Adaptive Fuzzy Control for Flexible Robotic Manipulator with a Fixed Sampled Period

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Abstract: In this paper, a backstepping sampled data control method is developed for a flexible robotic manipulator whose internal dynamic is completely unknown. To address the internal uncertainties, the fuzzy logical system (FLS) is considered. Moreover, considering the limited network bandwidth, the designed controller and adaptive laws only contain the sampled data with a fixed sampled period. By invoking the Lyapunov stability theory, all signals of the flexible robotic manipulator are semi-global uniformly ultimately bounded (SGUUB). Ultimately, an application to a flexible robotic manipulator is given to verify the validity of the sampled data controller.

Keywords: flexible robotic manipulator; sampled data control; adaptive fuzzy control; backstepping design

1. Introduction

For the past few years, robotics has experienced rapid development and has become widely used in a variety of fields, such as military, industry, aerospace and so on. With the thriving of robotics, the flexible robotic manipulator has also attracted wide attention, and diverse interesting control methods are being investigated, such as sliding control [1–5], feedback linearization method [6,7], and the passivity approach [8,9]. It should be noticed that modeling errors are inevitable in the actual modeling process, which will bring about a certain deviation in the dynamic characteristics of the model. Therefore, in order to allow the proposed method to be better applied in practice, the internal dynamics of the flexible robotic manipulator in this paper are considered unknown. To estimate the internal dynamics of the flexible robotic manipulator, the fuzzy logical system (FLS) is considered under the backstepping structure.

Recently, a number of backstepping adaptive fuzzy controllers have been designed subject to the flexible robotic manipulator [10–20]. In [18], by transforming the single-link flexible robotic manipulator system with unknown internal dynamics into an uncertain fifth-order nonlinear system, a backstepping adaptive fuzzy controller was developed for the fifth-order transformation model. However, it should be noticed that there exists an "explosion of complexity" problem in [18], which is mainly caused by the repeated appearance of the time derivative of virtual control signals. In [19], by introducing error compensation signals, a command filter is designed to conquer the "explosion of complexity" problem. In this paper, FLS is used to approximate the time derivative of virtual control signals to avert the "explosion of complexity" problem, which simplifies the backstepping structure and is more suitable for the practical case.

Along another key line of research, as a result the limit of the network bandwidth in practice, plenty of adaptive sampled data control approaches have been developed [20–26]. Rather than the normal adaptive fuzzy control method in [10–20], the difficulty of sampling control is mainly reflected in two aspects. Firstly, in order to save more network bandwidth, the network will transmit data after a fixed sampling period, which means the backstepping sampled data control method can only use the information at the sampling time. Secondly,

there always exist errors between the sampled data and the continuous data. How to ensure the stability of controlled objects by only using sampling data is a significant problem. Especially, under the backstepping structure, sampled data will be used in each step, which increases the difficulty of stability analysis.

In this paper, an adaptive fuzzy backstepping sampled data control method is developed subject to a flexible robotic manipulator. Firstly, by transforming the flexible robotic manipulator into a fifth-order non-strict feedback nonlinear system and using FLS to approximate the nonlinear function, the sampled data controller is designed. On the basis of Gronwall–Bellman inequality and Lyapunov stability theory, the designed sampled-data controller can guarantee that all closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB). The main contributions are summarized below:

(1) This paper is the first work concerning adaptive fuzzy backstepping sampled-data control for a flexible robotic manipulator system. In the control process, the designed controller only needs the sampled information of states rather the continuous case.

(2) By using the property of the fuzzy basic function, the algebraic loop issue caused by the non-strict feedback system is avoided. Furthermore, the designed adaptive laws also consider the sampled data, which is a great difference from previous works.

2. Preliminaries and Problem Formulation

The dynamics equation of a single-link manipulator with brushless direct current (DC) motor based on non-rigid joints is shown below [19]:

$$
\begin{aligned}
& J_1 \ddot{q}_1 + F_1 \dot{q}_1 + K(q_1 - \tfrac{q_2}{N}) + Mgd \cos q_1 = 0 \\
& J_2 \ddot{q}_2 + F_2 \dot{q}_2 - \tfrac{K}{N}(q_1 - \tfrac{q_2}{N}) = K_a I \\
& L\dot{I} + RI + K_c \dot{q}_2 = u
\end{aligned}
\tag{1}
$$

where J_1 and J_2 are the inertia coefficients, F_1 and F_2 are the viscous friction constants, K is the spring coefficient, M is the link mass, g is the acceleration of gravity, d is the center of gravity of the link, N is the gear ratio, K_a is the torque coefficient, I is the current of the armature, R is the armature resistance, L is the armature inductance, K_c is the back electromotive force constant, u is the armature voltage, and q_1 and q_2 are the angular position of the connecting rod and the motor shaft, respectively.

For the sake of applying the backstepping structure for a single-link flexible joint robot model (1), define the conversion variables as $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$, $x_4 = \dot{q}_2$, and $x_5 = I$. Thus, model (1) can be rewritten as

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x_1, x_2, x_3) + x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x_1, x_3, x_4, x_5) + x_5 \\
\dot{x}_5 &= f_3(x_4, x_5) + \tfrac{u}{L}
\end{aligned}
\tag{2}
$$

where $f_1(x_1, x_2, x_3) = -\tfrac{F_1}{J_1} x_2 - \tfrac{K}{J_1}(x_1 - \tfrac{x_3}{N}) - \tfrac{Mgd}{J_1} \cos x_1 - x_3$, $f_2(x_1, x_3, x_4, x_5) = -\tfrac{F_2}{J_2} x_4 + \tfrac{K}{J_2 N}(x_1 - \tfrac{x_3}{N}) + \tfrac{K_a}{J_2} x_5 - x_5$ and $f_3(x_4, x_5) = -\tfrac{R}{L} x_5 - \tfrac{K_c}{L} x_4$.

In this paper, to effectively reduce the burden of network bandwidth, the sampled data control method has been considered. The armature voltage u only needs to be updated at the sampling time. To state the concept clearly, another signal v is introduced to reflect the continuous change of the armature voltage u. In the subsequent calculation, the system states can only be obtained from the sampled data, which brings an enormous challenge to the analysis of the stability under the backstepping structure.

Before giving the control objective, the following preparations of FLS are first presented.

Lemma 1 ([18]). *If $f(x)$ is a continuous function of a compact set Ω, then for a constant $\varepsilon > 0$, the FLS $\theta^T \varphi(x)$ satisfies*

$$\sup_{x \in \Omega} \left| f(x) - \theta^T \varphi(x) \right| \leq \varepsilon$$

where ε is the fuzzy minimum approximation error, and there exists a positive constant ε^* that satisfies $|\varepsilon| \leq \varepsilon^*$; $\theta^T = [\theta_1, \theta_2, \cdots, \theta_N]$ is desired parameter vector of FLS, and $\varphi(x) = [\varphi_1(x), \cdots, \varphi_N(x)]^T$ is defined as

$$\varphi_l = \frac{\prod\limits_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum\limits_{l=1}^{N} \left(\prod\limits_{i=1}^{n} \mu_{F_i^l}(x_i) \right)} \tag{3}$$

Control Objective: The control objective of this work is to design an adaptive sampled-data control strategy for transformation model (2) by utilizing the sampled data of the states. Under the Lyapunov stability theory, all closed-loop signals are proved to converge a small residual set.

3. Adaptive Fuzzy Controller Design and Stability Analysis

In this section, an adaptive fuzzy sampled-data controller is designed for the backstepping structure, and the stability of the closed-loop system is proved by Lyapunov function stability theory.

In the first place, to design the adaptive sampled-data controller, the coordinate changes under the backstepping structure are defined as

$$z_1(t) = x_1(t), z_i(t) = x_i(t) - \alpha_{i-1} (i = 2, 3, 4, 5) \tag{4}$$

where α_{i-1} is the virtual control signal specified. To simplify the expression of the formula, we denote $z_j (j = 1, 2, 3, 4, 5)$ and x_j as $z_j(t)$ and $x_j(t)$, respectively.

As the sampled-data controller is only formed from the state at the sampling time, the adaptive fuzzy controller design process is divided into two parts. In case 1, we design the sampled-data controller and analyze the stability in the time interval (t_k, t_{k+1}), where the time sequence t_k denotes the k-th sampling activity. In case 2, we analyze the stability at the sampling time t_k.

Case 1.

Step 1. To design the sampled-data controller for transformation model (2), we chose the Lyapunov function as

$$V_1 = \frac{1}{2} z_1^2 \tag{5}$$

On the basis of the designed coordinate changes (4) and Lyapunov function (5), the time derivative of Lyapunov function can be expressed as

$$\dot{V}_1 = z_1(z_2 + \alpha_1) \tag{6}$$

By applying Young's inequality, one can obtain

$$z_1 z_2 \leq \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \tag{7}$$

For the backstepping structure, we design the virtual control signal α_1 as

$$\alpha_1 = -c_1 z_1 - \frac{1}{2} z_1 \tag{8}$$

Substituting (7) and (8) into (6) yields

$$\dot{V}_1 \le -c_1 z_1^2 + \frac{1}{2} z_2^2 \tag{9}$$

Step 2. In Step 2, we design the Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{1}{2} \widetilde{\theta}_1^T \widetilde{\theta}_1 \tag{10}$$

where $\widetilde{\theta}_1 = \theta_1 - \hat{\theta}_1$ is the approximation error between the ideal FLS parameter vector θ_1 and its estimation variable $\hat{\theta}_1$.

In accordance with the Lyapunov function (10), its time derivative form can be expressed as

$$\dot{V}_2 \le -c_1 z_1^2 + \frac{1}{2} z_2^2 + z_2 (f_1(x_1, x_2, x_3) + z_3 + \alpha_2 - \dot{\alpha}_1) - \widetilde{\theta}_1^T \dot{\hat{\theta}}_1 \tag{11}$$

By utilizing Lemma 1 to approximate the function $g_1(x_1, x_2, x_3) = f_1 + c_1 x_2 + \frac{1}{2} x_2$, one has

$$\begin{aligned}\dot{V}_2 \le &-c_1 z_1^2 + \frac{1}{2} z_2^2 + z_2(z_3 + \alpha_2 + \theta_1^T \varphi_1(x_1, x_2, x_3) + \varepsilon_1 \\ &+ \theta_1^T \varphi_1(x_1(t_k), x_2(t_k)) - \theta_1^T \varphi_1(x_1(t_k), x_2(t_k))) \\ &- \widetilde{\theta}_1^T \dot{\hat{\theta}}_1\end{aligned} \tag{12}$$

Using Young's inequality and the property of the fuzzy basic function $0 < \varphi_1^T(\cdot)\varphi_1(\cdot) \le 1$, we can obtain

$$z_2 z_3 + z_2 \theta_1^T \varphi_1(x_1, x_2, x_3) \le z_2^2 + \frac{1}{2} \theta_1^T \theta_1 + \frac{1}{2} z_3^2 \tag{13}$$

$$-z_2 \theta_1^T \varphi_1(x_1(t_k), x_2(t_k)) + z_2 \varepsilon_1 \le z_2^2 + \frac{1}{2} \theta_1^T \theta_1 + \frac{1}{2} \varepsilon_1^{*2} \tag{14}$$

Substituting (13) and (14) into (12) leads to

$$\begin{aligned}\dot{V}_2 \le &\, z_2(\alpha_2 + \hat{\theta}_1^T \varphi_1(x_1(t_k), x_2(t_k)) + \widetilde{\theta}_1^T \varphi_1(x_1(t_k), x_2(t_k))) \\ &- \widetilde{\theta}_1^T \dot{\hat{\theta}}_1 + \frac{5}{2} z_2^2 + \theta_1^T \theta_1 + \frac{1}{2} \varepsilon_1^{*2} + \frac{1}{2} z_3^2 - c_1 z_1^2\end{aligned} \tag{15}$$

We design the virtual control signal and adaptive law for Step 2 as

$$\alpha_2 = -c_2 z_2 - \hat{\theta}_1^T \varphi_1(x_1(t_k), x_2(t_k)) - \frac{5}{2} z_2 \tag{16}$$

$$\dot{\hat{\theta}}_1 = z_2(t_k) \varphi_1(x_1(t_k), x_2(t_k)) - \delta_1 \hat{\theta}_1 \tag{17}$$

Substituting (16) and (17) into (15) results in

$$\begin{aligned}\dot{V}_2 \le &-(z_2 - z_2(t_k)) \widetilde{\theta}_1^T \varphi_1(x_1(t_k), x_2(t_k)) + \delta_1 \widetilde{\theta}_1^T \hat{\theta}_1 \\ &+ \theta_1^T \theta_1 + \frac{1}{2} \varepsilon_1^{*2} + \frac{1}{2} z_3^2 - c_1 z_1^2 - c_2 z_2^2\end{aligned} \tag{18}$$

Based on Young's inequality, we further have

$$\begin{aligned}-(z_2(t_k) - z_2) \widetilde{\theta}_1^T \varphi_1(x_1(t_k), x_2(t_k)) + \delta_1 \widetilde{\theta}_1^T \hat{\theta}_1 \le \\ \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2}) \widetilde{\theta}_1^T \widetilde{\theta}_1 + \frac{\delta_1}{2} \theta_1^T \theta_1\end{aligned} \tag{19}$$

The time derivative of Lyapunov function can be rewritten as

$$\dot{V}_2 \le \frac{1}{2} z_3^2 - c_1 z_1^2 - c_2 z_2^2 + \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2}) \widetilde{\theta}_1^T \widetilde{\theta}_1 + d_1 \tag{20}$$

where $d_1 = (1 + \frac{\delta_1}{2}) \theta_1^T \theta_1 + \frac{1}{2} \varepsilon_1^{*2}$.

Step 3. In accordance with Step 2, we take the Lyapunov function as

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2}\widetilde{\theta}_2^T\widetilde{\theta}_2 \tag{21}$$

where $\widetilde{\theta}_2 = \theta_2 - \hat{\theta}_2$ is the approximation error between the ideal FLS parameter vector θ_2 and its estimation variable $\hat{\theta}_2$.

Recalling the definition of the coordinate changes (4), the time derivative of (21) can be represented as

$$\begin{aligned}\dot{V}_3 \leq &\frac{1}{2}z_3^2 - c_1z_1^2 - c_2z_2^2 + \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2})\widetilde{\theta}_1^T\widetilde{\theta}_1 \\ &+ d_1 + z_3(x_4 - \dot{\alpha}_2) - \widetilde{\theta}_2^T\dot{\hat{\theta}}_2\end{aligned} \tag{22}$$

To avoid using the time derivative of the virtual control signal, the FLS is considered to approximate the $-\dot{\alpha}_2$, which yields

$$\begin{aligned}\dot{V}_3 \leq &\frac{1}{2}z_3^2 - c_1z_1^2 - c_2z_2^2 + \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2})\widetilde{\theta}_1^T\widetilde{\theta}_1 + d_1 - \widetilde{\theta}_2^T\dot{\hat{\theta}}_2 \\ &+ z_3(z_4 + \alpha_3 + \theta_2^T\varphi_2(x_1, x_2, x_3) + \varepsilon_2 + \theta_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)) \\ &- \theta_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)))\end{aligned} \tag{23}$$

Invoking Young's inequality and the property of the fuzzy basic function $0 < \varphi_2^T(\cdot)\varphi_2(\cdot) \leq 1$, one has

$$z_3z_4 + z_3\theta_2^T\varphi_2(x_1, x_2, x_3) \leq z_3^2 + \frac{1}{2}\theta_2^T\theta_2 + \frac{1}{2}z_4^2 \tag{24}$$

$$-z_3\theta_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)) + z_3\varepsilon_2 \leq z_3^2 + \frac{1}{2}\theta_2^T\theta_2 + \frac{1}{2}\varepsilon_2^{*2} \tag{25}$$

Substituting (24) and (25) into (23), one can have

$$\begin{aligned}\dot{V}_3 \leq &\frac{5}{2}z_3^2 - c_1z_1^2 - c_2z_2^2 + \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2})\widetilde{\theta}_1^T\widetilde{\theta}_1 + d_1 - \widetilde{\theta}_2^T\dot{\hat{\theta}}_2 \\ &+ \theta_2^T\theta_2 + \frac{1}{2}z_4^2 + z_3(\alpha_3 + \theta_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k))) + \frac{1}{2}\varepsilon_2^{*2}\end{aligned} \tag{26}$$

Design the virtual control signal and adaptive law as

$$\alpha_3 = -c_3z_3 - \hat{\theta}_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)) - \frac{5}{2}z_3 \tag{27}$$

$$\dot{\hat{\theta}}_2 = z_3(t_k)\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)) - \delta_2\hat{\theta}_2 \tag{28}$$

Substituting (27) and (28) into (26), we can obtain

$$\begin{aligned}\dot{V}_3 \leq &-\sum_{i=1}^{3}c_iz_i^2 + \frac{1}{4}(z_2 - z_2(t_k))^2 + (1 - \frac{\delta_1}{2})\widetilde{\theta}_1^T\widetilde{\theta}_1 + d_1 - \widetilde{\theta}_2^T\dot{\hat{\theta}}_2 \\ &+ \theta_2^T\theta_2 + \frac{1}{2}z_4^2 - (z_3(t_k) - z_3)\widetilde{\theta}_2^T\varphi_2(x_1(t_k), x_2(t_k), x_3(t_k)) \\ &+ \delta_2\widetilde{\theta}_2^T\hat{\theta}_2 + \frac{1}{2}\varepsilon_2^{*2}\end{aligned} \tag{29}$$

Using Young's inequality, the following inequality holds:

$$\begin{aligned}-(z_3(t_k) - z_3)\widetilde{\theta}_2^T\varphi_2(x_1(t_k), x_2(t_k), x_2(t_k)) + \delta_2\widetilde{\theta}_2^T\hat{\theta}_2 \leq \\ \frac{1}{4}(z_3 - z_3(t_k))^2 + (1 - \frac{\delta_2}{2})\widetilde{\theta}_2^T\widetilde{\theta}_2 + \frac{\delta_2}{2}\theta_2^T\theta_2\end{aligned} \tag{30}$$

In accordance with (29) and (30), one has

$$\dot{V}_3 \leq -\sum_{i=1}^{3}c_iz_i^2 + \frac{1}{4}\sum_{i=2}^{3}(z_i - z_i(t_k))^2 + \sum_{i=1}^{2}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i + d_2 + \frac{1}{2}z_4^2 \tag{31}$$

where $d_2 = (1 + \frac{\delta_2}{2})\theta_2^T\theta_2 + \frac{1}{2}\varepsilon_2^{*2}$.

Step 4. In this step, we consider the Lyapunov function as

$$V_4 = V_3 + \frac{1}{2}z_4^2 + \frac{1}{2}\widetilde{\theta}_3^T\widetilde{\theta}_3 \tag{32}$$

where $\widetilde{\theta}_3 = \theta_3 - \hat{\theta}_3$ is the approximation error between the ideal FLS parameter vector θ_3 and its estimation variable $\hat{\theta}_3$.

Then, the time derivative of (32) can be easily calculated as

$$\begin{aligned}\dot{V}_4 \leq &-\sum_{i=1}^{3} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{3}(z_i - z_i(t_k))^2 + \sum_{i=1}^{2}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+ d_2 + \frac{1}{2}z_4^2 + z_4(f_2(x_1, x_3, x_4, x_5) + z_5 + \alpha_4 - \dot{\alpha}_3) \\ &- \widetilde{\theta}_3^T\dot{\hat{\theta}}_3\end{aligned} \tag{33}$$

To eliminate the impact of the nonlinear function $f_2(x_1, x_3, x_4, x_5)$ and the time derivative of virtual control signal $\dot{\alpha}_3$, it can be further obtained that

$$\begin{aligned}\dot{V}_4 \leq &-\sum_{i=1}^{3} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{3}(z_i - z_i(t_k))^2 + \sum_{i=1}^{2}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+ d_2 + \frac{1}{2}z_4^2 + z_4(z_5 + \alpha_4 + \theta_3^T\varphi_3(x_1, x_2, x_3, x_4, x_5) \\ &+ \varepsilon_3 + \theta_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) \\ &- \theta_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k))) - \widetilde{\theta}_3^T\dot{\hat{\theta}}_3\end{aligned} \tag{34}$$

Then, invoking Young's inequality and the property of the fuzzy basic function $0 < \varphi_3^T(\cdot)\varphi_3(\cdot) \leq 1$ again, we obtain

$$z_4 z_5 + z_4\theta_3^T\varphi_3(x_1, x_2, x_3, x_4, x_5) \leq z_4^2 + \frac{1}{2}\theta_3^T\theta_3 + \frac{1}{2}z_5^2 \tag{35}$$

$$-z_4\theta_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) + z_4\varepsilon_3 \leq z_4^2 + \frac{1}{2}\theta_3^T\theta_3 + \frac{1}{2}\varepsilon_3^{*2} \tag{36}$$

By inserting (35) and (36) into (34), it is not difficult to obtain

$$\begin{aligned}\dot{V}_4 \leq &-\sum_{i=1}^{3} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{3}(z_i - z_i(t_k))^2 + \sum_{i=1}^{2}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+ d_2 + \frac{5}{2}z_4^2 + \theta_3^T\theta_3 + \frac{1}{2}z_5^2 + \frac{1}{2}\varepsilon_3^{*2} + z_4(\alpha_4 \\ &+ \theta_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k))) - \widetilde{\theta}_3^T\dot{\hat{\theta}}_3\end{aligned} \tag{37}$$

Thereafter, we configure the virtual control signal and adaptive law as

$$\alpha_4 = -c_4 z_4 - \hat{\theta}_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) - \frac{5}{2}z_4 \tag{38}$$

$$\dot{\hat{\theta}}_3 = z_4(t_k)\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) - \delta_3\hat{\theta}_3 \tag{39}$$

Substituting the virtual control signal (38) and adaptive law (39) into (37) gives

$$\begin{aligned}\dot{V}_4 \leq &-\sum_{i=1}^{4} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{3}(z_i - z_i(t_k))^2 + \sum_{i=1}^{2}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+ d_2 + \theta_3^T\theta_3 + \frac{1}{2}\varepsilon_3^{*2} - (z_4(t_k) - z_4)\widetilde{\theta}_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) \\ &+ \delta_3\widetilde{\theta}_3^T\hat{\theta}_3\end{aligned} \tag{40}$$

Based on Young's inequality, the following inequality holds:

$$\begin{aligned}-(z_4(t_k) - z_4)\widetilde{\theta}_3^T\varphi_3(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k)) + \delta_3\widetilde{\theta}_3^T\hat{\theta}_3 \leq \\ \frac{1}{4}(z_4 - z_4(t_k))^2 + (1 - \frac{\delta_3}{2})\widetilde{\theta}_3^T\widetilde{\theta}_3 + \frac{\delta_3}{2}\theta_3^T\theta_3\end{aligned} \tag{41}$$

Combining (29) and (30) gives

$$\dot{V}_4 \leq -\sum_{i=1}^{4} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{4}(z_i - z_i(t_k))^2 + \sum_{i=1}^{3}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i + d_3 + \frac{1}{2}z_5^2 \tag{42}$$

where $d_3 = (1 + \frac{\delta_3}{2})\theta_3^T\theta_3 + \frac{1}{2}\varepsilon_3^{*2}$.

Step 5. In the final step, the Lyapunov function is chosen as

$$V_5 = V_4 + \frac{1}{2}z_5^2 + \frac{1}{2}\widetilde{\theta}_4^T\widetilde{\theta}_4 \tag{43}$$

where $\widetilde{\theta}_4 = \theta_4 - \hat{\theta}_4$ is the approximation error between the ideal FLS parameter vector θ_4 and its estimation variable $\hat{\theta}_4$.

The time derivative of (43) is given as

$$\begin{aligned}\dot{V}_5 \leq &-\sum_{i=1}^{4} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{4}(z_i - z_i(t_k))^2 + \sum_{i=1}^{3}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+d_3 + \frac{1}{2}z_5^2 + z_5(f_3(x_4, x_5) + \frac{u}{L} - \dot{\alpha}_4) - \widetilde{\theta}_4^T\dot{\hat{\theta}}_4\end{aligned} \tag{44}$$

Likewise, FLS is employed to approximate the function $g_3(x_1, x_2, x_3, x_4, x_5) = f_3(x_4, x_5) - \dot{\alpha}_4$, and (44) can be rewritten as

$$\begin{aligned}\dot{V}_5 \leq &-\sum_{i=1}^{4} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{4}(z_i - z_i(t_k))^2 + \sum_{i=1}^{3}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+d_3 + \frac{1}{2}z_5^2 + z_5(\theta_4^T\varphi_4(x_1, x_2, x_3, x_4, x_5) + \varepsilon_4 \\ &+\theta_4^T\varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k)) \\ &-\theta_4^T\varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k)) \\ &+\frac{u}{L}) - \widetilde{\theta}_4^T\dot{\hat{\theta}}_4\end{aligned} \tag{45}$$

Furthermore, Young's inequality and the property of the fuzzy basic function $0 < \varphi_4^T(\cdot)\varphi_4(\cdot) \leq 1$ are considered, and one has

$$z_5\theta_4^T\varphi_4(x_1, x_2, x_3, x_4, x_5) \leq \frac{1}{2}\theta_4^T\theta_4 + \frac{1}{2}z_5^2 \tag{46}$$

$$\begin{aligned}&-z_5\theta_4^T\varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k)) + z_4\varepsilon_3 \\ &\leq z_5^2 + \frac{1}{2}\theta_4^T\theta_4 + \frac{1}{2}\varepsilon_4^{*2}\end{aligned} \tag{47}$$

By inserting (46) and (47) into (45),

$$\begin{aligned}\dot{V}_5 \leq &-\sum_{i=1}^{4} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{4}(z_i - z_i(t_k))^2 + \sum_{i=1}^{3}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T\widetilde{\theta}_i \\ &+d_3 + 2z_5^2 + \theta_4^T\theta_4 + \frac{1}{2}\varepsilon_4^{*2} + z_5(\theta_4^T\varphi_4(x(t_k)) \\ &+\frac{u}{L} + \frac{v}{L} - \frac{v}{L}) - \widetilde{\theta}_4^T\dot{\hat{\theta}}_4\end{aligned} \tag{48}$$

where $x = [x_1, x_2, x_3, x_4, x_5]$.

The continuous controller and adaptive law are designed as

$$v = L(-c_5 z_5 - \hat{\theta}_4^T\varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k)) - \frac{5}{2}z_5) \tag{49}$$

$$\dot{\hat{\theta}}_4 = z_5(t_k)\varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k)) - \delta_4\hat{\theta}_4 \tag{50}$$

Plugging the continuous controller (49) and adaptive law (50) into (48), (48) can be transformed as

$$
\begin{aligned}
\dot{V}_5 \leq &-\sum_{i=1}^{5} c_i z_i^2 + \tfrac{1}{4}\sum_{i=2}^{4}(z_i - z_i(t_k))^2 + \sum_{i=1}^{3}(1 - \tfrac{\delta_i}{2})\widetilde{\theta}_i^T \widetilde{\theta}_i \\
&+ d_3 - \tfrac{1}{2}z_5^2 + \theta_4^T \theta_4 + \tfrac{1}{2}\varepsilon_4^{*2} + \delta_4 \widetilde{\theta}_4^T \hat{\theta}_4 + z_5(\tfrac{u}{L} - \tfrac{v}{L}) \\
&- (z_5(t_k) - z_5)\widetilde{\theta}_4^T \varphi_4(x_1(t_k), x_2(t_k), x_3(t_k), x_4(t_k), x_5(t_k))
\end{aligned}
\tag{51}
$$

Similar to the previous steps, by using Young's inequality, we have

$$
\begin{aligned}
&\delta_4 \widetilde{\theta}_4^T \hat{\theta}_4 + z_5(\tfrac{u}{L} - \tfrac{v}{L}) - (z_5(t_k) - z_5)\widetilde{\theta}_4^T \varphi_4(x(t_k)) \\
&\leq \tfrac{1}{4}(z_5 - z_5(t_k))^2 + (1 - \tfrac{\delta_4}{2})\widetilde{\theta}_4^T \widetilde{\theta}_4 + \tfrac{\delta_4}{2}\theta_4^T \theta_4 + \tfrac{1}{2}z_5^2 + \tfrac{1}{2}(\tfrac{u}{L} - \tfrac{v}{L})^2
\end{aligned}
\tag{52}
$$

With the help of (52), (51) can be rewritten as

$$
\dot{V}_5 \leq -\sum_{i=1}^{5} c_i z_i^2 + \frac{1}{4}\sum_{i=2}^{5}(z_i - z_i(t_k))^2 + \sum_{i=1}^{4}(1 - \frac{\delta_i}{2})\widetilde{\theta}_i^T \widetilde{\theta}_i + d_4 + \frac{1}{2}\left(\frac{u}{L} - \frac{v}{L}\right)^2
\tag{53}
$$

where $d_4 = (1 + \frac{\delta_4}{2})\theta_4^T \theta_4 + \frac{1}{2}\varepsilon_4^{*2}$.

Remark 1. *In the sampled data controller design, the virtual controllers (8), (16), (27), (38) and actual control signal (49) consist of continuous state information. To further save more network transmission times of the state's information, the armature voltage u of the flexible robotic manipulator is designed to update and calculate at the sampling time, which means the designed virtual controller and actual control signal only calculate at the sampled time.*

Theorem 1. *Under Assumption 1, for the bounded condition $V_{t_0} \leq Q$ at the initial time t_0, the proposed sampled data control method can guarantee that all signals of the single-link flexible joint robot model (1) are SGUUB.*

Proof. For the sake of proving the stability of the single-link flexible joint robot model (1), we denote variable $\chi(t) = [x_1(t), \cdots, x_5(t), \theta_1(t), \cdots, \theta_4(t)]^T$. For convenience, we define variable $\chi = \chi(t)$ in the sequel. Subsequently, we refer to the Gronwall–Bellman inequality of [24]; at the sampling period $[t_k, t_{k+1})$, the variable χ satisfies

$$
\|\chi - \chi(t_k)\| \leq \overline{\eta}(\sqrt{V_5(t_k)} + 1)(e^{\eta(t - t_k)} - 1)
\tag{54}
$$

where η and $\overline{\eta}$ are positive constants.

Therefore, when the control activity under the period $[t_k, t_{k+1})$ is in accordance with (54), we have $|x_i - x_i(t_k)| \leq \|\chi - \chi(t_k)\|$, $|\theta_j - \theta_j(t_k)| \leq \|\chi - \chi(t_k)\|$, $i = 1, 2, 3, 4, 5$, $j = 1, 2, 3, 4$. Thereafter, we have

$$
|u - v| \leq \ell\overline{\eta}(e^{\eta(t - t_k)} - 1)\sqrt{V_5(t_k)} + \ell\overline{\eta}(e^{\eta(t - t_k)} - 1)
\tag{55}
$$

where $\xi_1 = c_1 + \frac{1}{2}$, $\xi_i = c_i + \frac{5}{2}(i = 2, 3, 4)$, $\xi_5 = L(c_5 + \frac{5}{2})$ $\xi_6 = 1$, $\overline{\xi}_i = \prod_{j=i}^{5}\xi_j$, $\vartheta = \sum_{i=1}^{5}\overline{\xi}_i$, $\nu = \sum_{i=3}^{6}\overline{\xi}_i$ and $\ell = \vartheta + \nu$.

On the basis of (55) and considering (53), we arrive at

$$
\begin{aligned}
\dot{V}_5 \leq &-\varsigma V_5 + d_4 + \tfrac{1}{2L}(\chi^2 \overline{\eta}^2 (e^{\eta(t - t_k)} - 1)^2 V_5(t_k) \\
&+ \chi^2 \overline{\eta}^2 (e^{\eta(t - t_k)} - 1)^2 \sqrt{V_5(t_k)} + \chi^2 \overline{\eta}^2 (e^{\eta(t - t_k)} - 1)^2)
\end{aligned}
\tag{56}
$$

where $\varsigma = \min\limits_{i\in\Delta}(c_i, \frac{\delta_i}{2} - 1)(\Delta = \{1, 2, 3, 4, 5\})$, $\vartheta_c = \frac{1}{4}\sum\limits_{i=1}^{4}(1 + \hat{\xi}_i)$, $\hat{\xi}_i = \sum\limits_{j=1}^{i}\prod\limits_{k=j}^{i}\xi_k$,

$\nu_c = \frac{1}{4}(1 + \sum\limits_{i=3}^{4}(1 + \widetilde{\xi}_i))$, $\widetilde{\xi}_i = \sum\limits_{j=3}^{i}\prod\limits_{k=j}^{i}\xi_k$, $\ell_c = \vartheta_c + \nu_c$ and $\lambda = \ell_c + \ell$.

Under the proposed sampling theory, there exists a designed fixed sampling period h between each sampling activity. At the initial sampling interval (t_0, t_1), on the basis of the above analysis, we can obtain

$$\begin{aligned}
\dot{V}_5 &\leq -\varsigma V_5 + d_4 + \frac{1}{2L}(\lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2 Q \\
&\quad + \lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2 Q + \lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2)
\end{aligned} \tag{57}$$

Subsequently, it is simple to get

$$V_5(t_1^-) \leq e^{-ch}(V(t_0) - \frac{D}{\varsigma}) + \frac{D}{\varsigma} \tag{58}$$

where $D = d_4 + \frac{1}{2L}(\lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2 + \lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2 Q + \lambda^2\bar{\eta}^2(e^{\eta h} - 1)^2)$.

Repeating the process of (58) again, one has

$$V_5(t_{k+1}^-) \leq e^{-ch}(V(t_k) - \frac{D}{\varsigma}) + \frac{D}{\varsigma}, (\forall t \in (t_k, t_{k+1})) \tag{59}$$

Case 2.

At the second sampling time t_1, one directly gets

$$V_5(t_1) \leq e^{-ch}(V(t_0) - \frac{d_4}{\varsigma}) + \frac{d_4}{\varsigma} \tag{60}$$

Then, according to (60), one has

$$V_5(t_{k+1}) \leq e^{-ch}(V(t_k) - \frac{d_4}{\varsigma}) + \frac{d_4}{\varsigma}, (\forall t \in (t_k, t_{k+1})) \tag{61}$$

Therefore, for a fixed sampling period h, the Lyapunov function satisfies

$$V_5(t) \leq e^{-c(t-t_0)}(Q - \frac{d_4}{\varsigma}) + \frac{D}{\varsigma}(t \in [t_0, +\infty)) \tag{62}$$

When $t \to \infty$, we have $V_5(t) \leq \frac{D}{\varsigma}$, since Theorem 1 is proven.

The proof of Theorem 1 is completed. $\square$

Remark 2. *In order to guarantee the bounded $\frac{D}{\varsigma}$ is small enough, we can adjust the designed constants $c_i(i = 1, 2, 3, 4, 5)$ and $\delta_i(i = 1, 2, 3, 4)$ to increase the value of ς. In the same way, we can choose a smaller sampled period h to decrease the value of D to get the better control performance.*

4. Simulation Results

In this section, the parameters of the flexible robotic manipulator (1) are given in Table 1.

As in Section 3, FLS is considered to address the unknown nonlinear functions. The corresponding fuzzy membership functions (see Figure 1) are chosen as

$$\mu_{F_2^l}(x_i) = e^{[\frac{(x_i+3-l)^2}{4}]}(i = 1, 2), \ \mu_{F_3^l}(x_j) = e^{[\frac{(x_j+3-l)^2}{4}]}(j = 1, 2, 3)$$

$$\mu_{F_3^l}(x_k) = e^{[\frac{(x_k+3-l)^2}{4}]}(k = 1, 2, 3, 4), \ \mu_{F_4^l}(x_w) = e^{[\frac{(x_w+3-l)^2}{4}]}(w = 1, 2, 3, 4, 5)$$

$$(l = 1, 2, 3, 4, 5)$$

Table 1. The parameters of flexible robotic manipulator.

Symbol	Value	Unit	Symbol	Value	Unit
F_1	3	Nms/rad	F_2	3	Nms/rad
J_1	30	kg·m^2	J_2	30	kgm^2
R	10	Ω	K_a	3	Nm/A
L	6	H	K_t	0.976	Nm/A
d	0.4	m	m	0.102	kg
g	9.8	N/kg	K	20	\
N	1	\			
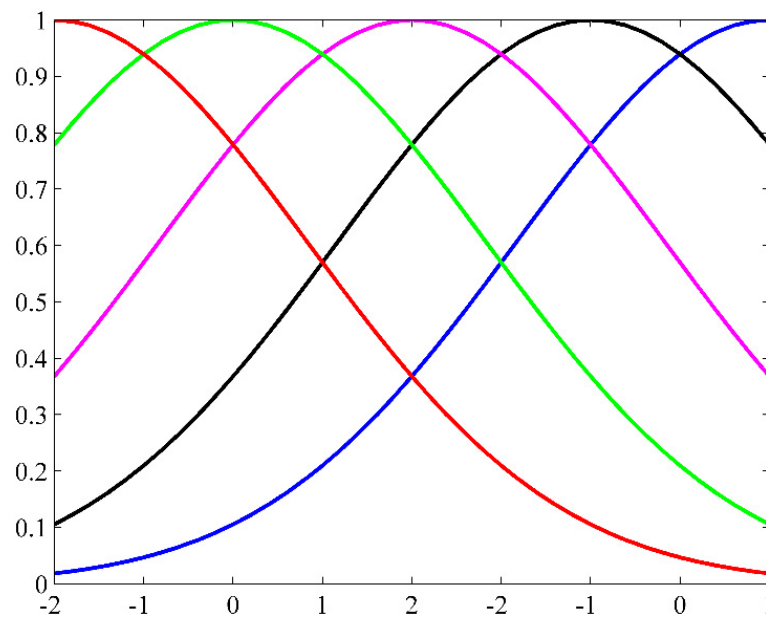
Figure 1. The fuzzy rules.

The initial values of the state are chosen as $x_1(0) = x_2(0) = x_3(0) = x_4(0) = x_5(0) = 0.1$, and the initial values for other variable are set as 1. Under the proposed backstepping sampled data control method, we design the virtual control signals (8), (16), (27), and (38), the adaptive updated laws (17), (28), (39), and (50), and actual control input (49), and the related parameters are designed as $c_1 = c_2 = c_3 = c_4 = 0.3$, $c_5 = 1$ and $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 5$. The fixed sampled data period is chosen as 0.2 s. The simulation results are shown in Figures 2–5.

Form Figures 2–8, we can see that the system states and the adaptive parameters converge on a small residual set. Besides, the controlled performance is further shown in Figures 2–7 by choosing the different sampling periods and the continuous case. According to Figures 2–6, we can conclude that at the inflection point of each state, the state changes faster when the sampling period is smaller. The reason for this phenomenon is that the sampled data controller is executed by the ZOH, which cannot adjust itself in time. In Figure 7, the control signals with a different sampling period and the continuous case are given. The control input signal is utilized for the controller plant in discrete form. Rather than the continuous form in [10–20], the designed control signal releases the constraints of the continuous state's information.
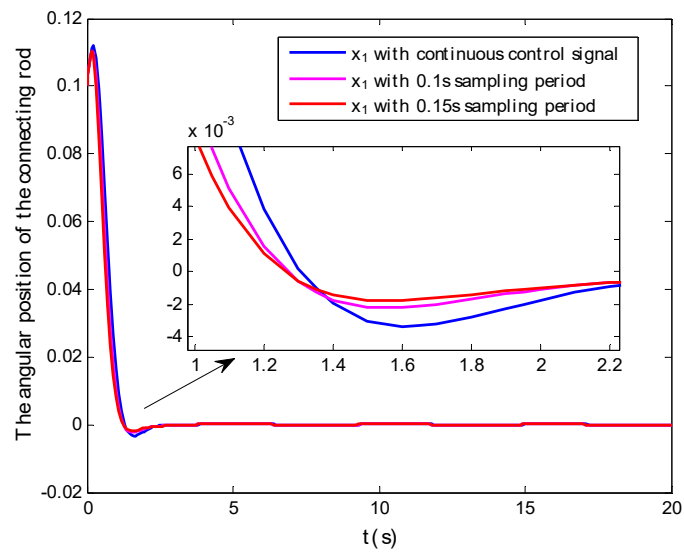
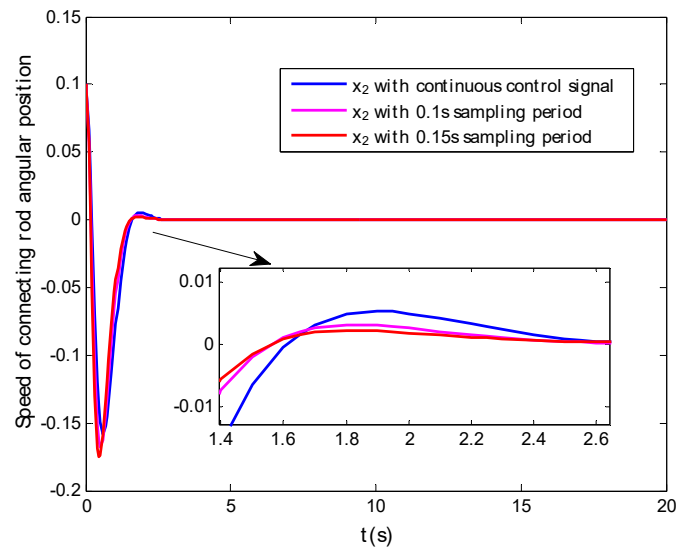**Figure 2.** The trajectories of state x_1 with different sampling periods.

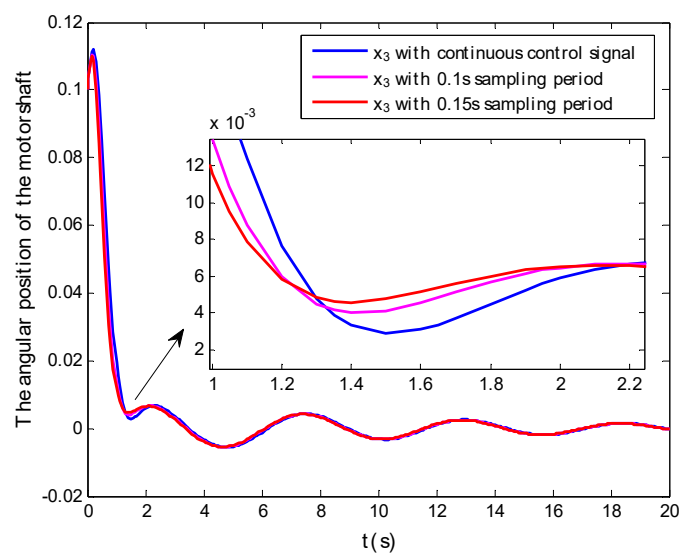**Figure 3.** The trajectories of state x_2 with different sampling periods.

Figure 4. The trajectories of state x_3 with different sampling periods.
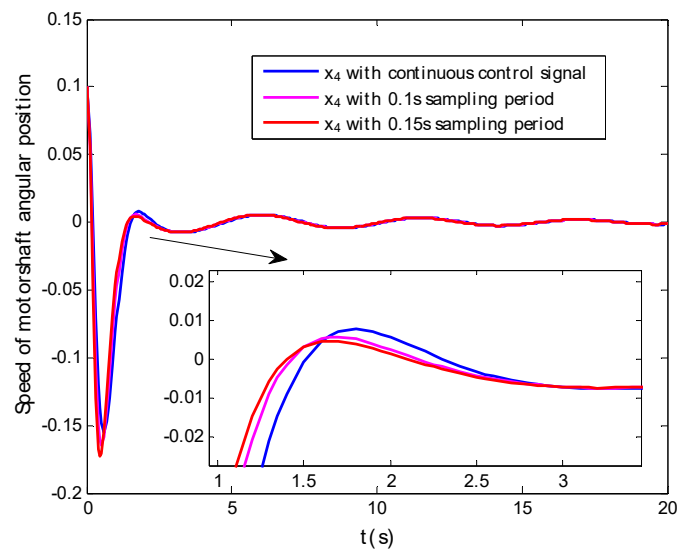
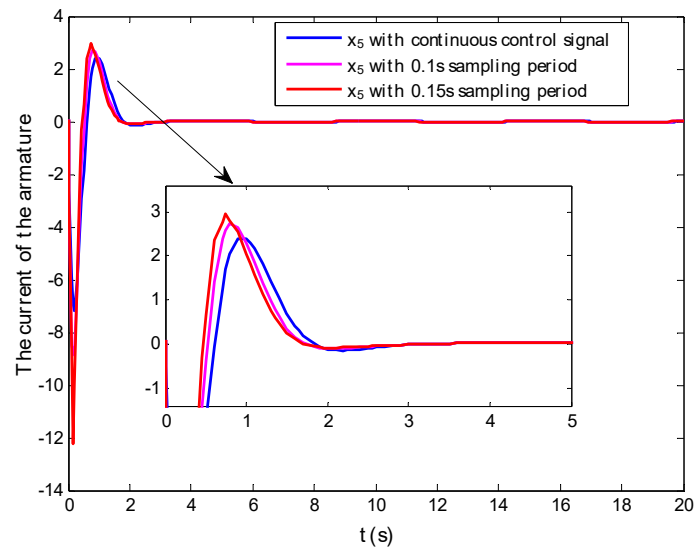**Figure 5.** The trajectories of state x_4 with different sampling periods.

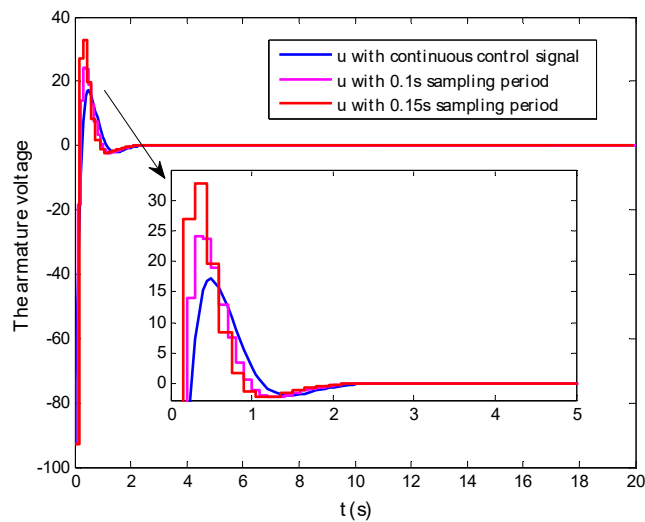**Figure 6.** The trajectories of state x_5 with different sampling periods.

Figure 7. The trajectories of sampled control signal u with different sampling periods.
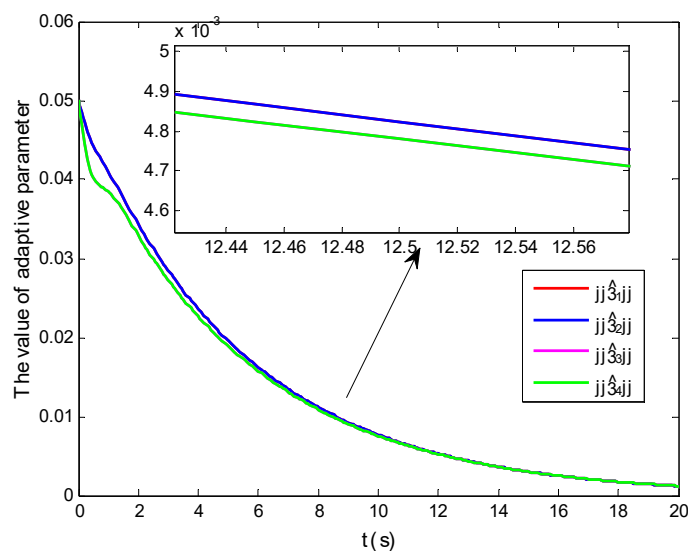
Figure 8. The trajectories of adaptive parameters $\hat{\theta}_i (i = 1, 2, 3, 4)$.

5. Conclusions

This paper studied the backstepping sampled data control method for a flexible robotic manipulator, whose internal parameters are unknown, and FLS was employed to eliminate the uncertainties. Furthermore, in the control design, we only used the sampled data of the states to design the control signal, which means we can use less information to achieve control objective than the normal ones. By considering the SGUUB stability theory, all signals of the flexible robotic manipulator are covered within a small neighborhood of the origin. The simulation results are also given to illustrate the validity of the sampled data control algorithms. Our future research direction is output feedback sampled data control for the flexible robotic manipulator.

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