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Analytical Expression for Line Voltage THD of Three-Phase Staircase Modulated Multilevel Inverters

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Abstract: In this article, a simple closed-form analytical expression for the line-voltage total harmonic distortion (LTHD) of three-phase staircase-modulated (SCM) multilevel inverters (MLI) is proposed. The revealed expression is valid for any conventional MLI topology with arbitrary number and parity of voltage levels. The proposed formulation is presented as an analytic function of the number of voltage levels (N) and the phase switching angles (PSA); thus, it is suited to MLIs of equal voltage source supply and of any topology. This function may be employed for accurate LTHD calculation and optimization. The results are verified against LTHD calculations and optimizations, obtained numerically from previous works. Both processor in loop (PIL)- and controller + hardware in loop (C-HIL)-based real-time experimental validations are included as well. A downloadable file containing the Maple and MATLAB functions of the proposed expression are also provided.

Keywords: multilevel inverters; staircase modulation; THD; optimization; line-voltage

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1. Introduction

The important role of three-phase (3ϕ) MLIs in different power conversion applications is clear [1–3]. One of the major challenges of MLI operation is output voltage THD minimization for a given desired modulation index, an approach commonly referred to as the optimal minimization of THD, or OMTHD [4–10]. While high switching frequency approaches, such as Sine-PWM, may provide better overall performance, SCM-based approaches, especially when combined with OMTHD, are often preferred due to their lower commutation burden, switching losses, and simpler control circuitry [2–12].

SCM implementation success relies strongly on the proper formulation of THD expression [3,4]. Commonly used frequency-domain numerical THD evaluations are usually 49 or 99 harmonics approximations, as recommended by IEEE 519 [13].

An analytical expression for the phase voltage THD (PTHD) of single-phase (1ϕ) SCM-based MLIs, valid for any value of N (odd and even), was recently introduced in [14]. However, in 3ϕ MLIs, line-voltage THD (LTHD) rather than PTHD is of concern, since the line-voltage directly influences the load current quality [3].

Even for a known value of N, the analytic formulation of LTHD is quite peculiar, since the line-voltage of 3ϕ SCM-MLI is a complex stepped waveform imposed by the PSA values. The derivation of an analytic expression for LTHD is presented in this article, utilizing the results presented in [14], but with a different derivation process. It is important to emphasize that the presence of analytical expression is essential for accurate LTHD minimization, eliminating the need for multiple numerical iterations, which are typically prone to underestimation and round-off errors due to the finite amount of harmonics taken into consideration.

Some LTHD formulations and, subsequently, LTHD optimizations were proposed in [4–12], focusing on specific odd-parity values of N only. An implicit LTHD expression, utilizing the unit-step function to represent an SCM waveform for a specific odd value of N was suggested in [11] and employed in [9] to perform the optimal minimization of LTHD in 7 level 3ϕ MLI. Another LTHD formulation, utilizing piecewise-recursive representation, was suggested in [4]; it was limited to odd values of N and relied on many piecewise terms. The so-called 3THD [4] is in fact a modified phase-voltage THD, in which triplen harmonics are not accounted for, making it an LTHD equivalent. The expressions proposed in [4–12] do provide accurate results, yet they were obtained numerically. Nevertheless, no generalized analytical LTHD expression has been proposed thus far.

In this study, generalized analytical LTHD expression is derived and verified. The revealed expression may be implemented using any commercial mathematic computational software. Given the desired value of N, which can be both odd and even, the proposed LTHD formula is an explicit closed-form analytical function of the PSA set, and may be used for accurate LTHD calculation or optimization.

2. The Proposed LTHD Expression

Generalized quarter-waves of SCM waveform with odd-symmetry, representing the normalized phase voltage waveforms of 3ϕ-MLIs, are presented in Figure 1c,d for odd and even values of N, corresponding to MLI topologies of the cascaded H-bridge (CHB) [15–17] and diode-clamped converter (DCC) [18–20] phase-legs illustrated in Figure 1a,b, respectively. Both waves in Figure 1 may be generally represented by a unified expression [14]:

$$v_a(\theta) = \frac{\sum_{k=1}^{M}(u(\theta-\alpha_k))}{N-1} + \frac{\lfloor \frac{N}{2} \rfloor - \frac{N-1}{2}}{N-1}, \quad \begin{matrix} 0^\circ \le \theta \le 90^\circ \\ 0^\circ \le \alpha_k \le 90^\circ \end{matrix}, \tag{1}$$

where $\lfloor\,\rfloor$ denotes the floor operator, $\theta = \omega_1 t$ with ω_1 signifies the base frequency and α_k with $k = 1 \ldots M$ is the kth PSA,

$$M = \left\lfloor \frac{N-1}{2} \right\rfloor \tag{2}$$

symbolizes the number of voltage steps per quarter-wave and

$$u(\theta-\alpha) = \begin{cases} 0, & \theta < \alpha \\ 1, & \theta \ge \alpha \end{cases} \tag{3}$$

denotes a unit-step function. The normalized line voltage is defined as

$$v_{ab}(\theta) = v_a(\theta) - \underbrace{v_a(\theta - 120^\circ)}_{v_b(\theta)}$$

Utilizing the property of a periodic odd-symmetry function $f(\theta) = -f(\theta \pm 180^\circ)$, the SCM line-voltage may be rewritten as follows:

$$v_{ab}(\theta) = \begin{cases} v_a(\theta+30^\circ) - v_a(30^\circ-\theta), & 0^\circ \le \theta < 30^\circ \\ v_a(\theta+30^\circ) + v_a(\theta-30^\circ), & 30^\circ \le \theta < 60^\circ \\ v_a(150^\circ-\theta) + v_a(\theta-30^\circ), & 60^\circ \le \theta \le 90^\circ \end{cases} \tag{4}$$

The time-domain LTHD definition is given by [3,21,22]:

$$LTHD(N,\alpha) = 100 \cdot \sqrt{\frac{2V_{Lrms}^2(N,\alpha)}{V_{L1}^2(N,\alpha)} - 1} \tag{5}$$

where $\alpha = \{\alpha_1, \alpha_2, \ldots \alpha_M\}$ is the PSA set in degrees, V_{Lrms} is the normalized line-voltage root mean square value and V_{L1} is the normalized line voltage fundamental component (i.e., the line modulation index m_a), which can be expressed (assuming balanced three-phase load) as [22]:

$$V_{L1} = m_a = \frac{4\sqrt{3}}{\pi \cdot (N-1)} \cdot \left(\sum_{k=1}^{M} \left(\cos\left(\frac{\pi \alpha_k}{180} \right) \right) + \left\lfloor \frac{N}{2} \right\rfloor - \frac{N-1}{2} \right) \tag{6}$$

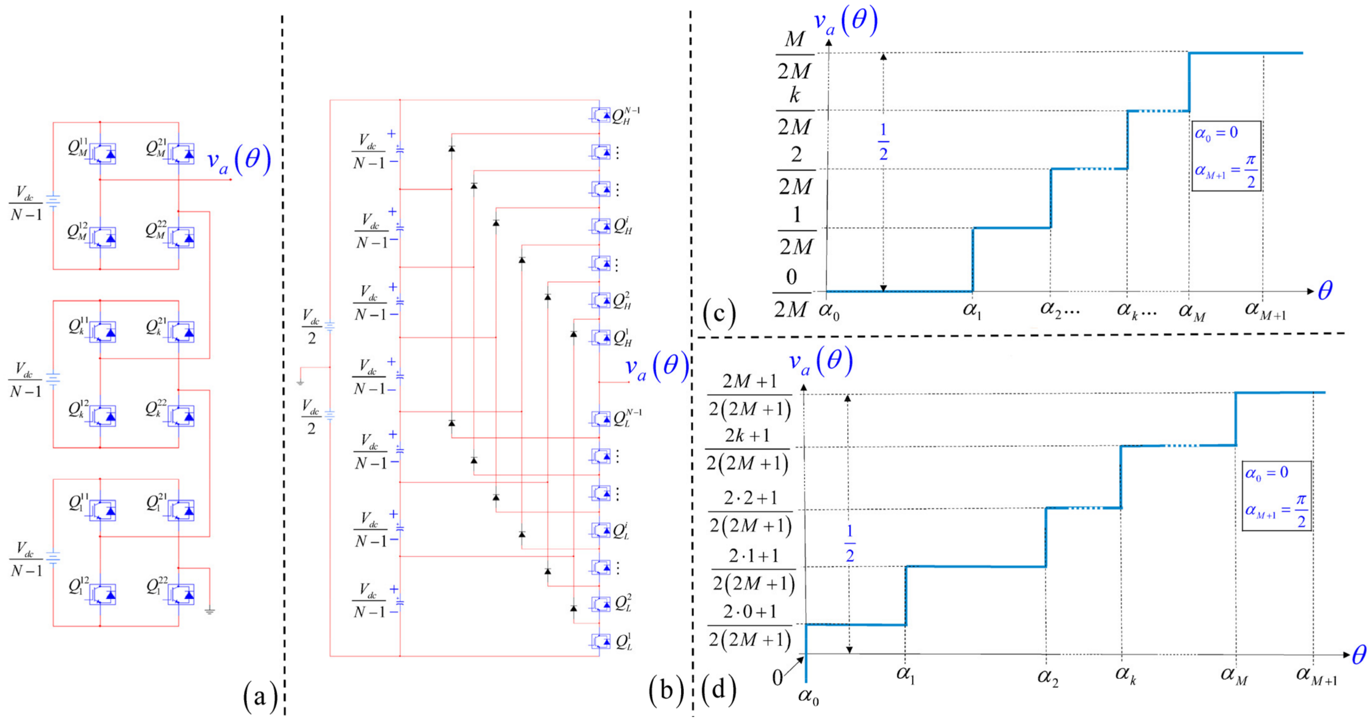

Figure 1. Generic MLI phase-leg: (**a**) cascaded H-bridge, (**b**) diode-clamped, and normalized odd quarter-wave symmetry SCM waveforms for: (**c**) odd-N case, and (**d**) for the even-*N* case.

The feasible range of the line-voltage modulation index depends on the parity of N as [22]:

$$\begin{array}{ll} 0 \leq V_{L1} \leq \frac{2\sqrt{3}}{\pi} = 1.1, & N \text{ odd} \\ \frac{2\sqrt{3}}{\pi(N-1)} \leq V_{L1} \leq \frac{2\sqrt{3}}{\pi}, & N \text{ even} \end{array} . \tag{7}$$

Substituting (1) into (4) and simplifying the use of the unit step function property $u(-\theta) = 1 - u(\theta)$ yields

$$v_{ab}(\theta) = \frac{1}{N-1} \begin{cases} f_1(\theta), & 0^\circ \leq \theta < 30^\circ \\ f_2(\theta), & 30^\circ \leq \theta < 60^\circ \\ f_3(\theta), & 60^\circ \leq \theta \leq 90^\circ \end{cases} \tag{8}$$

with

$$\begin{aligned} f_1(\theta) &= \sum_{k=1}^{M} (u(\theta - (30 - \alpha_k)) - u(-\theta + (\alpha_k - 30))) \\ f_2(\theta) &= \sum_{k=1}^{M} (u(\theta - (\alpha_k - 30)) - u(-\theta + (30 + \alpha_k))) + \left\lfloor \tfrac{N}{2} \right\rfloor \\ f_3(\theta) &= \sum_{k=1}^{M} (u(\theta - (30 + \alpha_k)) - u(\theta - (150 - \alpha_k))) + \left\lfloor \tfrac{N}{2} \right\rfloor \end{aligned} \tag{9}$$

An RMS value of (8) is obtained using (10), considering the following relation: $u(\theta-\alpha) = \frac{d}{d\theta}\max(\theta-\alpha, 0)$, vis.:

$$V_{Lrms}^2 = \frac{1}{90(N-1)}\left(\int_0^{30} f_1^2(\theta)d\theta + \int_{30}^{60} f_2^2(\theta)d\theta + \int_{60}^{90} f_3^2(\theta)d\theta\right) = \frac{N_1+N_2+N_3+N_4}{90\cdot(N-1)^2} \quad (10)$$

with

$$N_1 = \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\min(\alpha_i,\alpha_j) + \min(30,\alpha_i,\alpha_j) - \max(30,\alpha_i,\alpha_j)\right), \quad (11)$$

$$N_2 = -2\cdot\sum_{i=1}^{M}\sum_{j=1}^{M}\begin{pmatrix} \max(0, 30-\alpha_j, \min(30, 90-\alpha_i)) \\ +\max(30, 90-\alpha_j, \min(60, 30+\alpha_i)) \\ +\max(60, \min(120-\alpha_i, \alpha_j)) \end{pmatrix}, \quad (12)$$

$$N_3 = -2\cdot\sum_{k=1}^{M}\begin{pmatrix} M\cdot\min(60, 2\alpha_k-30, \alpha_k) \\ +\frac{N}{2}\cdot\max(\alpha_k-30, 2\alpha_k-90) \end{pmatrix}, \quad (13)$$

$$N_4 = 15\cdot\left(\frac{N}{2}+N-1\right)^2 + 375\cdot M^2 \quad (14)$$

Substituting (10) and (6) into (5) yields:

$$LTHD(N,\alpha) = 100\cdot\sqrt{\frac{N_1+N_2+N_3+N_4}{D} - 1} \quad (15)$$

with

$$D = \frac{2160}{\pi^2}\cdot\left(\sum_{k=1}^{M}\left(\cos\left(\frac{\pi\cdot\alpha_k}{180}\right)\right) + \frac{N}{2} - \frac{N-1}{2}\right)^2. \quad (16)$$

Equation (15) forms a generic N-level analytic LTHD expression, which is a function of PSA set α (in degrees) for any given N, which is either odd for a CHB MLI [9] or even for a DCC MLI [18]

3. Validation

The proposed N-level LTHD expression functions are implemented (without loss of generality) using Maple and MATLAB. The source code for these implementations is available for download in [23]. Evaluating (15) with basic $N = 2$ (corresponding to $M = 0$) returns the well known result of $100\cdot\sqrt{\pi^2/9-1} = 31.08\%$ [3]. LTHD evaluated by the proposed expression for 3 level and 4 level cases ($M = 1$) are presented vs. PSA in Figure 2, while 7 and 8 level case results ($M = 3$) are depicted in Figure 3. Numerically calculated LTHD results, accounting for 49 up to 999 harmonics, as recommended in [13], are also included in Figures 2 and 3 for comparison.

The results in Figures 2 and 3, which compare the proposed analytical approach with the conventional-frequency spectra-based numerical approximation, recommended by the IEEE 519 standard [13], demonstrate strong correlations between the two approaches while highlighting the typical underestimation of numerical methods, as previously reported in [22]. The outcomes obtained by (15) were further compared to previously obtained results, including square-wave modulation-based LTHD calculation [3] and LTHD optimization [9]. Employing equivalent PSA values, the LTHDs evaluated using the proposed analytical formulation for 3, 4, and 5 level SCM are summarized in Table 1, along with corresponding results from [3]. Excellent matching is evident for all cases.

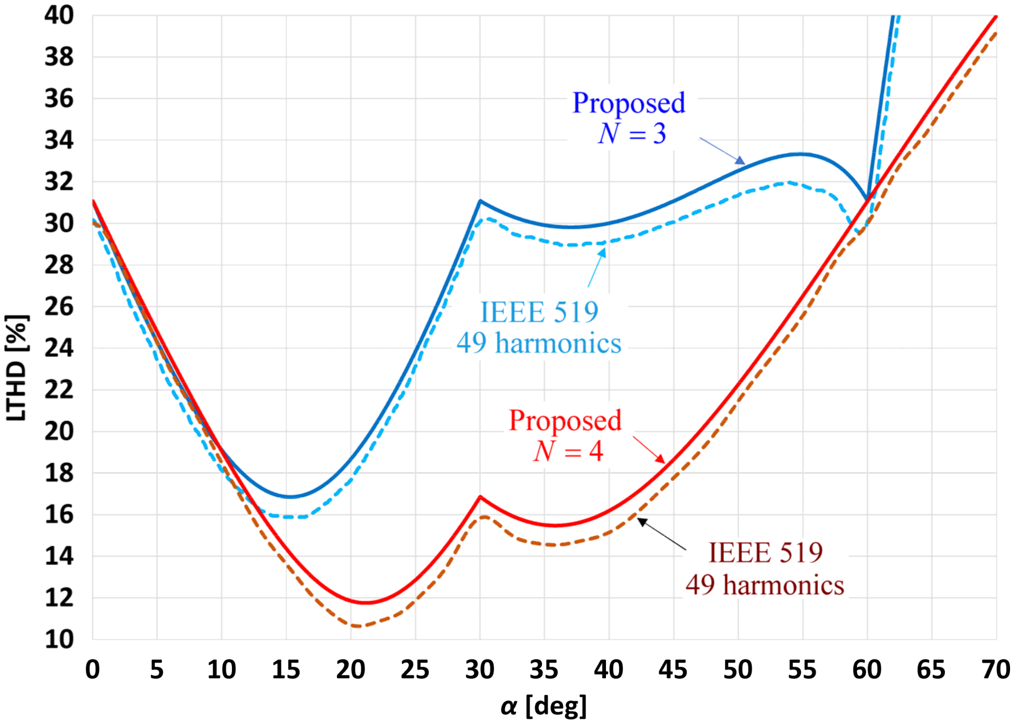

Figure 2. LTHD for N = 3, 4: proposed vs. 49 harmonics (IEEE 519 Std).

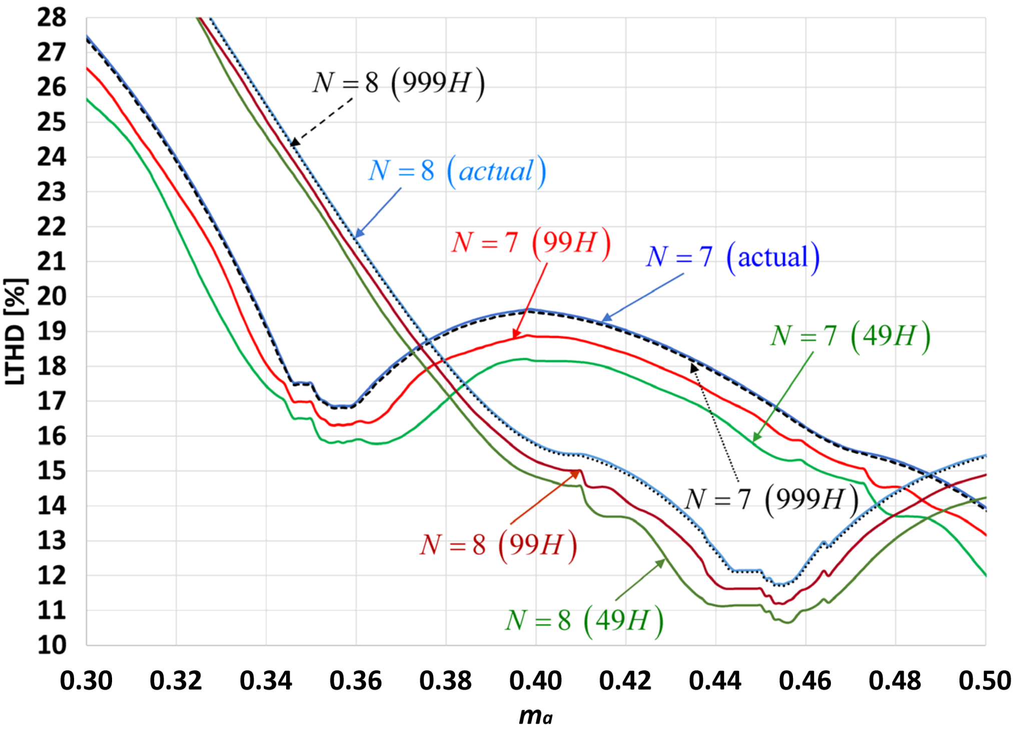

Figure 3. Minimum LTHD for N = 7, 8: proposed vs. recommended 49, 99, and 999 harmonics approximations.

Table 1. Proposed vs. [3] LTHD comparison, %.

N	α_1	α_2	Analytic (Symbolic)	Analytic (Result)	Ref. [3]
2	-	-	$\frac{100}{3}\sqrt{\pi^2-9}$	31.08419398	31.08
3	15	-	$\frac{5}{9}\sqrt{\frac{3150\pi^2}{\cos\left(\frac{\pi}{12}\right)^2}-32400}$	16.86330189	16.86
4	20	-	$\frac{50}{9}\sqrt{\frac{69\pi^2}{\left(\cos\left(\frac{\pi}{9}\right)+\frac{1}{2}\right)^2}-324}$	11.85809395	11.86
5	$\frac{15}{2}$	$\frac{45}{2}$	$25\sqrt{\frac{6\pi^2}{\left(\cos\left(\frac{\pi}{24}\right)+\cos\left(\frac{\pi}{8}\right)\right)^2}-16}$	9.431778601	9.43

The proposed formulation can also be used for finding absolute minimum LTHD (MLTHD) values, as illustrated for the 5 level case in Figure 4. These MLTHD values were obtained using MATLAB's genetic algorithm (GA) [22] for $2 \leq N \leq 13$ and are listed in Table 2, along with the optimum PSA set and normalized fundamental component (6). By utilizing the GA-based optimization method from [22] with the proposed LTHD formulation, optimal LTHD results vs. m_a (i.e., OMTHD) were obtained for several values of N and presented in Figure 5. The actual percentage of modulation error (ME), as previously suggested in [22], was calculated using the following expression [22]:

$$\varepsilon_m = 100 \cdot \left|\frac{m_T - m_a}{m_T}\right|, \tag{17}$$

where m_a is the actual modulation index (6) and m_T is the desired (target) modulation index. The ME results, corresponding to the optimal LTHD results from Figure 5, obtained while the ME (17) was constrained to a practical value of 1% [9], are presented in Figure 6, which shows excellent ME compliance with the desired tolerance.

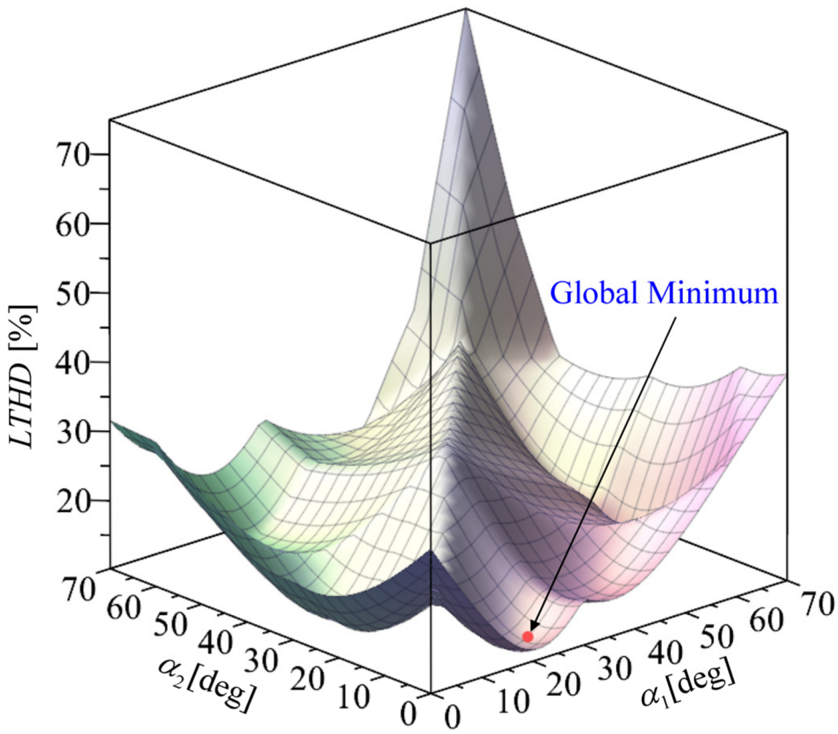

Figure 4. Analytical LTHD with global minimum for N = 5.

Table 2. Minimum LTHD Results, %.

N	*LTHD*	m_a	α_1	α_2	α_3	α_4	α_5	α_6
2	31.08	1.10	-	-	-	-	-	-
3	16.86	1.06	15.30	-	-	-	-	-
4	11.76	1.05	21.13	-	-	-	-	-
5	9.23	1.05	7.84	24.16	-	-	-	-
6	7.76	1.05	12.66	26.00	-	-	-	-
7	6.26	1.02	5.38	16.33	34.22	-	-	-
8	5.43	1.03	9.21	18.66	34.05	-	-	-
9	4.92	1.03	4.00	12.09	20.42	33.94	-	-
10	4.32	1.01	7.27	14.66	26.29	39.25	-	-
11	3.88	1.02	3.25	9.78	16.45	26.93	38.52	-
12	3.60	1.02	5.88	11.83	17.91	27.47	37.96	-
13	3.35	1.01	2.72	8.18	13.72	22.30	28.31	41.61

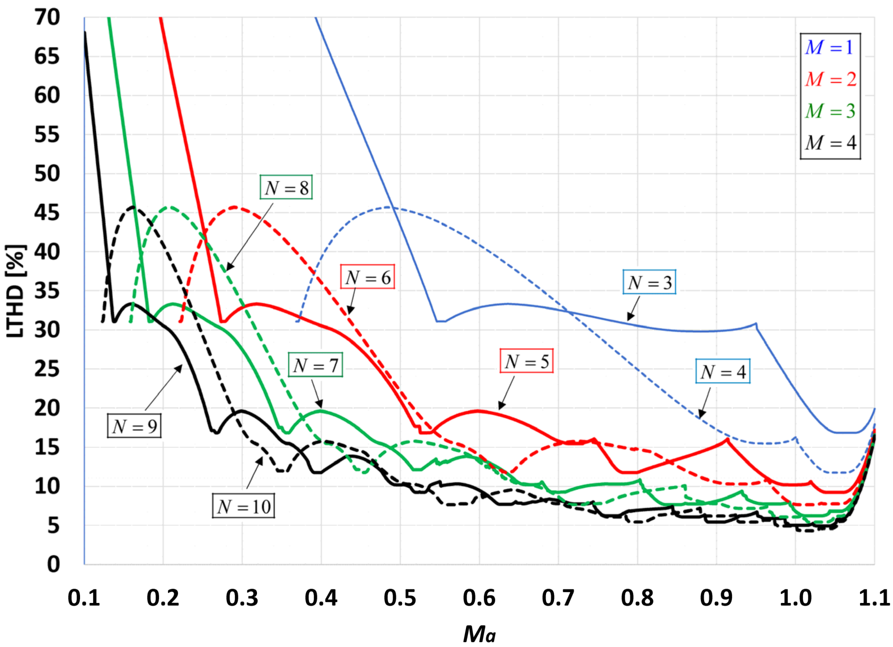

Figure 5. Calculated minimum LTHD vs. modulation index for various values of N.

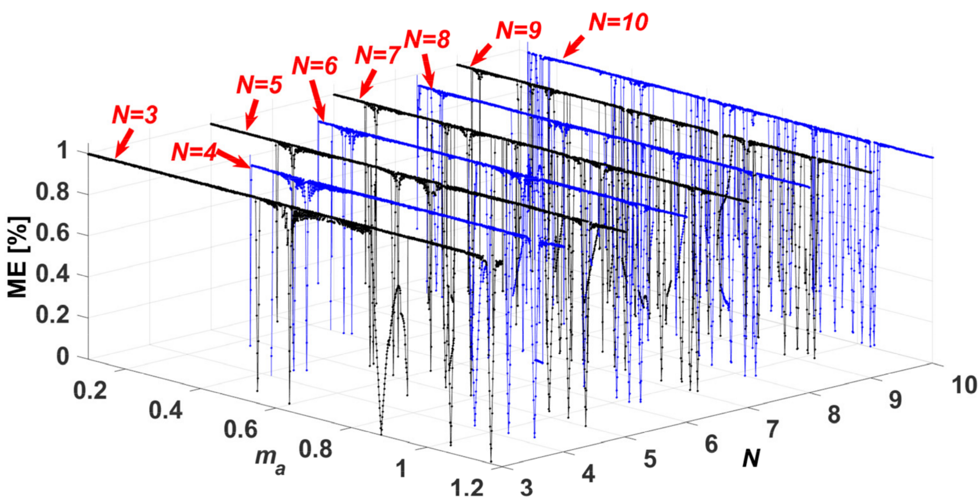

Figure 6. Calculated ME results for optimal LTHD vs. modulation index for various values of N.

To further test the proposed LTHD formulation, a case study of a 7 level OMTHD implementation for a 3ϕ CHB MLI, in which an LTHD-optimal analysis was carried out using the revealed LTHD expression, was performed. The results were then compared to previously reported results [9]. The optimum PSA results from [9] were digitally extracted and plotted against matching optimum PSA results, obtained using the proposed analytic LTHD expression and the optimization algorithm from [22]. The 7 level optimum PSA comparison results are depicted in Figure 7, which reveals an overall good correlation between the two approaches, considering the fact that an open-form analytic LTHD function was utilized in [9].

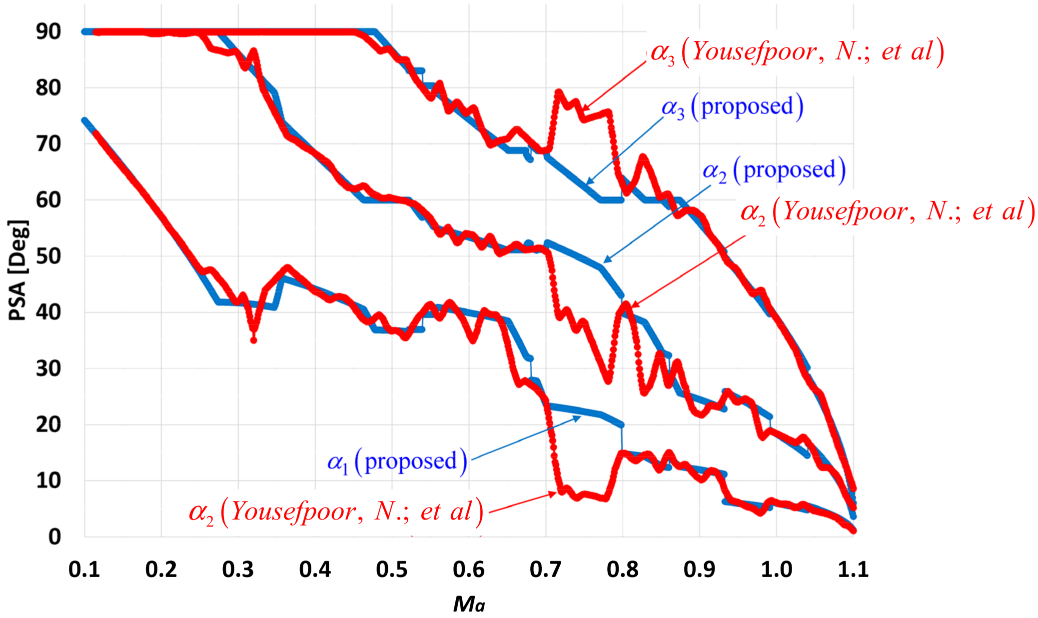

Figure 7. LTHD-optimal PSAs for N = 7: analytical (proposed) vs. obtained from [9].

The data from [9], were also used to calculate the corresponding optimal LTHD values, which were also plotted against the proposed results. As indicated in Figure 8, the proposed analytical results seem much smoother, more consistent, and free from calculation-error-related fluctuations. Moreover, the obtained LTHD results using the proposed formulation are significantly better, by an absolute value of up to 7% (cf. zoomed-n window in Figure 8). Using the same data extracted from [9], the ME values were also calculated using Equation (17) and compared against the proposed results, as indicated by Figure 9. These ME results reveal another advantage to using the proposed analytical closed-form expression, in which, unlike the open-form expression used in [9], the ME values remain restricted to the maximum preselected tolerance of 1% throughout the entire feasible range of the modulation index (cf. Figure 9).

Table 3 summarizes eight cases of optimal LTHD comparisons between the proposed results and corresponding results from three other sources [8–10], which includes the minimum LTHD calculation examples of $N = 5$, 7, and 9. The table includes (left to right) the following: case number; N value; target MI (m_T, for OMTHD only); data source; reported LTHD result; actual LTHD result (calculated using the proposed formulation); error due to under or overestimation (deviation from the reported value); error due to deviation from proposed optimal result; calculated ME; and PSA set. The proposed corresponding results are provided in the next line, below, for ease of comparison.

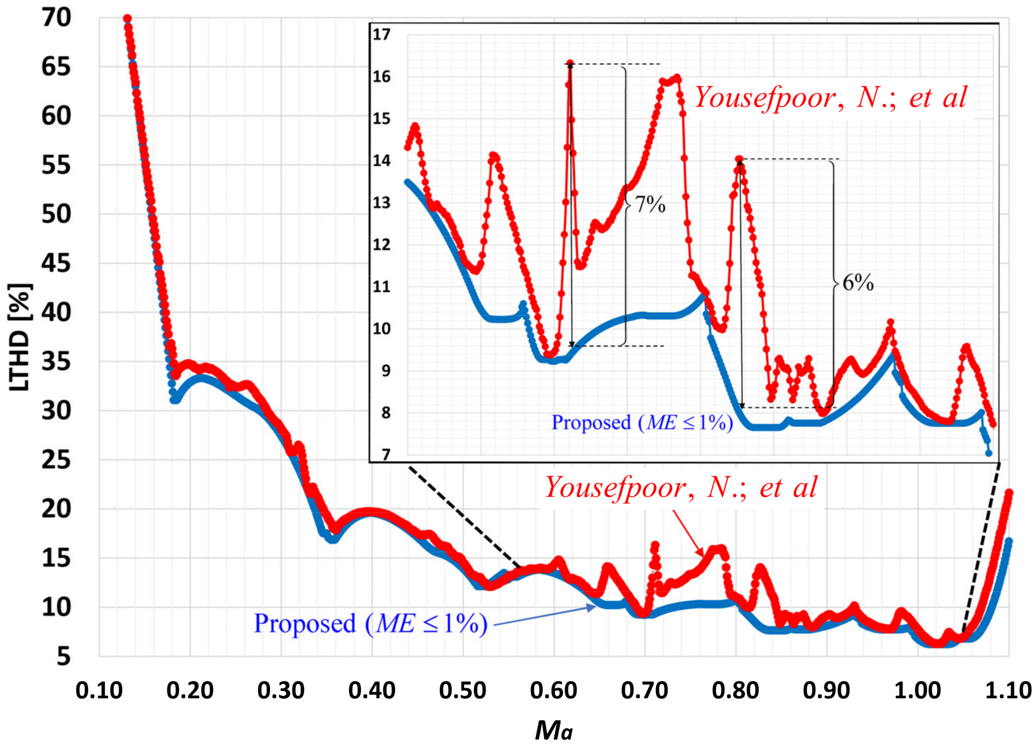

Figure 8. LTHD-optimal results (actual LTHD vs. *ma*) for N = 7: analytical (proposed) vs. obtained in [9].

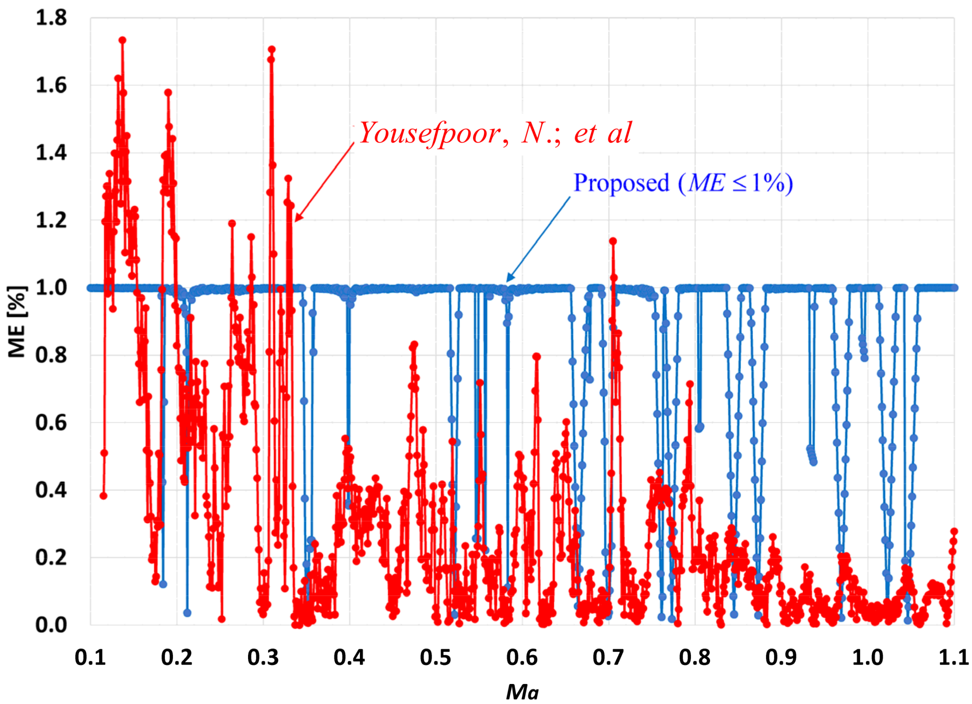

Figure 9. LTHD-optimal actual ME results for N = 7: analytical (proposed) vs. obtained in [9].

Table 3. Optimal LTHD calculations: comparison between proposed and previously claimed by other sources.

#	N	m_T	Source	Reported LTHD (%)	Actual LTHD (%)	Report Error (%)	Actual Error (%)	ME (%)	α_1 (°)	α_2 (°)	α_3 (°)	α_4 (°)
1	5	-	Ref. [8]	8.270	9.239	−10.486	0.099	-	7.61	24.40	-	-
			Proposed	9.230	9.230	0	0	-	7.84	24.16	-	-
2	7	-	Ref. [8]	5.200	6.258	−16.909	0.042		5.46	16.30	34.40	-
			Proposed	6.256	6.256	0	0		5.38	16.33	34.22	-
3	9	-	Ref. [8]	3.940	5.102	−22.771	3.593		5.33	12.70	20.40	33.70
			Proposed	4.925	4.925	0	0		4.00	12.08	20.42	33.94
4	7	0.77	Ref. [9]	10.310	10.313	−0.033	0.014	0.013	21.81	47.75	60.06	-
			Proposed	10.312	10.312	0	0	0.009	21.75	47.83	60.00	-
5	7	0.87	Ref. [10]	12.300	8.725	40.967	12.471	0.00	11.68	31.18	58.58	-
			Proposed	7.758	7.758	0	0	0.78	12.66	26.00	60.00	-
6	7	0.35	Ref. [10]	30.200	23.530	28.346	35.161	10	44.17	74.33	87.40	-
			Proposed	17.409	17.409	0	0	1.00	42.16	77.84	90.00	-
7	7	0.09	Ref. [10]	190.300	31.490	504.328	−71.509	379.66	55.85	63.43	83.02	-
			Proposed	110.523	110.523	0	0	1.00	76.23	90.00	90.00	-
8	7	0.74	Ref. [10]	14.800	10.252	44.366	1.648	0.01	22.77	49.38	64.57	-
			Proposed	10.085	10.085	0	0	1.00	22.78	50.75	64.65	-

For example, in case #3 in Table 3, according to the 9 level results extracted from [8], the reported minimum LTHD (MLTHD) is 3.94%, corresponding to the reported optimum set of PSAs of $\alpha_1 = 5.33^{\circ}, \alpha_2 = 12.7^{\circ}, \alpha_3 = 20.4^{o}$. Using these PSA values with $N = 9$ to calculate the LTHD results using the proposed formulation from Equation (15) yields an actual result of 5.102%, indicating a significant −22.77% report error, which is attributed to the underestimation phenomena associated with the 50 harmonics numerical approximation used in [8]. When applying the same LTHD minimization using the proposed analytical approach for both optimization and LTHD calculation, the actual minimum LTHD value for case #3, corresponding to an optimum PSA set of $\alpha_1 = 4.0^{\circ}, \alpha_2 = 12.08^{\circ}, \alpha_3 = 20.42^{\circ}$, is 4.925%, indicating a deviation of +3.593%, as indicated by the actual error in Table 3 (relative deviation from the actual value of 5.102%). Notably, a very small deviation in the optimum PSA results may lead to a significant deviation in the actual LTHD value, as indicated in Table 3. Another conclusion from Table 3 is the fact that the higher the value of N and the lower the value of the target modulation index, m_a, the higher the calculation error, as indicated by the 504% (!) report and–72% actual errors of case #7 in Table 3 (cf. [10]).

Real-world practical experiments involving multilevel converters are quite challenging due to the required alterations in operating conditions, as well as safety when higher voltages are involved. This has led to other reliable validation solutions, such as the hardware in loop (HIL) and controller + HIL (C-HIL), which has been gradually identified as an effective tool for power electronic converter development and digital control design, with near-practical testing conditions [24,25]. To test the proposed LTHD formulation practically, LTHD-based OMTHD utilizing the proposed analytical LTHD expression was validated using both processor in loop (PIL) real-time target hardware simulations (hosted by PSIM) and C-HIL based experiments, using a Typhoon HIL402 power-stage real-time emulator [24], hosting a Typhoon DSP180 digital controller card (TI's C2000 TMS32F28335 based), namely, real-time emulations of both controller and MLI [25]. Consequently, 80 kHz FFT-based LTHD calculations were obtained for the C-HIL experiments using a Keysight

X4024A oscilloscope The experimental setups, which are depicted in Figure 10, consisted of 3ϕ 7- and 8-level MLIs (CHB and DDC) with lookup table-based SCM controllers [22] and a 3ϕ induction motor, serving as a dynamic load to the MLIs. The experimental setup parameters are listed in Table 4.

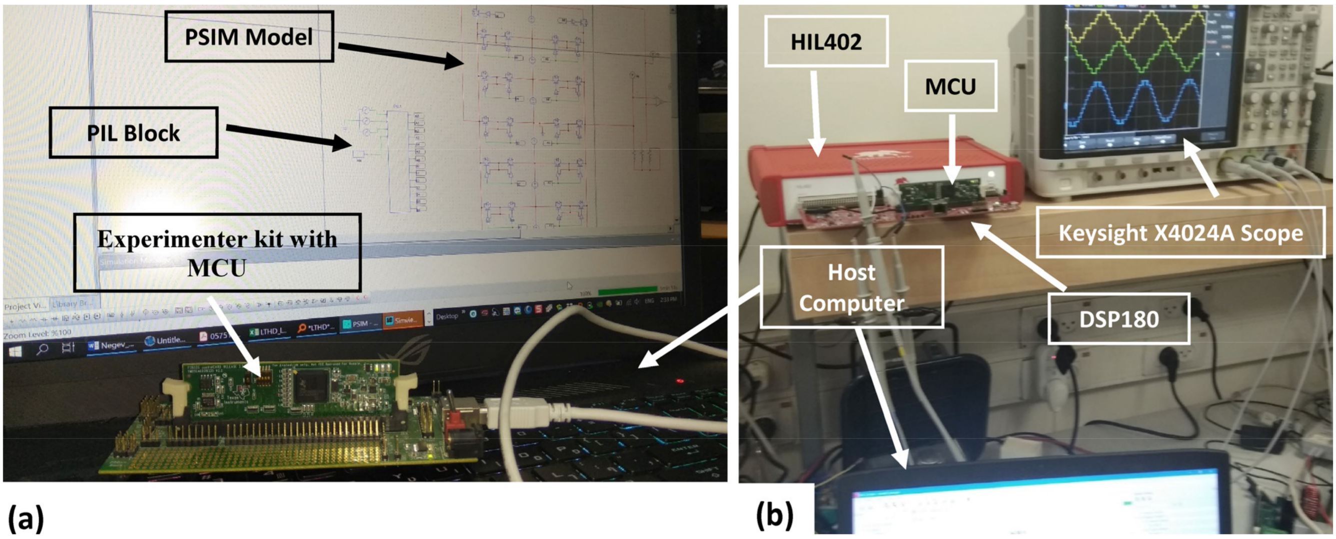

Figure 10. Real-time experimental setups: (**a**) processor in loop, (**b**) controller + hardware in loop.

Table 4. Setup parameters for real-time C-HIL base experimental verification.

Parameter	Symbol	Value/Model
Total DC-link Voltage	V_{dc}	400 V
IGBT+Diode	-	Infineon AUIRGDC0250 (modified PSIM SPICE model)
DC-link Partial Voltage Caps	C_k	4 mF, 1000 V (PIL)/400 V (HIL)
Rated Output Fundamental AC Voltage	V_{nom_LL}	200 V @50 Hz
Load Type	-	3ϕ squirrel-cage 20 kW @0.8, 50 Hz, 1460 RPM motor (model's parameters extracted from Simscape)

The first test was PIL-based, using the PSIM software of a 7 level 3ϕ CHB MLI, which was used to recreate the experimental results in [9] under similar conditions for a modulation index of 0.772. The minimum LTHD value was evaluated as 10.31%, matching the results reported in [9]. The results in Figure 11 show that the proposed approach yields less sensitive and smoother (i.e., more "analytical" in nature) values compared to the results obtained in [9].

The second test was based on the real-time C-HIL setup from Figure 10a. The experimental waveforms, shown in Figure 12, include the phase- and line-voltages, v_a, v_b, v_{ab} (scaled by 50:1), and the load current, i_a (scaled by 10:1). Line-voltage THD and fundamental AC RMS calculations (real-time FFT-based using the Keysight X4024A oscilloscope) are also included with the captured waveforms of Figure 12. The experimental validations for both the 7 level CHB and the 8 level DDC were repeated for both low and high values of modulation indices. The optimum PSA values for the desired modulation indices were calculated offline and used for the lookup table-based SCM controller [9,22]. These optimum results are included in the supplemental file provided, which contains the Maple and MATLAB functions of the proposed LTHD formulation, as well as pre-calculated sets of optimum PSAs for different values of N ($3 \leq N \leq 13$). It is also provided for readers' convenience as an online link in [23].

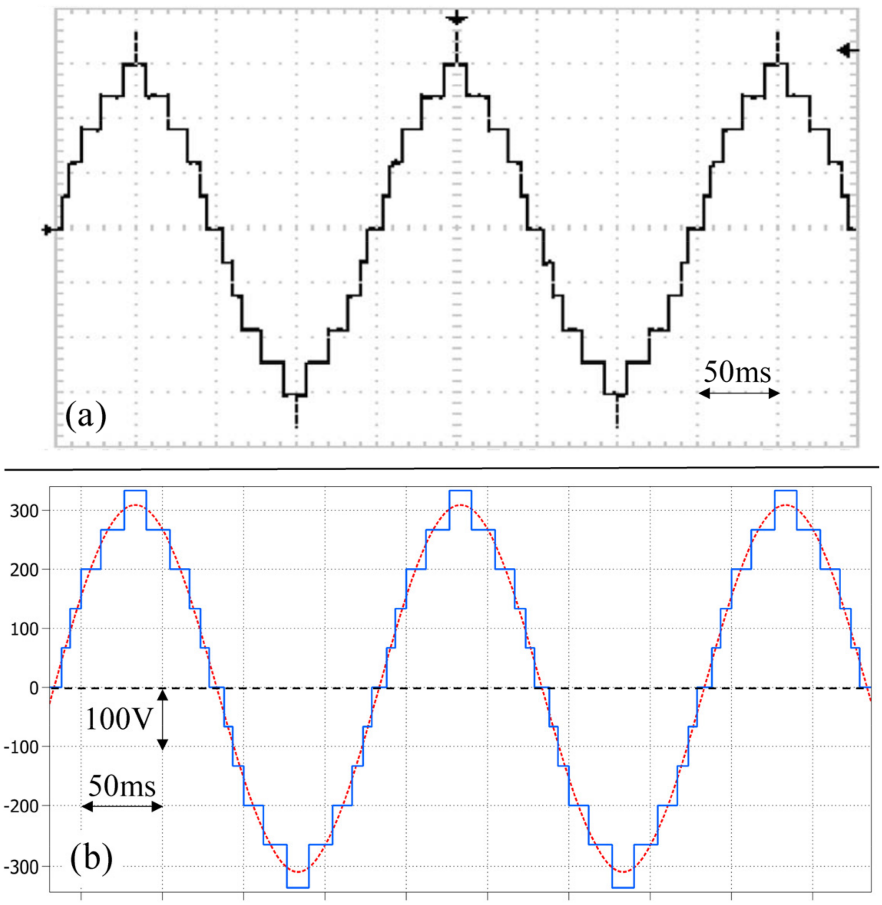

Figure 11. Line voltage SCM waveforms for $m_a = 0.772$. (**a**) Experimental results in [9], (**b**) corresponding processor-in-the-loop waveform with first harmonic indicated (dashed line).

The modulation index range feasibility is determined by Equation (7); therefore, its lowest possible value for the 8 level case is $m_T = 2\sqrt{3}/7\pi = 0.157 \approx 0.16$, while for the 7 level case, it was set to 0.1. A higher-range modulation index value of 0.9 was tested as well. As can be concluded from the experimental results in Figure 12, the practical LTHD values for the 7 level MLI were 97.14% and 8.27% for $m_T = 0.1$ and $m_T = 0.9$, respectively. These results are slightly higher than the calculated theoretical results (available in [23]), which were 96.45% and 8.13%, respectively. For the 8 level case, the practical LTHD results were 32.58% and 7.29% for $m_T = 0.16$ and $m_T = 0.9$, respectively. Once again, these were slightly higher than the corresponding theoretical results of 31.91% and 7.75%, respectively [23]. The actual ME values remained below 1.3% for all the experimental results. It is apparent that the higher the modulation index, the lower the LTHD and ME result. These results fully confirm the advantage of using simple closed-form analytical LTHD expressions for optimal SCM implementation in three-phase MLIs, where the quality of the line-voltage is most important, as indicated by the revealed line-voltage to load-current waveforms correlation and THD values (cf. Figure 12).

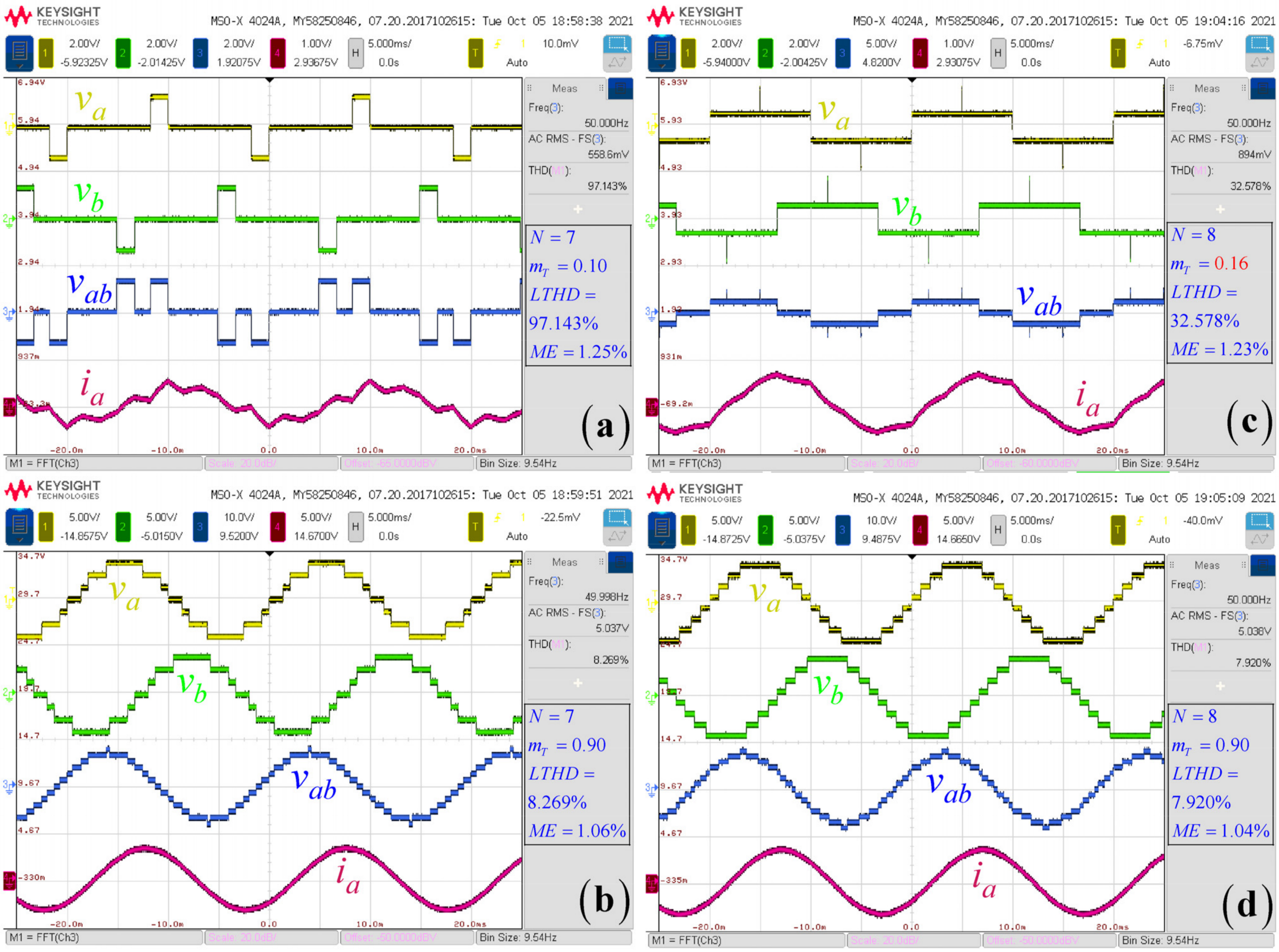

Figure 12. Real-Time experimental (C-HIL) results of optimal LTHD-based SCM (phase voltage, line voltage, and load current waveforms): (**a**) 7 Level CHB at $m_T = 0.1$, (**b**) 7 Level CHB at $m_T = 0.9$, (**c**) 8 Level DDC at $m_T = 0.16$, (**d**) 8 Level DDC at $m_T = 0.9$.

4. Conclusions

In three-phase MLIs with three conductor loads (no neutral connection), the line-voltage THD values are the main concern, as they exert a direct effect on the load's current harmonics. While previous papers did offer LTHD-based optimization for SCM-based control, no true analytical LTHD expression valid for any chosen topology and number of voltage levels has been presented until now. In this article, the analytical voltage LTHD expression of SCM 3ϕ MLIs, applicable to any number and parity of voltage levels, was revealed and verified by exploring both the LTHD calculation accuracy and LTHD-based optimization methods. The validation, carried out by comparing the results obtained using the proposed method to results previously obtained in other studies, demonstrated excellent matching and the complete elimination of underestimation errors, which are typically associated with frequency-spectra-based numerical approximations, as per IEEE 219 standards. It was shown that by utilizing the proposed closed-form analytical LTHD formulation to solve the OMTHD problem, applied directly to line-voltage waveforms, high-accuracy, low-sensitivity results were yielded within the whole range of practical modulation indices. To enhance the contribution of this work, downloadable links to N-level LTHD functions for both MATLAB and Maple software, in addition to a look-up table of the optimum switching angles for both phase and line voltage THD, using eight different values of N (3 to 10), were also included.

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