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Designing the Calibration Process of Weigh-In-Motion Systems

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Abstract: Weigh-In-Motion (WIM) systems provide information on the state of road traffic and are used in activities undertaken as part of traffic supervision and management, enforcement of applicable regulations, and in the design of road infrastructure. The further development of such systems is aimed at increasing their measurement accuracy, operational reliability, and their resistance to disturbing environmental factors. Increasing the accuracy of measurement can be achieved both through actions taken in the hardware layer (technology of load sensors, the number of sensors and their arrangement, technology used in the construction of the pavement, selection of the system location), as well as by implementing better system calibration algorithms and algorithms for pre-processing measurement data. In this paper, we focus on the issue of WIM system calibration. We believe that through the correct selection of the calibration algorithm, it is possible to significantly increase the accuracy of vehicle weighing in WIM systems, from a practical point of view. The simulation and experimental studies we conducted confirmed this hypothesis.

Keywords: WIM systems; system calibration; accuracy of measurement; load sensors; algorithms; measurement data

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1. Introduction

Weigh-in-Motion (WIM) systems have been known for over 50 years and have been widely used for over twenty years in the monitoring of the weight and axle load of motor vehicles [1,2]. Such systems make it possible to measure the gross vehicle weight and the static load of the individual axles of the vehicle traveling at normal road speed. For many years, they were used mainly in the road traffic monitoring process, as a source of valuable information on traffic volume, development of road transport, weight of transported loads, vehicle class (e.g., silhouette, axle spacing, vehicle length, etc.), and the structure of traffic flow (traffic lane, direction, transverse position, etc.). This information was mainly used in the process of road design, traffic management, and road infrastructure renovation planning. At the same time, WIM systems were used in the protection of the bridge infrastructure, in providing information on the weight and axle load of vehicles, and served as pre-selection systems, contributing to the improvement of the efficiency of the vehicle weight control process carried out by specialized services [3–5]. In recent years, in Europe and in other countries around the world, work has been undertaken to apply WIM systems in direct mass enforcement. Such systems have been called virtual WIM systems or e-WIM [6–9].

Both in the case of pre-selection WIM systems and e-WIM systems, the main problems are the accuracy of weighing results and the stability of this accuracy in changing operating conditions. These are particularly important in the case of e-WIM systems, as administrative actions are taken on the basis of the measurement results obtained from them. Currently, both national and international standards are being developed, specifying the conditions for both the approval of e-WIM systems for use and their subsequent operation [1,8,10–13]. In order to ensure the required accuracy and to ensure the maintenance of this level of accuracy, WIM systems are periodically calibrated [14,15]. The various methods of calibrating WIM systems are briefly discussed in Section 2. In our paper, we use the term calibration in the strict sense. Calibration is a comparison between a known measurement (the standard) and the measurement using the calibrated device (instrument). The accuracy of the standard should be many times greater than the accuracy of the calibrated device (measuring instrument), e.g., 10 times. In practice, this requirement is milder and is limited to the level 3–5 [16,17]. The need to have a standard of the measured quantity with a high accuracy is one of the reasons there are difficulties with the calibration of WIM systems. If we assume that the calibrated WIM system should measure the mass of the vehicle with an error of no more than 5%, it is necessary to have a mass standard with an accuracy of 0.5% and a value of several tons. The inability to measure constant force in many WIM systems makes this task more difficult. The second value measured in WIM systems is the static axle load. However, there is no load standard. It can be seen that the calibration of WIM systems is not a trivial task.

In this paper we present the results of research on the calibration process of WIM systems carried out using the method of pre-weighed vehicles. We called these vehicles test vehicles. We conducted the research using both simulation methods and the results of experiments carried out in real road traffic. All simulations were performed in the MATLAB R2021b simulation environment by MathWorks. In order to conduct experimental research, we built a multi-sensor WIM system which was installed in a lane of National Road no. 81. The subject of the research was the impact of various parameters of the calibration process, such as the number of test vehicles used, the distribution of vehicle weight in the measuring range of the WIM system, the number of runs of each test vehicle through the calibrated test bench, and the form of the algorithm used to estimate the parameters of the static characteristics of the WIM system being calibrated. The quality of the WIM system was assessed on the basis of the bias error value and the random variability of vehicle weighing results. The obtained test results and the conclusions formulated on their basis make it possible to design the calibration process of a WIM system from the point of view of its intended use and the required accuracy of this system. Improving the accuracy of the measurement system can be achieved both by actions taken in the hardware layer as well as in the system software (control, processing of measurement results). In this paper we showed that in relation to such specific measurement systems as WIM systems it is possible, from a practical point of view, to improve the accuracy of weighing through the correct selection and implementation of the system calibration algorithm. We considered four algorithms that differ in their statistical properties. For each of these algorithms, we indicated the quantitative influence of the algorithm parameters on the weighing errors. We presented recommendations regarding the use of individual algorithms depending on the purpose of the WIM system. We have shown on the basis of simulation and experimental tests that the correct selection of the calibration algorithm allows a reduction of the vehicle weighing error at least twice.

This problem was initially discussed in our earlier work [18]. In that paper, the subject of the study was only the impact of the quality of the road surface in which load sensors were installed on the weighing accuracy of the WIM system. Two WIM systems were compared, namely a system equipped with two load sensors and a system equipped with 16 such sensors.

This paper is organized as follows: in Section 2 we describe the known methods of calibrating WIM systems, Section 3 contains an overview of the algorithms for estimating the coefficients of the static characteristics of WIM systems (these algorithms differ in statistical properties), in Section 4 we discuss the simulation tests and present their results, Section 5 contains a discussion of the results of experimental research, and finally conclusions regarding the design of the calibration process of the WIM system are presented in Section 6.

2. Calibration Methods for WIM Systems

The basic methods of calibration of WIM systems that are used very often in practice include:

- calibration with standard weights, which involves loading the installed sensor with a constant and known force. This is accomplished by putting a reference weight on the sensor, thus ensuring the repeatability of the obtained results. However, it allows only the calibration of the sensor itself, and the calibration coefficients determined in this way do not take into account the effects caused by environmental conditions, road surface condition, vehicle-surface interaction, etc. [15,19,20]. The advantage of this method is the high accuracy of the applied standards. The disadvantage is of limiting its use only to load sensors that respond to static load;
- calibration with pre-weighed vehicles (test vehicles) that use vehicles with known axle configurations, gross weight and axle load values determined under static conditions. This method involves the use of multiple runs through the calibrated weighing station with several vehicles, with different gross vehicle weights which fall within the measuring range of the WIM system, various types of suspension and at a strictly defined speed [10]. During each run of the test vehicle through the calibrated WIM stand its speed and the static load of each axle are measured. The accurate values for the gross vehicle weight and axle load were previously measured on a platform scale (static measurement) and on a low-speed scale (vehicle speed limited to 5 km/h). The low-speed scale allows you to measure the load of individual axles. Axle load measurement results are corrected in such a way that the sum of the loads of all axles is equal to the gross weight of the vehicle, determined on the platform scale. The vehicle weight value and axle load values determined in this way are treated as standards. The results obtained during the calibration experiments on the WIM stand and the standard values are then processed in accordance with the calibration algorithm. The result of the processing are the sought values of the calibration coefficients. In Section 3, we presented selected calibration algorithms that were the subject of research in our paper. The method does not require specialized equipment. However, it is very time consuming and therefore expensive;
- calibration with specific devices, which include the use of a so-called instrumented vehicle designed to calibrate WIM systems, whose task is to measure the dynamic axle load of vehicles. This method enables the calibration of the weighing station with a small number of tests [21]. The method is very effective. Theoretically, only one run of the instrumented vehicle through the WIM station is sufficient to perform the calibration. The method is especially useful when calibrating multi-sensor WIM systems. Its disadvantage, however, is the need to have specialized and expensive equipment;
- auto-calibration (automatic self-calibration), which consists in continuous estimation of the WIM system calibration coefficient and modifying the weighing results in accordance with the currently determined value of this coefficient. This method is well suited for the calibration of WIM systems in which the weighing result is significantly non-stationary due to the strong influence of external factors, e.g., environmental factors. It is assumed that in the stream of weighed vehicles one can distinguish those whose selected quantities (e.g., gross vehicle weight or axle load) can be considered as a reference quantity [22–24]. This is a disadvantage of this method.

The choice of the calibration method depends primarily on the technology of load sensors used in the WIM system. The research presented in this paper concerns the most commonly used method of calibrating WIM systems, namely the method using pre-weighed vehicles. During the calibration carried out by this method, vehicles with a known weight and axle configuration (test vehicles) are used, repeatedly passing through the calibrated weighing site. A second variant of this method exists which uses random vehicles selected from the stream of vehicles passing through the calibrated WIM site once. These vehicles, after being weighed on a calibrated WIM system, are then weighed on a static scale, which should be located near the location of the WIM system. On this basis, the average difference between the weighing results from the WIM system and the results of static weighing is determined. The parameters of the WIM system (slope and zero shift of the static characteristic) are adjusted in such a way as to bring this difference as close to zero as possible.

The parameters of the calibration procedure with the use of pre-weighed vehicles include the number of used test vehicles, the number of runs of each of these vehicles, the GVW (Gross Vehicle Weight) of each test vehicle in relation to the measuring range of the WIM system, and the algorithm according to which the results of weighing test vehicles are processed. The purpose of this processing is to calculate the estimates of the parameters of the static characteristics of the calibrated WIM system (slope and zero shift), i.e., calibration coefficients. These coefficients are determined as mathematical formulas which allow for the calculation of values measured directly in the WIM system (dynamic axle load) into the desired values, namely the static axle load, and after further processing, into GVW.

3. Algorithms for Estimation of Calibration Factors Using the Pre-Weighed Vehicles Method

Assuming the linear model of the WIM system, the processing equation (static characteristics) of the WIM system, generally speaking, has the form (1).

$$D_{i,k} = \frac{1}{C_{ref}} \cdot M_k + b_{ref} + \varepsilon_i \quad (1)$$

where:

M_k actual mass of the k-th test vehicle or static load of a selected axle of this vehicle, determined as a result of weighing on a static scale (reference value),

ε_i additive random disturbance in the i-th weighing result,

$D_{i,k}$ direct result of the load signal processing from sensors of the WIM system (result before system calibration),

k test vehicle number,

I test vehicle run number,

$C_{ref},\ b_{ref}$ unknown coefficients of the static characteristics of the WIM system.

The values of the coefficients $C_{ref},\ b_{ref}$ are unknown. Due to random disturbances occurring during weighing, it is only possible to determine estimates of these coefficients. The model (2) is used for this purpose,

$$D_{i,k} = \frac{1}{C} \cdot M_k + b \quad (2)$$

whose coefficients C and b determined in the calibration process are treated as estimates of the unknown coefficients of the static characteristics of the WIM system.

The purpose of WIM system calibration is to experimentally determine the calibration coefficients C and b. These factors should be used to scale the results obtained from the measurement system (D) to obtain an estimate of the static axle load of the vehicle on the ground or its gross vehicle weight (M).

In practice, we often use the simplified model (2). The simplification of the model is based on the assumption that the static characteristics of the calibrated WIM system are not shifted against zero ($b_{ref} = 0$). If only one calibration (test) vehicle is used, such a simplification is necessary.

Both calibration coefficients C and b are selected by the optimization method in such a way that the weighing results M have specific metrological properties, e.g., unbiased, minimum variance, or simultaneous unbiased and minimum variance, which leads to minimization of the mean square error.

Depending on the adopted optimization criterion, we obtain various estimators of the coefficients $C_{ref},\ b_{ref}$, i.e., various algorithms for processing the measurement results obtained during calibration in order to determine the values of the coefficients C and b [2,18].

Below we present the estimates obtained as a result of minimization of errors in the weighing results, performed both for the special case ($b_{ref} = 0$) and for the general case ($b_{ref} \neq 0$). The minimized errors are:

- mean square error for the specific case ($b_{ref} = 0$);
- bias error;
- mean square error for the general case ($b_{ref} \neq 0$).

The basic measure of the accuracy of the measurement results is their variance, bias and mean square error. Variance describes the random variation in measurement results around the mean value, observed over a series of measurements. Variance can be minimized by increasing the number of measurements and averaging their results. The bias error describes the constant offset between the mean value from the series of measurements and the actual value of the measured quantity. Minimizing the bias generally requires the use of a different measurement method, different tools or a data processing algorithm. The bias error can take both positive and negative values. The mean square error is the sum of the variance and the square of the bias error. It is always positive. Therefore, two of the above-mentioned errors are sufficient to fully describe the accuracy of the measurement. In our paper, we chose the mean square error and the bias.

3.1. Minimizing the Mean Square Error of the Weighing Results for the Special Case ($b_{ref} = 0$)

Mean square error of dynamic weighing results $D_{i,k}$ for the k-th test vehicle, assuming $b_{ref} = 0$ is described by the relationship (3).

$$\Psi_k^2 = \frac{1}{n_k} \cdot \sum_{i=1}^{n_k} [D_{i,k} - p \cdot M_k]^2 \tag{3}$$

where:

n_k for $k = 1, 2, \ldots, N_{test}$—number of passes of the k-th test vehicle,
$p = \frac{1}{C}$—reciprocal of the calibration factor,
N_{test}—number of test vehicles.

The cumulative mean square error that takes into account the weighing results of all N_{test} test vehicles used in the calibration process is of the form (4).

$$\Psi^2 = \frac{1}{N_{test}} \cdot \sum_{k=1}^{N_{test}} \Psi_k^2 = \frac{1}{N_{test} \cdot n_k} \sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} [D_{i,k} - p \cdot M_k]^2 \tag{4}$$

The minimization of the mean square error (4) due to the coefficient p leads to the estimation algorithm in the form (5). The coefficient $C1$ calculated in this way is the estimation of the desired coefficient C_{ref} [18,25].

$$C1 = C = \frac{1}{p} = \frac{\sum_{k=1}^{N_{test}} n_k \cdot (M_k)^2}{\sum_{k=1}^{N_{test}} \left(M_k \sum_{i=1}^{n_k} D_{i,k}\right)} \tag{5}$$

3.2. Minimizing the Bias of Weighing Results

As the expected value of the weight estimator of the k–th test vehicle, the mean value $\underline{M}_k$, calculated from the set of weighing results obtained in subsequent runs (6), can be assumed:

$$\underline{M}_k = \frac{1}{n_k} \cdot \sum_{i=1}^{n_k} C \cdot D_{i,k} \tag{6}$$

Hence, the estimate of the bias of the weighing results of the k-th test vehicle is described by the Equation (7).

$$bias_k = \frac{1}{n_k} \cdot \sum_{i=1}^{n_k} C \cdot D_{i,k} - M_k \tag{7}$$

Taking into account that the number of test vehicles used in the calibration process is equal to N_{test}, the total bias error of the weighing results of all these vehicles is the sum of the errors obtained for all vehicles and amounts to (8):

$$bias = \sum_{k=1}^{N_{test}} n_k \cdot bias_k = \sum_{k=1}^{N_{test}} \left[\sum_{i=1}^{n_k} C \cdot D_{i,k} - n_k \cdot M_k \right] \tag{8}$$

Equating the bias (8) to zero, we obtain an algorithm for estimating the coefficient C in the form (9) [18,25].

$$C2 = C = \frac{\sum_{k=1}^{N_{test}} n_k \cdot M_k}{\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} D_{i,k}} \tag{9}$$

Alternatively, the bias (7) for each test vehicle can be demanded to be zero (10).

$$bias_k = 0 \rightarrow \frac{1}{n_k} \cdot \sum_{i=1}^{n_k} C \cdot D_{i,k} - M_k = 0 \rightarrow \sum_{i=1}^{n_k} C \cdot \frac{D_{i,k}}{M_k} = n_k \tag{10}$$

Summing up both sides of Equation (10) over the set of all test vehicles, we get:

$$\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} C \cdot \frac{D_{i,k}}{M_k} = \sum_{k=1}^{N_{test}} n_k \tag{11}$$

Considering that the bias can be both positive and negative, condition (11) only minimizes the mean error of the load in the set of weighed vehicles. Such a solution may be useful in the case of WIM systems that provide statistical data on the total load transported over a specific period of time. The estimation algorithm for the coefficient C, satisfying the condition (11) has the form (12) [18,25].

$$C3 = C = \frac{\sum_{k=1}^{N_{test}} n_k}{\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} \left(\frac{D_{i,k}}{M_k} \right)} \tag{12}$$

However, it should be remembered that the implementation of the postulate (10) cannot be met simultaneously for all test vehicles. The obtained form of the estimator (12) provides only an approximate fulfilment of condition (10).

3.3. Minimizing the Mean Square Error for the General Case ($b_{ref} \neq 0$)

In the general case, where $b_{ref} \neq 0$, the mean square error of weighing results $D_{i,k}$ for the k-th test vehicle is described by the relationship (13).

$$\Psi_k^2 = \frac{1}{n_k} \cdot \sum_{i=1}^{n_k} \left[D_{i,k} - (p \cdot M_k + b) \right]^2 \tag{13}$$

The cumulative mean square error including the weighing results of all test vehicles is of the form (14).

$$\Psi^2 = \frac{1}{N_{test}} \cdot \sum_{k=1}^{N_{test}} \Psi_k^2 = \frac{1}{N_{test} \cdot n_k} \sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} \left[D_{i,k} - (p \cdot M_k + b) \right]^2 \tag{14}$$

The minimization of the mean square error (14) due to the coefficients p and b leads to the estimation algorithm in the form [18,25]:

$$C4 = \frac{1}{p} = \frac{\left(\sum_{k=1}^{N_{test}} n_k\right)\left(\sum_{k=1}^{N_{test}} n_k \cdot (M_k)^2\right) - \left(\sum_{k=1}^{N_{test}} n_k \cdot M_k\right)^2}{\left(\sum_{k=1}^{N_{test}} n_k\right)\left(\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} M_k \cdot D_{i,k}\right) - \left(\sum_{k=1}^{N_{test}} n_k \cdot M_k\right)\left(\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} D_{i,k}\right)} \tag{15}$$

$$b4 = \frac{\left(\sum_{k=1}^{N_{test}} n_k \cdot (M_k)^2\right)\left(\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} D_{i,k}\right) - \left(\sum_{k=1}^{N_{test}} n_k \cdot M_k\right)\left(\sum_{k=1}^{N_{test}} \sum_{i=1}^{n_k} M_k \cdot D_{i,k}\right)}{\left(\sum_{k=1}^{N_{test}} n_k\right)\left(\sum_{k=1}^{N_{test}} n_k \cdot (M_k)^2\right) - \left(\sum_{k=1}^{N_{test}} n_k \cdot M_k\right)^2} \tag{16}$$

4. Simulation Experiments

The simulation experiments were carried out with the use of synthetic measurement data. The data were contained in two sets, calibration data and reference data. The calibration data served as the result of weighing a certain number of test vehicles. The data set parameters were:

- number of test vehicles $N_{test} = 3$, or changed in subsequent experiments when its impact was the object of assessment;
- the number of runs is the same for each test vehicle $n_k = N_{run} = 50$, for $k = 1, 2, 3, \ldots, N_{test}$;
- weight of each test vehicle M_k. It was assumed that the gross weight of all test vehicles was evenly distributed over the entire measuring range of the WIM system. The exceptions are experiments in which the distribution of the gross weight of the test vehicles was the object of research.

Synthetic reference data used in the assessment of errors in the calibrated WIM system were generated separately. The basis for such an assessment are the results of weighing reference vehicles of known weight and axle load, which repeatedly pass through the WIM station. Experiments are performed after the previously determined calibration coefficients have been entered into the system. The data set parameters were:

- number of reference vehicles N_{ref}, assumed as $N_{ref} = N_{test}$;
- the same number of runs for each reference vehicle $N_{run_{ref}} = 5000$;
- vehicle weight M_{ref}. In each case, the weight of the reference vehicles was evenly distributed within the measuring range of the WIM system.

The model of the WIM system adopted for the purposes of simulation studies was in the form (1). We assumed the following as parameters of the WIM system:

- the lower limit of the measuring range Z_{min} = 10,000 kg;
- the upper limit of the measuring range Z_{max} = 40,000 kg;
- values of the static characteristic coefficients C_{ref} and b_{ref};
- standard deviation of disturbances ε, we adopted a constant value of $\sigma = 0.05 Z_{min}$. We assumed that the disturbances ε have the expected value equal to zero and the standard deviation σ.

The simulation experiments consisted in generating both data sets, containing, respectively, synthetic data from the weighing of test vehicles and reference vehicles. These data were generated for the assumed values of the parameters of both data sets and also for the assumed parameters of the WIM system. We used the data contained in the calibration set to calculate the estimates of the calibration coefficients, in accordance with the relationships (5), (9), (12) and (15) and (16). The data contained in the reference set were used to determine the errors of the calibrated WIM system. Relative bias (17), standard deviation (18), and rms errors were used to assess the accuracy of WIM systems (19) [26].

$$bias = \frac{1}{N_{ref}} \sum_{k=1}^{N_{ref}} bias_k \tag{17}$$

where:

$$bias_k = \frac{\frac{1}{N_{run_{ref}}} \sum_{i=1}^{N_{run_{ref}}} C(D_{i,k} - b) - M_{ref,k}}{M_{ref,k}}$$

$M_{ref,k}$—weight of the k-th reference vehicle,

C, b—estimates of the coefficients of static characteristics of the WIM system determined in accordance with algorithms (5), (9), (12) and (15). In special cases $b_{ref} = 0$.

$$\sigma = \frac{1}{N_{ref}} \sum_{k=1}^{N_{ref}} \sigma_k \tag{18}$$

where:

$$\sigma_k = \frac{\sqrt{\frac{1}{N_{run_{ref}}-1} \sum_{i=1}^{N_{run_{ref}}} \left[C(D_{i,k} - b) - E\right]^2}}{M_{ref,k}}$$

$$E = \frac{1}{N_{run_{ref}}} \sum_{i=1}^{N_{run_{ref}}} C(D_{i,k} - b)$$

$$\delta = \frac{1}{N_{ref}} \sum_{k=1}^{N_{ref}} \delta_k \tag{19}$$

where:

$$\delta_k = \sqrt{bias_k^2 + \sigma_k^2}$$

Figure 1 shows the bias error of the coefficient estimates of Equation (1) depending on the shift b_{ref} and slope C_{ref} of the static characteristics of the calibrated WIM system. These characteristics were determined for each of the estimators considered for these coefficients. The error value has been referenced to C_{ref} and b_{ref}, respectively.

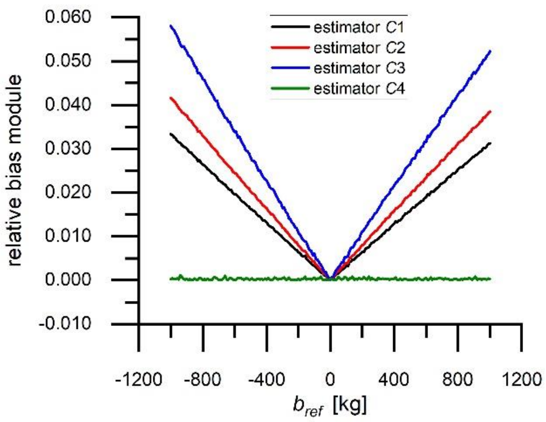
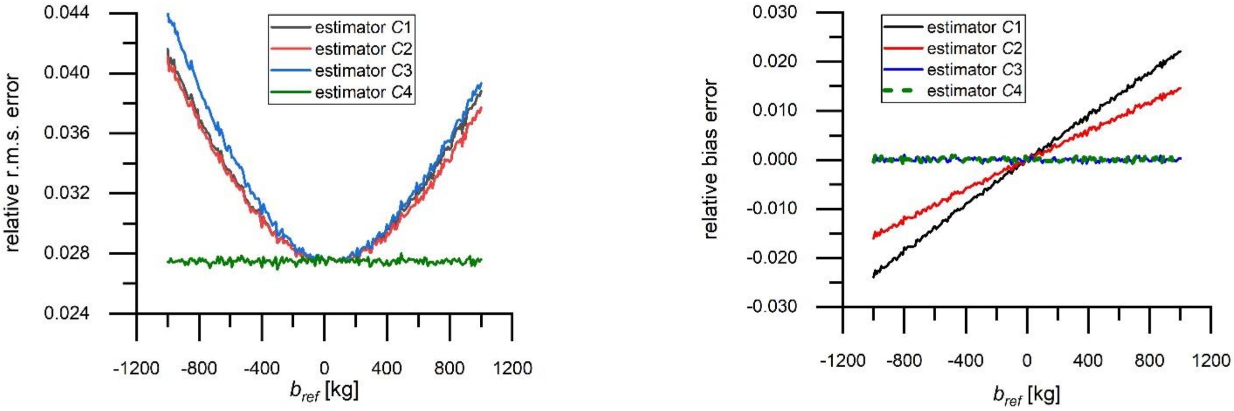


Figure 1. Effect of the shift b_{ref} of the WIM static characteristic on the relative bias error of the estimates of the C calibration factor for different calibration algorithms and $C_{ref} = 1$.

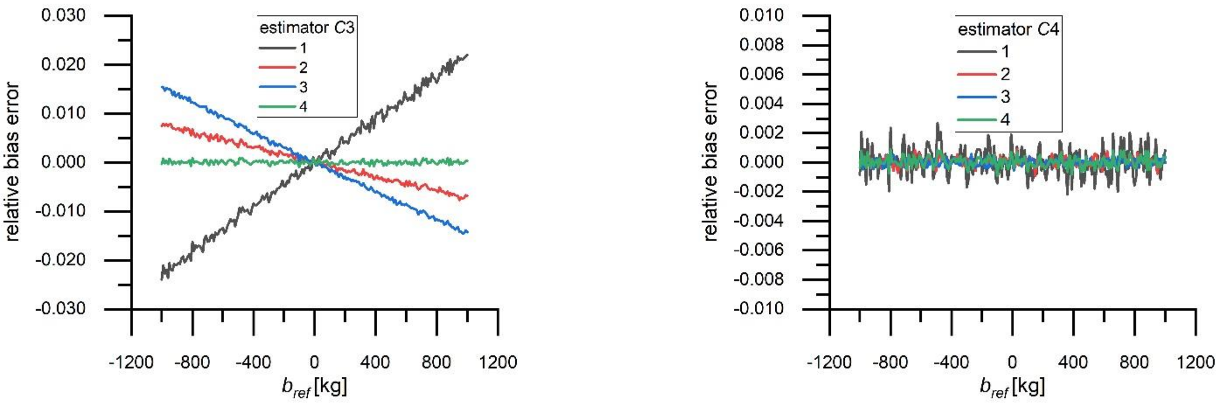
The presented characteristics confirm the qualitative properties of the four discussed estimators. The shift b_{ref} of the WIM static characteristic causes an erroneous (bias error) estimation of its slope for the algorithms $C1 - C3$. As expected, the $C4$ algorithm is not susceptible to such a disturbance. On the other hand, these characteristics make it possible to evaluate this phenomenon in terms of quantity. The shift b_{ref} with a value of 1000 kg, i.e., 10% of the lower limit of the measurement range, causes the error of slope estimation C_{ref} within the range of 3–6% depending on the algorithm used. The random variability of the calibration coefficient assessments measured by their relative standard deviation is similar for all four algorithms and amounts to approx. 2%. The errors in estimating the coefficients of the static characteristics of the WIM system transfer to the error of weighing vehicles. This influence is illustrated by the characteristics presented in Figure 2.



(**a**) (**b**)

Figure 2. Influence of the WIM static characteristic shift on: (**a**) relative r.m.s. errors and (**b**) relative bias error of weighing, $C_{ref} = 1$.

As can be seen from the characteristics presented in Figure 2, errors in estimating the calibration coefficients translate directly into weighing errors. The rms error for $b_{ref} = 1000$ kg is between 3% and 4.5%. It should be emphasized that in the absence of a WIM static characteristic shift ($b_{ref} = 0$), all four estimators ensure the same accuracy in weighing vehicles. The use of the estimator $C4$ and $b4$ ensures the insensitivity of weighing errors to the shift of the WIM static characteristic for each vehicle. The estimator $C3$ allows obtaining unbiased results in the set of all weighing results, regardless of the presence of a WIM characteristic shift. However, the results of weighing individual vehicles will be biased. This property of estimator $C3$ is illustrated by the characteristics presented in Figure 3.



(**a**) (**b**)

Figure 3. Bias of results of weighing on the WIM station calibrated according to the algorithm: (**a**) $C3$, (**b**) $C4$. (1–3) bias errors of weighing of three exemplary test vehicles and (4) sum of these bias errors, $C_{ref} = 1$.

The $C4$ algorithm allows us, under these conditions, to obtain virtually unbiased weighing results for each test vehicle. The algorithm $C3$ ensures the minimization of the sum of bias errors of the weighing results of all vehicles, but the weighing result of each of them is biased separately. This is because bias errors have different signs. Taking into account that the algorithm $C3$ is easier to implement, its use is justified, e.g., in the case of WIM systems collecting data for statistical purposes, i.e., systems providing information on the total load transported in a specific time interval.

Important elements in the designing of the calibration process include the selection of the number of N_{test} test vehicles used in the calibration process, their total weight, and the distribution of the weight of these vehicles within the adopted measuring range of the WIM system. In the conducted tests, we assumed that a fixed number of test vehicles is used ($N_{test} = 2$ to $N_{test} = 50$), the mass of which is evenly distributed in the range, the lower limit of which coincides with the lower limit of the measurement range of the WIM system (Z_{min} = 10,000 kg), and the upper limit of which is shifted in subsequent experiments from 15,000 kg to 40,000 kg with a step of 5000 kg. The accuracy assessment of such a calibrated WIM system was carried out in the full measuring range, i.e., 10,000 to 40,000 kg. For each value, the following were determined: rms error, relative bias, and relative standard deviation of the weighing results after calibrating the WIM system. Relative values were obtained by reference to the actual values of the measured quantity. The characteristics presented in Figures 4 and 5 illustrate the dependence of the rms error and the bias of the weighing results on the number of test vehicles, for the four tested estimators of calibration coefficient. The random variation of the weighing results does not depend on the method of weight distribution of the test vehicles, but rather on their number.

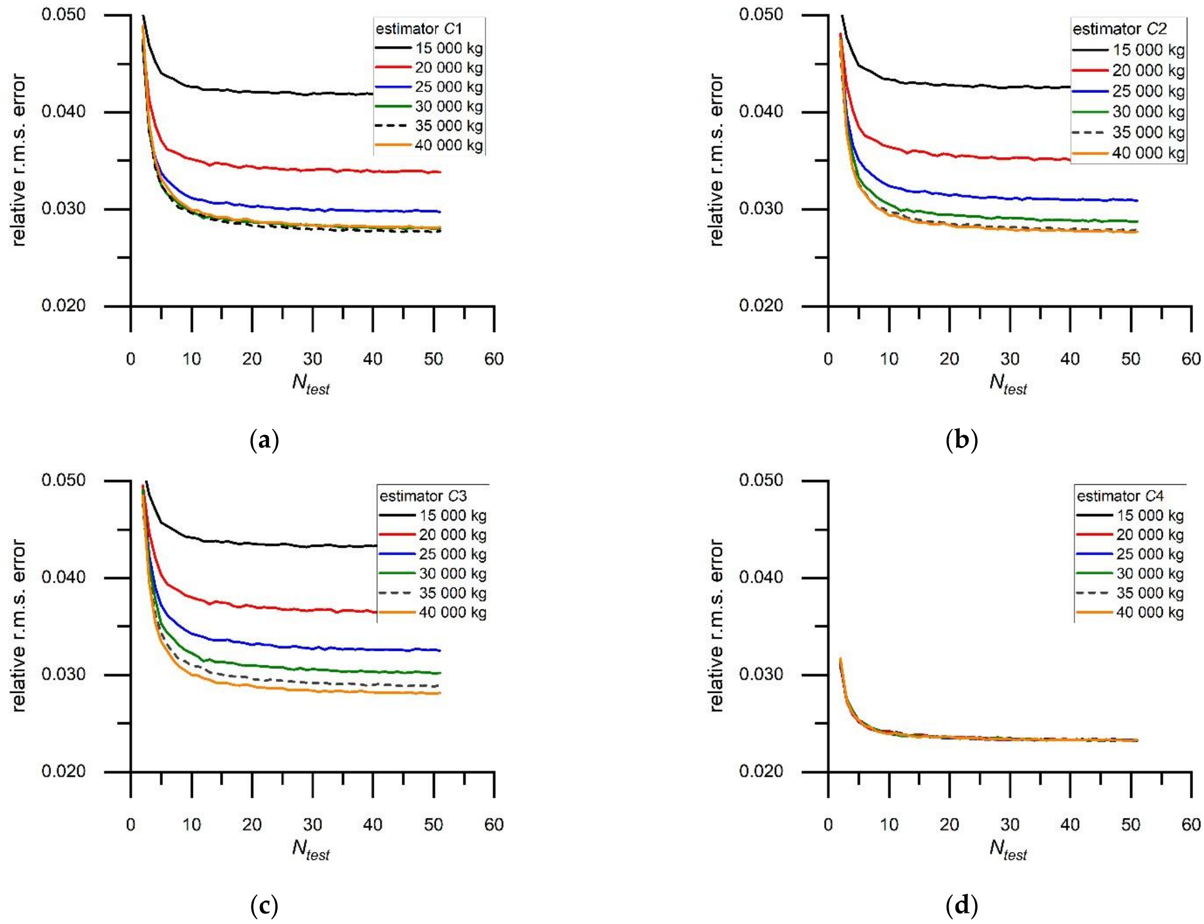


Figure 4. Influence of the weight distribution of test vehicles in the measurement range of the WIM system on the rms error of vehicle weighing results for various estimators of calibration coefficients ($C_{ref} = 1$, $b_{ref} = 0.1Z_{min}$), (**a**) estimator $C1$, (**b**) estimator $C2$, (**c**) estimator $C3$, (**d**) estimator $C4$.

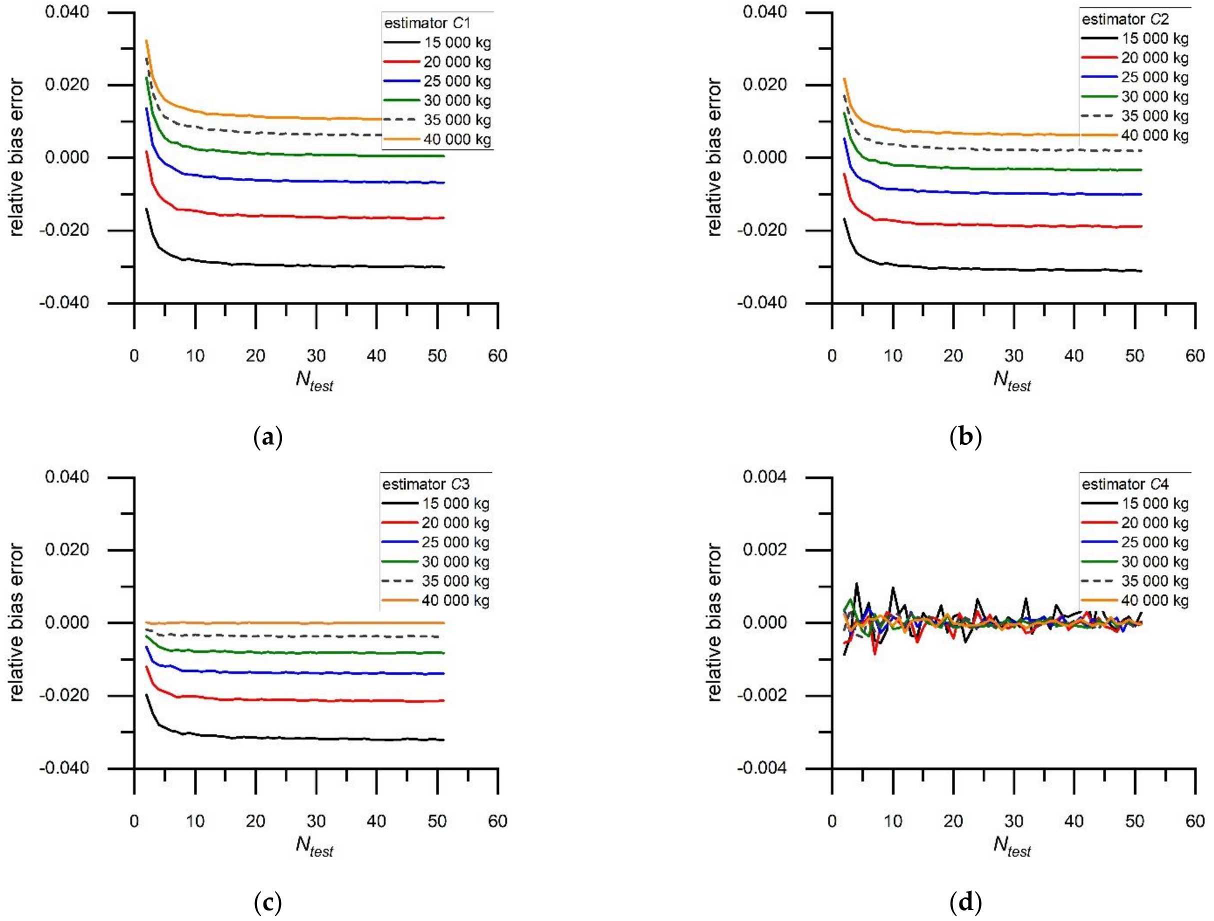


Figure 5. Influence of the weight distribution of test vehicles in the measurement range of the WIM system on the bias error of vehicle weighing results for various algorithms for estimating calibration coefficients ($C_{ref} = 1$, $b_{ref} = 0.1Z_{min}$). (**a**) estimator *C*1, (**b**) estimator *C*2, (**c**) estimator *C*3, (**d**) estimator *C*4.

Based on the characteristics presented in Figures 4 and 5, the following conclusions can be drawn:

- even distribution of the weight of the test vehicles in the whole measuring range of the WIM system allows the rms error of the weighing results to be minimized;
- in the case of algorithm *C*4, the weight distribution of the test vehicles does not affect the weighing accuracy. The *C*4 algorithm allows an unbiased weighing result to be obtained regardless of the method of weight distribution of the test vehicles in relation to the WIM system measurement range. These are the significant advantages of this algorithm;
- algorithm *C*3 allows for the minimization of the bias of the weighing results regardless of the number of test vehicles, but only if the gross weight of these vehicles is distributed over the entire measuring range of the calibrated WIM system;
- during calibration, ensure that the gross weight of the test vehicles is distributed over at least 75% of the upper limit of the measuring range;
- it is not justified to use more than 10 test vehicles. However, increasing the number of test vehicles from 2 to 10 makes it possible to reduce the weighing error in the WIM system calibrated in this way by almost half.

The characteristics presented in Figure 6 illustrate an exemplary selection of parameters of the calibration process (N_{test}, weight distribution of test vehicles) depending on the permissible rms error value of vehicle weighing. For example, the line of constant rms error = 0.04 was assumed. Such an error value may be achieved for different values of both parameters of the calibration process. However, as seen, narrowing the range in which the mass of test vehicles is distributed forces an increase in the number of these vehicles.

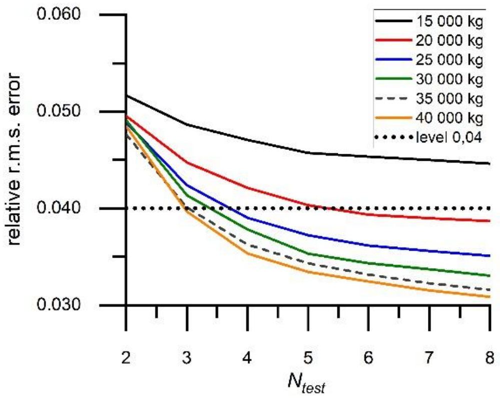


Figure 6. Illustration of the method for selecting parameters of the calibration process for the assumed, permissible error of weighing.

5. Experimental Research

The experiment carried out as part of this work consisted in repeatedly driving three test vehicles through a calibrated WIM station. The test vehicles were 5-axle sets consisting of a two-axle tractor unit and a 3-axle semi-trailer. The vehicles were weighed before and after the experiment. Weighing was carried out on a platform scale enabling the determination of the GVW and on a static scale for individual axles to determine their load. The parameters of the test vehicles are presented in Table 1.

Table 1. Parameters of test vehicles used in the calibration process.

Vehicle Number	GVW [kg]	Axle Load [kg]					Number of Runs
		1	2	3	4	5	
1	19,460	6044	5505	2604	2604	2703	47
2	25,060	6645	7229	3526	3805	3855	49
3	29,360	6436	7328	5149	5248	5199	51
Total							147

The test vehicles traveled through the WIM system at various but stable speeds. The speed of each run was measured. The design of the experiment is presented in Table 2.

Table 2. Plan of the calibration experiment.

Velocity [km/h]	Vehicle Number			Total
	1	2	3	
	Number of Runs			
65–75	13	13	14	40
55–65	12	13	13	38
45–55	11	11	12	34
35–45	11	12	12	35
Total	47	49	51	147

The collected measurement results made it possible to define 11 sub-experiments. Each sub-experiment consisted of a single run of each test vehicle at the speed contained in each of the four distinguished sub-ranges, so one sub-experiment consisted of a total of 12 runs of three test vehicles with each of the four distinguished speeds, i.e., within one sub-experiment $N_{run} = 4$. Therefore, 132 measurement results were used in these experiments. In order to maintain the identical structure of all 11 sub-experiments, the remaining 15 measurement results were omitted.

In order to assess the impact of the number of test runs on the accuracy of the calibrated WIM system, an 11-fold cross-validation was carried out. In each repetition, a maximum of ten sub-experiments could be used to estimate the calibration coefficients, i.e., the maximum value $N_{run} = 40$. Theoretically, the minimum value of $N_{run} = 4$ corresponds to the situation when only one sub-experiment was used in the calibration process. However, such a small number of measurement data leads to assessments with poor statistical properties. The eleventh sub-experiment was used to assess the errors of the WIM system ($N_{run_{ref}} = 4$). The entire process was repeated 11 times, with different choices of calibration experiments and assessing experiments.

The tests were carried out for a WIM system with correctly adjusted static characteristics and for a system where the static characteristic shift was artificially introduced $b_{ref} = 5000$ kg. The test results illustrate the characteristics presented in Figures 7 and 8. The bias shown in the figures is the mean value of the bias errors for each test vehicle.

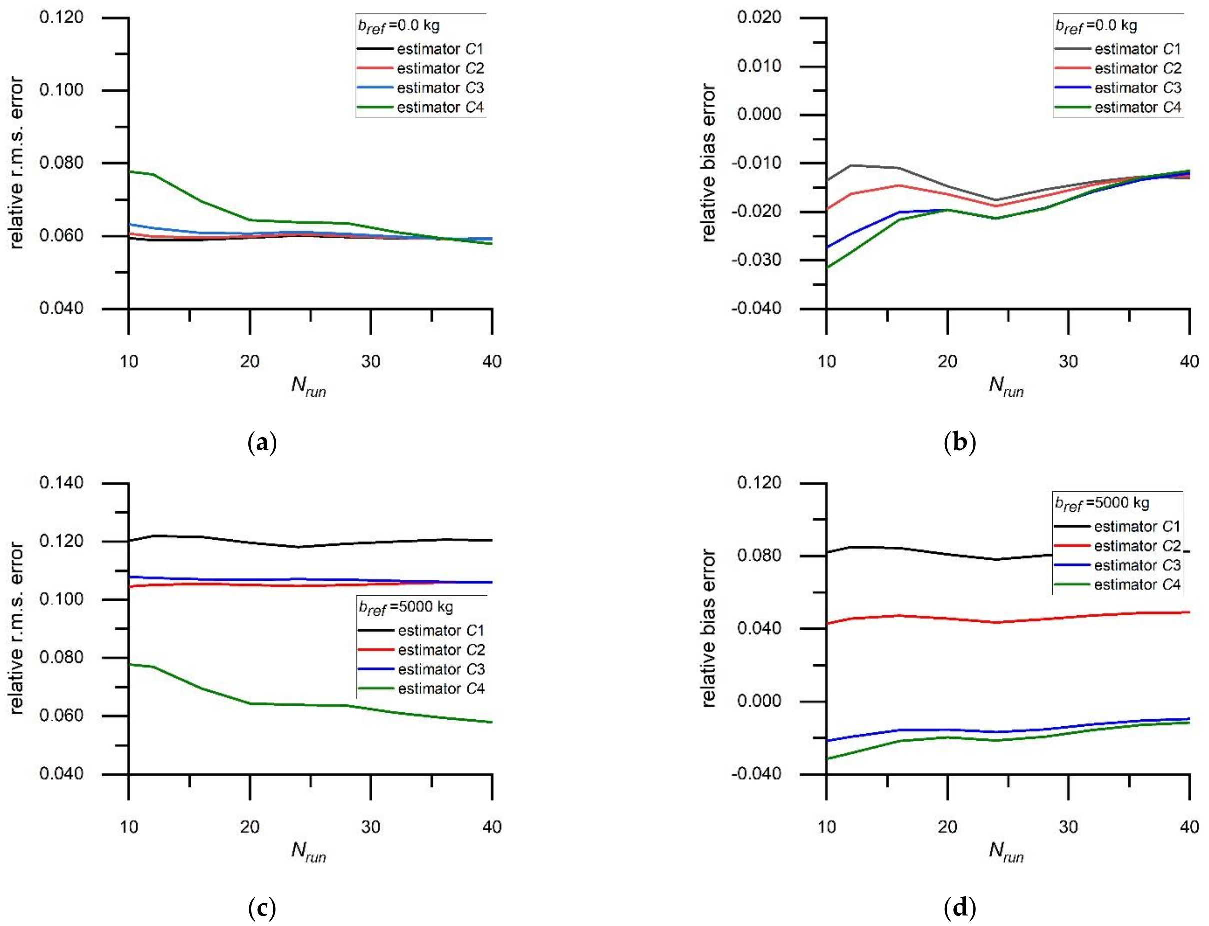


Figure 7. Dependence of weighing errors for the WIM system calibrated according to four discussing estimators on the number of runs of test vehicles N_{run}, respectively for (**a**,**b**) $b_{ref} = 0$ kg and (**c**,**d**) $b_{ref} = 5000$ kg.

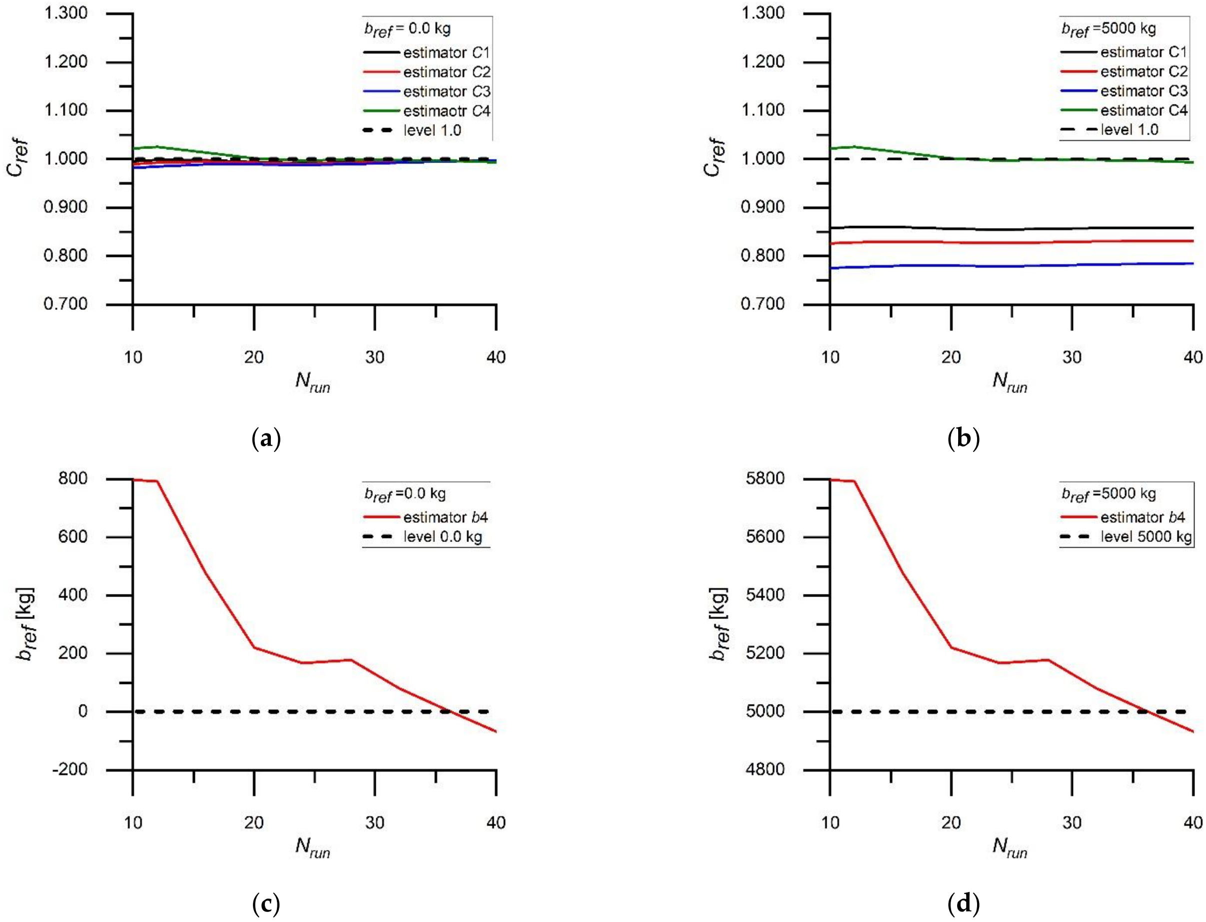


Figure 8. Estimation of the calibration coefficients (**a**,**b**)—C_{ref}; (**c**,**d**)—b_{ref}, obtained using different estimators, respectively, for $b_{ref} = 0$ kg and $b_{ref} = 5000$ kg, as a function of the number of test vehicle runs N_{run}.

The results of the experimental research confirm the conclusions previously formulated at the simulation stage. In the case when $b_{ref} = 0.0$, all estimators $C1$–$C4$ are practically unbiased (bias = 1.2%) and provide comparable weighing accuracy. The estimator $C4$ requires an increased number of passes, which is justified by the necessity to estimate two calibration coefficients.

When $b_{ref} \neq 0$, the WIM system calibrated according to the estimators $C1$ and $C2$ gives biased weighing results. The estimators $C3$ and $C4$ are unbiased, with the proviso that the $C3$ estimator ensures unbiased results in the entire population of weighed vehicles.

The characteristics presented in Figure 8 illustrate the course of evaluation of the calibration coefficients as a function of the number of runs of test vehicles N_{run}. If the static characteristic of the WIM system is not shifted, all the compared estimators $C1$–$C4$ correctly estimate the coefficient C_{ref}. When the static characteristic is shifted, the estimators $C1$–$C3$ "try" to compensate for the impact of this shift on the weighing error by changing the value of the estimated coefficient C_{ref}. Not surprisingly, the estimators $C4$ and $b4$ also correctly estimate both coefficients in this case. An important observation from a practical point of view is the slow progress of $b4$ to the true value b_{ref}. Correct use of this estimator requires a large number of test runs.

6. Conclusions

We presented a comparison of the properties of four estimators used to estimate the static characteristic coefficients of Weigh-In-Motion (WIM) systems. The estimation is based on the measurement results obtained during the implementation of the pre-weighed vehicles method. We presented the justification of the form of the compared estimators as well as the results of simulation and experimental tests carried out on the constructed WIM system. The results of the experimental research confirm the conclusions previously formulated at the simulation stage.

The compared estimators differ in their properties, which translates into different metrological properties of WIM systems calibrated with their use. There are also differences regarding the performance of these estimators. The $C4$ and $b4$ estimators are undoubtedly the most "demanding"—their use requires a greater number of test runs compared to the other estimators and the availability of at least two test vehicles. In return, they allow the best properties of the calibrated WIM system to be obtained—no bias in weighing results.

When the static characteristic of the WIM system is shifted in relation to zero, the remaining estimators (apart from $C4/b4$) "try" to compensate for the bias of the results by assuming a smaller slope of the static characteristic.

An interesting alternative to the $C4/b4$ estimator is the $C3$ estimator. It allows unbiased weighing results to be obtained also in the case of $b_{ref} \neq 0$. However, it should be remembered that in this case no bias refers to the entire population of weighing results, and not to the result obtained for a single vehicle. Such a feature can be valuable when the purpose of weighing is to evaluate the total load that has traveled through the selected road section in the assumed time interval.

The obtained results indicate that it is possible to significantly increase the accuracy of WIM systems not only through hardware activities related to, for example, the selection of the number of load sensors or their manufacturing technology, but also through a "soft" operation, which in this case is the proper selection of the calibration algorithm.

The calibration algorithms presented in the paper are based on the method of pre-weighed vehicles. The main disadvantage of this method is the need to have several trucks of different classes. During the experiment they pass many times through the calibrated WIM station. This is a time-consuming and therefore costly process. Algorithm (14) is particularly demanding in terms of the number of passes made. This is due to the need to determine two calibration coefficients, not one, as in the case of other algorithms. The time-consuming nature of this calibration method is its main disadvantage, especially in the case of the high variability of the metrological properties of the WIM system caused by changing environmental conditions.

The results presented in the study are promising. However, it seems advisable to extend the scope of experimental research. This is particularly important in relation to the $C3$ and $C4/b4$ estimators, as their use may bring particularly significant benefits.

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