalty increases, the self-operated platforms should settle with a low market share; if the self-operated platforms settle in the hybrid platforms, then a higher price competition is advantageous for all members and can facilitate supply chain coordination.

Keywords: self-operated e-commerce platform; hybrid e-commerce platform; market competition; settlement strategy; multi-leader–follower game



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1. Introduction

With the rapid development of e-commerce platforms, online retail has maintained strong growth in recent years. It is reported that there are more than 5 billion Internet users worldwide, and the number of people shopping online continues to increase. In 2024, global retail e-commerce sales are expected to exceed 6.3 trillion dollars, and this figure is expected to reach new heights in the next few years. In general, the market share is mainly concentrated in the hands of a few large e-commerce platforms with a large consumer group, such as Amazon.com and JD.com, which has led to the urgent need for many other self-owned platforms to expand their sales channels.

At present, there are two main sales modes for e-commerce platforms [1]. One is the platform mode, in which the manufacturers enter the platform to sell their products by paying commissions, and the prices are determined by the manufacturers. The other is the self-operated mode, in which the platform wholesales products from the manufacturers, determines the prices and sells them to consumers, making profits by earning the difference. In recent years, large e-commerce platforms such as JD.com and Amazon.com have begun to open their platform to third-party platforms, realizing a transformation from the self-operated platform to the hybrid platform (self-operated and platform modes). At the same time, in order to gain more market share, self-operated platforms with a small proportion can choose to enter the hybrid platform in the form of flagship stores. For example, Gome.com and JD.com adopt a self-operated mode for manufacturers such as Midea, Haier, and Huawei, and compete fiercely, but in 2020, Gome.com entered JD.com in the form of flagship stores, mainly focusing on major appliances, 3C electronics and other

categories. In this form of settlement, Gome.com sells products through its own online platform, as well as the JD.com platform. And for every transaction on JD.com, Gome.com must pay a commission to JD.com. In this context, the two platforms have formed a new cooperative relationship, but competition also exists at the same time. Therefore, whether a self-operated platform entering a hybrid platform will intensify the competition, and whether it can obtain a larger market share and profit, is worth in-depth study.

At present, some scholars have considered the situation where one platform settles in another platform. Ryan et al. considered a retailer with its own website selling its products through a settling format and analyzed the optimal strategy selection problem for each member [2]. Fan et al. investigated the impact of the settling of a self-operated platform to another platform on channel members for a supply chain consisting of a single manufacturer and two e-commerce platforms [3]. In summary, the research on e-commerce platform strategies under competition mainly focuses on the mode selection problem, while the research on settlement strategy selection mainly focuses on manufacturers and retailers, and most of them do not consider the competition between e-commerce platforms. However, with the rapid development of e-commerce, the competition between e-commerce platforms is becoming increasingly fierce. In this context, this paper takes self-operated e-commerce platforms as the research object to analyze their settlement strategy selection problem under competition. Our study aims to answer the following questions:

- (1) Under what conditions will self-operated platforms, such as Gome.com, choose to settle in hybrid platforms such as JD.com?
- (2) How does the settlement strategy affect members?
- (3) Can the settlement strategy achieve supply chain coordination?

To address the above questions, we consider a market consisting of one hybrid e-commerce platform called the large platform, one self-operated platform called the small platform, and one manufacturer. Both platforms adopt a self-operated mode, and the small platform can compete with the large platform through their own platform, or sign a contract with the large platform to settle and sell products through the market of the large platform. Considering factors such as market share, price competition intensity, commission rate, and customer loyalty, we construct multi-leader–follower game models in both the settled and non-settled scenarios, with the platforms as leaders and the manufacturer as the follower. Furthermore, mathematical techniques are used to simplify the models and then the settlement strategy of the self-operated platforms is analyzed through numerical experiments, and the impact of the settlement strategy on each member is discussed. Some valuable insights can be stated as follows. First, a lower commission rate is more suitable for the self-operated platforms to settle in. Second, when the commission rate is constant, the self-operated platforms should choose to settle when the price competition is weak or moderate, and when the price competition is strong and as the customer loyalty of the hybrid platform increases, the self-operated platforms should choose to settle when the market share of the hybrid platform is low. Third, when the self-operated platforms settle in the hybrid platform, a stronger price competition can increase the profits of each member and is conducive to achieving the overall coordination of the supply chain.

There is little in the literature on the settlement strategies of two platforms. Drawing on channel selection in marketing, we liken the problem of settlement strategies to the channel selection problem. Combined with the literature related to this paper [2,3], we aim to fill in the research gap in pricing and multi-channel layouts of self-operated platforms. Our study has three contributions. First, Ryan et al. considers the settlement selection of platforms [2] and does not consider the pricing strategies of manufacturers on different platforms, even though manufacturers' pricing strategies will inevitably affect the pricing of goods and related decisions on each platform. Based on this, this article focuses on the choice of platform settlement considering the pricing strategy of the manufacturers. Second, Fan et al. considers one platform adopting a platform model and another platform adopting a self-operated model, mainly considering the game between the manufacturers and self-operated platforms [3]. In this paper, both platforms adopt a self-operated model,

considering the game between the manufacturers and platforms as well as the competition game between the two platforms. Finally, in terms of model analysis, ref. [2] establishes a leader–follower game model between large and small platforms, while [3] establishes a Bertrand model. We take the platforms as the leaders and the manufacturer as the follower, and establish a multi-leader–follower game model, which is more complex than that presented in the first two studies. We discuss the convexity conditions of the models, obtain the analytical expressions of each optimal strategy, and use the numerical experimental results to provide an influence analysis of the different strategies.

This paper is structured as follows. The literature most relevant to this paper is reviewed in Section 2. In Section 3, multi-leader–follower game models are formulated between two platforms and the manufacturer to explore whether the small platform settles in the hybrid platform. Some equilibrium results are given in Section 4. In Section 5, we establish numerical experiments to analyze the results. Finally, the conclusions and future research directions are given in Section 6. All the proofs of the propositions are given in the Appendix A.

2. Literature Review

This study is closely related to the following three streams of literature: the operational strategies of e-commerce platforms, settlement strategies, and market competition and cooperation in supply chains.

2.1. Operational Strategies of E-Commerce Platforms

The operational strategies of e-commerce platforms under market competition have been studied in depth, and mainly focus on mode selection. Abhishek et al. considered the mode selection of e-commerce platforms to explore when platforms should adopt an agency selling mode instead of a traditional reselling mode and found that sales through electronic channels can affect strategic selling modes [4]. Chen et al. investigated how the chosen pricing model affects the online retailer's profit and found that the agency pricing model always leads to a lower retail price and higher consumer surplus [5]. Zenny studies the strategic contract (wholesale or agency) between a monopoly platform and two competing suppliers who sell goods through the platform and found that when the product substitutability is low (high) enough, the platform offers a low (high) royalty rate to induce suppliers to adopt an agency (wholesale) contract [6]. Li and Ai considered a cross-selling supply chain mode consisting of two competing manufacturers and two competing e-retailers, explored which sales mode e-retailers should rely on under horizontal competition for both suppliers and e-retailers, and showed that the sales mode choice of the e-retailers depends on the channel competition and revenue sharing [7]. Wei et al. explored how competing e-retailers choose the optimal online sales mode through a stylized theoretical model. The results show that if two e-retailers sell the products of a common manufacturer on their online platforms, the manufacturer's profit is the highest when both e-retailers adopt an agency sales model and charge the manufacturer the same referral fee, which is also the case for the entire supply chain [8]. Zhang and Hou studied how manufacturers should sell online in the context of some e-tailers launching private brands and competing with manufacturers' products in the terminal market. They found that when commissions are low (high), manufacturers should adopt agency sales (resales) [9]. In addition, there are also some studies related to the operational strategies of e-commerce platforms, such as discussing order fulfillment [10,11], online product reviews [12,13], promotion [14–16], and demand information sharing [17,18].

With the rapid development of e-commerce platforms, many large e-commerce platforms have begun to open their platforms to attract third-party sellers [19]. Will third-party sellers choose to settle in large e-commerce platforms? This is what this paper focuses on. It is worth noting that the current research on the operational strategies of e-commerce platform focuses on mode selection, while there are fewer studies on settlement strategies, and this paper is dedicated to enriching the research in this area.

2.2. Settlement Strategies

On the one hand, some of the literature has studied the opening strategy of e-commerce platforms. Mantin et al. explored the strategic rationale for retailers to introduce 3P marketplaces. The results show that by committing to having an active 3P marketplace, retailers create an “external option” that improves their bargaining position in negotiations with manufacturers, and that the presence of a 3P marketplace benefits consumers, but this benefit diminishes as retailers become more powerful [20]. Tian et al. investigated retailers that have started to serve as online marketplaces by providing a platform to directly connect sellers with buyers and found that this new format will mitigate the double-marginalization effect and benefit both the intermediary and suppliers through a revenue sharing scheme [21]. Hagiou et al. invited rivals to sell products or services on top of its core product and found that hosting eliminates the additional shopping costs to consumers when buying a specialist rival’s competing version of the multiproduct firm’s noncore product [22]. Song et al. explored why an online retailer would open its platform and why a third-party seller would join the platform when the third party carries products identical to those the retailer sells as well as products the retailer does not carry, and found that the spillover effect always (weakly) benefits the third party, but it does not necessarily hurt the retailer [23]. Sun and Liu examined the motivations for an online retailer that exclusively resells goods to open its platform to competing sellers and found that the retailer can reduce their price competition with the 3P seller by charging rent and commission [24].

On the other hand, some scholars have also studied the settlement strategy of merchants. Ryan et al. explored under what conditions online retailers choose to enter into contracts with marketplace firms to sell their products through the marketplace system. The results show that in a pure coordination equilibrium, retailers agree to participate in the marketplace contract, while the marketplace firm chooses not to sell the products itself. In a pure competition equilibrium, retailers do not participate in the marketplace system [2]. Wang et al. analyzed the different mode choices of suppliers to enter the e-commerce platform, and found that when the product reputation or the commission rate charged by the platform to the supplier is high, the supplier is more likely to choose the FBP model [25]. Fan et al. considered a decentralized supply chain consisting of a manufacturer and two online retailers and explored the value of horizontal cooperation among online retailers. The results show that horizontal cooperation has the value of promoting channel coordination and can be enhanced when market competition becomes more intense [3]. Xu et al. considered a platform and two representative suppliers (an incumbent supplier and a new entrant) to examine the strategic integration of a third-party product amidst the competition between a self-operating channel and a marketplace and found that an escalation in the commission rate resulted in diminished profits for the incumbent supplier [26].

In summary, current studies on settlement strategies mainly focus on merchants. This paper extends the research focus of settlement strategies by considering the settlement strategies of self-operated platforms.

2.3. Competition and Cooperation in Supply Chains

The key to the settlement selection strategy problem is that there is both competition and cooperation among supply chain members [20]. There is a rich area of the literature that considers both competition and cooperation in supply chains. For example, Zhou et al. studied the impact of intercompetitor licensing between an original equipment manufacturer and an independent remanufacturer on market competition between new and remanufactured products [27]. Katsaliaki et al. systematically reviewed existing research findings in the field of supply chain competition and examines the theories of competition and organizational relations in intra-firm and inter-firm supply chains [28]. In addition, competition and cooperation are also studied from the perspective of sustainability. Guo et al. considered a fashion supply chain consisting of one manufacturer and two competing retailers and investigated how retail competition and consumer returns affect green product development of fashion apparel, with results indicating that more competitive markets

lead to lower optimal green levels [29]. Deligiannis et al. aimed to explore the impact that service-driven always-a-share behavior may have on the competitive and cooperative inventory policies of its suppliers [30]. Under competition, each supplier's optimal active base inventory level is higher than its short-sighted base inventory level, and the other supplier's active base inventory level increases. Under mild conditions, the active base levels of both suppliers have at least one pure strategy Nash equilibrium solution. Dou and Choi investigated how channel relationships (i.e., competition or cooperation) among manufacturers affect government policies (e.g., emission tax prices), and found that while cooperation leads to better economic performance, competition may be a better channel relationship for improving sustainability and social welfare [31].

Closely related to our work is the exploration of competition and cooperation in supply chains from the perspective of settlement. Ryan et al. considered the sales of products by retailers with their own websites through the form of settlement, and analyzed the optimal strategy selection problem of each member [2]. Fan et al. studied the impact of a self-operated platform settling in another platform on channel members and focused on the game between manufacturers and platforms [3]. Xu et al. explored the strategic integration of a third-party product amidst the competition between a self-operating channel and a marketplace and studied the game between platforms and suppliers [26]. To sum up, the existing literature mainly studies the competitive and cooperative game between merchants and platforms. This paper considers self-operated platforms into hybrid platforms, focusing on competition and cooperation between platforms, which is a new type of competitive and cooperative relationship.

3. The Model

In this section, we establish the mathematical models for two cases of the small platform settling in (denoted by case E) or out of (denoted by case T) the hybrid platform, to analyze the settlement selection strategy of the small platform.

3.1. Model Description

Consider a dual supply chain consisting of a manufacturer, a hybrid e-commerce platform (for short, called a large platform), a self-operated e-commerce platform (for short, called a small platform), and consumers. The large and small platforms wholesale their products from the manufacturer at wholesale prices w_{TB} and w_{TL} or w_{EB} and w_{EL} . When the small platform does not settle in the large platform (case T), the large and small platforms sell products on their own platforms at selling prices p_{TB} and p_{TL} , respectively, as shown in Figure 1. When the small platform chooses to settle in the large platform in the form of a flagship store (case E), it pays the settlement fee k and commission rate δ to the large platform, which are exogenous variables. And then, the large and small platforms sell products to consumers at sales prices p_{EB} and p_{EL} , respectively, as shown in Figure 2. We assume that the small platform sells products at the same price in different channels [3]. The notations used later on are summarized in Table 1.

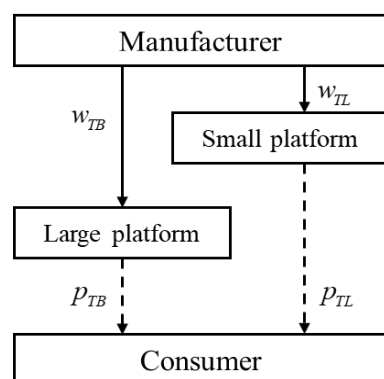


Figure 1. Supply chain structure of case T.

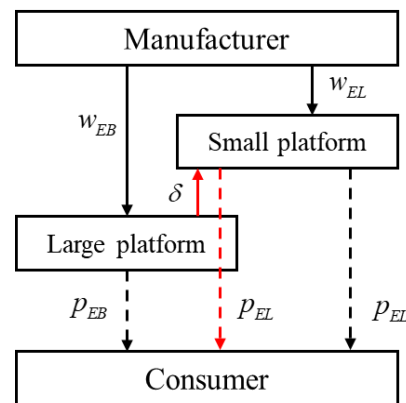


Figure 2. Supply chain structure of case E.

Table 1. Notations.

Notation	Description
Set	
I	Case set $i \in I = \{T, E\}$, T represents the case in which the small platform does not settle, E represents the case in which the small platform settles in the large platform
J	Market player set $j \in J = \{B, L\}$, B represents the large platform and L represents the small platform
Parameter	
k	Platform settlement fee
δ	Commission rate (charge according to the transaction volume), $\delta \in [0, 1]$
d	Potential market demand
λ	Market share of the large platform, $\lambda \in [0.5, 1]$
α	Price competition intensity, $\alpha \in [0, 0.5]$
θ	Customer loyalty to the self-operated products on the large platform, $\theta \in [0, 1]$
D_{ij}	Self-operated channel market demand for the platform j in case i
D_{BL}	Agent channel market demand for the small platform in case i
π_{ij}	Profit for the platform j in case i
π_{iM}	Profit for the manufacturer in case i
Variable	
w_{ij}	Wholesale price for the platform j in case i
p_{ij}	Retail price of the platform j in case i

3.2. Demand Functions

According to the literature [32–34], we use a linear demand function for describing the change in demand due to price competition. For case T, the demand functions for the small and large platforms are

$$D_{TL} = (1 - \lambda)d - p_{TL} + \alpha p_{TB}, \quad (1)$$

$$D_{TB} = \lambda d - p_{TB} + \alpha p_{TL}, \quad (2)$$

respectively. Here, d represents the potential market demand, $\lambda \in [0.5, 1]$ represents the potential market share occupied by the large platform, and $\alpha \in [0, 0.5]$ is the intensity of the price competition between the two platforms (the larger the value of α , the greater the price competition, assuming that $\alpha \in [0, 0.5]$ ensures the profitability of the large platform's self-operated products [3]).

For case E, the small platform obtains a new agent channel on the large platform, which can seize part of the market share of the large platform. And the large platform sells

both self-operated products and products of the small platform. Therefore, the demand functions are given by

$$D_{EL} = (1 - \lambda)d - p_{EL} + \alpha(p_{EB} + p_{EL}), \quad (3)$$

$$D_{EB} = \theta\lambda d - p_{EB} + 2\alpha p_{EL}, \quad (4)$$

$$D_{BL} = (1 - \theta)\lambda d - p_{EL} + \alpha(p_{EB} + p_{EL}), \quad (5)$$

respectively. Here, $\theta \in [0, 1]$ represents the proportion of consumers remaining on the large platform after the small platform settles (the loyalty of customers to the self-operated products of the large platform). $(1 - \theta)\lambda d$ denotes the potential market demand of the small platform on the large platform. α denotes the intensity of the price competition between the two platforms. Since this paper focuses on the relationship between the two platforms, referring to [3,32], only the price competition intensity parameter α between the two platforms is used, rather than the price competition parameters among three sales channels. In this case, the total demand of the small platform is $D_{EL} + D_{BL}$.

3.3. Model for Case T

In fact, large platforms such as JD.com and Amazon.com have a strong market position, absolute advantage and voice when negotiating with manufacturers [35]. Therefore, the game decision-making sequence is as follows: the large and small platforms first decide the retail prices, and then the manufacturer decides the wholesale price. Consequently, a multi-leader–follower game model is presented, in which a non-cooperative game is played between two platforms and a leader–follower game is played between the platforms and the manufacturer.

In case T, we establish a multi-leader–follower game (I), as follows:

$$\max_{p_{Tj} \geq w_{Tj}} \pi_{Tj}(\mathbf{p}_T, \mathbf{w}_T) = (p_{Tj} - w_{Tj})D_{Tj} \quad (6)$$

$$\text{s.t. } \max_{w_T \geq 0} \pi_{TM}(\mathbf{p}_T, \mathbf{w}_T) = w_{TL}D_{TL} + w_{TB}D_{TB} \quad (7)$$

where $j \in J = \{B, L\}$, $\mathbf{p}_T = (p_{TL}, p_{TB})$ represents the product price vectors for the platform j , $\mathbf{w}_T = (w_{TL}, w_{TB})$ is the wholesale price vector determined by the manufacturer in case T. (6) represents the profit function of the platform j . The constraint $p_{Tj} \geq w_{Tj}$ to ensure the non-negativity of the platforms' profits, prompting them to be willing to enter the market. (7) represents the profit function of the manufacturer.

3.4. Model for Case E

In case E, we establish a multi-leader–follower game (II), as follows:

$$\max_{p_{EL} \geq w_{EL}} \pi_{EL}(\mathbf{p}_E, \mathbf{w}_E) = (p_{EL} - w_{EL})(D_{EL} + D_{BL}) - \delta p_{EL}D_{BL} - k \quad (8)$$

$$\text{s.t. } \max_{w_E \geq 0} \pi_{EM}(\mathbf{p}_E, \mathbf{w}_E) = w_{EL}(D_{EL} + D_{BL}) + w_{EB}D_{EB} \quad (9)$$

$$\begin{aligned} \max_{p_{EB} \geq w_{EB}} \pi_{EB}(\mathbf{p}_E, \mathbf{w}_E) &= (p_{EB} - w_{EB})D_{EB} + \delta p_{EL}D_{BL} + k \\ \text{s.t. } (9) \end{aligned} \quad (10)$$

where $\mathbf{w}_E = (w_{EL}, w_{EB})$ is the wholesale price vector determined by the manufacturer in case E. (8) and (9) are the small platform's model, where (8) represents its profit function, and (9) represents the manufacturer's model; (9) and (10) are the large platform's models, and (10) represents the large platform's profit function. Constraints $p_{EL} \geq w_{EL}$, $p_{EB} \geq w_{EB}$ ensure the non-negativity of the platforms' profits, prompting them to be willing to enter the market.

4. Equilibrium Results

In this section, we present the equilibrium results of the game models for cases T and E. In fact, even if all the objective functions and constraints are linear functions, the leader–follower game model is NP-hard. Evenly, the multi-leader–follower game model is more difficult to solve. We use the reverse solution method. Specifically, at the first stage, we solve the lower-level models, and at the second stage, we substitute them into the upper-level models. Noting that both the lower-level and upper-level models are constrained optimization problems, we solve them based on the theoretical result that the stationary point of a convex optimization model must be its global optimal solution if it satisfies the constraints.

4.1. Equilibrium Results and Analysis for Case T

Referring to [35], we introduce the auxiliary variables $\tau_{ij} = p_{ij} - w_{ij}$ ($i \in \{T, E\}$, $j \in \{B, L\}$) to simplify the model. Further, the model (I) can be transformed into the following model (III):

$$\begin{aligned} \max_{\tau_{Tj} \geq 0} \quad & \pi_{Tj}(\tau_T, w_T) = \tau_{Tj} D_{Tj} \\ \text{s.t.} \quad & \max_{w_T \geq 0} \pi_{TM}(\tau_T, w_T) = w_{TL} D_{TL} + w_{TB} D_{TB} \end{aligned}$$

We first discuss the lower-level model, that is, the manufacturer's model. Moreover, the equilibrium results are shown in Proposition 1.

Proposition 1. *In case T, if $\alpha \in [0, 0.5]$, the manufacturer's model is a convex optimization problem with respect to w_T . Moreover, if $\tau_{TL} \leq \frac{(1-\lambda+\alpha\lambda)d}{1-\alpha^2}$ and $\tau_{TB} \leq \frac{(\lambda+\alpha-\alpha\lambda)d}{1-\alpha^2}$, the manufacturer's optimal wholesale prices for the small and large platform are*

$$w_{TL}^*(\tau_{TL}) = \frac{(1-\lambda+\alpha\lambda)d}{2-2\alpha^2} - \frac{\tau_{TL}}{2}, \quad w_{TB}^*(\tau_{TB}) = \frac{(\lambda+\alpha-\alpha\lambda)d}{2-2\alpha^2} - \frac{\tau_{TB}}{2},$$

respectively.

Proposition 1 reveals that optimal solutions exist for the manufacturer in case T if the auxiliary variables do not exceed some thresholds, which means that the retail price of the two platforms should not be too high. Moreover, the optimal wholesale prices are affected by the retail price of the two platforms. In particular, $w_{TL}^*(\tau_{TL})$ is independent of $w_{TB}^*(\tau_{TB})$, which shows that the manufacturer's wholesale price for the small and large platforms is affected only by its own channel price and not by the other channel price.

By substituting w_{TL}^* and w_{TB}^* into the upper-level models, that is, the platforms' models, we can obtain the equilibrium results, as shown in Proposition 2.

Proposition 2. *In case T, if $\alpha \in [0, 0.5]$ and $\lambda \in [0.5, 1]$, the small and large platforms' models are convex optimization problems with respect to τ_{Tj} . Furthermore, two platforms' optimal retail prices and the manufacturer's optimal wholesale prices are*

$$\begin{aligned} w_{TL}^* &= \frac{(2-2\lambda+3\alpha\lambda+\alpha^2-\alpha^2\lambda)d}{2(1-\alpha^2)(4-\alpha^2)}, \quad w_{TB}^* = \frac{(2\lambda+3\alpha-3\alpha\lambda+\alpha^2\lambda)d}{2(1-\alpha^2)(4-\alpha^2)}, \\ p_{TL}^* &= \frac{(6-6\lambda+5\alpha\lambda-3\alpha^2+3\alpha^2\lambda-2\alpha^3\lambda)d}{2(1-\alpha^2)(4-\alpha^2)}, \quad p_{TB}^* = \frac{(6\lambda+5\alpha-5\alpha\lambda-3\alpha^2\lambda-2\alpha^3+2\alpha^3\lambda)d}{2(1-\alpha^2)(4-\alpha^2)}. \end{aligned}$$

From Proposition 2, it can be found that the convexity and optimal solutions of the two platforms' models do not require conditions, only the basic range of parameters is required. Moreover, substituting the above equilibrium solutions into the profit function of the manufacturer and small and larger platforms, we can obtain the following optimal profits of the two platforms and the manufacturer:

$$\pi_{TL}^* = \frac{(2-2\lambda+\alpha\lambda)^2 d^2}{2(4-\alpha^2)^2}, \quad \pi_{TB}^* = \frac{(2\lambda+\alpha-\alpha\lambda)^2 d^2}{2(4-\alpha^2)^2},$$

$$\pi_{TM}^* = \frac{((2-2\lambda+3\alpha\lambda+\alpha^2-\alpha^2\lambda)(2-2\lambda+\alpha\lambda)+(2\lambda+3\alpha-3\alpha\lambda+\alpha^2\lambda)(2\lambda+\alpha-\alpha\lambda))d^2}{4(1-\alpha^2)(4-\alpha^2)^2},$$

respectively. Through simple calculations, we have the following results.

Proposition 3. *In case T, the sensitivity analysis results for optimal decisions are as follows:*

$$\frac{\partial p_{TL}^*}{\partial \lambda} < 0, \quad \frac{\partial p_{TB}^*}{\partial \lambda} > 0; \quad \frac{\partial w_{TL}^*}{\partial \lambda} < 0, \quad \frac{\partial w_{TB}^*}{\partial \lambda} > 0.$$

Proposition 3 shows that when the small platform does not settle in the large platform, the higher the market share of the large platform, the lower the price of the small platforms, and the higher the price of the large platform. Therefore, when the market share of the large platform is high, the small platform should adopt a low-price strategy to attract consumers, while the large platform can obtain greater profits by raising prices. The manufacturer has different wholesale prices for different platforms, which means that when the market share of the large platform is high, the manufacturer will reduce wholesale prices for the small platform in order to sell more products, but increase the wholesale prices for the large platform to increase profits.

4.2. Equilibrium Results and Analysis for Case E

Similarly to Section 4.1, by introducing the auxiliary variables $\tau_{ij} = p_{ij} - w_{ij}$ ($i \in \{T, E\}$, $j \in \{B, L\}$), the model (II) can be transformed into the following model (IV):

$$\begin{aligned} \max_{\tau_{EL} \geq 0} \quad & \pi_{EL}(\tau_E, w_E) = \tau_{EL}(D_{EL} + D_{BL}) - \delta(w_{EL} + \tau_{EL})D_{BL} - k \\ \text{s.t.} \quad & \max_{w_E \geq 0} \pi_{EM}(\tau_E, w_E) = w_{EL}(D_{EL} + D_{BL}) + w_{EB}D_{EB} \\ \max_{\tau_{EB} \geq 0} \quad & \pi_{EB}(\tau_E, w_E) = \tau_{EB}D_{EB} + \delta(w_{EL} + \tau_{EL})D_{BL} + k \\ \text{s.t.} \quad & \max_{w_E \geq 0} \pi_{EM}(\tau_E, w_E) = w_{EL}(D_{EL} + D_{BL}) + w_{EB}D_{EB} \end{aligned}$$

We first discuss the manufacturer's model, and the equilibrium results are given in Proposition 4.

Proposition 4. *In case E, if $\alpha \in [0, 0.5]$, the manufacturer's model is a convex optimization problem with respect to w_E . Moreover, if $\tau_{EL} \leq \frac{(1-\theta\lambda+2\alpha\theta\lambda)d}{2A}$ and $\tau_{EB} \leq \frac{(\alpha+\theta\lambda-2\alpha\theta\lambda)d}{A}$, the manufacturer's optimal wholesale prices for the small and large platform are*

$$w_{EL}^*(\tau_{EL}) = \frac{(1-\theta\lambda+2\alpha\theta\lambda)d}{4A} - \frac{1}{2}\tau_{EL}, \quad w_{EB}^*(\tau_{EB}) = \frac{(\alpha+\theta\lambda-2\alpha\theta\lambda)d}{2A} - \frac{1}{2}\tau_{EB},$$

respectively, where $A = 1 - \alpha - 2\alpha^2$.

From Proposition 4, we can find that cases T and E have similar results. Specifically, optimal solutions exist for the manufacturer in case E if the auxiliary variables do not exceed some thresholds, which means that the retail price of the two platforms should not be too high. Moreover, the optimal wholesale prices are affected by the retail price of two platforms. In particular, $w_{EL}^*(\tau_{EL})$ is independent of $w_{EB}^*(\tau_{EB})$, which shows that the manufacturer's wholesale price for the small and large platforms is affected only by its own channel price and not by the other channel price.

By substituting w_{EL}^* and w_{EB}^* into the upper-level models, we can find that π_{Ej} is concave in τ_{Ej} and then we can obtain each member's optimal solutions, as summarized in Proposition 5.

Proposition 5. In case E, if $\alpha \in [0, 0.5)$ and $\delta \in [0, 1]$, the small and large platforms' models are convex optimization problems with respect to τ_{Ej} . Furthermore, two platforms' optimal retail prices and the manufacturer's optimal wholesale prices are

$$\begin{aligned} w_{EL}^* &= \frac{M_3 - M_1}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)}, & w_{EB}^* &= \frac{M_4 - M_2}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)}, \\ p_{EL}^* &= \frac{M_1 + M_3}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)}, & p_{EB}^* &= \frac{M_2 + M_4}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)}, \end{aligned}$$

where M_1, M_2, M_3, M_4 are shown in Table A1.

From Proposition 5, it can be found that the convexity and optimal solutions of the two platforms' models do not require conditions, and only the basic range of parameters is required. Since the optimal solution expressions of each member in case E are too complex, for convenience, the more cumbersome expression is replaced by some letters, as shown in Table A1. Moreover, substituting the above equilibrium solutions into the profit function of the manufacturer and small and larger platforms, the optimal profits of the two platforms and the manufacturer are

$$\begin{aligned} \pi_{EL}^* &= \frac{2M_1(D_{EL}^* + D_{BL}^*) - \delta(M_1 + M_3)D_{BL}^*}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)} - k, & \pi_{EB}^* &= \frac{2M_2D_{EB}^* + \delta(M_1 + M_3)D_{BL}^*}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)} + k, \\ \pi_{EM}^* &= \frac{(M_3 - M_1)(D_{EL}^* + D_{BL}^*) + (M_4 - M_2)D_{EB}^*}{4A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)}, \end{aligned}$$

respectively, where

$$\begin{aligned} D_{EB}^* &= \frac{1}{2} \left(\theta\lambda d + \frac{2\alpha M_1 - M_2}{2A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)} \right), \\ D_{EL}^* &= \frac{1}{4} \left((3 + \theta\lambda - 4\lambda)d + \frac{\alpha M_2 - (1-\alpha)M_1}{A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)} \right), \\ D_{BL}^* &= \frac{1}{4} \left((4\lambda - 3\theta\lambda - 1)d + \frac{\alpha M_2 - (1-\alpha)M_1}{A(4-\delta)(8-8\alpha-4\alpha^2-\alpha^2\delta)} \right). \end{aligned}$$

Proposition 6. In case E, the sensitivity analysis results for optimal decisions are as follows:

$$\frac{\partial p_{EL}^*}{\partial \theta} < 0, \frac{\partial p_{EB}^*}{\partial \theta} > 0; \quad \frac{\partial w_{EL}^*}{\partial \theta} < 0, \frac{\partial w_{EB}^*}{\partial \theta} > 0.$$

Proposition 6 shows that when the small platform settles in the large platform, the price of the small platform decreases with the increase in the customer loyalty of the large platform, and the price of the large platform increases with the increase in the customer loyalty of the large platform. The wholesale price given by the manufacturers to the small platform decreases with the increase in the customer loyalty of the large platforms, while the wholesale price given to the large platform increases with the increase in the customer loyalty of the large platform. These results suggest that as the customer loyalty increases on the large platform, the smaller platform should lower prices to grab customers, the large platform can make greater profits by raising prices, and the manufacturer needs to adjust the wholesale prices according to the decisions of each platform.

5. Numerical Analysis

In this section, some numerical experiments will be conducted using MATLAB 9.9.0 to analyze the settlement selection strategy of the small platform and its impact on each supply chain member. Since the analytical expression of the profit function is too complex to obtain theoretical results, this paper employs numerical experiments to reveal management insights. To ensure the quality of the management insights obtained through the numerical experiments, this paper conducts several experiments within the parameter support region. We found that although the specific values of the results are different each time, the trend of the change is consistent. It is worth noting that management insights are obtained from

the change trend rather than the specific value, which indicates that the quality of the management insights in this paper can be maintained.

5.1. Analysis of Settlement Selection Strategy

This part analyzes the settlement selection strategy of the small platform by comparing their profits in two cases: non-settlement (case T) and settlement (case E). Specifically, managerial insights are obtained by analyzing the impact of parameters such as the price competition intensity α , market share λ , commission rate δ , and customer loyalty θ on the settlement selection strategy of the small platform. $d = 100$, $k = 0$ are set in our experiments. Since parameters α , λ exist in both cases, we analyze them by changing the values of δ and θ . In fact, the actual commission rate is usually not more than 30% and then we consider three types of commission rate: low ($\delta = 0.1$), medium ($\delta = 0.2$) and high ($\delta = 0.3$).

5.1.1. Selection of Settlement Strategy at Low Commission Rates ($\delta = 0.1$)

By selecting different values of customer loyalty θ , we can observe the impact of the price competition intensity α and market share λ on the choice of settlement strategy. As shown in Figure 3, the pink part indicates that the settlement mode is the optimal choice, and the blue part indicates that the non-settlement mode is the optimal choice (the same below). Observing the experimental results in Figure 3, it can be seen that if $\theta \in [0, 1]$, $\lambda \in [0.5, 1]$, when $\alpha \in [0, 0.485]$, the small platform chooses the settlement mode; when $\alpha \in (0.485, 0.5)$, with θ increase, when the market share of the large platform is low, the small platform chooses the settlement model, and when the market share of the large platform is high, the small platform chooses the non-settlement model.

5.1.2. Selection of Settlement Strategy at Low Commission Rates ($\delta = 0.2$)

Observing the experimental results in Figure 4, when $\theta \in [0, 1]$, $\lambda \in [0.5, 1]$, the blue area increases. When $\alpha \in [0, 0.46]$, the small platform chooses the settlement mode; when $\alpha \in (0.46, 0.5)$, as θ increases, the small platform chooses the settlement mode when the market share of the large platform is low; and the small platform chooses the non-settlement mode when the market share of the large platform is high.

5.1.3. Selection of Settlement Strategy at Low Commission Rates ($\delta = 0.3$)

Observing the experimental results in Figure 5, when $\theta \in [0, 1]$, $\lambda \in [0.5, 1]$, the blue area increases further. When $\alpha \in [0, 0.40]$, the small platform chooses the settlement mode; when $\alpha \in (0.40, 0.5)$, as θ increases, the small platform chooses the settlement mode when the market share of the large platform is low; and the small platform chooses the non-settlement mode when the market share of the large platform is high.

In summary, when the price competition between the large platform and the small platform is weak or moderate, the small platform can obtain higher profits after settling in, and other factors can be ignored to choose the settlement mode. When the price competition is strong, and as the customer loyalty of the large platform continues to increase, the small platform should pay attention to the market share of the large platform, and choose the settlement mode when the market share of the large platform is low to seize part of the market share, and choose the non-settlement mode when the market share of the large platform is high. The reason for this is that when the customer loyalty and market share of the large platform are high, the original channel market share of the small platform is less, and the new channel after choosing to settle cannot obtain more market share, and the choice of the settlement mode will reduce profits. Therefore, when the customer loyalty and market share of the large platform are high at the same time, it is not conducive to the small platform to settle in.

Through the analysis of the changes in various parameters under different commission rates, it is found that the higher the commission rate, the larger the area (that is, the blue area) that the small platform does not settle in, indicating that the settlement decision of

the small platform will be affected by the commission rate. When the commission rate is low, if the price competition is fierce, small platforms tend not to settle in large platforms. When the commission rate is high, if the price competition is strong, the small platform tends not to settle in the large platform. Therefore, small platforms should enter when the price competition is weak or moderate, or when the degree of competition is strong and the market share of the large platforms is low.

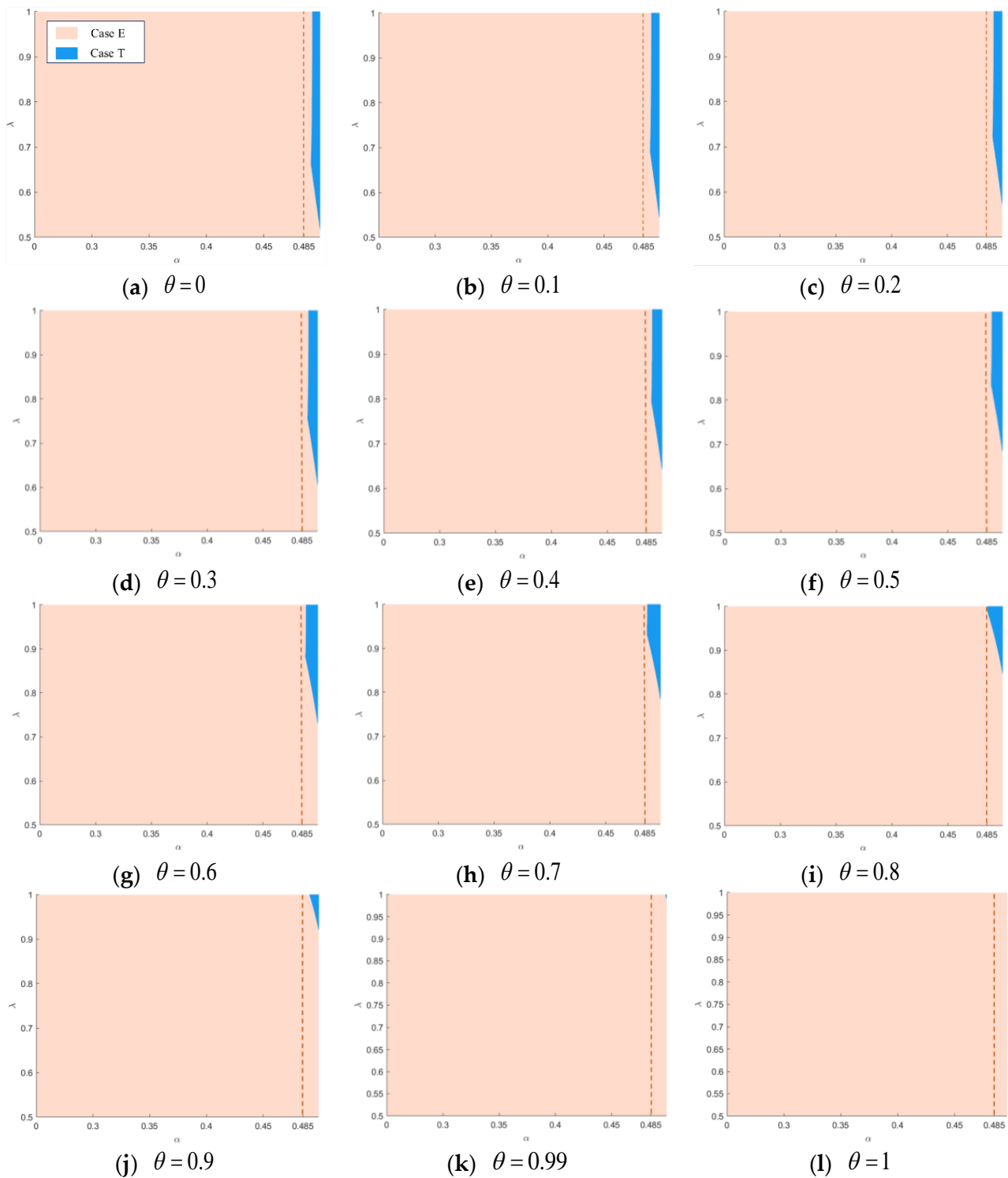


Figure 3. Analysis of settlement selection under low commission rate.

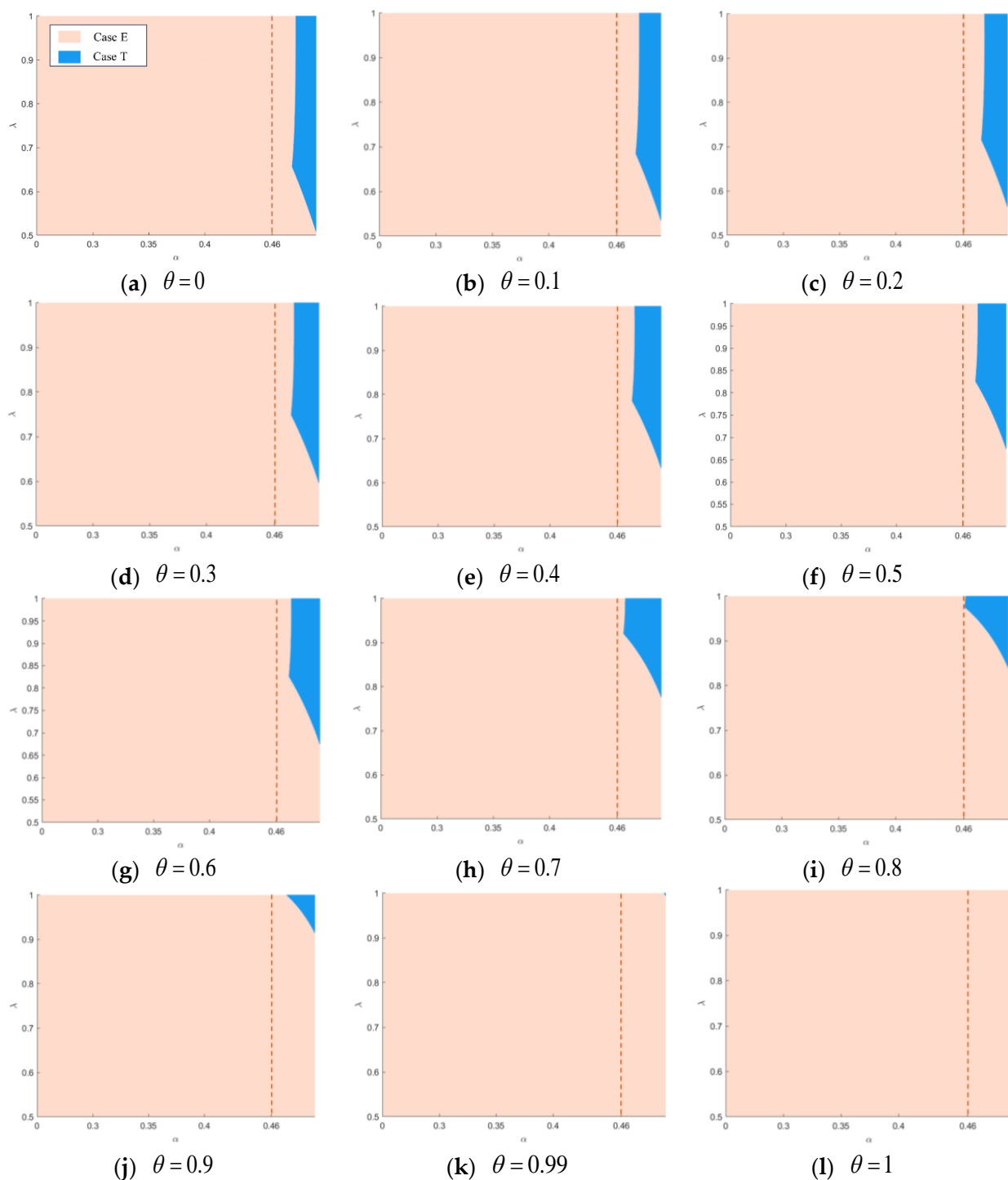


Figure 4. Analysis of settlement selection under medium commission rate.

5.2. Impact of Settlement Strategy on Supply Chain Members

The following analyzes the impact of the settlement strategy of the small platform on each member. First, we set the benchmark parameters as $d = 100$, $k = 80$, $\delta = 0.2$, $\theta = 0.6$, $\lambda = 0.7$, which meet the conditions for the small platform settlement. The experimental results on the price competition intensity are shown in Figure 6. It can be observed that the small platform's profit increases with the intensity of price competition after settling, and the overall profit is higher than that under the non-settlement mode. For the large platform, when the price competition intensity is low, the profit of the large platform in the non-settlement mode is higher than when it is in the settlement mode, and

its profits will increase exponentially as the price competition intensity increases. Finally, the win-win situation of the two platforms can be achieved when the profits after settlement exceed those without settlement. For the manufacturer, as the price competition intensity increases, the manufacturer will obtain a higher profit in the settlement mode, indicating that as the competition between the large and small platforms intensifies, both parties want to purchase more products for sale, and the manufacturer can increase the wholesale prices to earn higher revenues.

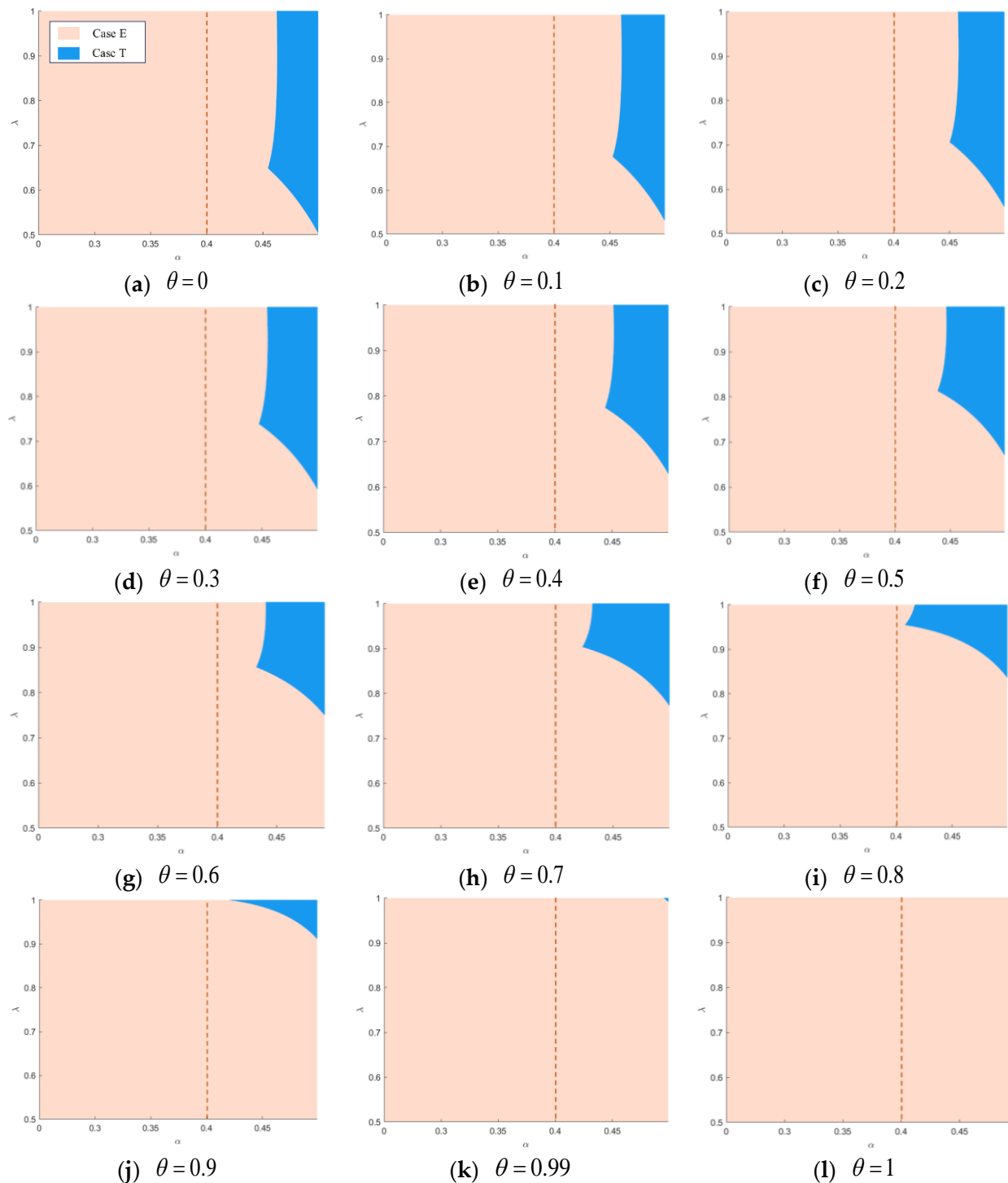


Figure 5. Analysis of settlement selection under high commission rate.

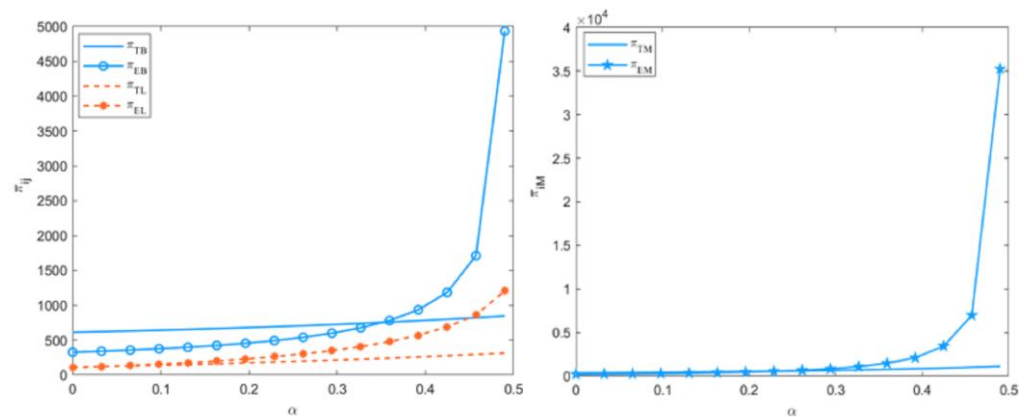


Figure 6. Impact of price competition intensity.

Next, we set the benchmark parameters as $d = 100$, $k = 80$, $\delta = 0.2$, $\theta = 0.6$, $\alpha = 0.36$, and obtain the experimental results regarding market share as shown in Figure 7. It can be observed that the profit of the small platform after settlement has always been higher than that after non-settlement, indicating that in the strong price competition, regardless of the market share of the large platform and the small platform, it will not affect the settlement of the small platform. As for the large platform, when its market share is low or moderate, the large platform's profits will increase when the small platform settles, but there will be a contrast when the market share is extremely high. This shows that when the market share of the large platform is high, the large platform may set high prices, but due to the general customer loyalty of their own products, a large number of consumers will choose the small platform products after settlement. Therefore, when the customer loyalty of the large platform is average and the market share is high, the settlement of the small platform is not conducive to the large platform. As for the manufacturer, when the price competition is strong, regardless of the market share of the large platform, it will obtain a higher revenue than before, and when the market share of the large platform is high, the profit obtained by the manufacturer is also high, which is beneficial for the small platform and the manufacturer, but not for the large platform.

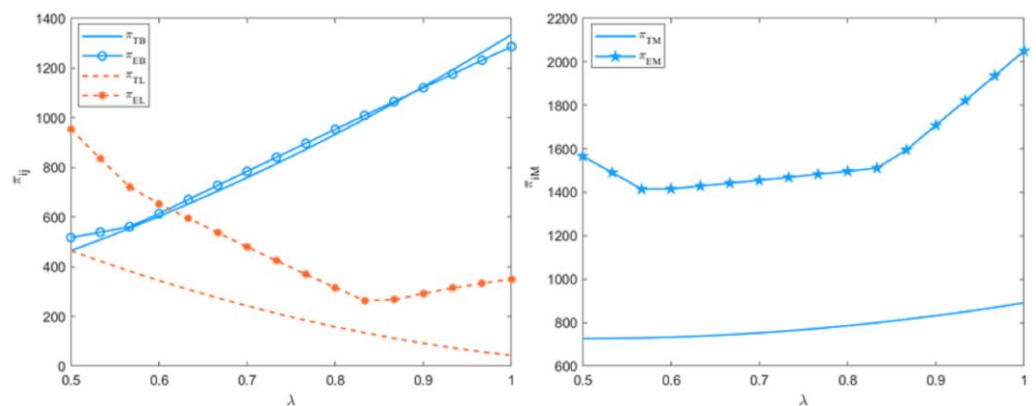


Figure 7. Impact of the market share.

5.3. Impact of Settlement Strategy on Supply Chain Coordination

In this part, we focus on the impact of settlement strategy on supply chain coordination. Consider the influence of α , λ , θ , and δ on each member. We assume that $d = 100$, $k = 80$, the parameter α is selected as 0.1 and 0.4, λ is selected as 0.6 and 0.8, θ is selected as 0.3 and 0.7, and δ is selected as 0.1 and 0.3, respectively. The experimental results are shown in Tables 2 and 3. The results show that for all members, the price competition intensity is a key factor affecting their profits, especially for the manufacturer and the large platform.

When the price competition intensity is weak, the profit of the small platform increases after it enters the market, but the profit of the large platform and the manufacturer is lower than that in the non-settlement mode, which cannot promote supply chain coordination. In addition, the settlement of the small platform under this condition is not conducive to other members. When the price competition is strong, if the loyalty and commission rate of the large platform are low, the settlement of the small platform will reduce the profit of the large platform, which is consistent with the actual situation. Therefore, supply chain coordination cannot be achieved at this time. On the other hand, if the loyalty or commission rate of the large platform is not low, the profit of each member will increase through the settlement of the small platform, achieving a win-win situation.

Table 2. Impact of weak competition intensity ($\alpha = 0.1$).

(θ, δ)	$\lambda = 0.6$				$\lambda = 0.8$			
	$\Delta\pi_L$	$\Delta\pi_B$	$\Delta\pi_M$	$\Delta\pi$	$\Delta\pi_L$	$\Delta\pi_B$	$\Delta\pi_M$	$\Delta\pi$
(0.3, 0.1)	140.15	−296.98	−96.86	-	364.92	−548.99	−78.67	-
(0.3, 0.3)	60.63	−217.78	−125.93	-	179.57	−359.83	−84.74	-
(0.7, 0.1)	23.48	−142.58	−63.86	-	2.08	−293.04	−133.28	-
(0.7, 0.3)	40.56	−138.66	−90.23	-	−33.94	−255.36	−146.84	-

Table 3. Impact of strong competition intensity ($\alpha = 0.4$).

(θ, δ)	$\lambda = 0.6$				$\lambda = 0.8$			
	$\Delta\pi_L$	$\Delta\pi_B$	$\Delta\pi_M$	$\Delta\pi$	$\Delta\pi_L$	$\Delta\pi_B$	$\Delta\pi_M$	$\Delta\pi$
(0.3, 0.1)	566.93	−23.60	1559.90	+	379.62	−123.02	1917.80	+
(0.3, 0.3)	388.02	327.30	1293.60	+	223.78	582.26	1707.20	+
(0.7, 0.1)	401.81	130.91	1670.50	+	284.66	111.03	1671.80	+
(0.7, 0.3)	530.52	237.00	1431.80	+	142.90	417.6515	1425.90	+

where $\Delta\pi_L = \pi_{EL} - \pi_{TL}$, $\Delta\pi_B = \pi_{EB} - \pi_{TB}$, $\Delta\pi_M = \pi_{EM} - \pi_{TM}$, $\Delta\pi = \Delta\pi_L + \Delta\pi_B + \Delta\pi_M$.

In summary, when the price competition is weak, the small platform settling in the large platform cannot promote supply chain coordination. When the price competition is strong, supply chain coordination can be achieved unless both the customer loyalty and commission rates are low, which supports the view presented in the literature [3].

5.4. Discussion

This paper considers two cases: self-operated platforms settling in hybrid platforms and self-operated platforms not settling in hybrid platforms. Moreover, through numerical experiments, we explore the conditions under which self-operated platforms settle in hybrid platforms and the impact of the settlement strategy of self-operated platforms on supply chain members and supply chain coordination.

Self-operated platforms settling in hybrid platforms are able to expand their sales channels, but this is not always profitable. First, lower commission rates are favorable for self-operated platforms settling in hybrid platforms. This is because with lower commission rates, hybrid platforms have a lower drawdown percentage, which favors self-operated platforms. Moreover, this result is also consistent with intuition and reality. Second, when commission rates are fixed and price competition is weak or moderate, self-operated platforms should settle in hybrid platforms. In particular, factors such as the customer loyalty and market share of the hybrid platforms can be ignored, which may be due to the fact that the intensity of price competition is not high and the market environment is more friendly. This is favorable for self-operated platforms to expand their sales channels. Third, when commission rates are fixed but price competition is intense, the customer loyalty and market share of hybrid platforms are the main factors affecting whether self-operated platforms should settle in hybrid platforms. Specifically, when the customer loyalty and market share of hybrid platforms are large, the self-operated platform should not settle in

hybrid platforms. This is because the settlement strategy of self-operated platforms does not increase the market share and revenue, but instead results in the loss of revenue due to the need to pay commission fees. When hybrid platforms have high customer loyalty and low market share, self-operated platforms can settle in hybrid platforms and thus be able to expand part of the market. In reality, the market share of hybrid platforms is usually high, so self-operated platforms should know the customer loyalty of hybrid platforms in advance before determining the settlement strategy.

The settlement strategy of self-operated platforms has a significant impact on self-operated platforms, hybrid platforms, and manufacturers. For simplicity, we denote the profit margin by $\Delta\pi = \pi_E - \pi_T$. For self-operated platforms, as the intensity of price competition increases, the settlement strategy leads to an increase in the profit margin; as the market share of the hybrid platforms increases, the settlement strategy leads to a decrease and then an increase in the profit margin. We find that although self-operated platforms are more suitable for settling when price competition is weak or moderate, appropriate price competition after self-operated platform settling instead promotes higher returns for self-operated platforms. For hybrid platforms, when the intensity of price competition is low, the settlement strategy of self-operated platforms may impair the profitability of hybrid platforms. This is due to the fact that the advantages of hybrid platforms cannot be realized and the market share can be easily captured by self-operated platforms. For manufacturers, self-operated platforms' settlement strategy will always increase their revenue. This is because the settlement of self-operated platforms expands sales channels and facilitates cooperation between self-operated platforms and hybrid platforms, which increases sales and, hence, manufacturers' profits.

Interestingly, the settlement strategy of self-operated platforms can facilitate supply chain coordination when the intensity of competition is high. This explains well the social phenomenon in which an appropriate intensity of market competition can promote the better development of firms and achieve a win-win situation. This may be due to the fact that when there is competition in the market, firms are less likely to set higher selling prices, consumer demand increases, and thus the benefits to all members of the supply chain increase, and supply chain coordination is realized.

6. Conclusions

This paper considers the strategic selection of self-operated platforms to settle in hybrid platforms under competition. We construct a multi-leader-follower game involving two e-commerce platforms and one manufacturer. In order to ensure the optimal solution, the convexity conditions of each mode are discussed, and the optimal strategy of each member is obtained on this basis. Furthermore, the impact of the settlement strategy of the small platform on each member is analyzed through numerical examples. Our key results are as follows.

First, we obtain the conditions for self-operated platforms to settle in hybrid platforms. When the price competition intensity is weak or moderate, the small platform should settle in the large platform. When the price competition is fierce, as the customers' loyalty to the large platform increases, the small platform should settle in when the market share of the large platform is low, and not when their market share is high. In addition, the settlement strategy of the small platform will be affected by the commission rate, and when other factors remain unchanged, the higher the commission rate, the more inclined the small platform is not to settle in, so the small platform will be more willing to sell products with a lower commission rate on the large platform. Second, through the analysis of the profits of each member, we find that the settlement strategy of self-operated platforms is detrimental to hybrid platforms when the competitive intensity is weak, but it is always favorable to manufacturers. Third, it can be seen that after the small platform adopts the settlement strategy, with the increase in the price competition intensity, the profits of the large platform and the manufacturer increase exponentially, so a win-win situation can be achieved for all parties. The strong price competition is conducive to promoting supply chain coordination.

In addition, some useful recommendations can be summarized for supply chain members. For hybrid platforms, opening up to third-party entry is beneficial in most cases and can lead to win–win cooperation. However, when the intensity of competition in the market is low, the settlement of self-operated platforms may jeopardize the revenue of hybrid platforms. Therefore, hybrid platforms should examine the market environment before deciding whether to develop third-party entry. For self-operated platforms, market research should be conducted before deciding whether to settle in hybrid platforms, as settling in hybrid platforms may expand sales channels but may still result in a loss of profits. For manufacturers, due to the increased market competition after self-operated platforms settle in hybrid platforms, both platforms hope to purchase more products to sell. At this time, manufacturers can appropriately increase the wholesale price to increase revenue.

Although our research has achieved some meaningful results, there are still some directions that deserve further exploration. First, this article considers the cases of a single hybrid platform, a single self-operated platform, and a single manufacturer. In future research, we can consider the cases of multiple hybrid platforms, multiple self-operated platforms, and multiple manufacturers that are more in line with realistic scenarios. Second, this paper does not consider the behavioral preferences of supply chain members. Risk preferences, quality preferences, etc. can be considered in the future. Finally, we consider linear demand functions; however, nonlinear demand is more realistic, which can be used as a future research direction.

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Appendix A

Table A1 shows the mathematical expressions of some key notations.

Table A1. Notations for Proposition 5.

Letter	Notation
L	$4(1 - \theta\lambda) - \delta(4\lambda - 3\theta\lambda - 1)$
M_1	$4Ad(L + \alpha\theta\lambda(4 - \delta)) + \delta d(1 - \theta\lambda + 2\alpha\theta\lambda)(\alpha^2(4 - \delta) + 4(1 - \alpha))$
M_2	$Ad(\alpha L(4 + \delta) + 8\theta\lambda(1 - \alpha)(4 - \delta)) + \alpha\delta d(1 - \alpha)(12 - \delta)(1 - \theta\lambda + 2\alpha\theta\lambda)$
M_3	$d(4 - \delta)(1 - \theta\lambda + 2\alpha\theta\lambda)(8 - 8\alpha - 4\alpha^2 - \alpha^2\delta)$
M_4	$2d(4 - \delta)(\alpha + \theta\lambda - 2\alpha\theta\lambda)(8 - 8\alpha - 4\alpha^2 - \alpha^2\delta)$
$M_3 - M_1$	$2d\theta\lambda(-8(1 - 4\alpha + 4\alpha^2) - 2\alpha\delta(5 - 12\alpha + 4\alpha^2) - \alpha^2\delta^2(1 - 2\alpha)) +$ $2d(8\delta\lambda A + 8(1 - \alpha + \alpha^2) - 2\delta(4 - 4\alpha - \alpha^2) + \alpha^2\delta^2)$
$M_1 + M_3$	$4\theta\lambda d(4(-3 + 8\alpha - 2\alpha^2 - 4\alpha^3) + \delta(4 - 7\alpha - 4\alpha^2 + 4\alpha^3)) - 16\delta\lambda dA +$ $4d(12(1 - \alpha - \alpha^2) - \alpha^2\delta)$
$M_4 - M_2$	$4\theta\lambda d(1 - 2\alpha)(4(2 - 3\alpha + \alpha^2) - \delta(2 - 5\alpha + 7\alpha^2) - \alpha\delta^2) + 4\alpha\delta\lambda dA(4 + \delta) +$ $4\alpha d(\alpha^2\delta^2 - \delta(9 - 9\alpha - 4\alpha^2) + 12 - 12\alpha)$
$M_2 + M_4$	$4\theta\lambda d(1 - 2\alpha)(4(6 - 5\alpha - 5\alpha^2) + \delta(-6 + 3\alpha + 7\alpha^2) + \alpha\delta^2(1 + \alpha)) - 4\alpha\delta\lambda dA(4 + \delta) -$ $4\alpha d(4(-5 + 5\alpha + 4\alpha^2) + \delta(-1 + \alpha + 4\alpha^2))$

Next, we present the proofs of all the propositions in the main paper.

Proof of Proposition 1. The Hessian matrix of π_{TM} with respect to w_T is

$$H = \begin{bmatrix} -2 & 2\alpha \\ 2\alpha & -2 \end{bmatrix}.$$

This matrix is negative semidefinite when $\alpha \in [0, 0.5]$ and, hence, the lower-level objective function π_{TM} is concave in w_T . Since the lower-level constraints are linear, the lower-level model is a convex optimization problem, which means that the stationary points of its objective function must be globally optimal solutions as long as the constraints are satisfied. By solving $\nabla_{w_T} \pi_{TM} = 0$, we obtain a unique stationary point:

$$w_{TL}^*(\tau_{TL}) = \frac{(1 - \lambda + \alpha\lambda)d}{2 - 2\alpha^2} - \frac{\tau_{TL}}{2}, \quad w_{TB}^*(\tau_{TB}) = \frac{(\lambda + \alpha - \alpha\lambda)d}{2 - 2\alpha^2} - \frac{\tau_{TB}}{2}.$$

To ensure the non-negative conditions $w_{TL}^* \geq 0$ and $w_{TB}^* \geq 0$, it is sufficient to satisfy

$$\tau_{TL} \leq \frac{(1 - \lambda + \alpha\lambda)d}{1 - \alpha^2}, \quad \tau_{TB} \leq \frac{(\lambda + \alpha - \alpha\lambda)d}{1 - \alpha^2}$$

This completes the proof. \square

Proof of Proposition 2. By substituting $w_{TL}^*(\tau_{TL})$ and $w_{TB}^*(\tau_{TB})$ into the upper-level model of (III), we have the following model:

$$\begin{aligned} \max_{\tau_{TL}} \pi_{TL}(\tau_{TL}, w_T^*(\tau_{TL})) &= \tau_{TL} D_{TL}^* \\ \text{s.t. } 0 &\leq \tau_{TL} \leq \frac{(1 - \lambda + \alpha\lambda)d}{1 - \alpha^2} \\ \max_{\tau_{TB}} \pi_{TB}(\tau_{TB}, w_T^*(\tau_{TB})) &= \tau_{TB} D_{TB}^* \\ \text{s.t. } 0 &\leq \tau_{TB} \leq \frac{(\lambda + \alpha - \alpha\lambda)d}{1 - \alpha^2} \end{aligned}$$

The objective function π_{Tj} is the second derivative with τ_{Tj} respect to its variables $\pi''_{Tj}(\tau_{Tj}) = -1 < 0$. Therefore, π_{Tj} is concave in τ_{Tj} . Since the constraints in both the small and the large platforms' models are linear, these two models are both convex optimization problems. Letting $\nabla \pi_{TL} = 0$, $\nabla \pi_{TB} = 0$, we obtain their stationary points:

$$\tau_{TL}^* = \frac{(2 - 2\lambda + \alpha\lambda)d}{4 - \alpha^2}, \quad \tau_{TB}^* = \frac{(2\lambda + \alpha - \alpha\lambda)d}{4 - \alpha^2}.$$

When $\alpha \in [0, 0.5]$, $\lambda \in [0.5, 1]$, the conditions

$$0 \leq \tau_{TL}^* \leq \frac{(1 - \lambda + \alpha\lambda)d}{1 - \alpha^2}, \quad 0 \leq \tau_{TB}^* \leq \frac{(\lambda + \alpha - \alpha\lambda)d}{1 - \alpha^2}$$

can ensure their feasibility and so they are all globally optimal solutions. Substituting τ_{Tj}^* into $w_{TL}^*(\tau_{TL})$, $w_{TB}^*(\tau_{TB})$, we can obtain the optimal wholesale prices and, furthermore, we can obtain p_{TL}^* , p_{TB}^* by $p_{Tj} = w_{Tj} + \tau_{Tj}$. This completes the proof. \square

Proof of Proposition 4. The Hessian matrix of π_{EM} with respect to w_E is

$$H = \begin{bmatrix} 4\alpha - 4 & 4\alpha \\ 4\alpha & -2 \end{bmatrix}.$$

This matrix is negative semidefinite when $\alpha \in [0, 0.5]$ and, hence, the lower-level objective function π_{EM} is concave in w_E . Since the lower-level constraints are linear, the

lower-level model is a convex optimization problem, which means that the stationary points of its objective function must be globally optimal solutions as long as the constraints are satisfied. By solving $\nabla_{w_E} \pi_{EM} = 0$, we obtain a unique stationary point:

$$w_{EL}^*(\tau_{EL}) = \frac{(1 - \theta\lambda + 2\alpha\theta\lambda)d}{4A} - \frac{1}{2}\tau_{EL}, \quad w_{EB}^*(\tau_{EB}) = \frac{(\alpha + \theta\lambda - 2\alpha\theta\lambda)d}{2A} - \frac{1}{2}\tau_{EB}.$$

To ensure the non-negative conditions $w_{EL}^* \geq 0$ and $w_{EB}^* \geq 0$, it is sufficient to satisfy

$$\tau_{EL} \leq \frac{(1 - \theta\lambda + 2\alpha\theta\lambda)d}{2A}, \quad \tau_{EB} \leq \frac{(\alpha + \theta\lambda - 2\alpha\theta\lambda)d}{A}$$

This completes the proof. \square

Proof of Proposition 5. By substituting $w_{EL}^*(\tau_{EL})$ and $w_{EB}^*(\tau_{EB})$ into the upper-level model of (IV), we have the following model:

$$\begin{aligned} \max_{\tau_{EL}} \pi_{EL}(\tau_E, w_E^*(\tau_E)) &= \tau_{EL}(D_{EL}^* + D_{BL}^*) - \delta(w_{EL}^* + \tau_{EL})D_{BL}^* - k \\ \text{s.t. } 0 &\leq \tau_{EL} \leq \frac{(1 - \theta\lambda + 2\alpha\theta\lambda)d}{2A} \\ \max_{\tau_{EB}} \pi_{EB}(\tau_E, w_E^*(\tau_E)) &= \tau_{EB}D_{EB}^* + \delta(w_{EL}^* + \tau_{EL})D_{BL}^* + k \\ \text{s.t. } 0 &\leq \tau_{EB} \leq \frac{(\alpha + \theta\lambda - 2\alpha\theta\lambda)d}{A} \end{aligned}$$

When $\alpha \in [0, 0.5)$, $\delta \in [0, 1]$, π_{EL} is concave in τ_{EL} , π_{EB} is concave in τ_{EB} . Since the constraints in both the small and the large platforms' models are linear, these two models are both convex optimization problems. Letting $\pi'_{EL}(\tau_{EL}) = 0$, $\pi'_{EB}(\tau_{EB}) = 0$, we obtain their stationary points:

$$\tau_{EL}^* = \frac{M_1}{2A(4 - \delta)(8 - 8\alpha - 4\alpha^2 - \alpha^2\delta)}, \quad \tau_{EB}^* = \frac{M_2}{2A(4 - \delta)(8 - 8\alpha - 4\alpha^2 - \alpha^2\delta)},$$

where M_1 , M_2 can be seen in Table A1.

To ensure the conditions $0 \leq \tau_{EL}^* \leq \frac{(1 - \theta\lambda + 2\alpha\theta\lambda)d}{2A}$ and $0 \leq \tau_{EB}^* \leq \frac{(\alpha + \theta\lambda - 2\alpha\theta\lambda)d}{A}$, it is sufficient to satisfy $M_3 - M_1 \geq 0$, $M_4 - M_2 \geq 0$, where M_3 , M_4 can be seen in Table A1. Letting $Z(\theta, \delta, \alpha, \lambda) = \frac{M_3 - M_1}{2d}$, we can proof $\frac{\partial Z}{\partial \theta} \leq 0$ easily. $Z(\theta, \delta, \alpha, \lambda)$ decreases with $\theta \in [0, 1]$; therefore, $Z(\theta, \delta, \alpha, \lambda) \geq Z(1, \delta, \alpha, \lambda)$. We can proof that $Z(1, \delta, \alpha, \lambda)$ decreases with $\delta \in [0, 1]$, and we can obtain

$$Z(\theta, \delta, \alpha, \lambda) \geq Z(1, 1, \alpha, \lambda) = 22\alpha^2 + 2\alpha\lambda(14 - 25\alpha - 6\alpha^2).$$

When $\alpha \in [0, 0.4375]$, $Z(1, 1, \alpha, \lambda)$ increases with $\lambda \in [0.5, 1]$, so

$$Z(1, 1, \alpha, \lambda) \geq Z(1, 1, \alpha, 0.5) = \alpha(14 - 3\alpha - 6\alpha^2) \geq 0.$$

When $\alpha \in (0.4375, 0.5)$, $Z(1, 1, \alpha, \lambda)$ decreases with $\lambda \in [0.5, 1]$, so

$$Z(1, 1, \alpha, \lambda) \geq Z(1, 1, \alpha, 1) = 4\alpha(7 - 7\alpha - 3\alpha^2) \geq 0.$$

Therefore, when $\alpha \in [0, 0.5)$, $\delta \in [0, 1]$, $\theta \in [0, 1]$, $\lambda \in [0.5, 1]$, we can obtain $Z(\theta, \delta, \alpha, \lambda) \geq 0$, which is $M_3 - M_1 \geq 0$. We can obtain $M_4 - M_2 \geq 0$ in the same way. Thus, they are all globally optimal solutions. Substituting τ_{EL}^* , τ_{EB}^* into $w_{EL}^*(\tau_{EL})$, $w_{EB}^*(\tau_{EB})$, we can obtain the optimal wholesale prices and, furthermore, we can obtain p_{EL}^* , p_{EB}^* by $p_{Ej} = w_{Ej} + \tau_{Ej}$. This completes the proof. \square

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