

## Article

# Research on a Control Strategy for a Distributed Economic Dispatch System in an Active Distribution Network, Considering Communication Packet Loss

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**Abstract:** The benefits of a distributed economic dispatch system include significant resilience, uniform processing and communication pressure, and good scalability, but, currently, there is limited in-depth research on the impact of packet loss on the communication links of a distributed economic dispatch system. To analyze how communication packet loss affects a distributed economic dispatch system and to develop appropriate scheduling approaches, in this paper, we design a fully distributed economic dispatch strategy for an active distribution network (ADN) in leader-follower mode, we use the Bernoulli stochastic process to describe the communication packet loss phenomenon, and we give sufficient conditions for the system in the form of linear matrix inequality (LMI) to be able to achieve stochastic stability. The communication packet loss probability affects the convergence performance of the system, and a high packet loss probability can cause the system to lose stability. By simulating various situations in the IEEE-14 and IEEE-39 systems, in this study, we confirm the validity and efficacy of the distributed economic dispatch control strategy and the system stability analysis method.



**Citation:** Le, J.; Qi, G.; Zhao, L.; Jin, R. Research on a Control Strategy for a Distributed Economic Dispatch System in an Active Distribution Network, Considering Communication Packet Loss.

*Electronics* **2022**, *11*, 3288. <https://doi.org/10.3390/electronics11203288>

Academic Editor:  
Antonio Moreno-Munoz

Received: 5 August 2022  
Accepted: 24 September 2022  
Published: 12 October 2022

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**Keywords:** economic dispatch; consensus algorithm; communication packet loss; Bernoulli stochastic process; Lyapunov stability; LMI

## 1. Introduction

One of the most fundamental problems of power systems is economic dispatch (ED), which involves determining the output of multiple generating units to meet the total load demand and other constraints, while minimizing the total generation cost. This issue has attracted a lot of attention and impressive results have been achieved in terms of dispatch models and optimization algorithms [1,2]. Due to the increasing complexity and scale of power systems and the rapid development of distributed generations (DGs) with high-permeability distributed power, the economic dispatch of a traditional, centralized distribution network cannot meet the requirements of high system reliability and enough topology flexibility, and there is significant pressure on the dispatch center for calculations and communication [3]. An ADN is a distribution network with energy sources that are internally dispersed or decentralized, as well as active control and operational capabilities. ED in an ADN is a subject that has been studied by many scholars. At present, distribution systems have gained increasing attention in ADN economic dispatch systems due to good scalability, uniform computational and communication pressure, and greater robustness [4–6].

A study by [7] demonstrated that an optimal solution to the economic dispatch problem could be obtained when the cost of each generation unit increased at a consistent rate. A consensus algorithm was designed, which could solve the economic dispatch problem in a distributed way on the premise of connected communication topology. The method was independent of the initial power distribution and could automatically respond to real-time load demand changes. A study by [8] proposed a leader-follower consensus distributed economic dispatch method, with the incremental cost of each generator as the consensus

variable. In this mode, a “leader” node was selected to obtain the information (system power difference), while the other generating unit nodes exchanged the consensus variable via the communication network and calculated the updated power output autonomously according to the consensus protocol. Finally, the optimal solution was obtained under the condition that the constraints were satisfied. Based on the leader-follow model, [9] selected different power generation nodes as “leaders” in the same topology by combining social network and graph theory knowledge, analyzed the influence of different “leader” choices on algorithm convergence speed, and revealed the relationship between leader position selection and algorithm convergence performance. A distributed economic dispatch method that could automatically respond to real-time load demand changes, independent of the initial power allocation, was proposed by [10]. The method obtained a system incremental cost and an optimal allocation that depended only on the total required load and the quadratic, primary, and constant term coefficients of each DG generation cost function. The authors of [11] proposed a distributed control strategy based on distributed power sources active power sharing and voltage balancing, using a distributed consistency protocol in a distribution network with main network power scheduling, in which the injected power track was a given power reference value for scheduling purposes. The authors of [12] proposed a distributed economic dispatch control strategy based on virtual generators, using the virtual generators as the leader to obtain power information at the point of common coupling (PCC) of the grid in real time. While it analyzed the impact of individuals passing incorrect values of the consensus variable to improve their own economic efficiency, it introduced suitable correction vectors to eliminate this impact, and therefore, the economic dispatch reached optimal values.

The optimization dispatch algorithm based on a consensus algorithm in the abovementioned studies is mostly composed of “consensus term + adjustment term”. An adjustment term is used to correct the imbalance value of supply and demand of the system; therefore, the consensus variables in the algorithm tend to be the same value. However, the power imbalance value is difficult to obtain and highly dependent on the performance of the leader, and most of the studies have been concerned with the economic dispatch under ideal communication conditions. Current research on multi-intelligent packet loss has usually assumed a Bernoulli distribution model with independent identical distribution or a Markov channel model and used the stochastic theory to analyze the stability [13–19]. The authors of [13] investigated stochastic consistency conditions for first-order and second-order discrete multi-intelligent systems with time delay and multiplicative measurement noise, but the method relied heavily on the order and structure of the system matrix and could not be applied in generalized linear intelligent body systems. Research by [14–16] analyzed the conditions that the leader-follower consensus algorithm could converge to consensus in the multi-agent system with switching and fixed topology and measurement noise and communication delay, but there was no consideration of random communication packet loss. In [17], the authors proposed that for a communication network with random packet loss, in order to ensure its consensus, the expected communication topology with a spanning tree, and also the packet loss probability must have an upper bound. The authors of [18] established a micro-network economic scheduling model based on the consensus algorithm and proposed a distributed economic scheduling algorithm that considered the reliability of the communication line on this basis and simulated the algorithm from the cases of communication packet loss rate of 1%, 2.5%, 5%, etc., and it analyzed the impact of communication line packet loss on the convergence performance of the algorithm. Meanwhile, they also proposed a fault correction mechanism based on the similarity measure method to reduce the impact of packet loss on the algorithm. A study by [19] comprehensively considered transmission delay, noise, time-varying topology, and the problem of plug and play, and proposed the consensus gain function and effectively suppressed the delay and noise of information transmission between agents. However, the model did not solve the problem of obtaining the power imbalance value and the dependence on

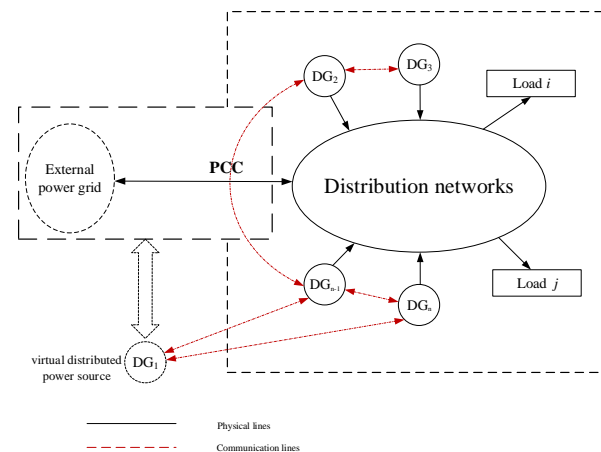
the leader performance, and no requirements were provided for the convergence of the associated management strategy.

As a result, considerable research has been carried out on distributed economic dispatch algorithms based on the consensus protocol, but, so far, there is no in-depth quantitative analysis of the impact of packet loss in the communication links of economic dispatch systems. To examine how a distributed economic dispatch system is impacted by communication packet loss and to provide suitable scheduling strategies, in this study, a distributed economic dispatch control strategy for an active distribution network in leader-follower mode is designed based on the stochastic theory using the incremental cost as the consensus variable and the Bernoulli stochastic process to describe the packet loss phenomenon, considering the coupling of the power difference between the total generated power and the load power with the consistency variable, while using the Lyapunov stability theorem and the Schur complementary theorem. Sufficient conditions are given for the LMI form to enable the system to achieve stochastic stability. Finally, the correctness and validity of the control strategy and sufficient conditions in the form of LMI to enable the system to achieve stochastic stability are verified by simulation.

## 2. Design of a Distributed Economic Dispatch System for ADN

### 2.1. Distributed Economic Dispatch Architecture

An active distribution network economic dispatch system requires the sum of the total active power output of all the DGs in the distribution network and the active power input from the external network to balance the active power demand of the total distribution network load. It also minimizes the total generation cost of the DGs in the distribution network while satisfying other constraints. In this paper, we adopt an ADN dispatch system, as shown in Figure 1, to achieve the above goals.



**Figure 1.** Active distribution network dispatch system.

The ADN is connected to the external network, and the DGs can communicate with each other as well as different loads. The PCC, the common connection point between the external grid and the active distribution grid, monitors the power flow exchange between the two in real time. The power difference between the total generated power and the load power is an important piece of information to enable a distributed economic dispatch control strategy, and, in this paper, we provide this information indirectly by measuring the exchanged power  $P_{PCC}$  at the PCC, which can be measured by the distribution network regulator.

The regulator of each DG makes independent decisions according to the control strategy based on local measurement information, such as the respective incremental cost  $\lambda_i$  and output power  $P_{DG,i}$  of the respective DG, combining the information sent by other agents via the communication link and the system power difference information, etc. Then, the regulator sends the power command  $P_{DG,i}$  to the DG that it regulates and sends its own

information to the other DG regulators, so that the active power balance of the distribution network is achieved through the cooperation of the DGs and the incremental cost of each DG is equal in the end, thus, realizing the distributed economic dispatch control strategy.

In order to facilitate the incorporation of the power exchange information between the distribution network and the external network into the designed distributed economic dispatch control strategy in a uniform form, in this paper, we provide the distribution network regulator with a “virtual distributed power source” and set the number of this “virtual distributed power source” to one. The virtual DG has no actual power output and does not need to send commands to the DG.

## 2.2. Distributed Economic Dispatch Mode

The objective function for minimizing the total cost of generation is:

$$\min F = \sum_{i=1}^n F_i(P_{DG,i}) \quad (1)$$

where  $n$  is the total number of DGs in the distribution network;  $P_{DG,i}$  and  $F(P_{DG,i})$  are the output power and power-cost function of the  $i$ th DG, respectively. Distributed energy sources such as wind turbines and photovoltaics can be formed into dispatchable DGs by adding energy storage devices, whose power-cost functions are mostly in the form of quadratic convex functions, i.e.,:

$$F_i(P_{DG,i}) = a_i P_{DG,i}^2 + b_i P_{DG,i} + c_i \quad (2)$$

where  $a_i$ ,  $b_i$ , and  $c_i$  are the quadratic, primary, and constant term coefficients of the  $i$ -th DG generation cost function, respectively, and  $a_i > 0$ . For a virtual DG, its power-cost function is constant at zero and has no real power output.

The equation constraint condition of the dispatch model is power balance, namely:

$$\sum_{i=1}^n P_{DG,i} = P_D + P_{LOSS} - P_{PCC} \quad (3)$$

where  $P_D$  is the total active distribution network load,  $P_{LOSS}$  is the active power lost of the distribution network, and  $P_{PCC}$  is the active power input from the external grid.

The linear inequality constraint for the economic dispatch model is:

$$P_{DG,i}^{min} \leq P_{DG,i} \leq P_{DG,i}^{max} \quad (4)$$

where  $P_{DG,i}^{min}$  and  $P_{DG,i}^{max}$  are the lower limit and upper limit of output of the  $i$ -th DG, respectively.

This constrained economic dispatch problem can be solved using the Lagrange multiplier method, and the Lagrange operator  $L$  is specified as follows:

$$L(P_{DG,i}, \lambda) = F + \lambda \Delta P \quad (5)$$

where  $\lambda$  is the Lagrange multiplier, which also indicates the generation incremental cost for each DG; the equation constrains  $\Delta P = P_D + P_{LOSS} - \sum_{i=1}^n P_{DG,i}$  and indicates the power imbalance difference for this power system.

Take the partial derivatives of the economic scheduling function  $L(P_{DG,i}, \lambda)$  with respect to  $P_{DG,i}$  and  $\lambda$  and set them equal to zero, then, the optimal solution is solved as follows [10]:

$$\lambda^* = (P + \sum_{i=1}^n \frac{b_i}{2a_i}) / (\sum_{i=1}^n \frac{1}{2a_i}) \quad (6)$$

$$P_{DG,i}^* = (\lambda^* - b_i) / (2a_i) \quad (7)$$

where  $P$  is the total network power demand in the ADN.

### 3. Distributed Economic Dispatch System Model Considering Communication Packet Loss

#### 3.1. Packet Loss Modeling for Multi-Intelligence Communications

In this paper, we use a distributed consensus protocol to complete the exchange of information between the DG agents.

The topology of a multi-intelligence network containing  $n$  agents can be represented by a graph  $G = (V, E)$ , where  $V = \{1, 2, \dots, n\}$  is the set of vertices;  $E \subseteq V \times V$  is the set of edges; and the ordered node pair  $(i, j) \in E$  denotes a directed edge starting at node  $i$  and ending at node  $j$ , i.e., agent  $j$  can receive messages from agent  $i$ . Define  $N^+_i = \{j \in V: (i, j) \in E\}$  and  $N^-_i = \{j \in V: (j, i) \in E\}$  as the set of information output and input neighbor nodes, respectively. For an undirected graph  $N^+_i = N^-_i$ , the set of neighbors of vertex  $i$  is uniformly denoted by  $N_i = \{j \in V: (i, j) \in E\}$  and  $|N_i|$  denotes the number of elements in the set, also known as the degree of node  $i$ , denoted as  $D_i$ .

For a first-order linear system, the dynamics of the multi-agent system consensus protocol (control strategy of node  $i$ ) can be expressed as:

$$x_i(k+1) = x_i(k) + u_i(k) \quad (8)$$

$$u_i(k) = \omega \sum_{v_j \in N_i(k)} a_{ij}(x_j(k) - x_i(k)) \quad (9)$$

where  $\omega$  is the feedback factor;  $N_i(k)$  denotes the input type domain of agent  $i$  at moment  $k$ , agent  $i$  receives information from the agent in that domain at moment  $k$ ;  $x_i(k)$  is the state of the  $i$ -th agent at moment  $k$ ;  $a_{ij} > 0$  is the weight set by node  $i$  for the state information received from neighboring node  $j$ . In this paper, we select the following scheme for setting information weights  $a_{ij}$ :

$$a_{ij} = \begin{cases} 1, & j \in N_i \\ 0, & \text{others} \end{cases} \quad (10)$$

The normal operation of a multi-agent system depends on the state of the communication network. In practice, the communication links between the agents suffer from packet loss, delays, noise, and restricted bandwidth.

In this paper, we conduct research on the communication packet loss situation. There are currently three control design approaches for communication packet loss, namely the zero-setting method, the zero-order retainer method, and the predictive control method [20]. When a communication packet loss occurs: The zero-setting method sets the information received on that communication line to zero; using the zero-order retainer method, the information received from that communication line remains at the last received value until new data are received; and with the predictive control method, the agent uses a type of predictive algorithm to generate data to replace the information received from that communication line.

Since the zero-setting method has the advantages of simple implementation and system analysis, no loss of communication efficiency, and small information storage space requirements, in this paper, we chose to use the zero-setting method.

The communication between intelligences is propagated through communication links, and the information exchange between different DGs is often independent of each other. In distributed power systems, external information attacks, aging failure of communication links, or interface instability can lead to communication packet loss, which can be regarded as a random occurrence; therefore, we consider a multi-intelligent system under asynchronous communication packet loss, where the packet loss of all communication lines is a random event with independent identical distribution (i.i.d.), and we adopt a Bernoulli distribution communication packet loss model based on the zero-setting method.

Taking agent  $j$  sending a message to agent  $i$  as an example, the stability of the communication line  $(v_j, v_i)$  is represented by  $\gamma_{ij}$ . When the line is stable,  $\gamma_{ij} = 1$  and, when the line

experiences packet loss,  $\gamma_{ij} = 0$ .  $\gamma_{ij}$  uses a Bernoulli distribution with an expectation of  $p_{ij}$ , i.e.,:

$$\begin{cases} P(\gamma_{ij} = 1) = p_{ij} \\ P(\gamma_{ij} = 0) = 1 - p_{ij} \end{cases} \quad (11)$$

In this paper, assuming that the model shown in Equation (11) is used for packet loss on each communication channel of the communication network, Equation (9) can be rewritten under the zero-setting method as:

$$u_i(k) = \omega \sum_{j \in N_i} \gamma_{ij}(k) a_{ij} (x_j(k) - x_i(k)) \quad (12)$$

**Assumption 1.** For any pair of agents  $i$  and  $j$ , the communication channel between them exists or disappears at the same time.

Assumption 1 guarantees that the communication topology is always biaxially symmetric, in a dynamic process such that the Laplace matrix  $\mathbf{L}(k)$  of the system is a symmetric matrix.

The elements of the system Laplacian matrix  $\mathbf{L}(k) = [l_{ij}(k)] \in R^{n \times n}$  corresponding to Equation (12) are:

$$l_{ij}(k) = \begin{cases} -\gamma_{ij}(k) a_{ij}, i \neq j \\ \sum_{j=1, v_j \in N_i}^n \gamma_{ij}(k) a_{ij}, i = j \end{cases} \quad (13)$$

### 3.2. Distributed Economic Dispatch Strategy

With the incremental cost  $\lambda$  of the distributed power as the consensus variable, the distributed economic dispatch strategy considering the virtual DG is:

$$\begin{cases} u_i(k) = -\omega \sum_{j=1}^n l_{ij}(k) \lambda_j(k) + d_i \varepsilon \Delta f_p(k) \\ \lambda_i(k+1) = \lambda_i(k) + u_i(k) \end{cases} \quad (14)$$

where  $\lambda_i(k)$  and  $u_i(k)$  are the incremental cost and control input of  $DG_i$  at moment  $k$ , respectively;  $d_i$  is the leader identifier,  $d_i = 1$  means that this virtual DG is the leader and for other distributed power supplies there is  $d_i = 0$ ;  $\omega$  takes values in the range  $0 < \omega < 1/\Delta$  and  $\Delta = \max\{|N_1|, |N_2|, \dots, |N_n|\}$  is the maximum degree of the graph;  $\varepsilon > 0$  is the power deviation factor. In general, too large a value of  $\varepsilon$  will lead to non-convergence and too small will lead to slow convergence.

Equation (14) is written in matrix form as follows:

$$\lambda(k+1) = (\mathbf{I}_n - \omega \mathbf{L}(k)) \lambda(k) + \text{diag}(d_i) \varepsilon \Delta f_p(k) \quad (15)$$

The control strategy shown in Equation (15) consists of two components:

- (1) Regarding the share component,  $\lambda(k+1) = (\mathbf{I} - \omega \mathbf{L}) \lambda(k)$  can reduce the difference between each DG incremental cost  $\lambda$  and achieve an equal incremental rate for each generation unit through multiple iterations.
- (2) Regarding the total adjustment component, the  $\text{diag}(d_i) \varepsilon \Delta f_p(k)$  signal received by the leader node is used to adjust the total generation of each DG to satisfy the limit of zero power deviation.

In this paper, the economic dispatch performance evaluation function  $f_p$  is defined as [12]:

$$f_p = \frac{1}{2} \left( P_D + P_{\text{LOSS}} - \sum_{i=1}^n P_{\text{DG},i} \right)^2 \quad (16)$$

Take  $\Delta f_p(k)$  to be the negative value of the derivative of the performance evaluation function  $f_p$  with respect to the leader’s output force, i.e.,:

$$\Delta f_p(k) = - \left. \frac{\partial f_p}{\partial P_{DG,1}} \right|_{P_{DG,1}(k)} = \left( P_D + P_{Loss} - \sum_{i=2}^n P_{DG,i} - P_{DG,1} \right) \Big|_{P_{DG,1}(k)} = P_{PCC}(k) \quad (17)$$

Based on the updated value of the incremental cost obtained in Equation (15), each generating unit can obtain an instruction for its own output:

$$P_{DG,i}(k) = \frac{\lambda_i(k) - b_i}{a_i} \quad (18)$$

Considering the power constraint for each DG output, Equation (18) can be changed to:

$$P_{DG,i}(k) = \begin{cases} P_{DG,i}^{min} & \frac{\lambda_i(k) - b_i}{a_i} \leq P_{DG,i}^{min} \\ \frac{\lambda_i(k) - b_i}{a_i} & P_{DG,i}^{min} \leq \frac{\lambda_i(k) - b_i}{a_i} \leq P_{DG,i}^{max} \\ P_{DG,i}^{max} & \frac{\lambda_i(k) - b_i}{a_i} \geq P_{DG,i}^{max} \end{cases} \quad (19)$$

Consider  $\Delta f_p(k)$  is coupled to  $\lambda$ , and substitute Equation (19) into Equation (15), Equation (15) can be rewritten in the following matrix form:

$$\begin{cases} \lambda(k+1) = \mathbf{G}(k)\lambda(k) + \mathbf{H} \\ \mathbf{G}(k) = \mathbf{I} - \omega\mathbf{L}(k) - \varepsilon\mathbf{M} \\ \mathbf{M} = \mathbf{L}_f \cdot \mathbf{K} \end{cases} \quad (20)$$

where  $\mathbf{L}_f = [d_1, d_2, \dots, d_n]$  is the leader selection matrix after coupling and  $\mathbf{K} = [k_1, k_2, \dots, k_n]$  is the system power difference coupling matrix,  $k_i = 1/2a_i$ .

The distributed economic dispatch control strategy starts by initializing  $\Delta f_p(k)$  and  $\lambda_0$ , then updates the incremental cost of each DG after the iteration according to Equation (14), and then obtains the output of each DG at that time by calculating the incremental cost of each DG obtained, and finally substitutes the updated  $\lambda$  and the output of each DG into Equation (14) to continue the iteration until the system converges.

#### 4. System Stability Analysis

The stability analysis of a distributed economic dispatch system under the probabilistic communication packet loss model is a stochastic stability analysis problem, and the necessary reasoning is first given below.

**Lemma 1 [21].** For symmetric positive definite matrices  $\mathbf{P} \in \mathbf{R}^{n \times n}$  and matrices  $\mathbf{E} \in \mathbf{R}^{r \times n}$ ,  $\mathbf{F} \in \mathbf{R}^{r \times n}$ , there are the following relations:

$$\mathbf{E}^T \mathbf{G} \mathbf{F} + \mathbf{F}^T \mathbf{G} \mathbf{E} < \mathbf{E}^T \mathbf{G} \mathbf{E} + \mathbf{F}^T \mathbf{G} \mathbf{F} \quad (21)$$

**Lemma 2.** (Schur complementary theorem) for the symmetric matrix described by Equation (22)

$$\mathbf{S} = \begin{bmatrix} \mathbf{S}_{11} & \mathbf{S}_{12} \\ \mathbf{S}_{12}^T & \mathbf{S}_{22} \end{bmatrix} \quad (22)$$

The following three conditions are equivalent:

- (1)  $\mathbf{S} < 0$ ;
- (2)  $\mathbf{S}_{11} < 0$ ,  $\mathbf{S}_{22} - \mathbf{S}_{12}^T \mathbf{S}_{11}^{-1} \mathbf{S}_{12} < 0$ ;
- (3)  $\mathbf{S}_{22} < 0$ ,  $\mathbf{S}_{11} - \mathbf{S}_{12} \mathbf{S}_{22}^{-1} \mathbf{S}_{12}^T < 0$

**Lemma 3** [22]. For a symmetric graph  $G$  with a given Laplacian matrix  $\mathbf{L}(k)$  and symmetric matrix  $\mathbf{P}$ , the expectation  $E\{\mathbf{L}(k)\mathbf{P}\mathbf{L}(k)\}$  can be derived from the following equation:

$$E\{\mathbf{L}(k)\mathbf{P}\mathbf{L}(k)\} = p^2 \cdot \mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p) \cdot \Xi(\mathbf{P})$$

where  $\mathbf{L}^{(0)}$  is the Laplacian matrix of the graph  $G$  corresponding to no consideration of communication packet loss, and  $\Xi(\mathbf{P})$  is a function of the matrix  $\mathbf{P}$  defined as follows:

$$\begin{aligned} \Xi(\mathbf{P}) &= \sum_{m=1}^n \sum_{q=1}^n (\mathbf{E}^T_{(m,q)}\mathbf{P}\mathbf{E}_{(m,q)} + \mathbf{E}^T_{(m,m)}\mathbf{P}\mathbf{E}_{(m,m)} - \text{sym}(\mathbf{E}^T_{(m,q)}\mathbf{P}\mathbf{E}_{(m,m)}))(a_{mq})^2 \\ &+ \sum_{j=1}^n \sum_{m=j+1}^n \text{sym}(2\mathbf{E}^T_{(j,j)}\mathbf{P}\mathbf{E}_{(m,m)} - \mathbf{E}^T_{(j,j)}\mathbf{P}\mathbf{E}_{(m,j)} - \mathbf{E}^T_{(j,m)}\mathbf{P}\mathbf{E}_{(m,m)})(a_{jm})^2 \end{aligned} \tag{23}$$

where the matrix  $\mathbf{E}_{(m,q)} \in R^{n \times n}$  is 1 only at  $(q, m)$  and 0 at all other positions. Each  $\mathbf{E}^T_{(m1,q1)}\mathbf{P}\mathbf{E}_{(m2,q2)}$  will produce a new matrix. This matrix has a unique non-zero element in row  $m1$  and column  $m2$  only, and the value of this non-zero element is equal to  $\mathbf{P}_{(q1, q2)} \cdot \text{sym}(\mathbf{B}) = \mathbf{B}^T + \mathbf{B}$ .

The Lyapunov function is designed as  $V(k) = \mathbf{Z}^T(k)\mathbf{P}\mathbf{Z}(k)$  and  $\mathbf{Z}(k+1) = \mathbf{G}(k)\mathbf{Z}(k)$ .  $\mathbf{P}$  is a real symmetric positive definite matrix, and the mathematical expectation of the difference between the functions  $V(k+1)$  and  $V(k)$  is:

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(k+1) - V(k)\} = E\{\mathbf{Z}^T(k+1)\mathbf{P}\mathbf{Z}(k+1) - \mathbf{Z}^T(k)\mathbf{P}\mathbf{Z}(k)\} \\ &= E\{\mathbf{Z}^T(k)\mathbf{G}^T(k)\mathbf{P}\mathbf{G}(k)\mathbf{Z}(k) - \mathbf{Z}^T(k)\mathbf{P}\mathbf{Z}(k)\} \\ &= \mathbf{Z}^T(k)E\{\mathbf{N}^T\mathbf{P}\mathbf{N} + \omega^2\mathbf{L}(k)\mathbf{P}\mathbf{L}(k) - \omega\mathbf{L}(k)\mathbf{P}\mathbf{N} - \omega\mathbf{N}^T\mathbf{P}\mathbf{L}(k) - \mathbf{P}\}\mathbf{Z}(k) \end{aligned} \tag{24}$$

where  $\mathbf{N} = \mathbf{I} - \varepsilon\mathbf{M}$ .

The mathematical expectation of the matrix  $\mathbf{L}(k)$  can be calculated from  $E\{\mathbf{L}(k)\} = p\mathbf{L}^{(0)}$ , from which the expectation of the difference of Lyapunov functions can be obtained as:

$$E\{\Delta V(k)\} = \mathbf{Z}^T(k)\{\mathbf{N}^T\mathbf{P}\mathbf{N} + \omega^2 p^2\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p)\Xi(\mathbf{P}) - \omega p\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} - \omega p\mathbf{N}^T\mathbf{P}\mathbf{L}^{(0)} - \mathbf{P}\}\mathbf{Z}(k) \tag{25}$$

The mathematical expectation of the difference between the functions  $V(k+1)$  and  $V(k)$ ,  $E\{\Delta V(k)\} < 0$ , is equivalent to the matrix inequality:

$$\mathbf{N}^T\mathbf{P}\mathbf{N} + \omega^2 p^2\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p)\Xi(\mathbf{P}) - \omega p\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} - \omega p\mathbf{N}^T\mathbf{P}\mathbf{L}^{(0)} - \mathbf{P} < 0 \tag{26}$$

By treating  $-\mathbf{L}^{(0)}$  as  $\mathbf{E}^T$  and  $\mathbf{N}$  as  $\mathbf{F}$  in Lemma 1, we can obtain:

$$\frac{1}{2}\omega p(-\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} + \mathbf{N}^T\mathbf{P}(-\mathbf{L}^{(0)})) < \frac{1}{2}\omega p\mathbf{N}^T\mathbf{P}\mathbf{N} + \frac{1}{2}\omega p\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} \tag{27}$$

Substituting Equation (27) into Equation (26), obtain:

$$\begin{aligned} &\mathbf{N}^T\mathbf{P}\mathbf{N} + \omega^2 p^2\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p)\Xi(\mathbf{P}) - \omega p\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} - \omega p\mathbf{N}^T\mathbf{P}\mathbf{L}^{(0)} - \mathbf{P} \\ &< (\frac{1}{2}\omega p + 1)\mathbf{N}^T\mathbf{P}\mathbf{N} + (\frac{1}{2}\omega p + \omega^2 p^2)\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p)\Xi(\mathbf{P}) - \frac{1}{2}\omega p(\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} + \mathbf{N}^T\mathbf{P}\mathbf{L}^{(0)}) - \mathbf{P} \end{aligned} \tag{28}$$

Then, a sufficient condition for Equation (26) to hold is obtained as:

$$(\frac{1}{2}\omega p + 1)\mathbf{N}^T\mathbf{P}\mathbf{N} + (\frac{1}{2}\omega p + \omega^2 p^2)\mathbf{L}^{(0)}\mathbf{P}\mathbf{L}^{(0)} + p(1-p)\Xi(\mathbf{P}) - \frac{1}{2}\omega p(\mathbf{L}^{(0)}\mathbf{P}\mathbf{N} + \mathbf{N}^T\mathbf{P}\mathbf{L}^{(0)}) - \mathbf{P} < 0 \tag{29}$$

Repeated use of Lemma 2 for Equation (29) yields:

$$\begin{bmatrix} -\mathbf{P} + \omega^2\mathbf{\Theta} - \frac{1}{2}\omega p\vartheta & \omega & \mathbf{\Psi} \\ * & -\mathbf{P}^{-1} & 0 \\ * & * & -\mathbf{P}^{-1} \end{bmatrix} < 0 \tag{30}$$



where  $\Theta = p(1 - p)\Xi(\mathbf{P})$ ,  $\vartheta = \text{sym}(\mathbf{L}^{(0)}\mathbf{P}\mathbf{N})$ ,  $\omega = \sqrt{\frac{1}{2}\omega p + 1\mathbf{N}^T\mathbf{P}}$ , and  $\Psi = \sqrt{\frac{1}{2}\omega p + \omega^2 p^2}\mathbf{L}^{(0)}\mathbf{P}$ .

Combining the above equations, if for given positive real numbers  $\varepsilon$ ,  $\omega$ , and  $p$ , there exists a real symmetric positive definite matrix  $\mathbf{P}$  such that Equation (30) holds, it follows from the Lyapunov stability theory that the system  $\mathbf{Z}(k+1) = \mathbf{G}(k)\mathbf{Z}(k)$  is stochastically asymptotically stable, and hence, the distributed economic dispatch system Equation (20) is stochastically asymptotically stable.

According to the LMI in Equation (30), when  $p \rightarrow 0$ , Equation (30) is approximated by the following linear matrix inequality:

$$\begin{bmatrix} -\mathbf{P} & \omega \\ * & -\mathbf{P}^{-1} \end{bmatrix} < 0 \tag{31}$$

It is verified in MATLAB that Equation (31) is not strict (marginal infeasibility), and when the communication packet loss probability is high, i.e.,  $p \rightarrow 0$ , the system cannot remain stable. In addition, as the communication packet loss probability increases, the efficiency of the information interaction between the DGs decreases and the equalization part of the control strategy (15) reduces the difference between the incremental costs  $\lambda$  of the DGs slower, which can lead to the convergence performance of the system (20) being affected.

### 5. Simulation Verification

The IEEE-14 node distribution network system was used for simulation calculations. The system structure and local communication network are shown in Figure 2 [8]. Five DGs are connected to the distribution network, located at nodes 2, 3, 7, 10, and 12. The total active load of the entire network is 28.7 MW and the specific information for each DG is given in Table 1. The adjudication condition for convergence of the system in the simulation is that the difference between any two DG incremental costs  $|\Delta\lambda| < 0.0001$  \$/(MWh).

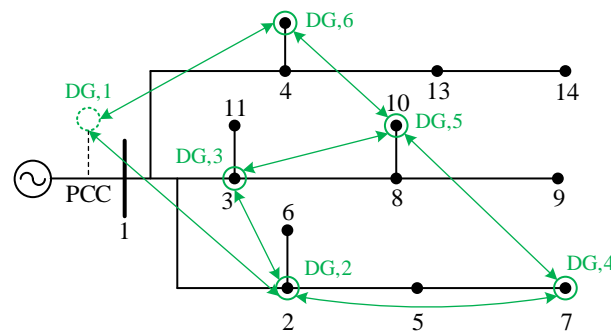


Figure 2. Topology of the simulation system and communication network.

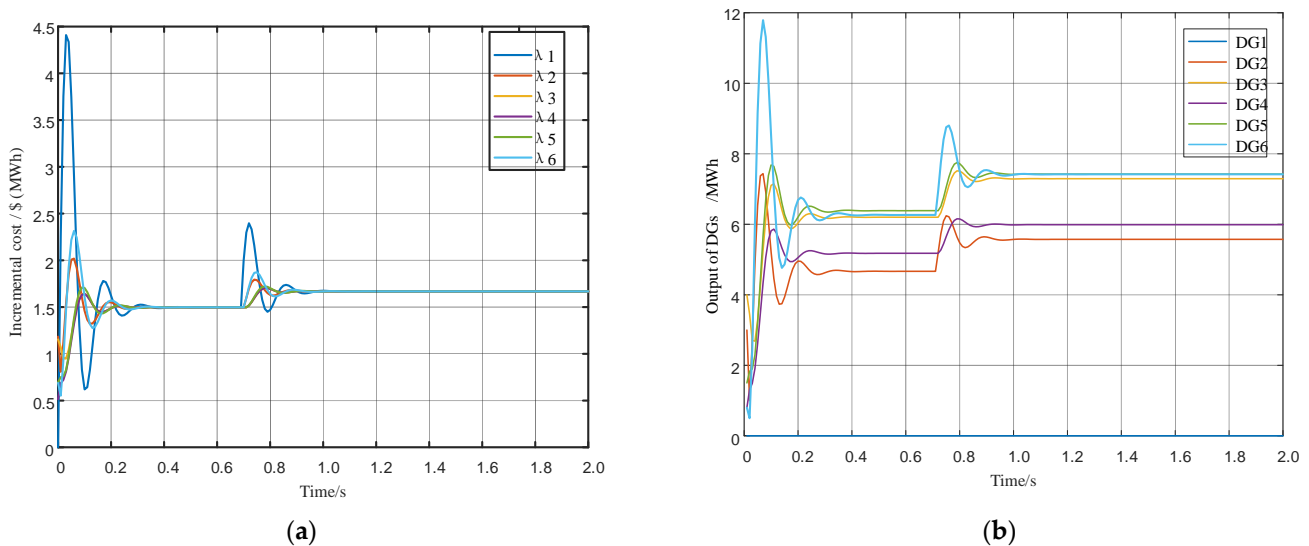
Table 1. Parameters of DGs in IEEE-14.

DG Number	$a_i/$ (\$/MW <sup>2</sup> h)	$b_i/$ (\$/MWh)	$c_i/$ (\$/h)	$p_{DG,i}^{min}/$ MW	$p_{DG,i}^{max}/$ MW	Initial Value/ MW
1	-	-	-	-	-	0
2	0.094	0.62	90	0.5	15	2.0
3	0.078	0.53	87	1.0	20	1.0
4	0.105	0.41	86	0.8	20	1.5
5	0.082	0.45	45	0.6	15	1.5
6	0.074	0.57	73	0.5	18	0.8

### 5.1. Scenario 1: Ideal Communication Conditions

Taking the feedback coefficient  $\omega = 0.2$  and the power deviation coefficient  $\varepsilon = 0.1$ , the eigenvalues of the control system shown in Equation (20) are  $0.7611 \pm j 0.3707$ ,  $0.0593$ ,  $0.3208$ ,  $0.6978$ , and  $0.6000$ , all within the unit circle, indicating that the system is stable.

The total active load of the distribution network increases from an initial 28.7 MW to 33.7 MW at time  $t = 0.7$  s. The response of the distributed economic dispatch system is shown in Figure 3.



**Figure 3.** Incremental costs and DG outputs under ideal communication condition: (a) Incremental cost; (b) output of DGs.

As can be seen from Figure 3, when the load is 28.7 MW, the DG incremental cost converges to the same optimal value  $\lambda^* = 1.497 \$/(\text{MWh})$  from each initial value, the sum of each DG output is balanced with the load in the distribution network; when the load increases to 33.7 MW, the DG output fluctuates briefly, and the DG output meets the new load demand. As the total load power increases, the incremental cost of each DG stabilization increases to the new optimal consensus value  $\lambda^* = 1.667 \$/(\text{MWh})$ . It is shown that the method in this paper can achieve the objective of economic dispatch of the active distribution network in a distributed manner during load changes, when the system converges to the optimal value in 0.63 s and 0.55 s

### 5.2. Scenario 2: Communication Packet Loss

- (1) Setting  $p = 0.9$ , i.e., packet loss probability is 0.1, Equation (29) holds and the system is able to maintain stability. The total active load of the distribution network increases from the initial 28.7 MW to 33.7 MW at  $t = 1$  s. Simulations are performed using the control strategies of [4] and this paper, respectively, and the responses of the distributed economic scheduling system are shown in Figures 4 and 5.

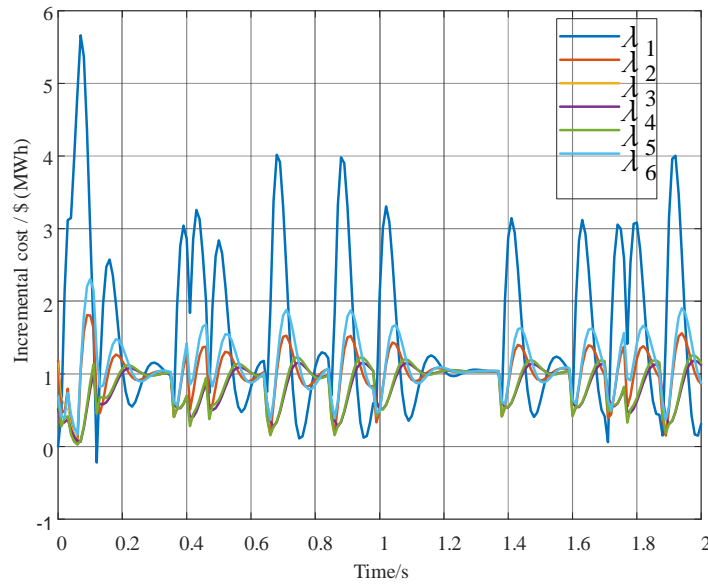


Figure 4. Response using the control strategy of [4] at  $p = 0.9$ .

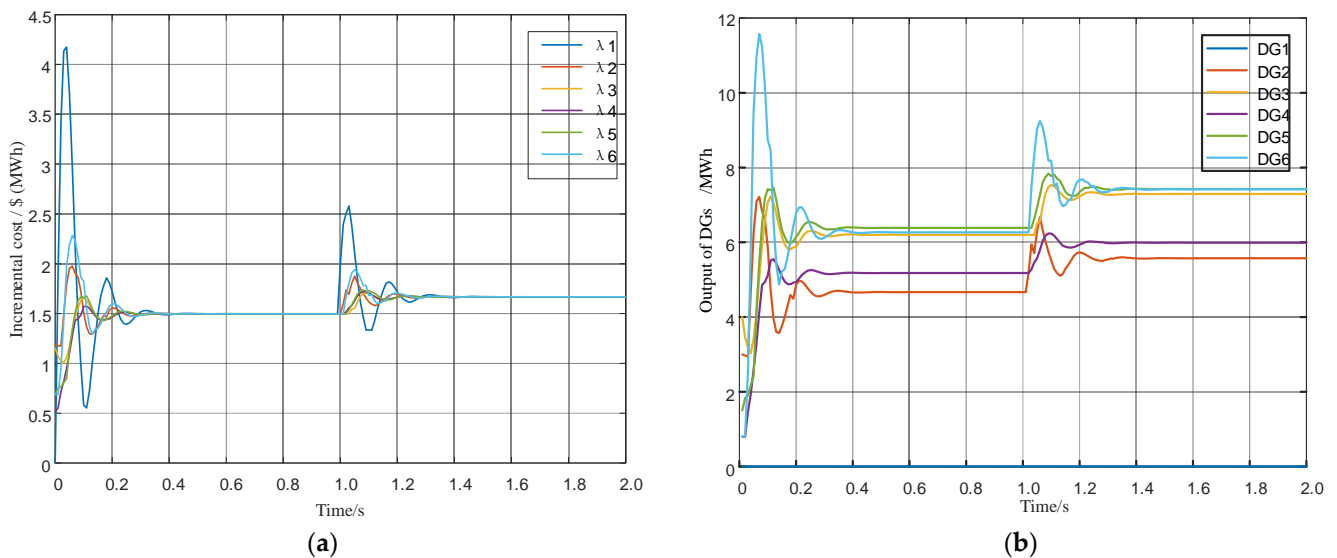


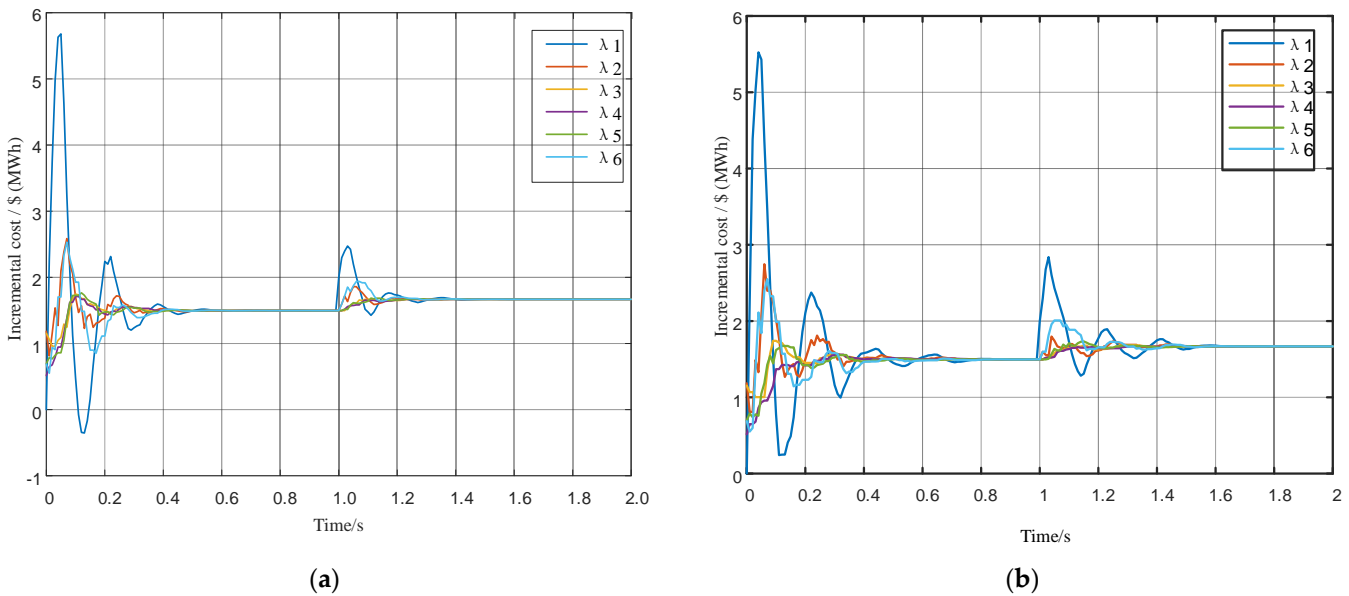
Figure 5. System response with this control strategy at  $p = 0.9$ : (a) Incremental cost; (b) output of DGs.

From the comparison of Figures 4 and 5a, the control strategy of [4] is not applicable at all when there is communication packet loss, and the packet loss condition will have a wide range of fluctuations and even break the steady state that has been achieved, while the control strategy of this paper can adapt well to the communication packet loss condition and achieve the control goal. As can be seen from Figure 5, for a load of 28.7 MW and a packet loss probability of 0.1, the DG incremental costs converge from their respective initial values to the same optimal value of  $\lambda^* = 1.497$  (\$/MWh), and the sum of each DG’s output is balanced with the load in the distribution network. The output of each DG briefly fluctuates when the demand reaches 33.7 MW, but soon stabilizes at a new ideal consensus value of  $\lambda^* = 1.667$  (\$/MWh).

It is shown that the system (20) considering communication packet loss can achieve the goal of economic dispatch of an active distribution network in a distributed manner, but due to the effect of communication packet loss, each DG will not be able to accurately receive the increment cost from neighboring DGs or pass out relevant information about

itself in a timely manner, and the time to converge to the optimal value before and after the load change increases to 0.65 s and 0.62 s, respectively.

- (2) Holding other parameters constant, the response of the system at  $p = 0.7$  and  $p = 0.5$  is shown in Figure 6.



**Figure 6.** Incremental costs for  $p = 0.7$  and  $p = 0.5$ : (a)  $p = 0.7$ ; (b)  $p = 0.5$ .

As can be seen from Figure 6, after the load increases to 33.7 MW, the incremental costs eventually stabilize to a new optimal consistent value of  $\lambda^* = 1.667$  (\$ /MWh), and the convergence time of the system to the final consistent value is 0.67 s, 0.92 s, and 1.09 s when  $p$  is 0.9, 0.7, and 0.5 respectively, indicating that the convergence speed of the distributed economic scheduling system becomes slower and slower as the communication packet loss probability increases. In addition, the variation in the incremental cost of each DG as the load changes increases with the probability of packet loss.

- (3) For an initial load of 28.7 MW, the time for the system to converge from the initial value to the optimum value for different packet loss probabilities is shown in Figure 7.

As can be seen from Figure 7, the time required for the system to converge to the optimum value increases rapidly with the probability of packet loss. In the communication topology shown in Figure 2, the system convergence time is <5 s when  $p > 0.2$ , i.e., the packet loss probability is less than 0.8; when the packet loss probability is higher than 0.8, the system convergence time increases significantly; when the packet loss probability exceeds 0.9, the convergence time is already close to 50 s and the system can no longer converge. The maximum packet loss probability  $p_{max} = 0.904$ , which holds by solving Equation (29) through the LMI toolbox in MATLAB, is consistent with the simulation results, verifying the validity of the sufficient condition (29) for the stochastic asymptotic stability of system (20).

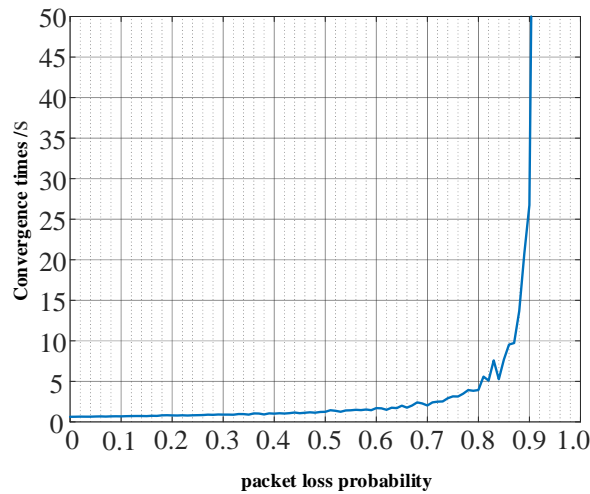


Figure 7. Convergence times of the system for different values of packet loss probability.

5.3. Scenario 3: IEEE-39 Node System

To further verify the effectiveness of the strategy proposed in this paper, a larger scale IEEE-39 node 10 DG system [23] was used for simulation analysis. Table 2 lists the basic parameters of each DG,  $G_1$  is a virtual distributed power supply, and the system communication network topology is shown in Figure 8

Table 2. Parameters of the DGs in the IEEE-39 system.

DG Number	$a_i/$ (\$/MW <sup>2</sup> h)	$b_i/$ (\$/MWh)	$c_i/$ (\$/h)	$P_{DG,i}^{min}/$ MW	$P_{DG,i}^{max}/$ MW	Initial Value/ MW
$G_1$	-	-	-	-	-	0
$G_2$	0.038	0.68	135.88	75	500	300
$G_3$	0.034	0.70	214.92	80	400	360
$G_4$	0.029	0.75	108.23	30	280	240
$G_5$	0.018	0.76	220.00	80	420	375
$G_6$	0.016	0.81	232.56	50	350	200
$G_7$	0.025	0.71	78.09	50	480	400
$G_8$	0.022	0.78	234.48	64	300	150
$G_9$	0.026	0.68	74.60	45	500	337
$G_{10}$	0.033	0.60	127.69	74	400	250
$G_{11}$	0.028	0.66	100.52	150	600	400

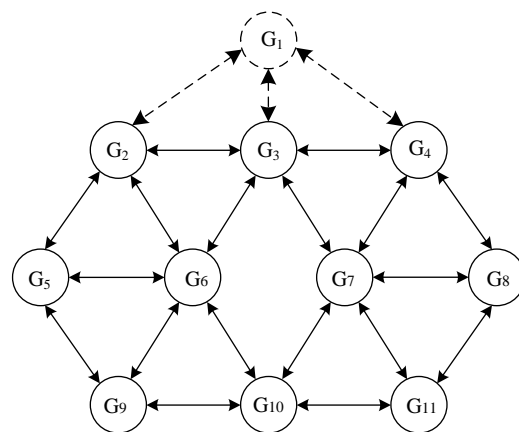


Figure 8. Communication network topology of the IEEE-39 system.

- (1) At  $p = 0.9$  and  $t = 2$  s, the system load increases by 200 MW from an initial 3420 MW, and decreases by 250 MW at  $t = 4$  s. The response of the distributed economic dispatch system is shown in Figure 9.

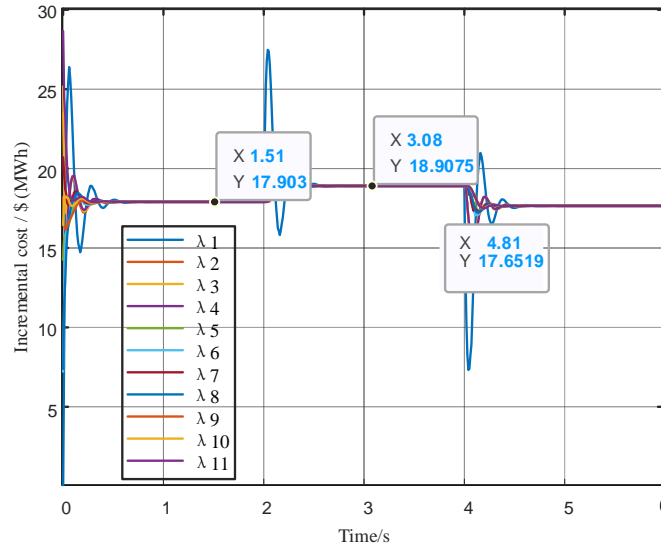


Figure 9. Incremental cost at  $p = 0.9$ .

As can be seen from Figure 9, when the load is 3420 MW and there is a packet loss probability of 0.1, the DG incremental costs converge to the same optimal value  $\lambda^* = 17.903$  \$/(MWh) from their respective initial values, and the sum of each DG output is balanced with the load in the distribution network. When the load fluctuates at  $t = 2$  s and  $t = 4$  s, there are brief fluctuations in each DG output, and the incremental costs stabilize to the new optimal consistent values  $\lambda^* = 18.9075$  \$/(MWh) and  $\lambda^* = 17.6519$  \$/(MWh). It is further shown that system (20) can achieve the goal of economic dispatch of the active distribution network in the event of packet loss in communication.

- (2) Regarding the impact of network connectivity, the distributed control systems can not achieve control objectives when the communication network is not connected, and communication packet loss will affect network connectivity to some extent. The degrees of each DG node in Figure 8 are: 3, 4, 5, 4, 3, 5, 5, 3, 3, 4, and 3, where the size of  $D_1$  of the leader node has a greater impact on the convergence performance of the system. The convergence times of the system from the initial value to the optimal value for different values of  $p$  and the number of connected communication links between the leader and other DGs are shown in Table 3 and Figure 10.

Table 3. Convergence times for different leader connectivity links and  $p$ .

Communication Topology	$D_1$	$t/s$			
		$p = 1$	$p = 0.9$	$p = 0.7$	$p = 0.5$
DG <sub>1</sub> with DG <sub>10</sub> and DG <sub>11</sub>					
Communication link	5	0.82	0.89	1.15	1.63
No change	3	1.11	1.24	1.60	2.36
DG <sub>1</sub> with DG <sub>2</sub> and DG <sub>4</sub>					
Communication link lost	1	3.81	4.21	5.60	8.73

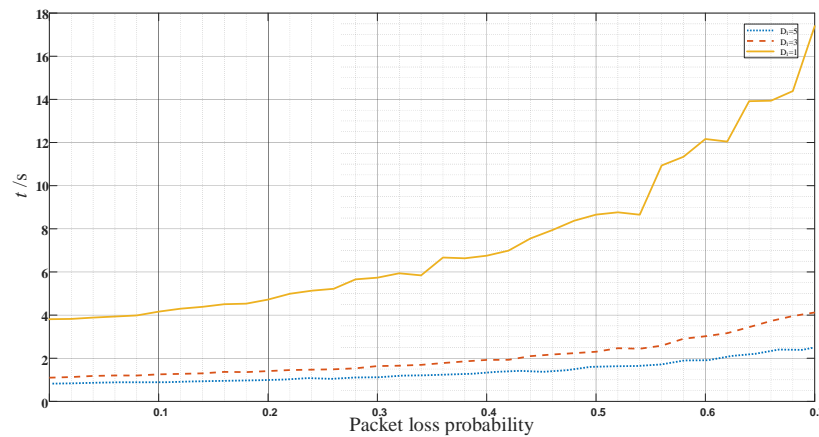


Figure 10. System convergence time with different leader connectivity links.

As can be seen from Table 3 and Figure 10, for the same packet loss probability, the system converges slower for both  $D_1 = 1$  and  $D_1 = 3$  than for  $D_1 = 5$ , and the smaller the value of  $D_1$ , the slower the system converges. The main reason is that, when the degree of the leader node is small, the DGs cannot quickly obtain information about the power difference of the system through the leader node, and thus, quickly adjust their own output. When the degree of the leader increases, the convergence speeds up. However, the increase in  $D_1$  leads to an increase in communication costs; therefore, it is important to choose the right  $D_1$  value to ensure the quality of communication and convergence performance while ensuring economic efficiency.

- (3) Regarding individual DG communication failure, let  $DG_3$ ,  $DG_6$ , and  $DG_9$  each have communication failures and packet loss with their respective adjacent DGs. The system’s convergence time is illustrated in Figure 11 below, and shows that the communication packet loss probability increases between the failing DG and the neighboring DGs.

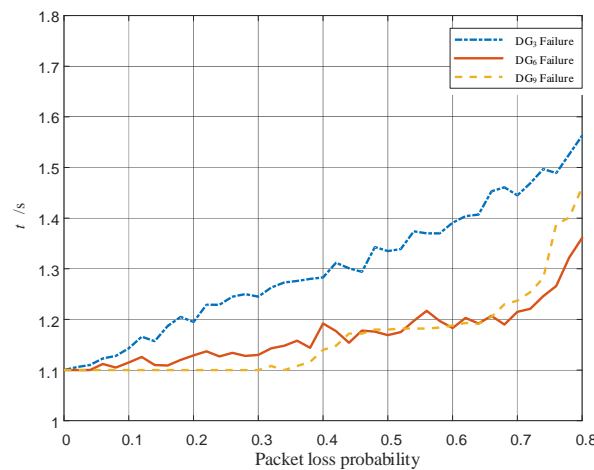


Figure 11. System convergence time for  $DG_3$ ,  $DG_6$ , and  $DG_9$  failures.

As can be seen from Figure 11, the system convergence performance is most affected when  $DG_3$  experiences a communication failure.

### 6. Conclusions

With incremental cost as the consensus variable, the Bernoulli’s stochastic packet loss process is used to describe the packet loss phenomenon, and based on the stochastic theory, a distributed economic dispatch strategy for the ADN in the leader-follower mode is designed, considering the coupling of the power difference between the total generation

power and the load power with the consensus variable, while using Lyapunov's stability theorem and the Schur complementary theorem. Sufficient conditions are given in the form of LMI that enable the system to achieve stochastic stability, which were not considered and are given in the references. Simulation calculations were carried out for several scenarios in different-sized ADNs and the results showed that:

The distributed economic dispatch control strategy based on communication packet loss can effectively meet the requirements of ADN power balance and minimum generation cost at the same time, and the ADN can maintain stable operation, which improves the practicality and reliability of the distributed economic dispatch control strategy of the ADN. (2) The validity of the sufficient conditions for the stochastic stability of the system in the form of LMI is verified by Scenario 2, and the convergence of the system under different communication packet loss probabilities can be judged by Equation (29) and the maximum packet loss probability  $p_{\max}$  of the system can be obtained. (3) The communication packet loss probability will affect the convergence performance of the system, and too high a packet loss probability will cause the system to lose stability; the selection of degree  $D$  affects the convergence performance of the system, and a suitable degree should be selected to ensure that each DG is not disturbed by too much information, while it collects enough information from neighboring DGs.

**Author Contributions:** Conceptualization, J.L. and G.Q.; methodology, J.L.; software, L.Z.; validation, G.Q., L.Z. and R.J.; formal analysis, J.L.; resources, J.L.; data curation, R.J.; writing—original draft preparation, G.Q.; writing—review and editing, J.L.; visualization, L.Z. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was supported by the National Key Research and Development Program, grant number 2022YFF0610601.

**Data Availability Statement:** The data presented in this study are available on request from the corresponding author.

**Conflicts of Interest:** The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Nomenclature

ADN	Active distribution network
LMI	Linear matrix inequality
ED	Economic dispatch
DG	distributed generation
PCC	Point of common coupling
$i, j$	Indices of the DGs
$n$	Total number of DGs
$\omega$	Feedback factor in consensus protocol
$P_{DG,i}$	Output power of the $i$ -th DG
$v_j$	Node $j$
$F(P_{DG,i})$	Power-cost function of the $i$ th DG
$a_i, b_i, c_i$	Quadratic, primary, and constant term coefficients of the $i$ -th DG generation cost function
$P_D$	Total active distribution network load
$P_{LOSS}$	Active power lost of the distribution network
$P_{PCC}$	Active power input from the external grid
$P_{DG,i}^{min}, P_{DG,i}^{max}$	Lower limit and upper limit of output of the $i$ -th DG
$G$	A graph
$V$	Set of vertices
$E$	Set of edges
$N_i(k)$	Input type domain of agent $i$ at moment $k$
$x_i(k)$	State of the $i$ th agent at moment $k$



$a_{ij}$	Weight set by node $i$ for the state information received from neighboring node $j$
$\gamma_{ij}$	Stability of the communication line $(v_j, v_i)$
$p_{ij}$	Probability of $\gamma_{ij} = 1$
$\mathbf{L}(k)$	System Laplacian matrix
$l_{ij}(k)$	Element of the system Laplacian matrix
$u_i(k)$	Control input of DG $_i$ at moment $k$
$\lambda_i(k)$	Incremental cost of DG $_i$ at moment $k$
$\varepsilon$	Power deviation factor
$d_i$	Leader identifier
$\lambda(k)$	Vector of $\lambda_i(k)$
$\mathbf{I}$	Unit matrix
$f_p$	Economic dispatch performance evaluation function
$\Delta f_p(k)$	Negative value of the derivative of $f_p$

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