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Adaptive-Mode PAPR Reduction Algorithm for Optical OFDM Systems Leveraging Lexicographical Permutations

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Abstract: In direct current optical orthogonal frequency division multiplexing (DCO-OFDM) systems, the high peak-to-average power ratio (PAPR) has been a significant challenge. Recently, lexicographical symbol position permutation (LSPP) using random permutations has been introduced as an efficient solution to reduce high PAPR. In this paper, we aim to evaluate the effectiveness of LSPP by comparing both adjacent and interleaved lexicographical permutation sequences with random lexicographical permutation sequences. Our findings demonstrate that random permutation yields superior PAPR reduction performance results when compared to adjacent and interleaved permutation. However, in scenarios with a limited number of sub-blocks, the use of adjacent and interleaved permutation becomes more favorable, as they can eliminate the possibility of generating identical permutation sequences, a drawback of random permutation. Additionally, we propose a novel algorithm to determine the optimal number of candidate permutation sequences that can achieve acceptable PAPR reduction performance while adhering to computational complexity constraints defined by the system requirements.

Keywords: CCDF; DCO-OFDM; lexicographical; PAPR; permutations; VLC

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1. Introduction

Visible light communication (VLC) has garnered significant interest from industry, standardization bodies, and the research community as an appealing alternative and complementary technology to radio frequency (RF) communication systems [1–3]. This is primarily due to its several distinctive advantages, including the utilization of abundant unregulated bandwidth, environmental friendliness, absence of electromagnetic compatibility concerns, and robust security and privacy features [4–6].

Direct-current optical orthogonal frequency division multiplexing (DCO-OFDM) has been introduced as a preferred modulation technique in VLC systems due to its ability to deliver high data rates and deal with adverse channel conditions such as fading and attenuation without the need for complicated equalization at the receiver [7]. Moreover, in DCO-OFDM both the even and odd subcarriers are used for data transmission, making it more spectrally efficient compared to asymmetrically clipped optical orthogonal frequency division multiplexing (ACO-OFDM), which uses only the odd subcarriers [8].

Nevertheless, due to the inherent nature of composite DCO-OFDM signals, which consist of independently modulated subcarriers, it is possible for certain subcarriers to exhibit constructive interference, resulting in significant fluctuations in the signal envelope in the time domain when there is a high correlation in the input sequence for the inverse fast Fourier transform (IFFT). As a consequence, a pronounced peak-to-average power ratio (PAPR) is commonly observed. This issue of high PAPR poses a formidable challenge in optical OFDM systems, which unfortunately has yet to be effectively addressed or resolved [9–13]. The challenge of high PAPR poses a formidable obstacle when it comes to preserving the integrity of the complete signal within the linear operating range of crucial transmitter elements. These essential components, including but not limited to the power amplifier, digital-to-analog converter (DAC), and light emitting diode (LED), are susceptible to the consequences of this challenge. Accordingly, the repercussions manifest in the form of signal distortion, wherein the signal undergoes either lower or upper clipping. Consequently, the distortion-induced degradation leads to an undesirable elevation in the bit error rate (BER).

A number of solutions have been proposed that could be deployed to deal with this problem. One such category consists of multiple signaling and probabilistic methodologies, such as the utilization of selected mapping (SLM) [14–16] and symbol position permutation (SPP) [17], which have become popular because they result in significant PAPR reduction without BER degradation. Both SLM and SPP accomplish PAPR reduction by generating and evaluating a range of diverse candidate signals from which the one exhibiting the least PAPR is then selected for transmission. The chosen signal is transmitted to the receiver along with its associated side information (SI) to enable the seamless and efficient recovery of the original data. In the SLM technique, the candidate signals are obtained by generating V random phase sequences, which are then multiplied by the optical OFDM symbol in the frequency domain to generate V candidate signals after taking the IFFT operations. In the SPP method, the required V candidate sequences are obtained from the permutation of Q sub-blocks obtained from the frequency domain optical OFDM symbol. In terms of complexity, the SPP method can be regarded as relatively simple compared to selected SLM, primarily thanks to its avoiding the need to multiply random phase sequences by the optical OFDM symbol. However, it should be noted that the effectiveness of PAPR mitigation in the SPP method is directly proportional to the number of candidate permutation sequences generated simultaneously. As a result, additional intricacies are introduced within the overall system architecture.

In a recent study, the authors of [18] presented an innovative PAPR mitigation technique called lexicographical symbol position permutation (LSPP). This novel approach focuses on DCO-OFDM systems, and involves the utilization of randomly generated lexicographical permutations to address the PAPR challenge. In this method, V random permutation candidate sequences from the possible $Q!$ permutation sequences are generated one at a time lexicographically based on the factorial number system.

Based on the work presented in [18], in this paper we use the complementary cumulative distribution function (CCDF) to compare the PAPR reduction performance capabilities of the three categories of lexicographical permutations, i.e., random, as used in [18], adjacent, and interleaved permutation sequences in DCO-OFDM systems.

Efficient resource allocation is crucial in a VLC system, and must take into account factors such as bandwidth and power requirements [19]. Furthermore, it is essential to consider the varying complexity levels required in different situations and application environments, encompassing both hardware and time complexity. The effectiveness of PAPR reduction achieved through the implementation of LSPP is directly linked to the quantity of candidate permutation sequences utilized. As a result, the complexity of this technique, particularly in terms of the necessary IFFT operations, increases proportionally with the number of candidates employed. To address this challenge and find a system that can be adapted to different situations and application environments, we use the global gain concept [20] to determine the most suitable number of candidate permutation sequences for PAPR reduction in DCO-OFDM systems while avoiding increases in computational complexity beyond system requirements.

The subsequent sections of this paper are organized as follows: Section 2 describes the LSPP scheme for DCO-OFDM systems, the PAPR challenge, and the global gain concept; Section 3 presents the algorithms used to generate adjacent and interleaved permutations and to obtain the most suitable number of candidate permutation sequences; and Section 4 provides a comprehensive presentation of the simulation results and subsequent discussion. Finally, Section 5 provides our concluding remarks.

2. Background

2.1. The PAPR Problem

The PAPR of the time domain signal subsequent to the IFFT operation, denoted as $x(n)$, serves as a valuable metric that quantifies the correlation between the highest power level (peak power) and the average power level present within the signal. This relationship is determined by dividing the peak power by the average power, and is mathematically represented by [11]

$$\mathrm{PAPR}\{x(n)\} = \frac{\max\{|x(n)|^2\}}{\langle |x(n)|^2 \rangle}, \tag{1}$$

where $\langle \cdot \rangle$ denotes the statistical expectation.

The CCDF is a commonly used tool for evaluating the reduction in PAPR and comparing the efficiency of various PAPR reduction techniques. It is described as the probability of the PAPR of an OFDM symbol surpassing a predetermined threshold value, denoted as PAPR_0. In mathematical terms, CCDF can be expressed as [21]

$$\begin{aligned} \mathrm{CCDF} &= 1 - \Pr\{\mathrm{PAPR} \le \mathrm{PAPR}_0\} \\ &= 1 - (1 - \exp(-\mathrm{PAPR}_0))^N, \end{aligned} \tag{2}$$

where N represents the number of subcarriers.

To achieve a more reasonable estimate of the PAPR, the frequency domain signal is typically over-sampled L times; consequently, (2) can be written as follows:

$$\mathrm{CCDF} = 1 - (1 - \exp(-\mathrm{PAPR}_0))^{\alpha N}, \tag{3}$$

where α is the over-sampling factor. When $L = 4$ (equivalent to $\alpha = 2.8$ [21]) is used, the peaks of the continuous time domain signal match those of the discrete time domain signal.

2.2. Lexicographical Symbol Position Permutation

Figure 1 illustrates the block diagram of the LSPP system used for PAPR mitigation in DCO-OFDM systems [18].

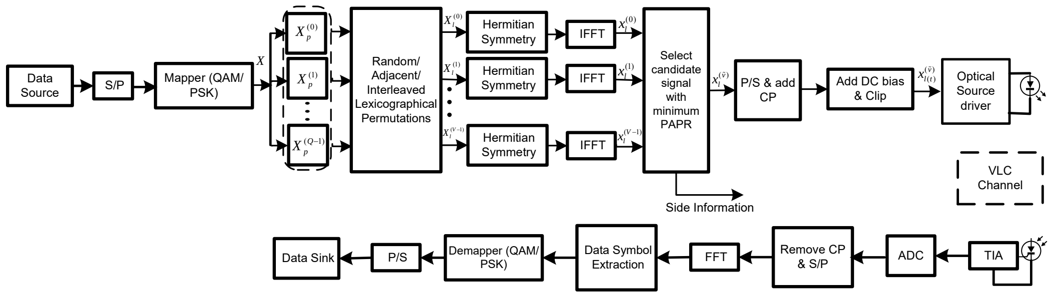

Figure 1. Lexicographical Symbol Position Permutation Scheme. S/P: serial-to-parallel converter, P/S: parallel-to-serial converter.

In this system, $X = X_k$, where $k = 0, 1, \ldots, (N/2 - 1)$ is the input data stream composed of N subcarriers that are already mapped onto a given constellation, such as M-ary quadrature amplitude modulation (QAM) or phase shift keying (PSK).

The input data stream X_k is divided into Q permutation sub-blocks, as follows [18]:

$$X_p^{(q)} = X_{Q\times 0+q}, X_{Q\times 1+q}, X_{Q\times 2+q}, \ldots, X_{Q\times (N/(2Q)-1)+q}, \tag{4}$$

where $q = 0, 1, \ldots, (Q-1)$ and $N/2Q$ is an integer used to ensure that the Q permutation sub-blocks are the same size.

For instance, considering an input data stream with a size of $N/2 = 32$, i.e., $X = X_0, X_1, \ldots X_{31}$ and $Q = 8$, the lexicographical permutation sub-blocks are subsequently presented as

$$\begin{aligned} X_p^{(0)} &= [X_0, X_8, X_{16}, X_{24}], \\ X_p^{(1)} &= [X_1, X_9, X_{17}, X_{25}], \\ &\vdots \\ X_p^{(7)} &= [X_7, X_{15}, X_{23}, X_{31}]. \end{aligned} \tag{5}$$

The next step is to generate the V lexicographical permutation sequences $X_l^{(v)}$, where $0 \leq v \leq (V-1)$, using Algorithm 1 for random lexicographical permutation sequences [18].

Algorithm 1 An algorithm for generating random lexicographical permutations

Input: Permutation sub-blocks, $X_p^{(q)}$, $0 \leq q \leq (Q-1)$; Select $V, V \in \{0, 1, \ldots, Q!\}$ set $i = 1$;
Output: Random Lexicographical permutation sequences $X_l^{(v)}$, $0 \leq v \leq (V-1)$

for $i = 1 : V$ **do**
  Generate the l^{th} permutation sequence index, l at random, $l \in \{0, 1, \ldots, (Q! - 1)\}$
  Set $X_l^{(v)} := []$
  while $X_p^{(q)} \neq []$ **do**
    $\omega := \left(\left|X_p^{(q)}\right| - 1\right)!$
    $j := \lfloor l/\omega \rfloor$
    $\vartheta := X_p^{(j)}$
    $l := l \bmod \omega$
    ϑ is appended to $X_l^{(v)}$
    ϑ is removed from $X_p^{(q)}$
  end while
  $X_{l(v)}$ is returned
end for

An optical OFDM system must use a real-valued signal to modulate the carrier intensity [22]. To achieve this condition, the input permutations must be constrained to Hermitian symmetry before IFFT, as shown in Figure 1. For this reason, the input for the IFFT is executed as follows:

$$X_{l(k)}^{(v)} = X_{l(N-k)}^{(v)*} \text{ for } 0 < k < N/2, \tag{6}$$

in which the two components $X_{l(0)}^{(v)}$ and $X_{l(N/2)}^{(v)}$ are set to zero, i.e., $X_{l(0)}^{(v)} = X_{l(N/2)}^{(v)} = 0$, and X^* represents the complex conjugate of X.

The resulting signal candidates are obtained by performing the N-point IFFT on the frequency domain input lexicographical permutation sequences, resulting in a series of signals

$$x_{l(n)}^{(v)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_{l(k)}^{(v)} \exp\left(j\frac{2\pi kn}{N}\right) \tag{7}$$

where $0 \leq n \leq N-1$ and $x_{l(k)}^{(v)}$ are the V lexicographical permutation sequences obtained from Algorithm 1 and j is the imaginary operator.

The time domain candidate signal with the minimum PAPR obtained from (7) is $x_{l(n)}^{(\tilde{v})}$ is then considered for transmission. In this case, $\tilde{v}$ represents the index of the candidate signal with minimum PAPR: $0 \leq \tilde{v} \leq (V-1)$.

A cyclic prefix (CP) is often added to the candidate signal with the minimum PAPR to address the potential issue of inter-symbol and inter-carrier interference in dispersive optical wireless communication channels, as it can mitigate these types of interference to provide more clear and reliable transmission.

A DAC is used to convert the discrete time domain candidate signal with the minimum PAPR $x_{l(n)}^{(\tilde{v})}$ into a continuous time domain signal $x_{l(t)}^{(\tilde{v})}$. In order to transmit a unipolar signal, a DC offset voltage $\mathrm{DC_{offset}}$ is added to the signal $x_{l(t)}^{(\tilde{v})}$. Finding the right balance is crucial; an excessive DC bias leads to escalated optical power, whereas a minimal DC bias results in an amplified clipping noise, with a detrimental effect on BER. Hence, an appropriate DC offset proportionate to the root mean square (RMS) of the signal $x_{l(t)}^{(\tilde{v})}$ is introduced; this can be mathematically expressed by [23]

$$\mathrm{DC_{bias}} = \eta\sqrt{\langle |x_{l(t)}^{(\tilde{v})}|^2\rangle}, \tag{8}$$

where the constant η holds significant importance and the $\mathrm{DC_{bias}}$ is defined as a bias that is quantified by the expression $10\log(\eta^2+1)\,\mathrm{dB}$.

Next, the remaining downward peaks are truncated, leading to the generation of a real and unipolar signal, which is then directed towards the LED for transmission within the VLC channel.

As shown in Figure 1, detection of the received optical signal is performed by a photodiode which converts the optical signal to an electrical signal. The analog signal is typically amplified by the transimpedance amplifier (TIA) before conversion to a digital signal [24]. After serial-to-parallel conversion and CP removal, the received frequency domain signal can be obtained using N-point fast Fourier transform (FFT). The index v of the selected candidate signal as well as the frequency domain symbol sequence can be extracted after performing the IFFT process. QAM or QPSK de-mapping is then performed, followed by parallel-to-serial conversion to obtain the received data bits.

It should be emphasized that, as in SLM, our proposed LSPP requires $\log_2(V)$ bits for the SI in each transmitted block of information. This SI can either be sent through an independent channel or integrated in the transmission and guarded with error control codes to avoid incorrect detection at the receiver. We presume accurate transmission and reception of the SI in this paper.

2.3. Global Gain

In order to determine the most suitable number of candidate permutation sequences, we use the global (net) gain which is defined as a particular case of the fitness/objective function-based approach [20], where the two factors of interest under consideration in this paper are the PAPR reduction performance and the computational complexity. We consider these two factors because in LSPP the number of candidate permutation sequences is directly proportional to the PAPR reduction performance, which can lead to an increase in both hardware and time complexity due to the need to perform the substantial number of IFFT operations required to generate the candidate signals.

Therefore, under certain VLC link configurations such as diffuse, directed line of sight, non-directed line of sight, and tracked, the PAPR reduction performance can be expressed in relative terms:

$$Z_1 = -10\log_{10}\left(\frac{\mathrm{PAPR_{after}}}{\mathrm{PAPR_{before}}}\right). \tag{9}$$

Similarly, the relative increase in computational complexity is provided by

$$Z_2 = -10\log_{10}\left(\frac{\text{Complexity}_{\text{after}}}{\text{Complexity}_{\text{before}}}\right). \tag{10}$$

Accordingly, if σ_a represents the weights of factors related to the significance level of PAPR reduction (a = 1) and the increase in computational complexity (a = 2) in the system, the aggregate fitness value is [25]

$$\Omega = \sum_{a=1}^{2} \sigma_a \cdot Z_u, \tag{11}$$

where

$$\sum_{a=1}^{2} \sigma_a = 1. \tag{12}$$

The appropriate number of candidate permutation sequences needed to achieve the desirable PAPR reduction performance while satisfying the computational complexity requirements of the system can then be chosen based on the aggregate fitness values.

3. Adjacent and Interleaved Lexicographical Permutation Sequences

The V lexicographical permutation sequences from among the possible $Q!$ permutation sequences generated from (4) can be random, interleaved, or adjacent. The adjacent and interleaved lexicographical permutation sequences are provided by

$$X_l^{(v)} = X^{(0)}_{\varsigma+0\times\Delta}, X^{(1)}_{\varsigma+1\times\Delta}, X^{(2)}_{\varsigma+2\Delta}, \ldots, X^{(V-1)}_{\varsigma+V\Delta-1}, \tag{13}$$

where $\Delta = 1$ for adjacent lexicographical permutations and $\Delta = 2$ for interleaved lexicographical permutations, while ς, $0 \leq \varsigma \leq (Q! - \Delta V)$ is the index of the first permutation in the lexicographical order and $0 \leq v \leq (V-1)$.

The adjacent and interleaved lexicographical permutations can be generated using Algorithm 2.

Algorithm 2 An algorithm to generate interleaved and adjacent lexicographical permutations

Input: Permutation sub-blocks, $X_p^{(q)}, 0 \leq q \leq (Q-1)$; Select V, $V \in \{0, 1, \ldots, Q!\}$; Set the index of the first permutation sequence, $\varsigma, 0 \leq \varsigma \leq (Q! - \Delta V)$; Select either, $\Delta = 1$ for adjacent or $\Delta = 2$ for interleaved permutations; Set $i = 1$;

Output: Either adjacent or interleaved lexicographical permutation sequences, $X_l^{(0)}, X_l^{(1)}, \ldots, X_l^{(V-1)}$

1: Set $\Delta = 1$ or $\Delta = 2$
2: **for** $i = 1 : V$ **do**
3: Set $X_l^{(v)} := []$
4: **while** $X_p^{(q)} \neq []$ **do**
5: $\omega := \left(\left|X_p^{(q)}\right| - 1\right)!$
6: $j := \lfloor \varsigma / \omega \rfloor$
7: $\vartheta := X_p^{(j)}$
8: $\varsigma := \varsigma \bmod \omega$
9: ϑ is appended to $X_l^{(v)}$
10: ϑ is removed from $X_p^{(q)}$
11: **end while**
12: $X_l^{(v)}$ is returned
13: $\varsigma = \varsigma + \Delta$ and go to step 3.
14: **end for**

3.1. Impact of the Number of Sub-Blocks on PAPR Reduction with Random LSPP

When employing random lexicographical permutation sequences V out of the maximum possible $Q!$ permutation sequences, with Q representing the number of sub-blocks, the probability of selecting a particular sequence m times is $\left(\frac{1}{Q!}\right)^m$, while the probability of selecting any other sequence from the remaining $V-m$ sequences is $\left(\frac{Q!-1}{Q!}\right)^{(V-m)}$. Consequently, the probability of m permutation sequences being identical is provided by

$$\text{Probability}_m = \left(\frac{1}{Q!}\right)^m \times \left(\frac{Q!-1}{Q!}\right)^{(V-m)}. \tag{14}$$

Therefore, when dealing with a small number of sub-blocks, the likelihood of randomly selecting lexicographical permutation sequences that are identical and posses the same PAPR values is very high. This implies that random lexicographical permutations may not be suitable for such scenarios. Instead, adjacent and interleaved lexicographical permutation sequences can be employed as alternatives.

3.2. The Most Suitable Number of Candidate Permutation Sequences

In order to determine the most suitable number of candidate permutation sequences, we can use the global gain defined in Section 2.3.

Figure 2 illustrates the procedure for obtaining the most suitable number of candidate permutation sequences, where Ω_{sys} is the global gain required for a particular system, σ_1 and σ_2 are the levels of importance attached to PAPR reduction performance and the computational complexity, respectively, and V_{suit} is the most suitable number of candidate permutation sequences for a particular case.

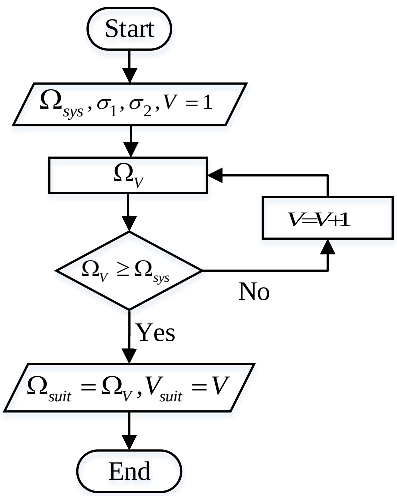

Figure 2. Algorithm for obtaining the most suitable number of candidate permutation sequences based on global gain.

Furthermore, we need to specify the value of the CCDF of the PAPR in order to compare the global gains. The typical values of the CCDF of the PAPR used in the available literature are between 10^{-2} and 10^{-4} [25]; therefore, in this paper we use 10^{-3} as a reference for global gain calculations.

In order to generate V lexicographical candidate permutation sequences, V $2N$-point IFFT operations are required [16]. Therefore, the complexity is provided by

$$\text{Complexity} = 2VN. \tag{15}$$

4. Results and Discussion

In this section, we present the results obtained using computer simulations in MATLAB®. First, we present the performance of different categories of lexicographical permutation sequences, specifically, random [18], interleaved, and adjacent, as described in Section 3. The comparison focuses on both PAPR reduction and BER degradation performance. Second, we present the results obtained with our proposed algorithm for determining the most suitable number of candidate permutation sequences described in Section 3.2.

In our simulations, we considered a DCO-OFDM communication system with a 16-QAM constellation and total number of subcarriers $N = 256$. We generated approximately 10^6 data blocks; each block was oversampled by an oversampling factor of $L = 4$, which is commensurable to the true value of the PAPR [21]. In addition, we obtained the results for the DCO-OFDM communication system using the same constellation without any PAPR reduction technique. Table 1 lists all the parameters used during the simulations.

Table 1. Simulation parameters.

Simulation Parameter	Value
Modulation scheme	16-QAM
Number of sub-carriers	$\mathrm{N} = 256$
Number of data blocks	10^6
Number of candidate sequences	$\mathrm{V} = 8, 16, 32$
Number of sub-blocks	$\mathrm{Q} = 16$
Over-sampling factor	$\mathrm{L} = 4$
$\mathrm{DC_{offset}}$	13 dB

4.1. Performance of Lexicographical Symbol Position Permutation Categories

Figure 3 illustrates the performance of the three categories of lexicographical symbol position permutation sequences in terms of PAPR reduction performance using the CCDF defined by (2). The simulation results of the original DCO-OFDM system are additionally incorporated for the sake of comparison and as a point of reference. It can be noted that the PAPR reduction performance achieved by LSPP using random lexicographical permutations [18] is slightly better compared to LSPP using interleaved or adjacent lexicographical permutations. For example, at a CCDF of 10^{-3} the PAPR values are respectively 11.26 dB, 8.62 dB, 8.70 dB, and 8.80 dB for the original DCO-OFDM system, and LSPP using random, interleaved, and adjacent permutations when $V = 8$ and $Q = 16$. This represents a respective difference of 0.08 dB and 0.18 dB for LSPP using interleaved and adjacent permutation sequences compared to LSPP using random permutations. When V is increased from 8 to 16 and 32, the performance of LSPP using random permutation sequences remains slightly better compared to LSPP using interleaved or adjacent permutation sequences. This better performance of LSPP when using random permutation sequences can be attributed to random permutation sequences being less correlated compared to interleaved and adjacent lexicographical permutations. Similarly, interleaved lexicographical permutation sequences are less correlated compared to adjacent lexicographical permutation sequences, justifying the difference of about 0.1 dB in PAPR reduction performance (with the former performing better).

It should be noted that in order to generate the random lexicographical permutation sequences Q should be sufficiently large to ensure that the possibility of generating repeated permutation sequences is minimized. Generation of repeated random lexicographical permutation sequences can be avoided by setting a constraint in Algorithm 1; however, this could increase the time complexity. Therefore, using adjacent or interleaved lexicographical permutation sequences could be preferable for low values of Q.

Figure 4 shows the BER performance comparison of LSPP employing random, interleaved, and adjacent lexicographical permutation sequences across a channel affected by additive white Gaussian noise (AWGN). As depicted in Figure 4, it can be noted that the three categories of LSPP achieve nearly identical BER performance, with minimal degradation in BER compared to the unaltered DCO-OFDM system lacking any PAPR mitigation techniques. These results are not surprising, as LSPP is a distortionless technique regardless of the category of lexicographical permutation sequences used, and as such no BER degradation is expected except in cases where the SI is received in error.

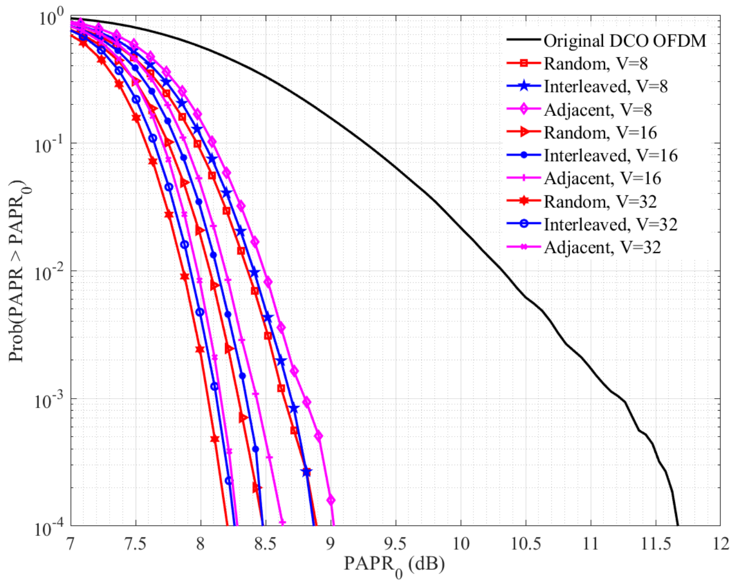

Figure 3. Comparing the PAPR performance of DCO-OFDM versus the three types of lexicographical permutations (random, interleaved, and adjacent) for $Q = 16$ sub-blocks and number of candidate signals $V = 8, 16$, and 32.

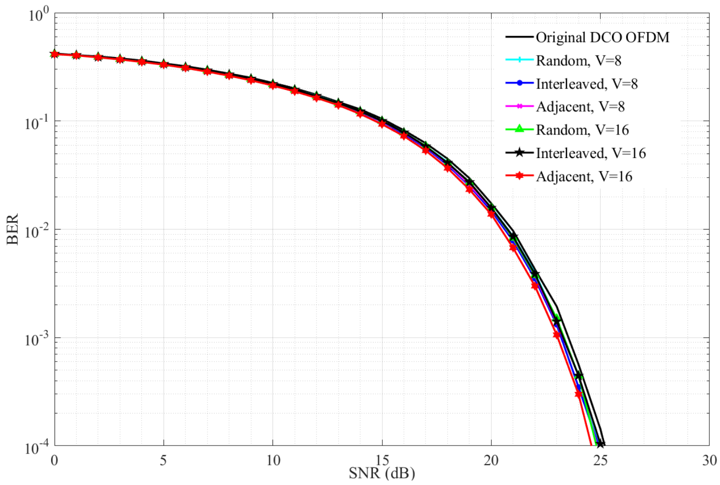

Figure 4. Comparison of the BER performance of DCO-OFDM versus the three types of lexicographical permutations (random, interleaved, and adjacent) for $Q = 16$ sub-blocks and number of candidate signals, $V = 8, 16,$ and 32.

4.2. The Most Suitable Number of Candidate Permutation Sequences to Avoid Increase in Computational Complexity

We considered LSPP using random lexicographical permutations when determining the most suitable number of candidate permutation sequences to avoid increasing computational complexity beyond system requirements, as it provides better PAPR reduction performance compared to using adjacent or interleaved lexicographical permutations.

Figure 5, plots Z_1 from (9) and Z_2 from (10) on the left vertical axis and right vertical axis, respectively, for different numbers of candidate permutation sequences V. As V increases, the absolute values of both Z_1 and Z_2 increase, while the rate of this increase decreases as V is increased beyond about eight candidate sequences. For example, from $V = 2$ up to $V = 8$, Z_1 and Z_2 increase by 135% and 190%, respectively, while for V from 8 to 16 Z_1 and Z_2 increase by 19% and 37%, respectively. This rate of increase decreases further as V is increased. The implication is that as V is increased, the benefit achieved in terms of PAPR reduction performance first increases rapidly up to a certain V, then the rate of increase decreases significantly as V is increased further.

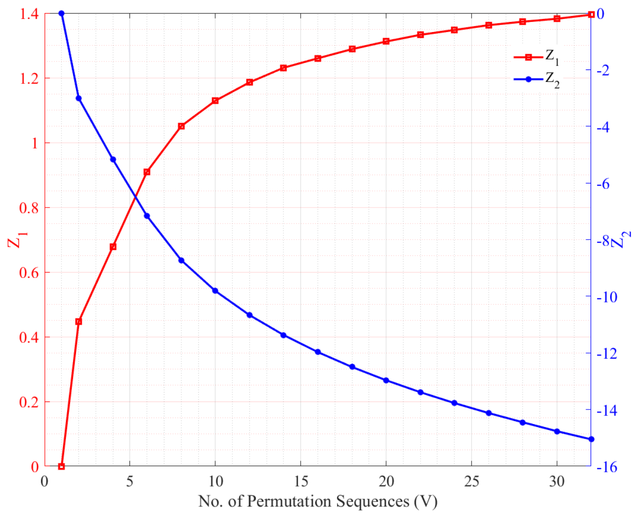

Figure 5. Z_1 and Z_2 for different candidate sequences.

Figure 6 presents the results for the global gain (Ω) for different numbers of candidate permutation sequences under different levels of importance attached to PAPR reduction performance (σ_1) along with the resulting increases in computational complexity (σ_2). It is clear that the values of σ_1 and σ_2 affect the resulting Ω.

In Figure 7 and Table 2, the global gain values for three different cases are presented: (1) when $\sigma_1 = \sigma_2 = 0.5$, i.e., equal significance is attached to both PAPR reduction performance and increased computational complexity; (2) when $\sigma_1 = 0.8$ and $\sigma_2 = 0.2$, i.e., PAPR reduction performance is considered more significant compared to increased computational complexity; and (3) $\sigma_1 = 0.2$ and $\sigma_2 = 0.8$, i.e., increased computational complexity is considered more significant than PAPR reduction.

From (9) and (10), it is apparent that a higher value of Z_1 is desirable, as it represents a lower PAPR value; similarly a higher value of Z_2 is desirable, as it represents lower complexity. Therefore, from (12), a higher value of the global gain Ω is desirable. However, as shown in Figure 7, it is necessary to select the most suitable value of the global gain based on the respective significance attached to computational complexity and PAPR reduction performance.

As PAPR reduction performance becomes more significant, the global gain is reduced due to the increase in computational complexity generated by the number of IFFT operations necessary to generate the candidate signals. Therefore, the most suitable number of permutation candidate sequences to achieve acceptable PAPR reduction performance should be selected without exceeding the level of computational complexity that the system can tolerate.

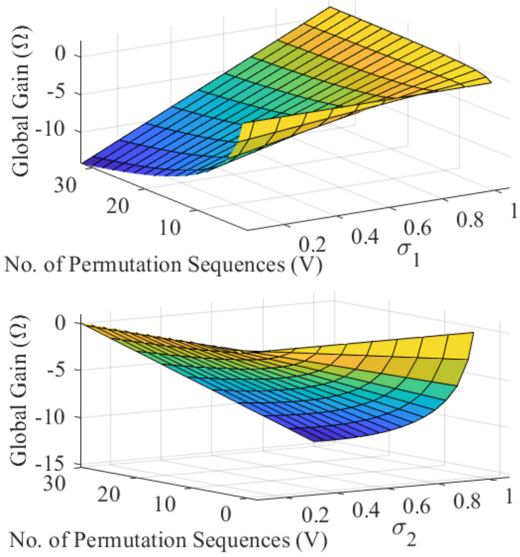

Figure 6. Global gain for different numbers of candidate sequences at different levels of importance attached to PAPR reduction performance (σ_1) and computational complexity (σ_2).

Table 2. Global gain Ω at different values of random lexicographical permutation sequences V for three different cases.

V	PAPR (dB)	Complexity	Global Gain, Ω (dB)		
			Case 1	Case 2	Case 3
2	9.90	1024	−0.205	−2.309	−1.257
4	9.14	2048	−0.529	−4.648	−2.589
8	8.67	4096	−0.948	−7.010	−3.979
16	8.30	8192	−1.399	−9.381	−5.390
32	8.05	16,384	−1.894	−11.762	−6.828

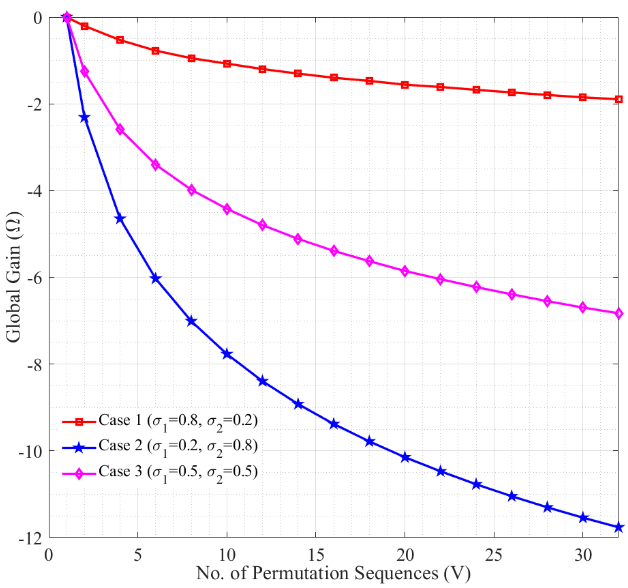

Figure 7. Global gain for different numbers of random lexicographical permutation sequences V at a $CCDF = 10^{-3}$ and number of subcarriers $N = 256$.

5. Conclusions

We performed a comparative study of the different categories of lexicographical permutation sequences, i.e., random, interleaved, and adjacent, in terms of PAPR reduction performance. Using computer simulations, we have shown that random lexicographical permutation sequences provide better PAPR reduction results compared to interleaved or adjacent lexicographical permutation sequences. For a small number of sub-blocks, adjacent or interleaved lexicographical permutation sequences may be preferable, however, as there is no possibility of generating the same permutation sequence, which is possible when using random lexicographical permutations. Although this possibility can be minimized by setting a constraint to stop generation of the same permutation sequences, this results in increased time complexity.

Our results show that all three categories of lexicographical permutation sequences provide almost the same BER performance compared to the original DCO-OFDM without any PAPR reduction technique applied.

Furthermore, we propose a new approach based on global gain for determining the most suitable number of candidate permutation sequences to achieve a reasonable PAPR reduction performance without leading to unacceptable levels of computational complexity for the system.

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