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Digital Twin for Math Education: A Study on the Utilization of Games and Gamification for University Mathematics Education

Jung Yeop Lee [1], Chong Un Pyon [2] and Jiyoung Woo [1,*]

[1] SCH Media Labs, Soonchunhyang University, Asan 31538, Republic of Korea; elises@sch.ac.kr
[2] Kalifa University, Abu Dhabi 999041, United Arab Emirates
* Correspondence: jywoo@sch.ac.kr

Abstract: Gamification has emerged as a promising strategy for engaging students and enhancing learning outcomes across various disciplines, including education. It involves incorporating game-like elements into non-game contexts to motivate and engage users. Extensive research has demonstrated the positive impact of gamification on student engagement, motivation, and learning outcomes in diverse subjects, including mathematics. Nevertheless, there is a paucity of research on the effectiveness of gamification in mathematics education. To bridge this gap, we investigated the effects of gamification in college mathematics education, with a specific emphasis on students from liberal arts backgrounds who may lack foundational mathematical knowledge. In this work, we designed and implemented a gamified mathematics course tailored for liberal arts students in a Korean university. Leveraging digital twin technology for game and gamification, we created an immersive learning environment that allows students to visualize mathematical concepts through games, pair gamification factors on physical and virtual worlds, and engage in interactive problem-solving activities. The effectiveness of this gamified course in improving students' engagement, motivation, and learning outcomes was thoroughly evaluated. Digital twin technology, in conjunction with gamification, holds immense potential to revolutionize the way mathematics is taught and learned, making it more accessible, interactive, and engaging for students from diverse educational backgrounds.

Keywords: digital twin; math education; game-based learning; gamification; higher education

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1. Introduction

Mathematics education has always been a challenge for educators, as many students often struggle to engage with the subject matter and apply the knowledge they have gained. In Korean universities, the challenge is compounded by the unique structure of high-school education, where liberal arts students often miss important courses such as calculus, which are essential for learning discrete mathematics at the university level. This lack of foundational knowledge can make it difficult for students to succeed in their university courses, leading to frustration and disengagement with the subject.

Sustainability in university convergence courses is crucial to ensure that students are adequately prepared to meet the demands of the modern workforce [1]. As technology advances and industries become more interconnected, students are expected to have a broad range of skills and knowledge in various fields, including mathematics. However, the traditional lecture-based approach to teaching mathematics often fails to engage students and inadequately prepares them for real-world problem-solving challenges [2].

This study was motivated by the need to create a digital model that can personalize and facilitate students' mathematics learning. Test scores typically measure student performance; however, this alone does not quantify what students are interested in and their engagement level in class. Digital twins are a way to create a manipulable simulation system for physical or virtual reality [3]. The first goal of our project was to capture the various student engagements that take place in class, quizzes, tests, group work, etc., into a

gamification model that can be improved and reused as a system to motivate students and improve their academic performance.

In recent years, gamification has emerged as a promising approach to engage students and enhance learning outcomes in various fields, including education [4]. Gamification refers to using game-like elements, such as points, badges, leaderboards, and rewards, in non-game contexts, to motivate and engage users [5]. Several studies have shown that gamification can effectively improve student engagement, motivation, and learning outcomes in various subjects, including mathematics [4–7].

However, research on the effectiveness of gamification in mathematics education is limited, particularly in the context of higher education. In this study, we aim to fill this gap by examining the effects of gamification in college mathematics education in Korea, specifically focusing on students from liberal arts backgrounds who may lack foundational knowledge in mathematics. We hypothesized that by introducing gamification elements into mathematics courses, we could improve students' engagement, motivation, and learning outcomes, particularly for those without a strong foundation in mathematics. Furthermore, by providing students with a fun and engaging way to experience mathematics, we hope to increase their interest and motivation to learn, leading to sustained learning outcomes in the long run.

This study also led to using mathematical principles at work through various board games in the classroom; for example, how probability works in a board game such as Rummikub. Game-based and gamification-based learning were implemented as a digital twin of the math educational learning management system in part. Professors played the actual game with their students, lectured on the mathematical principles inherent in the game, took a quiz to learn those mathematical principles, and then played the game again to learn the mathematical principles that allow them to play it more effectively. With this kind of game-based learning, games can become a teaching tool that can make the content of a class more interesting [8]. This game-based and gamification-based learning approach distinguishes it from other studies that have utilized gamification or games only.

In this study, we conducted a systematic literature review to examine the current state of research on gamification in mathematics education. We also designed and implemented a gamified mathematics course for liberal arts students at a Korean university and evaluated its effectiveness in improving students' engagement, motivation, and learning outcomes. This study shows that a model using gamification can be utilized in university education based on the systematic modeling of learning assessment methods.

2. Literature Review

Even before the concept of gamification was introduced, games were utilized as learning tools [9]. Goal-setting systems that games have, such as missions and quests, have become a great way to motivate players [10]. Gee argued that well-designed video games can be powerful learning machines [11]. Educational games can provide students with a motivating and stimulating environment and immediate feedback to promote learning [12]. However, since the 2000s, gamification, the practice of extracting characteristics from games and applying them to non-game environments or digital applications to engage users and provide numerical measures of their achievement, has gained increasing attention [13].

Csikszentmihalyi proposed the flow theory, which states that the right level of difficulty, combined with the user's skills, can lead to engagement [14]. This engagement and motivation are influenced by self-determination theory, which states that fulfilling basic human needs for autonomy, competence, and relatedness can increase motivation and engagement [15]. This is an important theoretical framework used to explain the effectiveness of gamification.

Gamification has gained popularity and increases motivation and engagement in various educational settings. While there is a growing body of research on gamification in math education, it has mostly been utilized in schooling below K-12 in the U.S. [16–18]. This is because younger students are more likely to be interested in game-like elements, and

virtual digital twin systems work more effectively in this environment [19]. Most of these studies have summarized that gamification can increase student motivation, engagement, and achievement. However, the question remains whether this motivation is consistent across all grade levels and whether engagement and achievement increase regardless of math problem-solving ability. Furthermore, whether gamification works in math education for college students, who are typically adults, also needs to be validated.

Several studies of game-based learning that apply already released digital games show that games are developed based on mathematical principles and can be applied to students' math education. Advanced mathematical topics such as probability theory and modular arithmetic can be introduced to students at middle or pre-college schools to cultivate their capacity for logical thinking and problem-solving skills [20,21]. In some cases, researchers have been sponsored by the government to develop applications and apply them to math education. Through these applications, students are guided to learn basic mathematical concepts with the goal of improving their knowledge, awareness, and attitudes towards geometry [22]. There are also examples of gamification and game-based learning, where the roles of professor and student are reversed by incorporating new teaching methods such as flipped teaching. In this study, researchers present a conceptual interpretation of amalgamating inquiry-based learning and the flipped classroom model with gamification/game processes as a means to strengthen and improve the way mathematics learning and teaching content, strategies, and approaches are enacted in primary, secondary, and tertiary education [23]. However, there is also a disadvantage in that the effects of gamification and educational methodologies overlap, making it challenging to examine the effectiveness of each method in the overall educational effect.

At the university level, students are not as easily motivated by the tools used in gamification because the factors that drive adults to learn are more complex and personalized [24]. Despite the availability of literature reviews on gamification and its effects, no work to date has focused exclusively on higher education [25]. Of course, it was necessary to narrow the scope of the study by combining studies that showed the effectiveness of game-based learning at the college level [12], those that showed the effectiveness of gamification in math education at the K-12 level, and others [16–18]. Therefore, it is necessary to see if the gamification model works universally well in higher education for university-level students.

3. Teaching and Learning Model

It is generally accepted that matching student-centered instruction to learners with higher conceptual levels and teacher-centered instruction to learners with lower conceptual levels improves student achievement and motivation in the classroom [26]. Educational gamification can be considered student-centered because of its high user engagement rate.

In 2016, Soon-Chun-Hyang University (SCHU) was selected for the PRIME Project of the Ministry of Education of Korea, so SCHU established a convergence college named Media Labs. Media Labs is an integrated college for liberal arts and natural sciences and consists of 11 convergent departments, such as the Department of Cultural Contents, New Media, Big Data, Internet of Things, and Smart Car.

In the curriculum of this college, there are mandatory general courses for students from 11 departments. Because these courses are common to 11 departments, students from the 11 departments can make a group and enroll together. These five courses are "Convergence and Infinite Imagination," "Story Making," "Yummy Mathematics," "Industry Inside," and "Media Inside." To prepare for the Fourth Industrial Revolution, convergence colleges must incorporate mathematical problem-solving and logical thinking capabilities into their curriculum. Therefore, a mathematical course as a mandatory general course was planned for all college students.

Most of the students in Media Labs College are middle- and high-class students who constitute the top 25% of the College Scholastic Aptitude Test in Korea. However, many students have lost interest in mathematics. A basic mathematical course was required for

these students so that they could review mathematics about the high-school curriculum and trigger interests and provide motivation for mathematics courses at the college level, such as "Discrete Mathematics."

From this viewpoint, "Yummy Mathematics" was opened as a mandatory general course for the freshmen of the convergence college, "Media Labs." It opened for the first time in the fall semester of 2017. The next spring semester in 2018 was opened to students in the liberal arts, and the upcoming fall semester was open to students in the natural sciences.

In the fall semester of 2017, the first course was opened as a game-based course. It followed the previous curriculum, where students were educated about the embedded mathematical content after playing games such as *Katan*, *Mafia 42*, and *Rummikub*. In the case of the previous game-based course, we found that it is possible to raise interest in mathematics during game-playing. However, their interests rapidly decreased while students learned mathematical content. A gamification model is required to provide motivation and provoke interest for the whole course. Gamification is an effective method to provide various tools such as a mission, badge, progress chart of learning, and leaderboard to encourage the interests of learners.

It was necessary to overcome the difficulties of the previous course. The first course in the fall semester of 2017 was open to all students, regardless of liberal arts or natural sciences backgrounds. It was evaluated as failing to satisfy both student groups. Students in liberal arts were repulsed by and scared of unfamiliar content such as physics, vector, matrix, etc., which are not in the high-school curriculum, whereas students from the natural sciences regarded game-based content as somewhat easy.

Because this course is a mandatory general course for the college for continued improvement, we intend to introduce various game-based materials and develop the curriculum by focusing on students in natural sciences for the next semester.

3.1. Gamification Model

We introduced various gamification models to evaluate the learning status of learners based on the Analysis, Design, Development, Implementation, and Evaluation (ADDIE) model that is used for the design of the education learning model. We designed a cyclical development model that induces intrinsic motivation in learners, analyzed the effectiveness of the processes, and applied the analysis results for further improvement of the course. Four gamification components, mission, leaderboards, progress bars, and badges, were used in the motivation stage.

The ADDIE model is a methodology for designing an education model in five stages: Analysis, Design, Development, Implementation, and Evaluation. It is most frequently used for the design of instructional systems and often used as a methodology for education gamification. The details are shown in Table 1.

Table 1. Lecture methodology with ADDIE model.

Analysis	Design	Development	Implementation	Evaluation
Requirement analysis Learners' analysis Environment analysis Tasks and assignment analysis	Specification of objectives of missions Design of evaluation tools Structuration Selection of education strategies and media	Development of education materials Pilot test and modification Development	Implementation and installation Maintenance and management	Evaluation of the results of education training

According to this model, liberal arts students in SCHU Media Labs did not have much interest in mathematics in their high school (Analysis). Therefore, we designed an ADDIE model for studying fundamental mathematics that is necessary for the enrollment of convergent courses. In this model, we studied mathematics by gamification components (Design), developed teaching material that reflect these components (Development), carried out pilot education (Implementation), and performed an effectiveness verification (Evaluation). All

gamification components are built in the learning management system to implement the digital twin for math education.

- Mission

Four games were used in this course. Each game was composed of three modules: playing a game, mathematics lectures related to the game, and evaluation of the form of the quiz. As one game had one mission, there were a total of six missions over the course. It took two weeks to accomplish one mission. Therefore, the course was 15 weeks, including orientation, midterm, and final term for one week each.

- Leaderboards

In each mission, the best student, who achieved the highest game score or the highest quiz results, was selected and displayed on the leaderboards on the bulletin of the online class.

- Progress Charts

The results of the mission of each student were transferred through the learning management system (LMS) system on campus. Thus, each student could check their learning progress using SMS messages.

- Badges

There were various badges for presenters: high-ranking students in quizzes, leadership in a team, outstanding teams in assignments, and students of full attendance. This led learners to collect more badges, which could be reflected in their grades. Badges were produced as a sticker using Formtec and distributed in every class and recorded and presented in the math education digital twin. Table 2 lists the type of badges and their roles.

Table 2. Types of badges and their roles.

Type of Badge	Explanation
Voice Controller	Badges for a presenter. This badge is given to a learner who presents well.
Question Leader	Badges for a questioner. This badge is given to an outstanding questioner in each class.
Time Keeper	Badges for a student of full attendance. They are given to a learner who attends the whole course throughout the semester.
Problem Solver	Badges for high-ranking students. They are given to a team or a person who achieves high-level quiz results
Paper Handler	Badges for the outstanding teams in assignments. They are given to an outstanding team in an assignment.

3.2. Game-Based Learning

In this course, four games, *Katan, Rummikub, Mafia 42,* and *Pocketball* were played, and the embedded mathematical principles and useful applications in the games were educated. *Katan* and *Rummikub* are board games. *Mafia 42* is a mobile game, and *Pocketball* is an arcade game. Only Mafia is played in a virtual world; the other games are played in the physical world.

In addition to the game module, "Matrix and simultaneous equations" and "Statistics in life" modules were inserted to add objects to be compared in effectiveness verification and led to learning practical mathematics. The module of "Matrix and simultaneous equations" was composed of frequently asked questions in a personality test of company employment tests so that it could help students with company employment. The "Statistics in life" module was composed of statistical information in daily life based on the frequently used statistics terminologies in newspapers. About the teaching and learning model, 100% of the planning was implemented.

3.3. Instruction for Teaching and Learning Model

- A development plan for instruction for teaching and learning model

The "Yummy Mathematics" curriculum was made by a committee for convergence mathematics, composed of faculties of education gamification and lecturers of engineering departments. The committee supplemented the previous curriculum conducted in the fall semester of 2017. They combined gamification contents to fit students of liberal arts by lowering the level of difficulty and improving the design of presentation files.

- A detailed plan for instruction for teaching and learning model

We prepared four games, *Mafia 42, Rummikub, Katan,* and *Pocketball*, for the training aids separately and distributed manuals for each game in advance. The professor who developed the teaching instructions taught directly; the instructions were unnecessary. However, all teaching materials, such as presentation files, activity sheets, quizzes, and gamification manuals, were packaged for future lecturers. The previous game-based learning teaching instructions were revised, and their level of difficulty was improved. The entire course was designed as one game by the gamification model, which used four games: missions, leaderboards, progress bars, and badges. All presentation files of lectures were improved in design and unified in format by a professional design company. Regarding instruction for the teaching and learning model, 100% of the planning was implemented.

3.4. Digital Twin Implementation

A digital twin is defined as a virtual model of a process, product, or service. In 2002, Michael Grieves introduced the concept and model of the digital twin for the first Society of Manufacturing Engineers conference. He introduced the digital twin as the conceptual modeling of product lifecycle management [27]. After that, the digital twin has been adopted as a conceptual basis in the aerospace domain in 2010 [28].

A digital twin is defined as the form of a cyber–physical system, such as a smart factory where a physical factory and a digital twin coexist in the same cyberspace [29]. It has been mainly studied in the manufacturing domain. Then, it was further expanded into the digital city, healthcare, etc. The main characteristic of a digital twin is to emulate the actual system." In addition, the digital twin can be further characterized as a digital technology that is both the consequence and an enabler of the homogenization of data [30]. The digital twin has strength through the pairing of the virtual and physical worlds, resulting in achieving analysis of data and monitoring of systems.

While the digital twin has been implemented and studied extensively in manufacturing, the education domain still needs to be included. Education can be regarded as a process or service so that the digital twin can be applicable. In this work, we implemented the physical gamification factors in the virtual learning management system. The mission accomplishment and the number of badges obtained during the classes are instantly updated to leaderboards, progress charts, and the number of badges in the digital learning management system. This pairing of physical systems and virtual systems is the implementation of digital twins in gamification-based math education.

All learning processes were performed in the physical world. Students participated in the classroom and participated in the game with classmates under the guidelines of gamification. The achievement of gamification factors in the classroom was paired with the virtual system and thus enabled the data collection and monitoring of the learning process. We designed only a digital game, Mafia, for game-based learning, but other games were played in the physical world. Even though we did not align all math learning processes virtually and physically simultaneously, we could boost the learning performance by implementing the digital twin for gamification factors.

4. Effectiveness Analysis

4.1. Data Collection

SCHU Media Labs held a pilot operation on the course for first-year students in college. Four sections and 218 students were enrolled in the spring semester of 2019. This number is large compared to other previous works. The age of the students ranges between 19~24. Their gender distribution is skewed toward females, as shown in Table 3. This is because their majors were categorized into art and media, and literature. The student participants were distributed according to their departments, as shown in Table 3. Six faculty members from SCHU College were involved in the class. Regarding the pilot operation, 100% of the planning was implemented.

Table 3. The number of student participants and their gender distribution according to their departments.

Division	Department	Number of Participants	Female, %	Male, %
Art and media	Department of Performing Arts	25	50.00%	50.00%
	Department of Digital Animation	20	85.71%	14.29%
	Department of Media and Communications	52	64.81%	35.19%
Literature	Department of English Studies	47	74.00%	26.00%
	Department of Chinese Studies	43	74.42%	25.58%
	Department of Korean Culture and Contents	31	75.76%	24.24%
Total		218	70.46%	29.54%

We designed three tests: a prior survey, an effective analysis of game and gamification education, and an effective analysis of participant characteristics.

4.2. Prior Survey for Participant Assessment

A preliminary survey was conducted at the beginning of the semester to gain insight on the students before the course commenced. The purpose of this preliminary survey was to analyze students' interest in mathematics and their perception of its effectiveness. Table 4 presents the selected questions related to their interest in mathematics and recognition of its effectiveness, along with the corresponding averages and deviations.

Table 4. The questions about math interests and learning skills and survey response results.

Survey Category	No.	Question	Average	Standard Deviation
Math interests and learning skills	5	I like mathematics.	2.23	1.13
	6	I think mathematics is used in daily life.	3.6	1.0
	9	I have the experience of giving mathematics up throughout my school years because it is difficult.	3.21	1.34
	18	My average grade for mathematics in high school is ( )	4.14	1.73

In the results of the prior survey, we found that the percentage of students who strongly liked mathematics was relatively small, and half of the students considered giving up mathematics during their school years. South Korea has an education policy that assigns

both school records and CSAT scores using nine grades, with 1 being the highest and 9 being the lowest grade. Regarding the mathematics grades of high-school students, the average grade was the 4th level, with the maximum being the 6th level. The 4th level corresponds to the top 23–40% of students based on their grades, representing a middle to high level. It was found that students are aware of the necessity of mathematics in general but find it challenging to learn and often lose interest. There was no significant difference between departments regarding interest and grades in mathematics.

The survey was conducted at the beginning of the semester to assess students' perceptions of games and gamification before the course began. The results are presented in Table 5.

Table 5. The questions about the interest in games and gamification and survey response results.

Survey Category	No.	Question	Average	Standard Deviation
Games and gamification	1	I like to play games	3.49	1.17
	2	My life or my studies have been disturbed by playing games	1.99	1.12
	3	I play games for ( ) hours per day on average	1.11	1.34
	4	I have heard about the concept of gamification	1.85	0.37

Students usually answered positively according to their preference for games. However, it was found that the average time spent playing games was limited to one hour, but there was a higher standard deviation. Furthermore, the recognition of gamification is generally low. We investigated the differences between departments and divisions regarding mathematics interests, learning skills, and game/gamification interests using Sheffe analysis. Sheffe analysis is a statistical method used to identify significant differences in means.

We could not conclude whether there was a difference between departments regarding preferences for games and gamification. However, we can confidently state that there were no significant differences between departments in mathematics learning skills. Table 6 shows that all departments belonged to the same group, and the statistical result was significant, with an alpha of 0.05 for mathematics learning skills.

Table 6. Sheffe analysis of game/gamification awareness and math learning skill.

Department	Game and Gamification Awareness			Math Learning Skill		
	Mean	Standard Deviation	Group	Mean	Standard Deviation	Group
Performing Arts	2.56	1.21	a	2.92	1.15	a
Digital Animation	2.47	1.22	a	2.90	1.18	a
Media and Communications	2.44	1.21	a	3.10	1.13	a
English Studies	2.43	1.29	a	3.02	1.09	a
Chinese Studies	2.35	1.13	a	3.13	1.12	a
Korean Culture and Contents	2.54	1.25	a	3.06	1.14	a
p-Value	0.858			0.0473 **		

** $p < 0.05$.

4.3. Post-Survey to Assess the Effectiveness of Yummy Mathematics

For perceived outcomes, we measured the perceived outcome of games and gamification, the overall evaluation, and compared the perceived outcomes before and after taking the Yummy Mathematics course.

4.3.1. Perceived Effectiveness

After the courses ended, Yummy Mathematics evaluated four components of the class, the mission, badge, leaderboard, and progress bar, from the perspectives of gamification and game-based learning (GBL). We developed questionnaires focusing on whether the gamification components were implemented as planned, their effectiveness in generating interest in math, and the perceived usefulness of Yummy Mathematics. Responses regarding the effectiveness of gamification are shown in Table 7.

Table 7. The questions about the effectiveness of gamification and their averages and deviations.

Gamification Component	No.	Question	Average	Standard Deviation
Related to mission	1	Missions are renewed every two weeks.	4.31	0.75
	2	Learning by mission raises interest in learning mathematics more than usual learning.	2.71	1.19
	3	Learning by mission is more useful to learn mathematics than usual learning.	2.57	1.17
	4	I did my best to get higher scores in missions.	3.72	1.08
		Subtotal	3.33	1.04
Related to badges	1	Badges are awarded in every class.	3.87	0.86
	2	Learning by badges raises interest in learning mathematics more than usual learning.	2.97	1.12
	3	Learning by badges is useful for learning mathematics.	2.61	1.15
	4	I did my best to get badges.	3.74	1.05
		Subtotal	3.3	1.05
Related to leaderboards or papers attached to badges	1	Current awarding states of badges are displayed on LMS (or papers attached by badges) every two weeks.	2.91	0.97
	2	It can motivate students' willingness to do better to see the current awarding states of badges on LMS.	2.67	1.05
	3	Learning through leaderboards is useful to participate in classes.	2.69	1.02
		Subtotal	2.76	1.01
Progress bar	1	Mission scores are informed by SMS every two weeks.	2.07	1.1
	2	I was stimulated to do better by being informed about progress or learning.	2.33	1.09
	3	Notices about the progress of learning are useful to participate in classes.	2.33	1.1
		Subtotal	2.24	1.1
		Total	3.0	1.05

Among the four gamification components (mission, badges, leaderboards, and progress bars), participants rated the effectiveness of missions the highest (3.33/5), followed by badges (3.3/5). However, the effectiveness of leaderboards and learning progress was rated lower. Therefore, it can be concluded that leaderboards and badges are significantly useful for motivating course participants. The results are visually shown in Figure 1. Our finding supports Gibson et al.'s findings that explain that badges when used with points and leaderboards can be a powerful means of achievement. However, our results do not match the following works. Santos et al. [31] reported that the badge was recognized as the most important aspect of gamification. O'Donovon et al. [32] reported leaderboards rank highest in motivating learners.

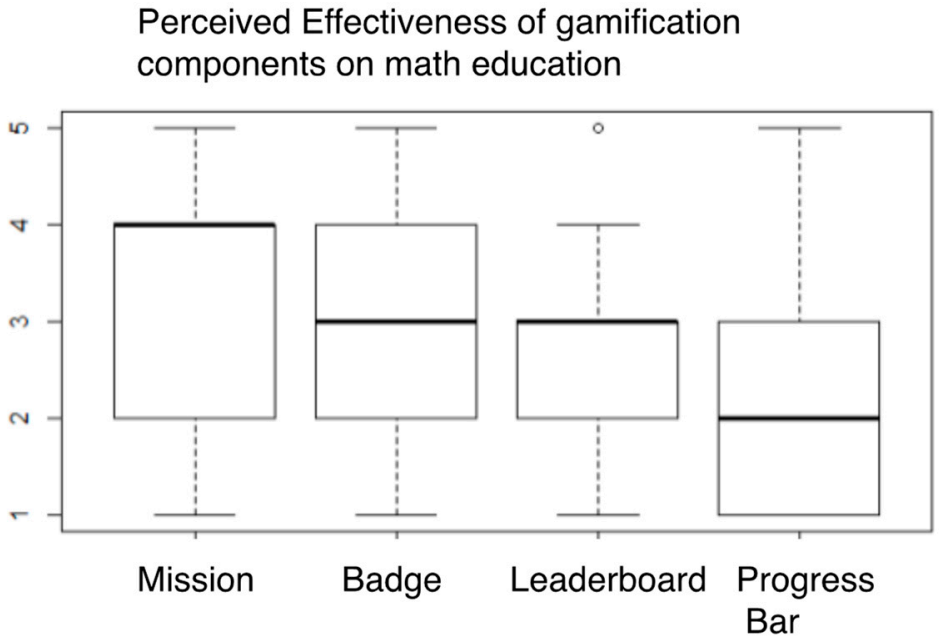

Figure 1. Evaluation of gamification components.

In addition, these two components establish immediate goals and provide rewards for achievement, whereas leaderboards and progress bars are associated with long-term goals. This suggests that participants were more effectively engaged when focusing on short-term goals and rewards rather than long-term ones. This was supported by Kapp's [6] and Raymer's [33] work, which details that feedback's frequency, intensity, and immediacy are also important for sustaining engagement throughout the learning processes.

Not every class module introduces a game. Generally, class modules with games, such as *Katan*, *Rummikub*, *Mafia 42*, and *Pocketball*, received higher interest from participants. However, other courses without games, such as matrix, sequence of numbers, and simultaneous equations, received lower evaluation scores. Regarding effectiveness, there was no significant difference between classes with and without games. However, the effectiveness is slightly higher in modules with games.

Regarding interest levels, there was a slightly more significant difference between the modules. In particular, *Rummikub* and *Katan* seemed to be motivating for participants. We found that interest levels were significantly different across modules. Table 8 shows the order of interest levels. Therefore, it is necessary to consider replacing modules without games, such as matrices, simultaneous equations, and statistics, with modules that incorporate games.

Table 8. Interest levels and efficiency according to the class modules.

Education Modules		Interest Level	Perceived Effectiveness	Total
Game and gamification module	Katan (modeling and strategic decision-making)	3.78	3.14	3.46
	Rummikub (sets and probability)	4.08	3.32	3.7
	Mafia game (proposition and reasoning)	3.46	2.79	3.13
	Pocketball (space and change)	3.32	2.91	3.12
	Subtotal	3.66	3.04	3.35
Gamification-only module	Statistics (statistics in life)	2.84	2.8	2.82
	Sequence of numbers and simultaneous equations	3.12	3.08	3.1
	Subtotal	3.24	2.96	3.10

We emphasize the importance of games in math education using the graph shown in Figure 2. Modules that incorporate gamification exhibit low interest levels and efficiency. Overall, interest levels were higher than perceived effectiveness when games were introduced in math class modules. Previous works found that game-based learning is effective in education. Park et al. found that game methods are effective in as much as 58% (learning outcomes) and 21% (enjoyment) of vocabulary learning [34]. All et al. found that game-based learning is more effective (15.8%) than lecture-based learning in fight safety education in their study of pre-test influences on the effectiveness of digital-game-based learning [35]. In our case, interest level and perceived effectiveness for game-based modules are (3.66–3.24)/3.24 = 13% higher than non-game-based modules. However, the perceived effectiveness for game-based modules is (3.04–2.96)/2.96 = 2.8% higher than the non-game-based modules. The effectiveness of game-based math learning was minimal compared with the previous two works.

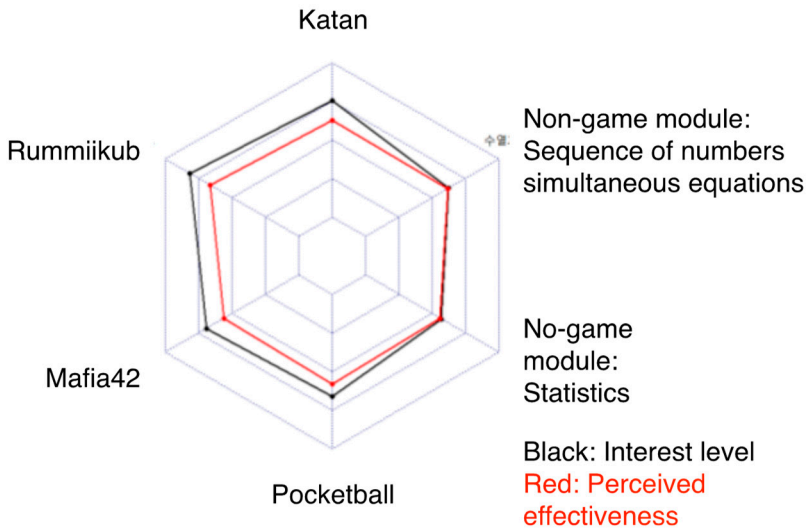

Figure 2. Evaluation of perceived effectiveness of game on math education.

We designed a survey to measure personal accomplishments and perceived effectiveness in two stages. The first stage measured the level of engagement, and the second stage measured the perceived effectiveness of Yummy Mathematics in math education. We established various metrics for perceived effectiveness, such as increased interest, perceived importance of math education, and decreased repulsion toward math.

Table 9 shows that student participation in the classes was high. However, they still found mathematics classes challenging. Moreover, because the students were all in liberal arts, they struggled to study mathematics, even though they participated passionately. Conversely, the rate of students who liked mathematics or were interested in mathematics was still evaluated as low. In conclusion, students somewhat appreciated the new approach of the "Yummy Mathematics" class but did not enjoy the mathematics class itself and felt stressed.

Table 9. Questions related to engagement and perceived effectiveness.

Category	Question	Average	Standard Deviation
Engagement	I participated in the "Yummy Mathematics" class passionately.	3.58	1.02
	I felt difficulty in the "Yummy Mathematics" class.	3.58	1.06
	I think the "Yummy Mathematics" class is fun.	2.61	1.15
	The new challenge of the "Yummy Mathematics" class was interesting.	3.1	1.3
Perceived effectiveness	"Yummy Mathematics" class was useful to me.	2.61	1.13
	"Yummy Mathematics" class helped me to understand new mathematics.	2.74	1.19
	I came to like mathematics by "Yummy Mathematics" class.	2.17	1.13
	I realized mathematics is used in daily life through the "Yummy Mathematics" class.	3.22	1.14
	I realized the importance of mathematics through the "Yummy Mathematics" class.	2.67	1.17
	The "Yummy Mathematics" class reduces my repulsion toward mathematics.	2.47	1.14
	Total	2.88	1.14

Compared with the previous work of Berkling and Thomas [36], where it was reported that student feedback score was not even half of 5 points, our result exceeds half of 5 points (2.65) for perceived effectiveness. When combined with engagement, the evaluation average was 2.88, as shown in Table 9.

Next, we compared the pre- and post-perceived outcomes as follows. The new approach in "Yummy Mathematics" improved the favor of math, perceived importance, and perceived usefulness. Hence, it could be considered successful in evoking the necessity of mathematics. Generally, the post-evaluation was better than the pre-evaluation. However, the increase in interest in mathematics was minimal. Furthermore, the perception that mathematics is difficult was not improved, and the expected fun factor of this class was not met. The details of the comparison results are listed in Table 10.

Finally, we investigated the relationship between achievement and other perceived effectiveness on game and gamification factors and achievement on gamification factors. Figure 3 shows the results. As we can see, the number of badges is mainly correlated to the final test score. The perceived effectiveness on mission points is ranked second. This is somewhat different from our previous finding that perceived effectiveness on badges was lower than on mission points.

Table 10. Questions related to pre-to-post-effectiveness analysis.

Topic	Question	Average	Increment
Mathematics favor	I like mathematics.	2.23	+0.035
	I enjoy studying mathematics.	2.34	
	subtotal	2.29	
	I came to like mathematics through "Yummy Mathematics" class.	2.17	
	The "Yummy Mathematics" class reduces my repulsion about mathematics.	2.47	
	subtotal	2.32	
Importance	I need to study mathematics related to my major.	2.42	+0.25
	I realized the importance of mathematics through the "Yummy Mathematics" class.	2.67	
Usefulness	I need to study mathematics related to my major.	2.42	+0.19
	"Yummy Mathematics" class was useful to me.	2.61	
Difficulty	Mathematics is difficult for me.	3.69	−0.11
	"Yummy Mathematics" class was difficult for me.	3.58	
Expectation for fun	This course seems to be fun.	3.00	−0.39
	"Yummy Mathematics" is fun for me.	2.61	
Expectation for effectiveness	I expect that studying math using games will make it easier.	2.86	+0.24
	The new attempts of "Yummy Mathematics" were interesting.	3.10	

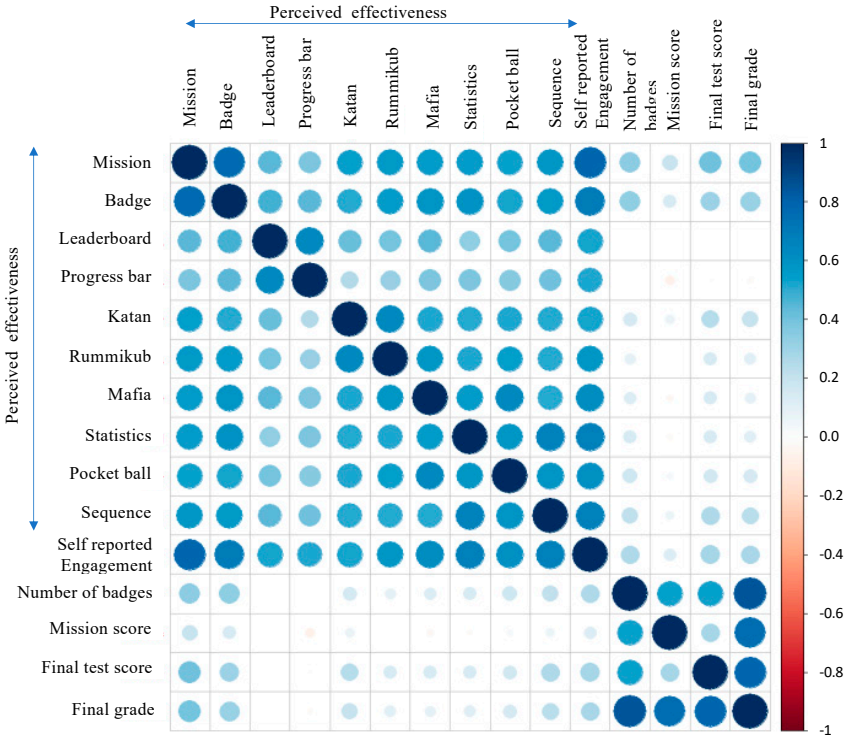

Figure 3. Evaluation of the component of gamification according to the math learning skill grades.

4.3.2. Effect Analysis of Students' Characteristics

In this section, we evaluated the control effect of student characteristics on the effectiveness of Yummy Mathematics. First, we investigated the effect of favor of game playing on the perceived effectiveness of each game. We divided participants into three groups

according to how much they like to play games, as shown in Table 11. We found that the difference between the middle and low groups is insignificant. Still, the difference between the high and middle groups is significant even though the statistical test results indicate that there is no significance between groups. Table 12 shows the differences of the interest level and perceived effectiveness on each game between groups.

Table 11. Grouping of participants according to favor of game playing.

Group	Range (5 Point Scale)	Number of Participants
High	4 < a< = 5	48
Middle	2 < a< = 4	126
Low	1 < =a < =2	44

Table 12. Perceived effectiveness of the game toward the math education according to levels of favor of game playing.

Game	No.	Question	Level of Favor of Game Playing			F Ratio	*p*-Value
			High	Middle	Low		
Katan (modeling and strategic decision-making)	1	Missions in Katan were interesting	4.07	3.68	3.66	2.52	0.08
	2	Missions in Katan were useful in math learning	3.35	3.09	2.95	1.50	0.23
		Subtotal	3.71	3.36	3.30		
Rummikub (sets and probability)	1	Missions in Rummikub were interesting	4.11	3.98	4.11	0.49	0.62
	2	Missions in Rummikub were useful in math interesting	3.52	3.25	3.09	1.66	0.19
		Subtotal	3.82	3.73	3.60		
Mafia game (proposition and reasoning)	1	Missions in Mafia were interesting	3.63	3.38	3.25	1.35	0.26
	2	Missions in Mafia were useful in math interesting	3.04	2.69	2.59	2.07	0.13
		Subtotal	3.34	3.14	2.92		
Pocketball (space and change)	1	Missions in Pocketball were interesting	3.57	3.18	3.32	2.06	0.13
	2	Missions in Pocketball were useful in math learning	3.11	2.78	2.82	1.35	0.26
		Subtotal	3.34	3.21	3.07		
		Total	3.54	3.39	3.23		

Next, we investigated the characteristics of the students in terms of their math learning skills and self-motivation.

We analyzed the relationship between the evaluation of gamification and pre-mathematics grades among the four classes. There is a slight difference between the grades, but it is not statistically significant enough to be fairly applied to students of all levels. In contrast to the evaluation of gamification components, it was found that students' grade achievements were significantly affected by their ability to study. In particular, higher-level students favor the "Yummy Mathematics" class. For students at high levels (A and B), gamification is very useful to motivate them. The results are shown in Figure 4. The components of gamification, such as badges, leaderboards, the progress of learning, mission, etc., are gears to encourage students who have been doing well. However, many students who hardly

earn badges gave up the courses after the 5th or 6th week. More tools should be designed for lower-level students to participate in the class to solve this problem. These supplements will be reflected in the gamification methodology for teaching instructions for the next academic year.

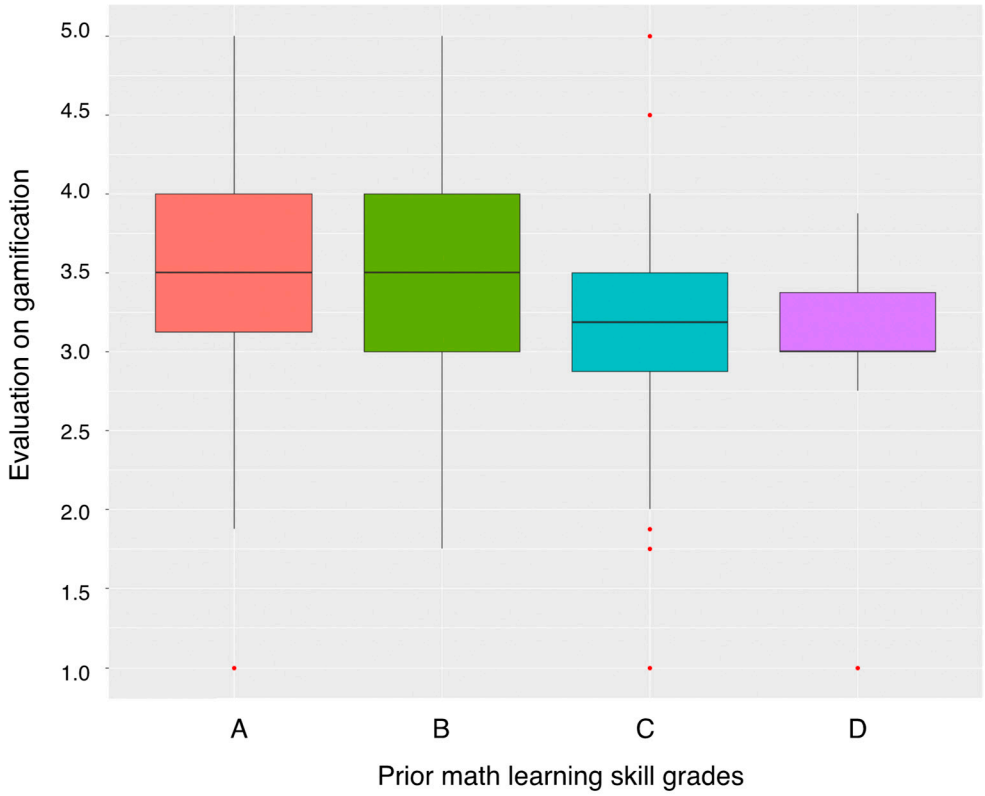

Figure 4. Evaluation of the component of gamification according to the math learning skill grades.

Next, we evaluated the effect of interest in math on personal achievement. Students were divided into high-, middle-, and low-interest groups based on their responses to questions related to interest in mathematics in the pre-survey. The classification criteria for the high, middle, and low groups were determined by calculating each student's average math study-related questions and classifying them based on the minimum, 1st quartile, 2nd quartile, 3rd quartile, and maximum values. We selected the following questions to divide the students into interest groups as shown in Table 13.

Table 13. Question related to self-motivation.

Division	Question
Favor	I like mathematics.
	I enjoy studying mathematics.
Usefulness	I think mathematics is used in daily life.
	I need to study mathematics related to my major.

The interest group was defined based on the average of the answers to the four questions as follows: high (>3, 67), middle (>2, 101), and low (otherwise, 50). We found that high motivation (interest) results in high perceived effectiveness. The differences between the groups were significant for all variables. The results of the differences in engagement and perceived effectiveness level according to groups divided by interest level in math are shown in Table 14.

Table 14. Correlation analysis between motivation level and evaluation of personal achievement.

Area	Question	High	Middle	Low	F-Statistics	*p*-Value
Engagement	I participated in the "Yummy Mathematics" class passionately.	3.98	3.6	3.34	5.48	***
	I felt difficulty in the "Yummy Mathematics" class.	3.08	3.55	4	11.33	***
	I think the "Yummy Mathematics" class is fun.	3.1	2.54	2.35	6.41	***
	The new challenge of the "Yummy Mathematics" class was interesting.	3.46	3.01	2.87	3.03	*
Perceived effectiveness	"Yummy Mathematics" class was useful to me.	3.15	2.5	2.31	8.67	***
	"Yummy Mathematics" class helped me to understand new mathematics.	3.33	2.72	2.26	12.31	***
	I came to like mathematics by "Yummy Mathematics" class.	2.98	2.07	1.73	8.66	***
	I realize mathematics is used in daily life by the "Yummy Mathematics" class.	3.63	3.15	2.98	4.51	***
	I realized the importance of mathematics by the "Yummy Mathematics" class.	3.27	2.65	2.21	12.2	***
	"Yummy Mathematics" class reduces my repulsion about mathematics.	3.1	2.47	1.98	14.52	***

* $p < 0.1$, *** $p < 0.001$.

Even though students recognize the effectiveness of the course, there is a definite difference in personal achievement based on their ability to study and motivation levels. Gamification is a useful tool to motivate students at a high level. The components of gamification, such as badges, leaderboards, learning progress, and missions, serve as gears to encourage students who have been performing well [22]. Therefore, there are many cases where students earn badges occasionally or cannot distinguish themselves and drop the courses in the 5th or 6th week. More tools should be designed for lower-level students to actively participate in class and address this issue. These supplements will be reflected in the gamification methodology for teaching instructions in the next academic year.

5. Conclusions

In this work, we designed and implemented a gamified mathematics course tailored for liberal arts students in a Korean university. Leveraging digital twin technology, we created an immersive learning environment that allows students to visualize mathematical concepts through games, manipulate physical and virtual objects, and engage in interactive problem-solving activities such as missions and badges. The effectiveness of this gamified course in improving students' engagement, motivation, and learning outcomes was thoroughly evaluated.

Initially, a preliminary survey was conducted to gather information about students' interest in math and their perception of its effectiveness. The results showed that only a small proportion of the students liked mathematics, and many had experienced difficulties and lost interest in the subject. The different departments had no significant differences in math interests and grades. A post-survey was conducted to evaluate the perceived effectiveness of "Yummy Mathematics." Participants rated the effectiveness of gamification components, such as missions, badges, leaderboards, and progress bars. The missions and badges were considered the most effective, while the leaderboards and progress bars were rated lower. Participants showed a higher interest in classes incorporating games, and there was a slight improvement in perceived effectiveness when games were included. However,

the increase in interest in mathematics was minimal, and participants did not find the class as fun as they expected. The study also examined the effects of student characteristics on the effectiveness of Yummy Mathematics. It was found that students' math learning skills and self-motivation affected their evaluation of gamification. Students with higher math grades showed a stronger preference for the class, while this course was less effective for students at lower levels.

As we initially hypothesized, we can see that students who were originally high achievers in math courses were also better motivated by gamification. This shows that gamification is not a one-size-fits-all methodology in higher education. While high-achieving students are highly motivated by earning badges and climbing leaderboards, these findings suggest that a more complex motivational mechanism is needed for lower-achieving college students.

Through this research, we aimed to contribute to advancing mathematics education not only in Korea but also beyond its borders. The utilization of digital twin technology in conjunction with gamification holds immense potential to revolutionize mathematics education, making it more accessible, interactive, and engaging for students from diverse educational backgrounds.

This study aimed to investigate the viability of integrating gamification into higher mathematics education, encompassing university-level and beyond. However, concerning motivation, the impact of gamification elements such as badges and leaderboards on adults exhibited certain distinctions compared to the effects observed in prior studies conducted on elementary or secondary level students by other researchers. Although the underlying reasons for these disparities may be multifaceted, it is primarily attributed to the potential divergence in psychological motivation between adults and minors. Nevertheless, this study did not explicitly identify the precise psychological factors contributing to these differences. Future research endeavors should focus on conducting more interdisciplinary studies to devise more sophisticated gamification tools capable of effectively stimulating adult motivation.

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References

1. Blackley, S.; Wilson, S.; Sheffield, R.; Murcia, K.; Brown, P.; Tang, K.-S.; Cooper, M.; Williams, J. How have COVID-19-related changes to tuition modes impacted face-to-face initial teacher education students? *Issues Educ. Res.* **2021**, *31*, 421–439.
2. Caton, M.T. Black male perspectives on their educational experiences in high school. *Urban Educ.* **2012**, *47*, 1055–1085. [CrossRef]
3. Jones, D.; Snider, C.; Nassehi, A.; Yon, J.; Hicks, B. Characterising the Digital Twin: A systematic literature review. *CIRP J. Manuf. Sci. Technol.* **2020**, *29*, 36–52. [CrossRef]
4. Dicheva, D.; Dichev, C.; Agre, G.; Angelova, G. Gamification in education: A systematic mapping study. *J. Educ. Technol. Soc.* **2015**, *18*, 75–88.
5. Barata, G.; Gama, S.; Jorge, J.; Goncalves, D. Engaging engineering students with gamification. In Proceedings of the 5th International Conference on Games and Virtual Worlds for Serious Applications (VS-GAMES), Poole, UK, 11–13 September 2013. [CrossRef]
6. Kapp, K.M. *The Gamification of Learning and Instruction: Game-Based Methods and Strategies for Training and Education*; John Wiley & Sons: Hoboken, NJ, USA, 2012.
7. Sailer, M.; Hense, J.U.; Mayr, S.K.; Mandl, H. How gamification motivates: An experimental study of the effects of specific game design elements on psychological need satisfaction. *Comput. Hum. Behav.* **2017**, *69*, 371–380. [CrossRef]
8. Cheng, C.-H.; Su, C.-H. A Game-based learning system for improving student's learning effectiveness in system analysis course. *Procedia Soc. Behav. Sci.* **2012**, *31*, 669–675. [CrossRef]

9. Jayakanthan, R. Application of computer games in the field of education. *Electron. Libr.* **2002**, *20*, 98–102. [CrossRef]
10. Baranowski, T.; Buday, R.; Thompson, D.I.; Baranowski, J. Playing for real: Video games and stories for health-related behavior change. *Am. J. Prev. Med.* **2008**, *34*, 74–82. [CrossRef] [PubMed]
11. Gee, J.P. Learning by design: Good video games as learning machines. *E Learn. Digit. Media* **2005**, *2*, 5–16. [CrossRef]
12. Bodnar, C.A.; Anastasio, D.; Enszer, J.A.; Burkey, D.D. Engineers at play: Games as teaching tools for undergraduate engineering students. *J. Eng. Educ.* **2016**, *105*, 147–200. [CrossRef]
13. Zichermann, G.; Cunningham, C. *Gamification by Design: Implementing Game Mechanics in Web and Mobile Apps*; O'Reilly Media, Inc.: Sebastopol, CA, USA, 2011.
14. Csikszentmihalyi, M.; Larson, R. *Flow and the Foundations of Positive Psychology: The Collected Works of Mihaly Csikszentmihalyi*; Springer: Berlin/Heidelberg, Germany, 2014; pp. 227–238.
15. Deci, E.L.; Ryan, R.M. The What and "why" of goal pursuits: Human needs and the self-determination of behavior. *Psychol. Inq.* **2000**, *11*, 227–268. [CrossRef]
16. Byun, J.H.; Joung, E. Digital game-based learning for K–12 mathematics education: A meta-analysis. *Sch. Sci. Math.* **2018**, *118*, 113–126. [CrossRef]
17. Jagušt, T.; Botički, I.; So, H.-J. Examining competitive, collaborative and adaptive gamification in young learners' math learning. *Comput. Educ.* **2018**, *125*, 444–457. [CrossRef]
18. Cunha, G.C. Araujo, Luciana Pelissari Barraqui, and Sergio Antonio Andrade De Freitas. In Proceedings of the Evaluating the Use of Gamification in Mathematics Learning in Primary School Children IEEE Frontiers in Education Conference (FIE), San Jose, CA, USA, 3–6 October 2018.
19. Katz, B.; Jaeggi, S.; Buschkuehl, M.; Stegman, A.; Shah, P. Differential effect of motivational features on training improvements in school-based cognitive training. *Front. Hum. Neurosci.* **2014**, *8*, 242. [CrossRef] [PubMed]
20. Tan, C.W.; Ling, L.; Yu, P.D.; Hang, C.N.; Wong, M.F. Mathematics Gamification in Mobile App Software for Personalized Learning at Scale. In Proceedings of the 2020 IEEE Integrated STEM Education Conference (ISEC), Princeton, NJ, USA, 1 August 2020. [CrossRef]
21. Tan, C.W.; Yu, P.-D.; Lin, L. Teaching computational thinking using mathematics gamification in computer science game tournaments. In *Computational Thinking Education*; Springer: Singapore, 2019; pp. 167–181. [CrossRef]
22. Tzoumpa, D.; Seralidou, E.; Alougdelis, A.; Douligeris, C. Pythagorean Academy"–A game for the teaching of junior high school mathematics using mobile devices. In Proceedings of the 2022 7th South-East Europe Design Automation, Computer Engineering, Computer Networks and Social Media Conference (SEEDA-CECNSM), Ioannina, Greece, 23–25 September 2022. [CrossRef]
23. Lameras, P.; Moumoutzis, N. Towards the gamification of inquiry-based flipped teaching of mathematics a conceptual analysis and framework. In Proceedings of the 2015 International Conference on Interactive Mobile Communication Technologies and Learning (IMCL), Thessaloniki, Greece, 19–20 November 2015. [CrossRef]
24. Hancock, D.R.; Bray, M.; Nason, S.A. Influencing university students' achievement and motivation in a technology course. *J. Educ. Res.* **2002**, *95*, 365–372. [CrossRef]
25. Rojas, O.; Elizabeth, M.; Chiluiza, K.; Valcke, M. Gamification in higher education and stem: A systematic review of literature. In Proceedings of the 8th International Conference on Education and New Learning Technologies (EDULEARN), Barcelona, Spain, 4–6 July 2016; Iated-int Assoc Technology Education A& Development: Valencia, Spain, 2016.
26. Goshevski, D.; Veljanoska, J.; Hatziapostolou, T. A review of gamification platforms for higher education. In Proceedings of the 8th Balkan Conference in Informatics, Skopje, Macedonia, 20–23 September 2017; pp. 1–6. [CrossRef]
27. Grieves, M.W. *Virtually Intelligent Product Systems: Digital and Physical Twins*; American Institute of Aeronautics and Astronautics: Reston, VA, USA, 2019. [CrossRef]
28. Piascik, R.; Vickers, J.; Lowry, D.; Scotti, S.; Stewart, J.; Calomino, A. *Technology area 12: Materials, structures, mechanical systems, and manufacturing road map*; National Research Council: Washington, DC, USA, 2010.
29. Mylonas, G.; Kalogeras, A.; Kalogeras, G.; Anagnostopoulos, C.; Alexakos, C.; Muñoz, L. Digital 7twins from smart manufacturing to smart cities: A survey. *IEEE Access* **2021**, *9*, 143222–143249. [CrossRef]
30. Grieves, M.; Vickers, J. Digital twin: Mitigating unpredictable, undesirable emergent behavior in complex systems. In *Transdisciplinary perspectives on complex systems: New findings and approaches*; Springer Nature: Basel, Switzerland, 2017; pp. 85–113. [CrossRef]
31. Santos, C.; Almeida, S.; Pedro, L.; Aresta, M.; Koch-Grunberg, T. Students' perspectives on badges in educational social media platforms: The case of SAPO campus tutorial badges. In Proceedings of the 2013 IEEE 13th International Conference on Advanced Learning Technologies, Beijing, China, 15–18 July 2013; pp. 351–353.
32. O'Donovan, S.; Gain, J.; Marais, P. A case study in the gamification of a university-level games development course. In Proceedings of the South African Institute for Computer Scientists and Information Technologists Conference, East London, South Africa, 7–9 October 2013; pp. 242–251.
33. Raymer, R. Gamification: Using game mechanics to enhance eLearning. *ELearn* **2011**, *2011*, 19. [CrossRef]

34. Park, J.; Kim, S.; Kim, A.; Mun, Y.Y. Learning to be better at the game: Performance vs. completion contingent reward for game-based learning. *Comput. Educ.* **2019**, *139*, 1–15.
35. All, A.; Plovie, B.; Castellar, E.P.N.; Van Looy, J. Pre-test influences on the effectiveness of digital-game based learning: A case study of a fire safety game. *Comput. Educ.* **2017**, *114*, 24–37.
36. Berkling, K.; Thomas, C. Gamification of a Software Engineering course and a detailed analysis of the factors that lead to it's failure. In Proceedings of the 2013 International Conference on Interactive Collaborative Learning (ICL), Kazan, Russia, 25–27 September 2013; pp. 525–530.