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Cascade Synthesis of Observers of Mixed Variables for Flexible Joint Manipulators Tracking Systems under Parametric and External Disturbances

Svetlana A. Krasnova, Aleksey S. Antipov *, Dmitry V. Krasnov and Anton V. Utkin

V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences, 65 Profsoyuznaya Street, Moscow 117997, Russia; skrasnova@list.ru (S.A.K.)
* Correspondence: scholess18@mail.ru; Tel.: +7-(495)-198-17-20 (ext. 1577)

Abstract: This paper considers a tracking system developed for a full-actuated manipulator with flexible joints under the following assumptions: torques are control actions, and current loop dynamics are not considered; the mass-inertial characteristics of the manipulator and other parameters are not exactly known; the external matched and unmatched disturbances act on the system, and matched disturbances are not smooth; the derivatives of the reference actions are achievable but are unknown functions of time; the set of sensors is not complete. Based on the representation of the control plant model in a block form of input–output with respect to mixed variables (functions of state variables, external influences and their derivatives), we have developed a combined control law for the case where the control matrix contains additive uncertain elements. In addition, we have designed the mixed variable observers of the smallest possible dimension with piecewise linear corrective actions for two cases: (i) only the generalized coordinates of the manipulator are measured; (ii) only the angular positions and velocities of the motors are measured. It is shown that in a closed-loop system with dynamic feedback, a given tracking error stabilization accuracy is provided in the conditions of incomplete information. We presented the results of numerical simulation of these algorithms for a single-link manipulator.

Keywords: flexible joint manipulator; tracking system; parametric and external disturbances; mixed variables observer; cascade synthesis; robust linear control with saturation

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1. Introduction

Mechatronic systems, which contain a mechanical system with an electric actuator, are quite diverse and very interesting for researchers. Using the Euler–Lagrange equations, we can obtain an adequate mathematical model of a mechanical system. In the general case, this model includes cross-couplings and significant nonlinearities. However, the nonlinear matrices of this model have characteristic features, which makes it possible to distinguish mechanical systems into a separate class of automatic control plants [1,2]. Plants of this class are often used for the approbation and verification of new theoretical methods. From a practical viewpoint, it is of interest to develop universal and simple algorithms that will ensure the operability of a closed-loop system with a complex of various uncertainties in various working scenarios.

The problems with controlling mechanical systems in different formulations have been considered within the framework of various approaches [1–4], primarily using classical methods: optimal control [5,6], PI, and PID controllers [7–9]. Optimal control methods provide minimization or maximization of the selected criteria. However, they are sensitive to uncertainties. For the synthesis of PI and PID controllers, it is not necessary to know the mathematical model of the plant. However, accurate tuning of the regulator coefficients is required. It depends on the operating conditions of the plant.

At present, fuzzy controllers and methods of neural network control have become popular [10–13]. These methods can provide high performance under changing conditions. However, an adequate training set is needed to tune the neural network.

The best performance is usually achieved by combining different approaches. The main efforts of specialists are aimed at the development of robust and adaptive control under the conditions of parametric uncertainty of the plant model and under the influence of external uncontrolled disturbances. A separate problem is the task of constructing state observers. This task arises when it is impossible to install a complete set of sensors for various reasons [14]. As a rule, these problems are considered separately, when known approaches can be applied to a system with parametric or external disturbances of a particular type [15–17].

For example, sliding mode control, both classical and higher order, is an effective way to suppress matched disturbances, i.e., acting on the same channels with control. The high performance of such control systems has been demonstrated in many publications [18–22]. The founders of this direction were Russian scientists Emelyanov S.V. and Utkin V.I. At present, researchers in many countries of the world have used sliding mode control. However, as an application, researchers often consider only a mechanical system without the dynamics of the actuators. In this case, they use the moments developed by the actuators as discontinuous controls. In such systems, disturbances are matched with control moments. However, discontinuous moments cannot be realized using an actuator.

In some papers, where real installations are described, rigorous proofs are given in the theoretical part for a system with discontinuous control moments. However, in the experimental part, they are replaced by continuous analogs (saturation function or hyperbolic tangent [23–26]). Thus, researchers have demonstrated the performance of a closed-loop system with continuous analogs, only by the results of experiments. There were no correct mathematical proofs.

If we consider the complete mathematical model of the electromechanical system, the unknown forces and moments acting on the mechanical subsystem become the unmatched disturbances. These disturbances are separated from the true control of several integrators. As is known, it is impossible to ensure the invariance of the entire state vector with respect to unmatched disturbances [27]. The control goal can be set only for a group of controlled variables. Simultaneously, the remaining phase variables will be forced to track the external disturbances. A typical task for systems with unmatched disturbances is the synthesis of a tracking system, where it is necessary to ensure that the output variables track the given signals. In addition, the remaining state variables are bounded. Currently, the control systems with unmatched disturbances are not fully studied. In such control systems, two types of disturbances can be distinguished. Note that the applied methods for their suppression or compensation depend on the type of disturbances.

The first case is when the non-smooth disturbances (dry friction forces, shock loads, etc.) act on the mechanical subsystem. This case is the most difficult to tackle. One of the effective ways is the suppression of disturbances using nonlinear smooth and bounded S-shaped local feedbacks (control moments), which can be implemented in practical applications [28]. These methods have been developed by the scientists of the V.A. Trapeznikov Institute of Control Sciences of Russian Academy of Sciences.

In this paper, we consider the second case, which is simpler. Here, the smooth disturbances act on the mechanical subsystem. However, we allow the non-smoothness of disturbances, which are matched with the control. In addition, a complete set of sensors is absent. As a control plant, we consider full-actuated flexible joint manipulators with n degrees of freedom without taking into account the dynamics of the current loop. The controls are torques applied to the actuator's shafts. For the above reasons, we cannot use the sliding mode control for this system. This paper poses the problem of synthesizing dynamic feedback, which ensures that the output variables (generalized coordinates of the manipulator) track the given smooth trajectories. Note that the generation of achievable trajectories is an independent problem that is solved at the planning stage. This problem depends on the working scenario [29], and is not considered in this paper.

The problems of control and observation are very complicated for plants with cross-couplings and nonlinearities. These problems do not always have a solution under parametric and external disturbances. For example, the construction of a state observer for a Lagrangian system requires the complete certainty of its parameters [30].

Note that unmatched disturbances include the derivatives of the reference actions. For systems with unmatched smooth disturbances, the universal approach to the synthesis of the tracking system is a flatness approach. Namely, this approach consists in the transformation into a canonical [31,32] or block-canonical [33,34] form "input–output". The mixed variables of these forms are constructed by successive differentiation of outputs (tracking errors). The main advantage of this method is that in the canonical basis of mixed variables, all uncertainties will act through the same channels as controls, i.e., become matched. The "input–output" form is observable with respect to tracking errors. Therefore, if tracking errors are measured, we can use this form as the basis for constructing a mixed variable observer. Thus, both the control (tracking) problem and the observation problem will be solved on the same coordinate basis of mixed variables. This greatly simplifies the structure of the controller, since there is no need to perform direct and inverse changes of variables in real time.

Next, two strategies are possible. The first one is to restore mixed variables using an observer, and by applying observer synthesis methods for systems with an uncertain input [31–36]. Moreover, special control laws are used to suppress external matched disturbances. To obtain a realizable control law, instead of discontinuous controls, one should use their continuous analogs [23–26,28,32,37,38]. If the control matrix is uncertain in the canonical system, the hierarchical control method can be applied to tune the controller [18,39]. In this approach, the controller tuning is performed on the basis of hierarchical inequalities, in which the lower estimates for the choice of gain factors are obtained for the "worst" case. Such estimates tend to be very conservative and provide overestimated gains.

The second strategy that we will implement in this paper is to reconstruct both mixed variables and disturbances using an observer. Next, it will be possible to apply a combined control consisting of two components. Using the first component, it is possible to compensate for the effect of matched disturbances by their estimates. The second component stabilizes the tracking errors. Using estimates of mixed variables, the second component can be constructed in the form of a conventional linear feedback. This strategy has two limitations. First, most of the known approaches for the synthesis of disturbance observers are designed to estimate the smooth uncertainties of a particular class. They require an additional expansion of the state space due to the dynamics of disturbances [32,40–45]. Second, to compensate for disturbances, it is necessary that the control matrix is exactly known in the canonical system. Under certain conditions, this problem can be solved if we also introduce the nominal control matrix, which will depend on the measured or observed signals. Simultaneously, the uncertain part of the control matrix is included in disturbances [32]. This paper will consider such an implementation. Moreover, the use of special observers of mixed variables and disturbances will allow us to expand the class of disturbances to be estimated. This expansion includes non-smooth bounded time functions with a bounded rate of change. However, in contrast to smooth disturbances of a particular class, which can be restored with an asymptotically decaying error [32], we can estimate non-smooth disturbances only with a given accuracy. As a result, we can ensure the ε-invariance of the output variables with respect to external and parametric matched and unmatched disturbances. The contributions of this paper are described below.

1. For full-actuated flexible joint manipulators, the mathematical model of which has a dynamic order $4n$ and excludes the current loop, we have developed a basic law of combined control. This law ensures the stabilization of the tracking error with a given accuracy. We showed that decomposition synthesis with step-by-step formation of linear local feedback makes it possible to directly control the parameters of the

process of the tracking error's convergence in the desired way. The conditions are formalized; under these conditions, it is possible to compensate for disturbances in the case when the control matrix is not exactly known.

2. Assuming that only the positions of the manipulator links and their reference trajectories are measured, we have developed a procedure for cascade synthesis of an observer of mixed variables and non-smooth disturbances of the minimum possible dynamic order $4n$. To synthesize the observer, we simultaneously used the principles of constructing a classical state observer and a differentiator of external signals. To stabilize the observational errors and their derivatives, we used piecewise linear corrective actions with saturation. This made it possible to simplify the tuning of the observer to avoid large peaks and noise in evaluation signals, in contrast to high-gain observers and sliding mode observers, respectively.
3. For the case when the manipulator does not have sensors and only the angular positions and velocities of the gearbox shafts are measured, we have developed a two-loop observer. In the first loop, the positions of the manipulator links are estimated with a given error, which can be made arbitrarily small. These estimates, with reference trajectories, enter the second circuit (the observer of mixed variables indicated above) and are used in the corrective actions in the second loop. For a parametrically uncertain system, we formalized the conditions under which it is possible to construct a physically realizable observer of output variables of a minimum dimension. Observation problems are considered in a deterministic formulation, i.e., in the absence of noise.

The paper has the following structure. Section 2 describes the control plant and poses the problems to be solved. Section 3 presents the main results of the paper described above. Section 4 demonstrates the results of numerical simulation of the developed algorithms for a single-link manipulator with a flexible joint.

2. Control Plant Model Problem Statement

This paper considers a full-actuated manipulator with flexible joints as an automatic control plant. It has n degrees of freedom. Next, the following $4n$-order equations describe the mathematical model of this control plant without considering the dynamics of the current loop [1,32]:

$$\dot{x}_1 = x_2,\ \dot{x}_2 = H^{-1}(x_1)[K(x_3 - x_1) - C(x_1, x_2)x_2 - G(x_1) + f_1(t)]; \tag{1}$$

$$\dot{x}_3 = x_4,\ \dot{x}_4 = J^{-1}(u - Dx_4 - K(x_3 - x_1) + f_2(t)). \tag{2}$$

Equation (1) is the Lagrangian subsystem, which is used to describe the dynamics of a manipulator with n rigid links. These links form kinematic pairs of the fifth class. They are elastically connected to the shafts of the gearboxes (2), on which the electric actuators are installed. In system (1) and (2), all vectors and matrices have dimensions n and $n \times n$, correspondingly. State variables are vectors of generalized coordinates $x_1 = (x_{11}, \ldots, x_{1n})^{\mathrm{T}}$ and velocities $x_2 = (x_{21}, \ldots, x_{2n})^{\mathrm{T}}$ of the manipulator, as well as vectors of angular positions $x_3 = (x_{31}, \ldots, x_{3n})^{\mathrm{T}}$ and velocities $x_4 = (x_{41}, \ldots, x_{4n})^{\mathrm{T}}$ of gearbox shafts. The set of sensors is not complete; we will consider various measurement options during the presentation.

In subsystem (1), $H(x_1)$ is a non-linear symmetric matrix of inertia, $H^{-1}(x_1) > 0$ for all $x_1 \in X \subset R^n$, where X is an open bounded workspace for changing generalized coordinates; $C(x_1, x_2)$ is the matrix of centripetal and Coriolis forces; $G(x_1)$ is the gravity vector. In subsystem (2), $K = \mathrm{diag}(\overline{K}_i)$, $J = \mathrm{diag}(J_i)$, $D = \mathrm{diag}(D_i)$ are diagonal matrices with positive elements, where $\overline{K}_i$ are torsional stiffness coefficients, J_i are reduced moments of inertia on the gear shafts, and D_i are viscous damping coefficients, respectively, $i = 1, \ldots, n$.

It is difficult to obtain an exact model of a real control plant. During the operation, the mass-inertial characteristics of the manipulator and other parameters may change.

Therefore, we admit the parametric uncertainty of the matrices of system (1) and (2). In addition, we consider vectors $f_1(t),\ f_2(t)$ in the model. These are uncontrollable forces and moments, which are interpreted as external bounded disturbances.

We assume that the vector of torques applied to the actuator shafts is control u. For the current loop, which is not taken into account here, u is the vector of reference actions. Therefore, due to physical constraints, controls must be chosen from the class of continuous functions.

Parametric and external disturbances that act on the second equation of subsystem (2) are matched. Thus, under certain conditions, we can compensate them directly using the control. Note that parametric and external disturbances that act on the second equation of subsystem (1) are unmatched. Its compensation requires special system transformation, which we will show below.

For system (1) and (2), we pose a problem for designing the control in the form of dynamic feedback, providing that the output variables $x_1(t)$ track the reference actions $g(t) = (g_1(t),\ \ldots,\ g_n(t))^T$ in a closed-loop system. It is assumed that $g_j(t),\ j = 1,\ldots,n$ are known smooth and achievable functions of time. However, they enter the control system from an autonomous source, and their analytical description is not known. Accordingly, their derivatives are also achievable up to the fourth order inclusive but unknown functions of time.

Let us list the necessary and sufficient conditions for transforming system (1) and (2) into the block-canonical form "input–output", where the vector u is the input, and the output is the vector of generalized coordinates of the manipulator x_1 or the vector of tracking errors $e_1 = x_1 - g$:

- Elements of matrices $H^{-1}(x_1),\ C(x_1, x_2)$ and vectors $G(x_1),\ f_1(t)$ must be twice differentiable with respect to all arguments;
- Matrices $H^{-1}(x_1),\ K,\ J^{-1}$ must be non-singular in all admissible ranges of uncertain parameters, in particular

$$B(x_1) = H^{-1}(x_1)KJ^{-1},\ \mathrm{det}B(x_1) \neq 0,\ x_1 \in X. \tag{3}$$

Expression (3) describes the structural properties of system (1). If (3) is valid, then system (1) is controllable and observable with respect to the output x_1.

Let us assume that these conditions are met for system (1) and (2). Hence, we can transform it into the block-canonical form "input–output". This form will consist of four blocks and its relative degree is four. As we will show in the next section, the control matrix is presented as (3) in this form. Suppose that the control matrix can be represented as the sum of two terms $B = B_0 + \Delta B$, where B_0 is a nominal matrix with known parameters, and ΔB is an uncertain matrix with bounded elements. Next, the requirement (3) is equivalent to the fulfillment of the condition

$$\mathrm{rank}B_0(x_1) = \mathrm{rank}(B_0(x_1) + \Delta B(x_1)) = n,\ x_1 \in X. \tag{4}$$

We do not detail the structural properties of other uncertain matrices, since they do not play a significant role in the method used. These matrices allow both additive and multiplicative uncertainty. All uncertain elements are bounded in absolute value. We assume that the parametrically uncertain model (1) and (2) are adequate, i.e., intervals of the parameter uncertainty have "reasonable" boundaries relative to the parameters of the nominal system.

To avoid performing cumbersome real-time calculations, as well as to ensure invariance with respect to matched disturbances under the conditions of uncertain control matrix (4), we will use a combined control, as well as observers of mixed variables and disturbances of the smallest possible dimension. Let us consider the two options for measuring the state variables of system (1) and (2):

1. Only the positions of the manipulator links $x_1(t)$ are measured;

2. The manipulator has no sensors, and only angular positions $x_3(t)$ and velocities $x_4(t)$ of gear shafts are measured.

It is assumed that there is no noise in the measurements. Thus, we consider the problem in a deterministic formulation.

In the next section, we have developed a procedure for designing a tracking system with dynamic feedback for plant (1) and (2). A basic control law is obtained, which is formed by mixed variables of the block-canonical form "input–output". This control law ensures the stabilization of the vector of tracking errors under conditions of complete information. For the mentioned measurement options, we have developed the corresponding observers of mixed variables. These observers will make it possible to implement the basic control law under conditions of incomplete information.

3. Theoretical Results

3.1. Basic Control Law

The procedure for synthesizing the basic control law consists in quadruple differentiation of the output of the nonlinear system (1) and (2), obtaining a block-canonical form, and linearizing it with feedback. The purpose of the standard technique is to obtain the canonical form of the system [31,32]. In contrast to this technique, we will introduce linear stabilizing local feedbacks using non-degenerate changes in variables in each block. Our goal is to stabilize the tracking error vector. Hence, it is taken as the output. Thus, we introduce the following non-degenerate changes in variables

$$\begin{aligned} &e_1 = x_1 - g,\ e_2 = x_2 - \dot{g} + K_1 e_1, \\ &e_3 = H^{-1}(x_1) K x_3 + w_2 + K_2 e_2, \\ &e_4 = H^{-1}(x_1) K x_4 + w_3 + K_3 e_3, \end{aligned} \tag{5}$$

where $e_i \in R^n$ are mixed variables. They are a combination of system state variables (1) and (2), external disturbances and their derivatives, as well as derivatives of the reference actions up to the fourth order inclusive

$$\begin{aligned} &w_2 = H^{-1}(x_1)[-K x_1 - C(x_1, x_2) x_2 - G(x_1) + f_1(t)] - \ddot{g}(t) + K_1(x_2 - \dot{g}(t)), \\ &w_3 = \tfrac{d}{dt}(H^{-1}(x_1) K) x_3 + \tfrac{d}{dt}(w_2 + K_2 e_2). \end{aligned} \tag{6}$$

After we perform the change in variables (5) and (6) system (1) and (2) with (3), namely, $H^{-1} K J^{-1} = B = B_0 + \Delta B$, will be represented in a block-canonical form closed with linear local feedbacks

$$\begin{aligned} &\dot{e}_1 = -K_1 e_1 + e_2,\ \dot{e}_2 = -K_2 e_2 + e_3, \\ &\dot{e}_3 = -K_3 e_3 + e_4,\ \dot{e}_4 = B_0(x_1) u + e_5. \end{aligned} \tag{7}$$

In system (7), the vector $e_5(t)$ depends on $f_1(t)$, $\dot{f}_1(t)$, $\ddot{f}_1(t)$, $f_2(t)$, $g(t)$, $g^{(i)}(t)$, $i = 1, \ldots, 4$. It is important that this vector acts on the same channels as the control, i.e., it is matched. This vector $e_5(t)$ includes not only external, but also parametric disturbances. We assume that the elements of $e_5(t)$ are bounded in absolute value by known constants:

$$\begin{aligned} &e_5 = \Delta B(x_1) u + H^{-1}(x_1) K J^{-1}(-D x_4 - K(x_3 - x_1) + f_2(t)) + \\ &+ \tfrac{d}{dt}(H^{-1}(x_1) K) x_4 + \tfrac{d}{dt}(w_3 + K_3 e_3), \\ &e_5 = (e_{51},\ \ldots,\ e_{5n})^{\mathrm{T}},\ \left| e_{5j}(t) \right| \le \overline{E}_{5j},\ j = 1, \ldots, n,\ t \ge 0. \end{aligned} \tag{8}$$

Note that we also interpreted the part of the control with an uncertain matrix as an uncertainty. This fact, as well as a priori assumption (4), will allow us to completely linearize the closed-loop system in the new coordinate basis of mixed variables (5). It will be performed using a combined control consisting of two parts. The first part is the linear stabilizing component, which is standard and easily implemented. The second part compensates for the unknown vector $e_5(t)$. Thus, the basic law of combined control under conditions of complete information has the form

$$u = -B_0^{-1}(x_1)(K_4 e_4 + e_5) \tag{9}$$

and leads to a stable closed-loop virtual system

$$\begin{array}{l} \dot{e}_1 = -K_1 e_1 + e_2,\ \dot{e}_2 = -K_2 e_2 + e_3, \\ \dot{e}_3 = -K_3 e_3 + e_4,\ \dot{e}_4 = -K_4 e_4, \\ K_i = \mathrm{diag}(k_{ij}),\ k_{ij} = \mathrm{const} > 0,\ i = 1, \ldots, 4, j = 1, \ldots, n. \end{array} \tag{10}$$

In Lagrangian systems, the matrix $B_0(x_1)$ depends only on the output variables that, under the assumptions made, are measurable or recoverable. In systems of a general type, the control matrix also depends on other state variables. In this case, for the feasibility of the combined control of the form (9), we must introduce additional requirements: all its arguments must either be measured or be observed relative to the measured variables [33].

The variables of system (10) converge to zero sequentially "from bottom to top" $|e_{ij}(t)| \underset{t\to+\infty}{=} O(\exp(-k_{ij}t)),\ i = 4, 3, 2, 1,\ j = 1, \ldots, n$. Accordingly, exponential stabilization of tracking errors is provided in the closed-loop system (1), (2), (9):

$$\begin{array}{l} |x_{1j}(t) - g_{1j}(t)| \underset{t\to+\infty}{=} O(\exp(-k_{1j}t)) + \\ +\frac{1}{k_{1j}}\Big(O(\exp(-k_{2j}t) + \frac{1}{k_{2j}}\Big(O(\exp(-k_{3j}t) + \frac{1}{k_{3j}}(\exp(-k_{4j}t)\Big)\Big),\ j = 1, \ldots, n. \end{array} \tag{11}$$

As you can see, system (10) is more convenient for stability analysis compared to the classical canonical system, since here you can choose the coefficients that directly affect the rate of stabilization of the output variable.

However, one must understand that the result (11) is too idealized. We will show below that, under conditions of incomplete information and the presence of uncontrolled disturbances, we can obtain estimates of unmeasured signals only with a given accuracy. These estimates are needed for feedback synthesis. We will obtain them by using state observers of the smallest possible dimension. Accordingly, the tracking problem will also be solved with some accuracy depending on the accuracy of estimating unmeasured signals.

With the selected gains k_{ij} and considering the permissible ranges of external and parametric disturbances, as well as the initial values of the system state variables (1) and (2), let us estimate the area of initial values of the mixed variables (5): $|e_{ij}(0)| \le E_{ij}$, $i = 1, \ldots, 4,\ j = 1, \ldots, n$. Let $\gamma(t) = (\gamma_1(t),\ \ldots,\ \gamma_n(t))^{\mathrm{T}}$ be the uncompensated small estimation errors, $|\gamma_j(t)| \le \overline{\gamma}_j,\ \overline{\gamma}_j = \mathrm{const} > 0,\ j = 1, \ldots, n,\ t \to \infty,$ which will appear in the last equation of system (10), closed by dynamic feedback on evaluation signals. Next, we can find the boundaries of the regions of the mixed variable changes in the following form:

$$|e_{4j}(t)| \le E_{4j} + \frac{\overline{\gamma}_j}{k_{4j}} = \overline{E}_{4j},\ |e_{ij}(t)| \le E_{ij} + \frac{\overline{E}_{i+1,j}}{k_{ij}} = \overline{E}_{ij},\ i = 3,\ 2,\ 1,\ j = 1, \ldots, n,\ t \ge 0. \tag{12}$$

Moreover, we have an estimate of the tracking error in the steady state:

$$|e_{1j}(t)| = |x_{1j}(t) - g_{1j}(t)| \le \frac{\overline{\gamma}_j}{k_{1j}k_{2j}k_{3j}k_{4j}},\ t \to \infty,\ j = 1, \ldots, n. \tag{13}$$

Expression (13) must be considered when choosing gains both in the controller and in the state observer, the synthesis of which is presented in the next subsection.

3.2. Cascade Synthesis of Mixed Variable Observer

To implement control law (9), we must know the vector of generalized coordinates $x_1(t)$ and mixed vector variables $e_4(t)$, $e_5(t)$.

First, let us consider the variant, when only signals $x_1(t)$, $g(t)$ are measured in system (1) and (2). Consequently, the tracking error $e_1(t) = x_1(t) - g(t)$ is known. Here, we can design a mixed variable observer for estimation $e_4(t)$, $e_5(t)$. It is constructed based on system (7). This system with an uncertain input e_5 is observable with respect to the output $e_1(t)$. We apply the method of cascade synthesis of piecewise linear corrective actions [31,33,45]. This will allow us to recover not only the unmeasured state variables of the system (7), but also its uncertain input e_5 by using the state observer of minimum dimension $4n$. To simplify the tuning of the observer, we modified the scheme proposed in [33] and combined the principles of constructing both the state observer and the differentiator of external signals [46] in one algorithm.

Thus, the observer–differentiator for system (7) has the form

$$\begin{aligned} &\dot{z}_i = -K_i z_i + v_i,\ i = 1, 2,\ 3, \\ &\dot{z}_4 = B_0(x_1)u + v_4, \end{aligned} \tag{14}$$

where $z_i \in R^n$ are the state variables, and $v_i \in R^n$ are the corrective actions of the observer. Let us introduce the vector of observational errors $\varepsilon_i = e_i - z_i \in R^n,\ i = 1, \ldots, 4$. Using (7) and (14), we compose a virtual system

$$\begin{aligned} &\dot{\varepsilon}_i = -K_i \varepsilon_i + e_{i+1} - v_i,\ i = 1,\ 2,\ 3 \\ &\dot{\varepsilon}_4 = e_5 - v_4, \end{aligned} \tag{15}$$

where mixed variables $e_i(t),\ i = 2, \ldots, 5$ are considered as external bounded disturbances (8), (12).

Let us note the fundamental differences between system (15) and the observational error equations obtained using an ordinary state observer of the form

$$\begin{aligned} &\dot{z}_i = -K_i z_i + z_{i+1} + v_i,\ \dot{\varepsilon}_i = -K_i \varepsilon_i + \varepsilon_{i+1} - v_i,\ i = 1, 2,\ 3, \\ &\dot{z}_4 = B_0(x_1)u + v_4,\ \dot{\varepsilon}_4 = e_5 - v_4. \end{aligned}$$

First, there is an external disturbance $e_{i+1}(t)$, $i = 1,\ 4$ in each i-th block of system (15). Second, the coefficient matrix of system (15) has a lower triangular shape. Therefore, we cannot ensure its stability using conventional linear corrective actions $v_i = l_i \varepsilon_i$. Thus, we use the methods of cascade synthesis and the separation of motions to stabilize system (15).

According to the cascade synthesis, the overall movement of observational errors $\varepsilon_i(t)$ is separated. It is achieved due to the corrective actions, which aim to suppress the disturbances. Such corrective actions may be linear controls with high gains, discontinuous controls, and their continuous analogs. Here, the accuracy of stabilization of the derivatives of the observational errors $\dot{\varepsilon}_i(t)$ is controlled. The observer–differentiator is tuned to ensure consistent stabilization of vector variables with a given accuracy

$$\begin{aligned} &\varepsilon_1(t) \approx \vec{0},\ \dot{\varepsilon}_1(t) \approx \vec{0} \Rightarrow v_1(t) \approx e_2(t), \\ &\varepsilon_i(t) \approx \vec{0},\ \dot{\varepsilon}_i(t) \approx \vec{0} \Rightarrow v_i(t) \approx e_{i+1}(t),\ i = 2,\ 3,\ 4, \end{aligned} \tag{16}$$

which provides a solution to the observation problem. As you can see, in the steady state, the evaluation signals of mixed variables will be not only the observer's variables (14), namely, $z_i(t) \approx e_i(t),\ i = 1, \ldots, 4$, but also their corrective actions (16). Thus, we can use the corrective actions of the i-th block to form corrective actions in the $(i+1)$-th block,

$i = 1, \ldots, 4$. For system (15), we introduce piecewise linear corrective actions as functions of both the measured output $e_1(t)$ and observer variables

$$\begin{aligned} & v_1 = M_1 \text{sat}(L_1(e_1 - z_1)),\ v_i = M_i \text{sat}(L_i(v_{i-1} - z_i)),\ i = 2,\ 3,\ 4, \\ & M_i = diag(m_{ij}),\ L_i = \text{diag}(l_{ij}),\ m_{ij} = \text{const} > 0,\ l_{ij} = \text{const} > 0, \end{aligned} \tag{17}$$

where

$$\begin{aligned} & v_1 = M_1 \text{sat}(L_1 \varepsilon_1) = (m_{11} \text{sat}(l_{11} \varepsilon_{11}),\ \ldots,\ m_{1n} \text{sat}(l_{1n} \varepsilon_{1n}))^{\text{T}}, \\ & m_{1j} \text{sat}(l_{1j} \varepsilon_{1j}) = \begin{bmatrix} +m_{1j},\ \varepsilon_{1j} > 1/l_{1j}, \\ m_{1j} l_{1j} \varepsilon_{1j},\ |\varepsilon_{1j}| \le 1/l_{1j}, \\ -m_{1j},\ \varepsilon_{1j} < -1/l_{1j},\ j = 1, \ldots, n. \end{bmatrix} \end{aligned} \tag{18}$$

Corrective actions $v_i,\ i = 2,\ 3,\ 4$ have a form similar to (18), namely

$$\begin{aligned} & v_i = M_i \text{sat}(L_i(v_{i-1} - z_i)) = (m_{i1} \text{sat}(l_{i1}(v_{i-1,1} - z_{i1})),\ \ldots,\ m_{in} \text{sat}(l_{in}(v_{i-1,n} - z_{in})))^{\text{T}}, \\ & m_{ij} \text{sat}(l_{ij}(v_{i-1,j} - z_{ij})) = \begin{bmatrix} m_{ij} \text{sign}(l_{ij}(v_{i-1,j} - z_{ij})),\ |v_{i-1,j} - z_{ij}| > 1/l_{ij}, \\ m_{ij} l_{ij}(v_{i-1,j} - z_{ij}),\ |v_{i-1,j} - z_{ij}| \le 1/l_{ij},\ j = 1, \ldots, n. \end{bmatrix} \end{aligned}$$

Piecewise linear corrective actions with saturation (17) and (18) are a continuous non-smooth hybrid of linear and discontinuous functions and have two adjustable parameters. The first parameter m_{ij} is the amplitude. It must be chosen to ensure the convergence of the observational error $\varepsilon_{ij}(t)$ to a small neighborhood of zero, where the corrective action v_{ij} is linear (18). We will call these areas "linear zones". The second parameter l_{ij} is high gain, and its value is inversely proportional to the radius of the linear zone, on which the stabilization accuracy depends.

Under nonzero initial conditions in system (15), the transient process of observational errors and their derivatives lasts for some time. Stabilization of each next variable is possible only after stabilization of all previous variables in the specified order (16), i.e., the total transient time of the next vector variable is greater than the previous one. Thus, the total evaluation time T (i.e., the stabilization time of all specified variables) is the time of the transient process $\dot{\varepsilon}_4(t) \approx \vec{0}$, which can be represented as eight intervals on a cumulative basis.

Using the measurements $e_1(t) = x_1(t) - g(t)$ in the observer (14) and, consequently, in the system (15), the following initial values can be set:

$$\begin{aligned} & z_1(0) = e_1(0) \Rightarrow \varepsilon_1(0) = \vec{0}; \\ & z_i(0) = \vec{0} \Rightarrow \varepsilon_i(0) = e_i(0),\ i = 2,\ 3,\ 4,\ |\varepsilon_{ij}(0)| \le E_{ij},\ j = 1, \ldots, n. \end{aligned} \tag{19}$$

From a theoretical viewpoint, the initial values in the observer can be set arbitrarily. However, as is known, the closer the initial conditions of the observer and the observed system are to each other, the faster the convergence of the observer's variables to the corresponding unmeasured variables will be. Therefore, we use known data to set the initial values in the first expression (19). Next, the values $\varepsilon_1(t)$ will immediately be in the linear zone, and the total estimation time will include not eight, but seven intervals on a cumulative basis $0 < t_1 < t_2 < t_3 < t_4 < t_5 < t_6 < t_7 = T$. At the indicated time intervals, we must consistently ensure the fulfillment of (16), namely

$$|\varepsilon_{1j}(t)| \le 1/l_{1j},\ t \ge 0; \tag{20}$$

$$\left|e_{2j}(t) - v_{1j}(t)\right| \le \alpha_{1j},\ t \ge t_1; \tag{21}$$

$$\left|v_{i-1,j}(t)\right| \le 1/l_{ij} \Leftrightarrow \left|\varepsilon_{ij}(t)\right| \le \alpha_{i-1,j} + 1/l_{ij},\ t \ge t_{2i-2}; \tag{22}$$

$$\left|e_{i+1,j}(t) - v_{ij}(t)\right| \le \alpha_{ij},\ t \ge t_{2i-1},\ i = 2,\ 3,\ 4,\ j = 1, \ldots, n. \tag{23}$$

The fulfillment of inequalities (20) and (22) (falling into linear zones) is ensured by choosing the appropriate amplitudes m_{ij}. The fulfillment of inequalities (21) and (23) is ensured by the choice of high gains l_{ij}.

Under conditions of uncertainty, the choice of the gains of corrective actions (17) and (18) is based on inequalities. To determine the minimum allowable values of gains, we use sufficient stability conditions and estimates of system (15) solutions on the appropriate time intervals with respect to (12) and (19).

Outside the linear zone, the following component-by-component estimates are valid for the first equation of system (15) and (18):

$$\varepsilon_{1j}\dot{\varepsilon}_{1j} = \varepsilon_{1j}(e_{2i} - m_{1i}\mathrm{sign}(\varepsilon_{1i}) - k_{1j}\varepsilon_{1j}) \le |\varepsilon_{1i}|(|e_{2i}| - m_{1i} - k_{1j}|\varepsilon_{1j}|),\ j = 1, \ldots, n.$$

Hence, we have inequalities for the choice of amplitudes that provide (20)

$$m_{1j} > \overline{E}_{2j} \ge \left|e_{2j}\right| \Rightarrow \varepsilon_{1j}\dot{\varepsilon}_{1j} < 0 \Rightarrow \left|\varepsilon_{1j}(t)\right| \le 1/l_{1j},\ j = 1, \ldots, n,\ t \ge 0. \tag{24}$$

Sufficient conditions for choosing the remaining amplitudes that provide (22) are similar to (24) due to the same structure of the blocks of system (15). When determining their minimum allowable values, we will consider the convergence time of the observational errors, namely, convergence $\varepsilon_{ij}(t_{2i-3})$ in the indicated neighborhoods of zero (22) during the time $t_{2i-2} - t_{2i-3}$, $i = 2,\ 3,\ 4,\ j = 1, \ldots, n$. The corresponding conservative estimates are

$$\begin{aligned}
&\left|\varepsilon_{ij}(t_{2i-3})\right| \le E_{ij} + \tfrac{\overline{E}_{i+1,j} + m_{ij}}{k_{ij}},\ j = 1, \ldots, n,\ i = 2,\ 3,\\
&m_{ij} \ge \tfrac{\left|\varepsilon_{ij}(t_1)\right|}{t_{2i-2} - t_{2i-3}} + \overline{E}_{i+1,j} \Rightarrow m_{ij} \ge \tfrac{E_{ij}k_{ij} + \overline{E}_{i+1,j}(k_{ij}(t_{2i-2} - t_{2i-3}) + 1)}{k_{ij}(t_{2i-2} - t_{2i-3}) - 1};\\
&\left|\varepsilon_{4j}(t_5)\right| \le E_{4j} + (\overline{E}_{5j} + m_{4j})t_5,\\
&m_{4j} \ge \tfrac{\left|\varepsilon_{4j}(t_5)\right|}{t_6 - t_5} + \overline{E}_{5j} \Rightarrow m_{4j} \ge \tfrac{E_{4j} + \overline{E}_{5j}t_6}{t_6 - 2t_5}.
\end{aligned} \tag{25}$$

The constraints (25) must be considered when setting intermediate intervals of estimation time:

$$t_2 > t_1 + 1/k_2,\ t_4 > t_3 + 1/k_3,\ k_i = \min\{k_{ij}\},\ j = 1, \ldots, n,\ i = 2,\ 3,\ t_6 > 2t_5.$$

Using (20) and (22), we obtain the inequalities for the choice of high gains l_{ij}. This choice is based on the estimates of solutions of the closed-loop system (15), (17), and (18) in the linear zones on the intervals $[0;\ t_1]$, $[t_{2i-2};\ t_{2i-1}]$, $i = 2,\ 3,\ 4$, and must ensure the fulfillment of inequalities (21) and (23):

$$\begin{aligned}
&\left|\varepsilon_{1j}(t_1)\right| \le \tfrac{\left|e_{2j}(t)\right|}{m_{1j}l_{1j}} + \tfrac{m_{1j} - \overline{E}_{2j}}{m_{1j}l_{1j}}\exp(-(m_{1j}l_{1j} + k_{1j})t_1) \Rightarrow\\
&\left|e_{2j}(t) - v_{1j}(t)\right| \le \alpha_{1j},\ t \ge t_1 \Leftrightarrow (m_{1j} - \overline{E}_{2j})\exp(-(m_{1j}l_{1j} + k_{1j})t_1) \le \alpha_{1j};
\end{aligned}$$

$$
\begin{aligned}
&\left|\varepsilon_{ij}(t_{2i-1})\right| \le \tfrac{\left|e_{i+1,j}(t)\right|}{m_{ij}l_{ij}} + \alpha_{i-1,j} + \tfrac{m_{ij}-\overline{E}_{i+1,j}}{m_{ij}l_{ij}} \exp(-(m_{ij}l_{ij}+k_{ij})(t_{2i-1}-t_{2i-2})) \Rightarrow \\
&\left|e_{i+1,j}(t) - v_{ij}(t)\right| \le \alpha_{ij},\ t \ge t_{2i-1} \Leftrightarrow \\
&(m_{ij} - \overline{E}_{i+1,j}) \exp(-(m_{ij}l_{ij}+k_{ij})(t_{2i-1}-t_{2i-2})) \le \alpha_{ij},\ i = 2,\ 3;
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
&\left|\varepsilon_{4j}(t_7)\right| \le \tfrac{\left|e_{5j}(t)\right|}{m_{4j}l_{4j}} + \alpha_{3j} + \tfrac{m_{4j}-\overline{E}_{5j}}{m_{4j}l_{4j}} \exp(-m_{4j}l_{4j}(t_7-t_6)) \Rightarrow \\
&\left|e_{5j}(t) - v_{4j}(t)\right| \le \alpha_{4j},\ t \ge t_7 \Leftrightarrow (m_{4j} - \overline{E}_{5j}) \exp(-m_{4j}l_{4j}(t_7-t_6)) \le \alpha_{4j},\ j = 1, \ldots, n.
\end{aligned}
$$

Let us set the desired accuracy of stabilization of observational errors, for example, as follows:

$$
\left|\varepsilon_{1j}(t)\right| \le 1/l_{1j} \le \beta_{1j};\ \left|\varepsilon_{ij}(t)\right| \le \underbrace{\alpha_{i-1,j}}_{\le \beta_{ij}/2} + \underbrace{1/l_{ij}}_{\le \beta_{ij}/2} \le \beta_{ij},\ i = 2,\ 3,\ 4. \tag{27}
$$

Next, the inequalities for choosing high gains l_{ij}, under which conditions (21), (23), (26), and (27) are simultaneously satisfied, have the form:

$$
\begin{aligned}
l_{1j} &\ge \max\left\{ \tfrac{1}{\beta_{1j}};\ \tfrac{1}{m_{1j}} \left(\tfrac{1}{t_1} \ln \tfrac{2(m_{1j}-\overline{E}_{2j})}{\beta_{2j}} - k_{1j} \right) \right\}; \\
l_{ij} &\ge \max\left\{ \tfrac{2}{\beta_{ij}};\ \tfrac{1}{m_{ij}} \left(\tfrac{1}{t_{2i-1}-t_{2i-2}} \ln \tfrac{2(m_{ij}-\overline{E}_{i+1,j})}{\beta_{i+1,j}} - k_{ij} \right) \right\},\ i = 2,\ 3; \\
l_{4j} &\ge \max\left\{ \tfrac{2}{\beta_{4j}};\ \tfrac{1}{m_{4j}(t_7-t_6)} \ln \tfrac{m_{4j}-\overline{E}_{5j}}{\alpha_{4j}} \right\},\ j = 1, \ldots, n.
\end{aligned}
\tag{28}
$$

We emphasize once again that the minimum possible values (24), (25), and (28) for choosing the gains of corrective actions were calculated from sufficient conditions for the "worst" case. Therefore, the obtained estimates are very conservative. An attempt to obtain less conservative estimates analytically without specifying the initial conditions will lead to excessively cumbersome constructions. An effective method for the additional "fine" tuning of the parameters of the observer is the numerical simulation of a closed-loop system with dynamic feedback.

It follows from inequalities (21)–(23) and (27), in that the evaluation signals of mixed variables $v_i(t) \approx e_{i+1}(t)$, $i = 1, 2, 3$ contain less error than the evaluation signals $z_i(t) \approx e_i(t)$, $i = 2, 3, 4$. Therefore, let us use the following estimates $v_3(t) \approx e_4(t)$, $v_4(t) \approx e_5(t)$, $t \ge T$. Next, we will implement the control law (9) in a closed-loop system (1) with measurements $x_1(t)$, $g(t)$ and the observer–differentiator (14) and (17) in the following form

$$
u = -B_0^{-1}(x_1)(K_4 v_3 + v_4). \tag{29}
$$

Here, the total estimation error will be $\overline{\gamma}_j \le \alpha_{4j} + 0.5k_{4j}\beta_{4j}$, $j = 1, \ldots, n$ in (13).

In Figure 1, we present the block diagram of the proposed method.

Note that we constructed the observer (14) based on system (7). Therefore, this observer gives estimates of mixed variables that are directly used in the control law. This greatly simplifies the structure of the controller, since there is no need to perform forward and backward changes of variables in real time. Moreover, the observer (14) can improve the performance of the closed-loop system even under the conditions of complete certainty and measurements. This is due to the fact that the analytical form of expressions e_4 (5), e_5 (8) is very cumbersome, and the calculation of these formulas in real time may require much more time than the calculation of the dynamic model (14).

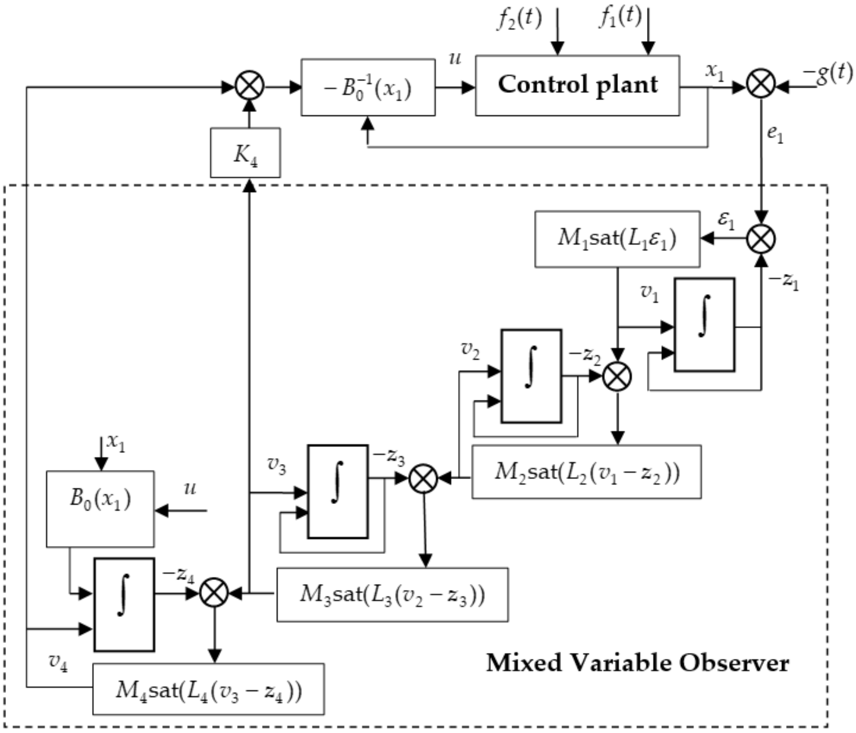


Figure 1. Block diagram of closed-loop system (1), (2), (29).

3.3. Two-Loop Observer Design

Now, we consider the second measurement option, when the manipulator has no sensors in system (1) and (2). The only measured variables are positions $x_3(t)$ and velocities $x_4(t)$ of the gearbox shafts, as well as $g(t)$. To implement the control law (9), vector variables $x_1(t), e_4(t), e_5(t)$ are needed. Here, we propose to leave the effective and easy-to-implement observer of mixed variables (14) in the feedback loop. It is supplemented by an observer–differentiator of the minimum possible dynamic order n for estimating the n generalized coordinates $x_1(t)$. These estimates are needed both as an output in the observer (14) and for computing the control matrix $B_0^{-1}(x_1)$.

Let us show that such an observer is physically realizable if the second equation of system (2) has no uncertainties, namely, the matrix elements $J,\ D,\ K$ are known and $f_2(t) \equiv \vec{0}$. Next, we can use this equation as the basis for solving the observation problem for $x_1(t)$. Using directly known signals $x_3(t)$, $x_4(t)$, $u(t)$, we will compose the dynamic model of the observer in the following form:

$$\dot{z} = J^{-1}(u - Dx_4 - Kx_3) + v, \tag{30}$$

where $z \in R^n$ is the state vector, $v \in R^n$ is the vector of corrective actions of the observer-differentiator. Let us introduce the observational error $\varepsilon = x_4 - z \in R^n$. Using (2) and (30), we write the differential equation for it

$$\begin{aligned} &\dot{\varepsilon} = Ax_1 - v,\ z(0) = x_4(0) \Rightarrow \varepsilon(0) = \vec{0}, \\ &A = J^{-1}K = \text{diag}(a_j),\ a_j = \text{const} > 0,\ j = 1, \ldots, n. \end{aligned} \tag{31}$$

In the virtual system (31), $x_1(t)$ is considered to be an external disturbance. The assumption about the boundedness of the areas of change in the positions of the links of the manipulator, namely,

$$\left|x_{1j}(t)\right| \leq X_{1j},\ j = 1, \ldots n,\ t \geq 0, \tag{32}$$

is natural, and the values $X_{1j} = \text{const} > 0$ depend on its configuration.

We must choose corrective actions to ensure the stabilization of the observational error and its derivative (31) with a given accuracy in a given time

$$\left|\varepsilon_j(t)\right| \le \beta,\ \left|\dot{\varepsilon}_j(t)\right| = \left|a_j x_{1j}(t) - v_j(t)\right| \le \alpha,\ t \ge T_0,\ j = 1, \ldots, n. \tag{33}$$

Next, we can use the corrective actions of the observer–differentiator (30) in feedback instead of the controlled variables $A^{-1}v(t) \approx x_1(t),\ t \ge T_0$.

To solve the problem, as well as in the previous subsection, we use piecewise linear corrective actions with saturation with two adjustable parameters

$$\begin{aligned} &v = M\text{sat}(L(x_4 - z)) = (m_1\text{sat}(l_1\varepsilon_1),\ \ldots,\ m_n\text{sat}(l_n\varepsilon_n))^{\text{T}}, \\ &M = diag(m_j),\ L = \text{diag}(l_j),\ m_j = \text{const} > 0,\ l_j = \text{const} > 0,\ j = 1, \ldots, n. \end{aligned} \tag{34}$$

Outside the linear zone and using (18), system (31) and (34) have the following component-by-component form $\dot{\varepsilon}_j = a_j x_{1j} - m_j \text{sign}(\varepsilon_j),\ j = 1, \ldots, n$. Similar to (24) and considering (32), the inequalities for the choice of gains of corrective actions, under which the fulfillment of the first task (33) is ensured, have the form

$$\begin{aligned} &m_j > a_j X_{1j} \ge a_j\left|x_{1j}\right| \Rightarrow \varepsilon_j\dot{\varepsilon}_j \le \left|\varepsilon_j\right|(a_j\left|x_{1j}\right| - m_j) < 0 \Rightarrow \\ &\Rightarrow \left|\varepsilon_j(t)\right| \le 1/l_j \le \beta \Rightarrow l_j \ge 1/\beta,\ j = 1, \ldots, n,\ t \ge 0. \end{aligned} \tag{35}$$

In the linear zone and using (18), system (31) and (34) have the following form $\dot{\varepsilon}_j = a_j x_{1j} - m_j l_j \varepsilon_j,\ j = 1, \ldots, n$. Similar to (26), let us analyze the estimates of its solutions for the interval $t \in [0;\ T_0]$:

$$\begin{gathered} \left|\varepsilon_j(T_0)\right| \le \tfrac{a_j\left|x_{1j}(t)\right|}{m_j l_j} + \left(\tfrac{1}{l_j} - \tfrac{a_j\left|x_{1j}(t)\right|}{m_j l_j}\right)e^{-m_j l_j T_0} \le \tfrac{a_j X_{1j}}{m_j l_j} + \tfrac{m_j - a_j X_{1j}}{m_j l_j}e^{-m_j l_j T_0}, \\ m_j l_j\left|\varepsilon_j(T_0)\right| - a_j X_{1j} \le (m_j - a_j X_{1j})e^{-m_j l_j T_0}, \\ \left|a_j x_{1j}(t) - v_j(t)\right| \le \alpha,\ t \ge T_0 \Leftrightarrow (m_j - a_j X_{1j})a_j X_{1j} \le \alpha,\ j = 1, \ldots, n. \end{gathered}$$

As we can see, the observational errors converge in the following neighborhoods of zero:

$$\left|\varepsilon_j(t)\right| \le \frac{a_j X_{1j} + \alpha}{m_j} \cdot \frac{1}{l_j} \le \frac{1}{l_j} \le \beta,\ j = 1, \ldots, n,\ t \ge T_0. \tag{36}$$

Note that if the amplitude values were taken to be large enough $m_j >> a_j X_{1j}$, then inequalities (36) can be used to reduce the lower bound for choosing high gains compared to (35). In the previous subsection, we did not use this fact; we will demonstrate it below. Thus, using (36), the minimum values for choosing high gains l_j, under which both requirements (33) are satisfied, can be estimated as follows:

$$l_j \ge \frac{1}{m_j}\max\left\{\frac{a_j X_{1j} + \alpha}{\beta};\ \frac{1}{T_0}\ln\frac{m_j - a_j X_{1j}}{\alpha}\right\},\ j = 1, \ldots, n,. \tag{37}$$

Hence, corrective actions (34) of the observer–differentiator (30) will reproduce unmeasured signals

$$v_j(t)/a_j \approx x_{1j}(t),\ \ \left|x_{1j}(t) - v_j(t)/a_j\right| = \left|\eta_j(t)\right| \le \alpha/a_j,\ j = 1, \ldots, n,\ t > T_0. \tag{38}$$

Evaluation signals (38) additively contain undamped parasitic signals $\eta_j(t)$ that cannot be compensated for. However, for $t > T_0$, they can be made arbitrarily small in modulo by increasing the high gains l_j (37). Signals (38) with the reference actions $g(t)$ enter the second circuit (observer (14)) and are used to form corrective actions (17):

$$v_{1j} = m_{1j}\text{sat}(l_{1j}(v_j/a_j - g_j - z_{1j})) = m_{1j}\text{sat}(l_{1j}(e_{1j} - z_{1j} + \eta_j)),\ j = \overline{1,\ n}.$$

The error $\eta_j(t)$ will increase the area of convergence $\varepsilon_1(t)$ (20)

$$|\varepsilon_{1j}(t)| \le \alpha/a_j + 1/l_{1j},\ t \ge T_0 + t_0,\ j = 1, \ldots, n$$

and the total time for solving the estimation problem, which in the two-loop observer-differentiator (30) and (14) is equal to $T_0 + t_0 + T$, where t_0 is the time in which the variable $\varepsilon_{1j}(t)$ enters the linear zones. In the steady state for $t \ge T_0 + t_0 + T$, estimation errors $|e_{4j}(t) - v_{3j}(t)| \le \alpha_{3j}$, $|e_{5j}(t) - v_{4j}(t)| \le \alpha_{4j}$ can be tuned by increasing the high gains l_{3j} and l_{4j} (28).

The total dynamic order of a two-loop observer (14), (17), and (30), (34) is $5n$. In a closed-loop system (1) with measurements $x_3(t)$, $x_4(t)$, $g(t)$ and dynamic feedback, the control law (9) will be implemented in the form

$$u = -B_0^{-1}(A^{-1}v)(K_4 v_3 + v_4). \tag{39}$$

In Figure 2, we show the block diagram of the closed-loop system with a two-loop observer.

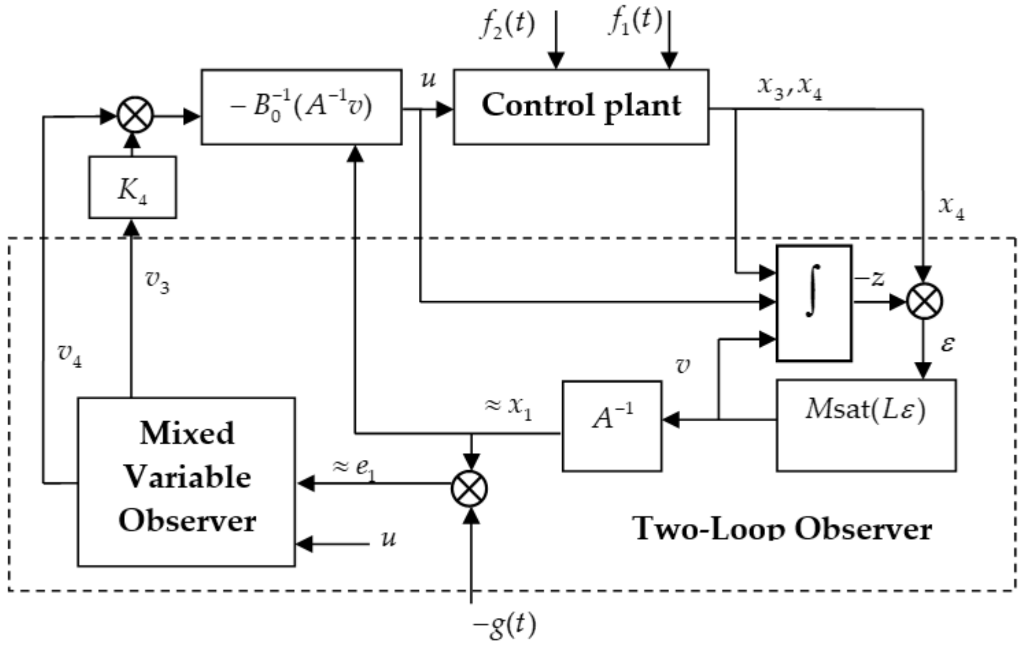


Figure 2. Block diagram of a closed-loop system (1), (2), (39).

Here, the total estimation error in (13) will be slightly larger than in the system with the measurement of output variables $x_1(t)$ and control (29).

4. Numerical Simulation Results

Numerical simulation was performed in the MATLAB-Simulink. We used the Euler method with a constant step 10^{-4} for numerical integration. As an example for the application of the developed algorithms, we considered a single-link rigid manipulator with a swivel joint elastically connected to the gearbox shaft. Its scheme is shown in Figure 3 [47].

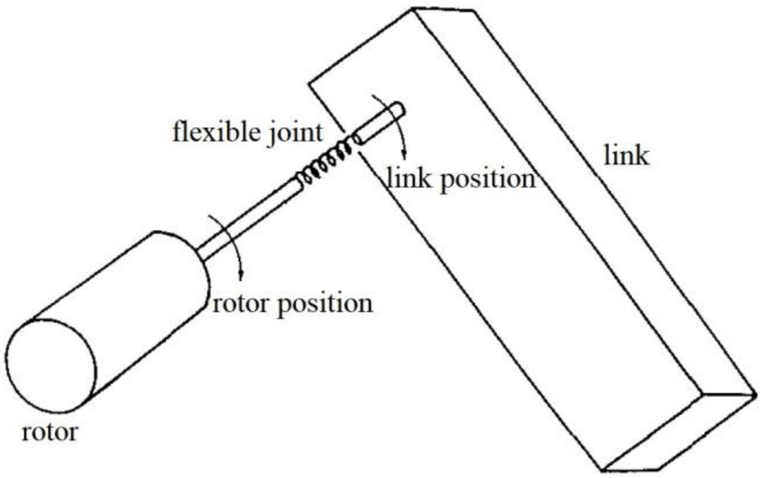


Figure 3. Scheme of the single-link manipulator.

For a given electromechanical plant, Equations (1) and (2) have the fourth dynamic order and the following form

$$\dot{x}_1 = x_2,\ \dot{x}_2 = a_{21}(x_3 - x_1) - a_{22}\sin(x_1) + f_1(t),$$
$$\dot{x}_3 = x_4,\ \dot{x}_4 = -a_{43}(x_3 - x_1) - a_{44}x_4 + a_{45}u + f_2(t), \quad (40)$$

where $x_i \in R$ are the state variables, and $x_i(t) = 0$, $i = 1, \ldots, 4$, a_{ij} are the positive design coefficients [47]:

$$a_{21} = k_l/J_l,\ a_{22} = \overline{m}\overline{g}h/J_l,\ a_{43} = k_l/J_m,\ a_{44} = d/J_m,\ a_{45} = k_m/J_m.$$

They are described in Table 1 with the other parameters of the plant.

Table 1. The variables and parameters of the plant.

Notation	Description, Measurement Unit
x_1	The angular position of manipulator's link, rad
x_2	The angular velocity of manipulator's link, rad/s
x_3	The angular position of DC motor's shaft, rad
x_4	The angular velocity of DC motor's shaft, rad/s
$f_1(t)$	Uncontrolled unmatched disturbance, rad/s^2
$f_2(t)$	Uncontrolled matched disturbance, rad/s^2
k_l	Gear rigidity, N·m/rad
k_m	Gain, N·m/A
$\overline{m}$	Manipulator's link mass, kg
$\overline{g} = 9.81$	Acceleration of gravity,m/s^2
h	Manipulator's link length, m
J_l	The moment of inertia of manipulator's link, kg·m
J_m	The moment of inertia of DC motor, kg · m^2
u	The armature current of DC motor, A

We selected the following values of the plant's parameters for the simulation:

$$a_{21} = 27.78,\ a_{22} = 49.05,\ a_{43} = 20,\ a_{44} = 4.5,\ a_{45} = 30. \quad (41)$$

The output (controllable) variable is $x_1(t)$. It is the angular position of the manipulator's link, $|x_1(t)| < \pi$ (rad), $t \geq 0$. The reference actions are smooth and achievable, namely, $|g(t)| < \pi$, $\left|g^{(i)}(t)\right| < X_i$, $i = 1, \ldots, 4$, $t \geq 0$, where $\left|x_1^{(i)}(t)\right| \leq X_i$ are the manipulator's design constraints. Parameters are not exactly known and may change within known ranges $0 < a_{ij,\min} \leq a_{ij}(t) \leq a_{ij,\max}$ during the operation.

In closed-loop system, (40) and (9), the control's goal was to ensure that the output variable will track the given signal

$$g(t) = 0.15\sin(t) + 0.05. \tag{42}$$

For model (40), using non-degenerate changes of scalar variables (5), we obtained form (7). In technical plants, the control resource is usually bounded $|u(t)| \leq U$, $t \geq 0$. We assumed that the mixed variables (5) and (8) are bounded. Thus, the inequality $|k_4 e_4(t) + e_5(t)|/b_0 \leq U$, $t \geq 0$ is satisfied for the chosen gains k_i, $i = 1, \ldots, 4$ (9). Here, in contrast to the general case (3), the factor before a control is a number $b = a_{21}a_{45} > 0$ and does not depend on $x_1(t)$. Therefore, here, the tracking error $e_1(t)$ can be directly measured instead of having to separately measure the signals $x_1(t)$ and $g(t)$. For this scalar case and under parametric uncertainty, requirement (4) has the form $a_{21} > 0$, $a_{45} > 0$, $b = b_0 + \Delta b > 0$, and $b_0 > 0$. In the case of arbitrary signs of the multipliers, the following conditions must be met to maintain controllability in all intervals of uncertain parameters

$$a_{21} \neq 0,\ a_{45} \neq 0 \Rightarrow b = b_0 + \Delta b \neq 0,\ b_0 \neq 0, \operatorname{sign}(b_0) = \operatorname{sign}(b_0 + \Delta b).$$

In the virtual system (7) and in the control law (9), the following gains were chosen:

$$k_i = 5,\ i = 1, \ldots, 4. \tag{43}$$

To test the performance of the developed algorithms, we implemented three experiments with the values of the plant parameters (41) and the reference action (42). In the first experiment, system (40) with static feedback (9) was simulated under the assumption that the exact values of all system parameters, internal and external signals, and their derivatives are known.

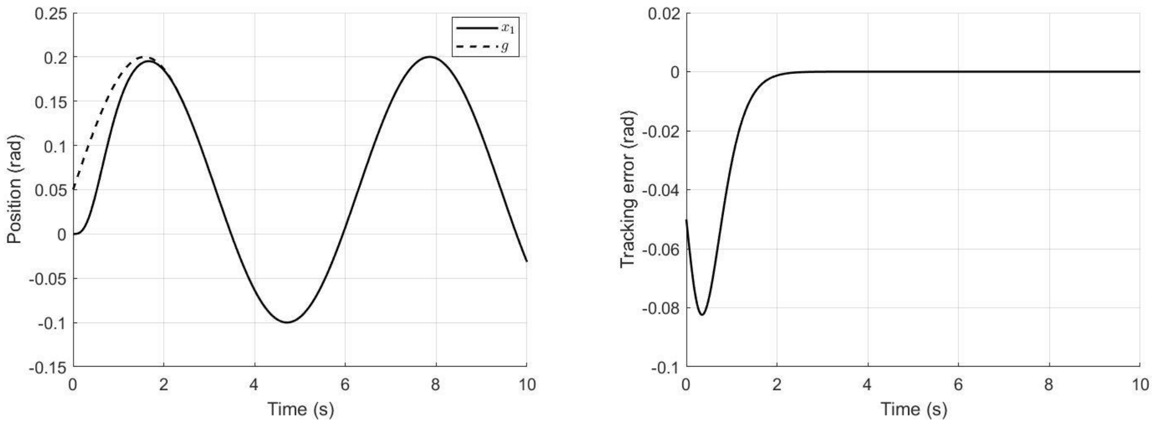
In the second experiment, we assumed that only tracking error $e_1(t)$ measurements are available, and the parameters of the plant (40) are not exactly known. In particular, parameters a_{21}, a_{45} are in the range $a_{21} = a_{21,0} + \Delta a_{21}$, $a_{45} = a_{45,0} + \Delta a_{45}$, where $a_{21,0} = 27$, $a_{45,0} = 29.5$ are the nominal values, $|\Delta a_{21}(t)| \leq 0.78$, $|\Delta a_{45}(t)| \leq 0.5$;$f_1(t) = 0.05$, $f_2(t) = 0.025t$ is the sawtooth function with principal period 2 s. To obtain estimates of mixed variables $e_4(t)$ and $e_5(t)$ (8) required for the control law (29), we implemented the observer of mixed variables (14), (17), and (18). Based on the given estimation accuracy $|e_4(t) - v_3(t)| \leq \alpha_3 = 0.01$ and $|e_5(t) - v_4(t)| \leq \alpha_4 = 3$, and using inequalities (24), (25), and (28), we selected the following gains:

$$\begin{aligned} &l_1 = 100,\ l_2 = 850,\ l_3 = 515,\ l_4 = 5, \\ &m_1 = 5,\ m_2 = 10,\ m_3 = 30,\ m_4 = 200. \end{aligned} \tag{44}$$

In the third experiment, only signals $g(t)$, $x_3(t)$, and $x_4(t)$ were measured. We assumed that the fourth equation of system (40) does not contain uncertain parameters, and $f_2(t) \equiv 0$. To obtain estimates of $x_1(t)$, $e_4(t)$, and $e_5(t)$, we constructed two-loop observer (14), (17), (44), and (30), (34). The corrective action (34) of observer (30) presented the estimate of $x_1(t)$. Based on the given estimation accuracy (38) $\alpha = 0.001$ (38), and using inequalities (35) and (37), we accepted the following gains:

$$m = 20,\ l = 100. \tag{45}$$

Figures 4–14 show the simulation results for experiments 1, 2, and 3, respectively. For each experiment, we present the plots of the reference action $g(t)$ and output variable $x_1(t)$, which track the reference action (Figures 4a, 6a and 11a, respectively). In addition, we demonstrate plots of the tracking error $e_1(t) = x_1(t) - g(t)$ (Figures 4b, 6b and 11b) and control $u(t)$ (Figures 5, 9 and 14).



(**a**) (**b**)

Figure 4. Experiment 1. In (**a**), plots of the reference signal $g(t)$ and the output $x_1(t)$; in (**b**), plot of the tracking error $e_1 = x_1(t) - g(t)$.

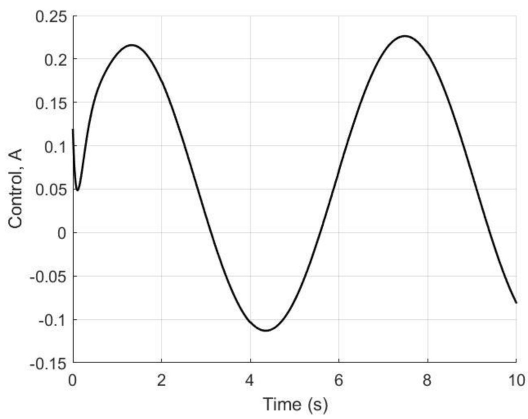
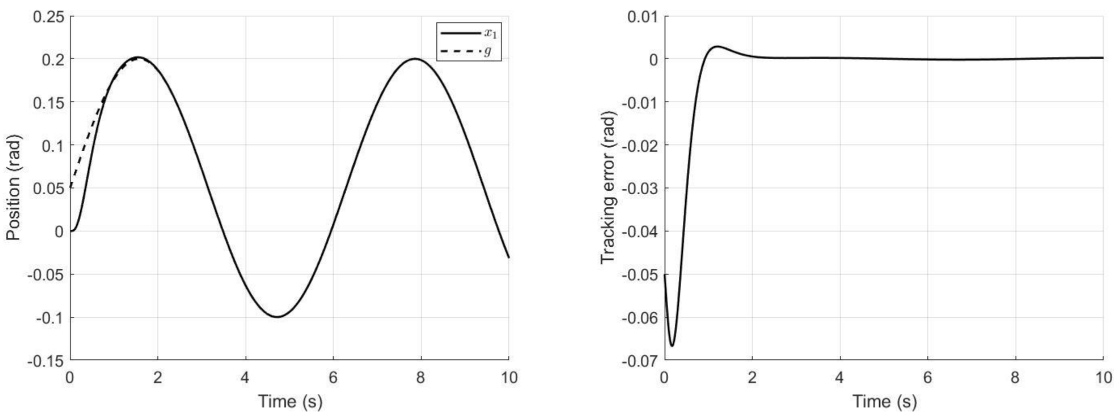
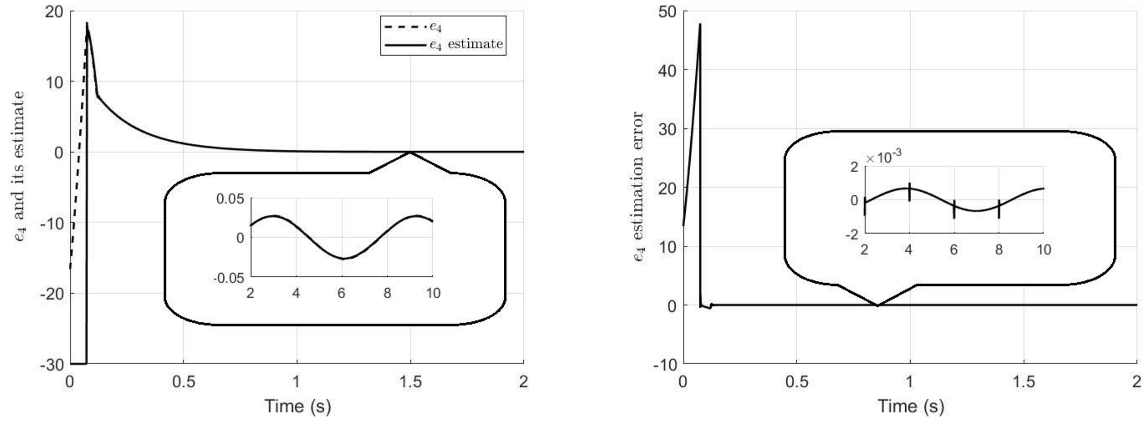


Figure 5. Experiment 1. Plot of the control $u(t)$.



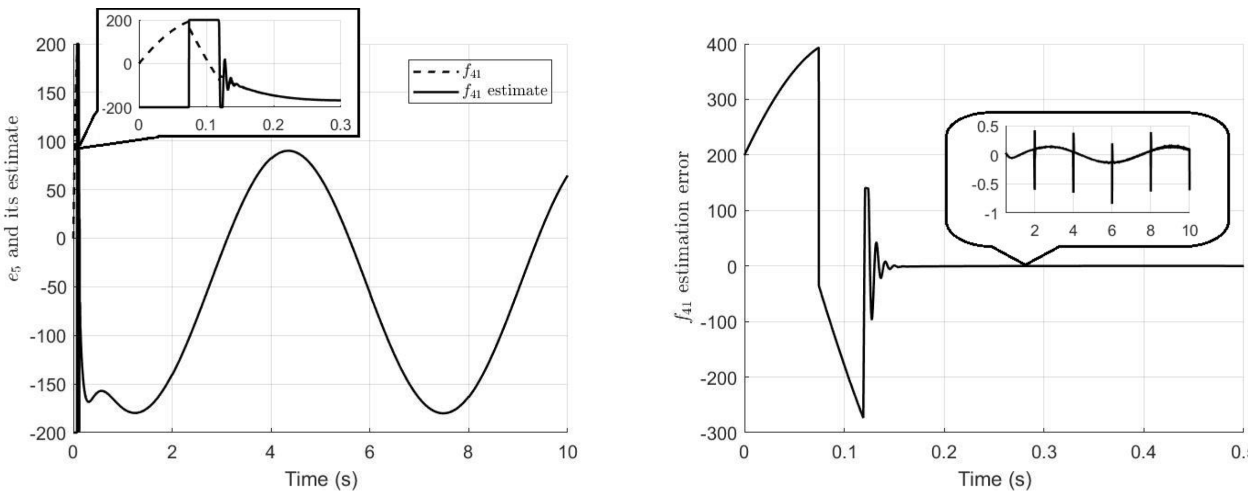
(**a**) (**b**)

Figure 6. Experiment 2. In (**a**), plots of the reference signal $g(t)$ and the output $x_1(t)$; in (**b**), plot of the tracking error $e_1 = x_1(t) - g(t)$.



(**a**) (**b**)

Figure 7. Experiment 2. In (**a**), plots of the mixed variable $e_4(t)$ and its estimate $v_3(t)$; in (**b**), plot of the estimation error $e_4(t) - v_3(t)$.



(**a**) (**b**)

Figure 8. Experiment 2. In (**a**), plots of the mixed variable $e_5(t) = f_{41}(t)$ and its estimate $v_4(t)$; in (**b**), plot of the estimation error $f_{41}(t) - v_4(t)$.

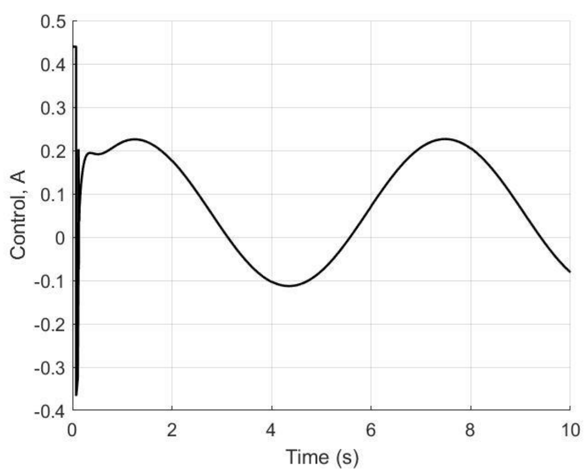


Figure 9. Experiment 2. Plot of the control *u(t)*.

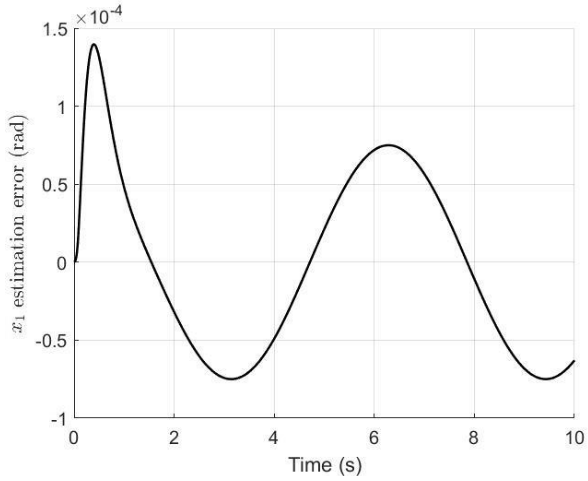
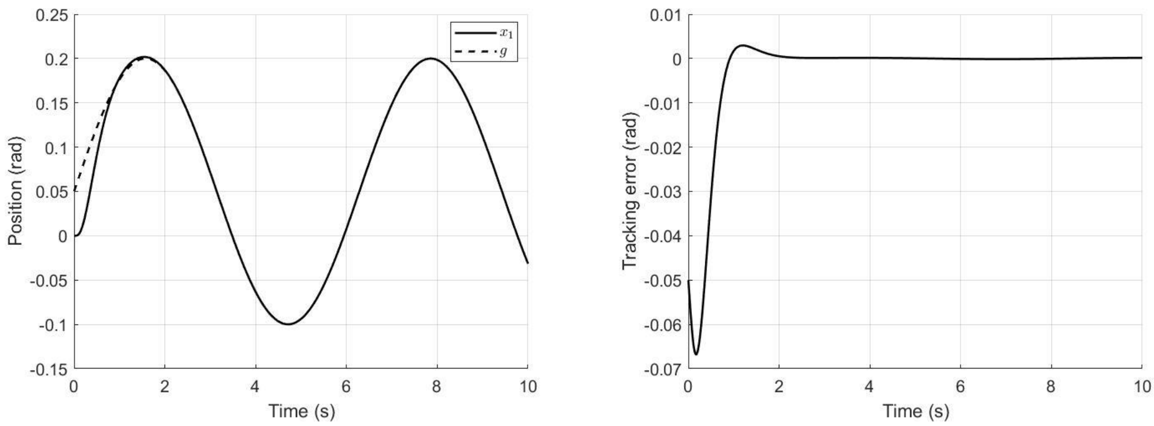
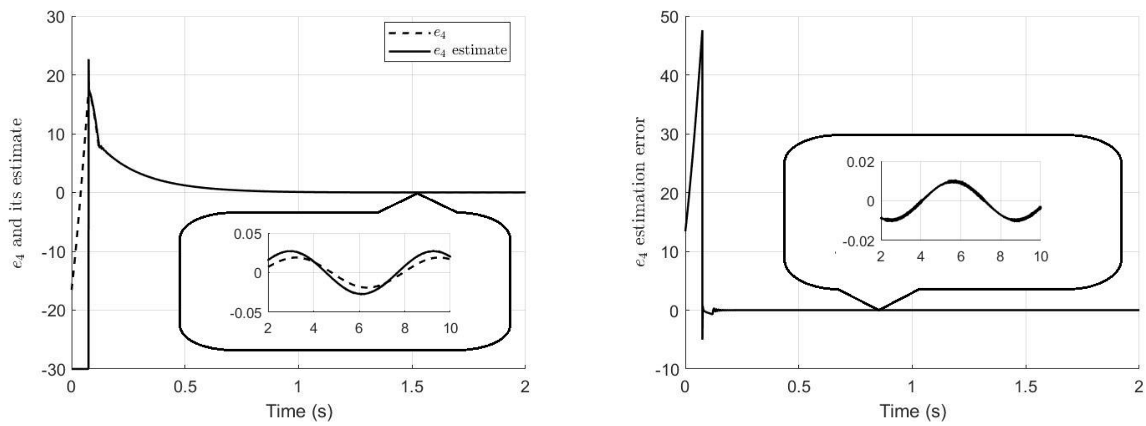


Figure 10. Experiment 3. Plot of the estimation error $x_1(t) - v(t)/a_{43}$.



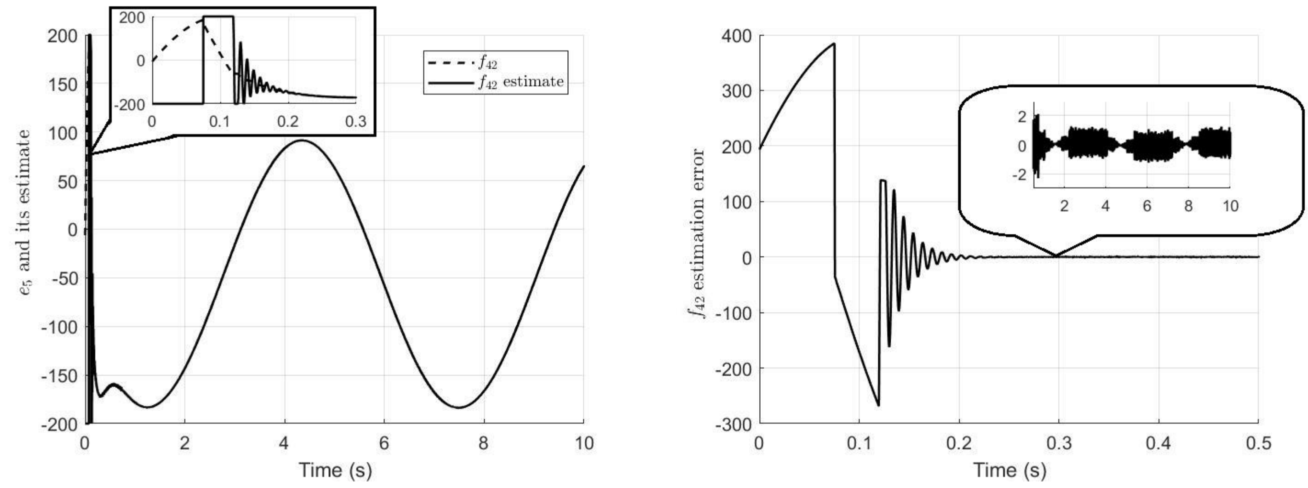
(**a**) (**b**)

Figure 11. Experiment 3. In (**a**), plots of the reference signal $g(t)$ and the output $x_1(t)$; in (**b**), plot of the tracking error $e_1 = x_1(t) - g(t)$.



(**a**) (**b**)

Figure 12. Experiment 3. In (**a**), plots of the mixed variable $e_4(t)$ and its estimate $v_3(t)$; in (**b**), plot of the estimation error $e_4(t) - v_3(t)$.



(**a**) (**b**)

Figure 13. Experiment 3. In (**a**), plots of the mixed variable $e_5(t) = f_{42}(t)$ and its estimate $v_4(t)$; in (**b**), plot of the estimation error $f_{42}(t) - v_4(t)$.

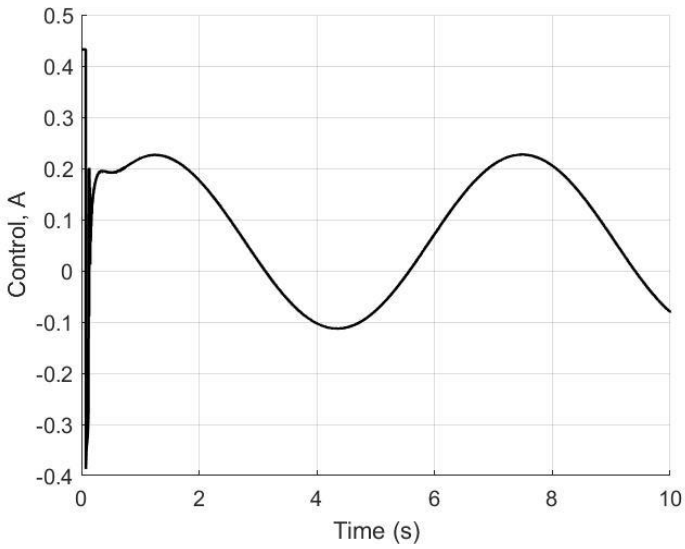


Figure 14. Experiment 3. Plot of the control $u(t)$.

For experiments 2 and 3, Figures 7a and 12a show plots of the mixed variable $e_4(t)$ and its estimate $v_3(t)$ obtained using observer (14), respectively. Figures 7b and 12b present the plots of estimation errors $e_4(t) - v_3(t)$. In Figures 8a and 13a, we show the plots of mixed variables $e_5(t)$ for experiment 2 $e_5(t) = f_{41}(t)$ and for experiment 3 $e_5(t) = f_{42}(t)$, respectively, as well as their estimates $v_4(t)$. Figures 8b and 13b show plots of estimation errors $f_{41}(t) - v_4(t)$, $f_{42}(t) - v_4(t)$ for experiments 2 and 3, respectively. In addition, for experiment 3, Figure 10 presents the plot of the estimation error $x_1(t) - v(t)/a_{43}$ of the angular position of the manipulator $x_1(t)$ by corrective action $v(t)$, (34), and (45) of the observer (30).

Table 2 shows the values of performance indicators of closed-loop systems in experiments 1, 2, and 3. We calculated the settling time t_1 (s): $|e_1(t)| \leq 0.005$ (rad), $t \geq t_1$; the overshoot of the tracking error $e_{1,\max}$ (rad): $e_{1,\max} \geq |e_1(t)|$, $t \geq 0$, tracking accuracy in the steady state Δ_1 (rad); maximal value of the control $u_{\max}$ (A): $u_{\max} \geq |u(t)|$, $t \geq 0$.

Table 2. Values of performance indicators of closed-loop systems.

Indicator	Experiment's Number		
	1	2	3
t_1, s	1.6107	0.7871	0.7850
$e_{1,\max}$, rad	0.0824	0.0667	0.0668
Δ_1, rad	$5.95 \cdot 10^{-6}$	$2.04 \cdot 10^{-4}$	$1.42 \cdot 10^{-4}$
$u_{\max}$, A	0.2263	0.4394	0.4321

It follows from Figures 4, 6 and 11 that the control's goal is achieved in all experiments: the tracking errors converge in small neighborhoods of zero. We obtained the following values for the quality indicators of signal $e_4(t)$, $f_{41}(t)$ estimation in experiment 2 and signal $x_1(t)$, $e_4(t)$, $f_{42}(t)$ estimation in experiment 3. For experiment 2:

$$\begin{aligned} &|e_4(t) - v_3(t)| \leq 47.8255,\ t \geq 0, |f_{41}(t) - v_4(t)| \leq 393.3656,\ t \geq 0, \\ &|e_4(t) - v_3(t)| \leq 0.01,\ t \geq 0.1401 \text{ (s)}, |f_{41}(t) - v_4(t)| \leq 3,\ t \geq 0.1472 \text{ (s)}, \\ &|e_4(t) - v_3(t)| \leq 0.0011,\ t \geq 2 \text{ (s)}, |f_{41}(t) - v_4(t)| \leq 0.8436,\ t \geq 2 \text{ (s)}. \end{aligned} \quad (46)$$

For experiment 3:

$$\begin{aligned} &|x_1(t) - v(t)/a_{43}| \leq 1.5 \cdot 10^{-4},\ t \geq 0, \\ &|e_4(t) - v_3(t)| \leq 47.4419,\ t \geq 0, |f_{42}(t) - v_4(t)| \leq 384.4208,\ t \geq 0, \\ &|e_4(t) - v_3(t)| \leq 0.01,\ t \geq 0.1396 \text{ (s)}, |f_{42}(t) - v_4(t)| \leq 3,\ t \geq 0.1468 \text{ (s)}, \\ &|e_4(t) - v_3(t)| \leq 6.85 \cdot 10^{-4},\ t \geq 2 \text{ (s)}, |f_{42}(t) - v_4(t)| \leq 0.1674,\ t \geq 2 \text{ (s)}. \end{aligned} \quad (47)$$

We can see from Figures 7 and 8 that the presence of non-smooth disturbances $f_2(t)$ in experiment 2 leads to peaks of signals at moments of time when $f_2(t)$ has points of discontinuity. Hence, it deteriorates the accuracy of signal estimation (46) compared with experiment 3 (47), where $f_2(t) = 0$. Simultaneously, in the transient process, the estimation time and maximum estimation errors are comparable, being (46) and (47) for experiments 2 and 3. Note that due to the presence of measurement $e_1(t)$, the estimated signals are smoother in experiment 2 compared to experiment 3 (Figures 8b and 13b).

Sufficiently large estimation errors in the transient process (up to 393.3656 (46), (47)) increase the maximum value of the control action by about two times for experiments 2 and 3 compared to experiment 1 (Table 2). Simultaneously, the tracking accuracy of a given signal deteriorates by about 34 times in experiments 2 and 3 compared to experiment 1 (where the asymptotic convergence of the tracking error (11) is theoretically achieved). However, the estimation errors converge rather quickly (in less than 0.2 s (46), (47)). The values of the settling time and overshoot for the tracking error do not exceed the values obtained for the basic control law in experiment 1 (Table 2). Note that the comparison with the basic control law is unconstructive, since its formation requires accurate knowledge not only of all values of the plant's parameters, but also of external influences and their derivatives. The organization of such measurements is unrealizable in practice. In addition, the quality indicators obtained for uncertain systems with observers correspond to the specified technological requirements.

To test the robustness of the developed algorithms, with respect to changes in parameters and external influences, we performed experiments 4 and 5. In these experiments, in comparison with experiments 2 and 3, we selected other laws of change in the reference action $g(t)$, external disturbances $f_1(t)$, and parameter $a_{21}(t)$:

$$g(t) = 0.1\cos(t) - 0.15\sin(t).$$

$$f_1(t) = 0.05\cos(0.5t),\ a_{21}(t) = 27 + 0.78\sin(t),$$

where the nominal value $a_{21,0} = 27$ of the parameter $a_{21}(t)$ was assumed to be known. Other conditions in experiments 4 and 5 were the same as in experiments 2 and 3, respectively. Moreover, we used the same observer and controller coefficients as in these experiments.

Figures 15 and 16 show the simulation results for experiments 4 and 5, respectively. Figures 15a and 16a demonstrate the plots of the reference action $g(t)$, and output variable $x_1(t)$, which tracks the reference action. In Figures 15b and 16b, we show plots of tracking errors $e_1(t) = x_1(t) - g(t)$. For experiments 4 and 5, Table 3 presents the values of the same indicators of quality regulation as in Table 2.

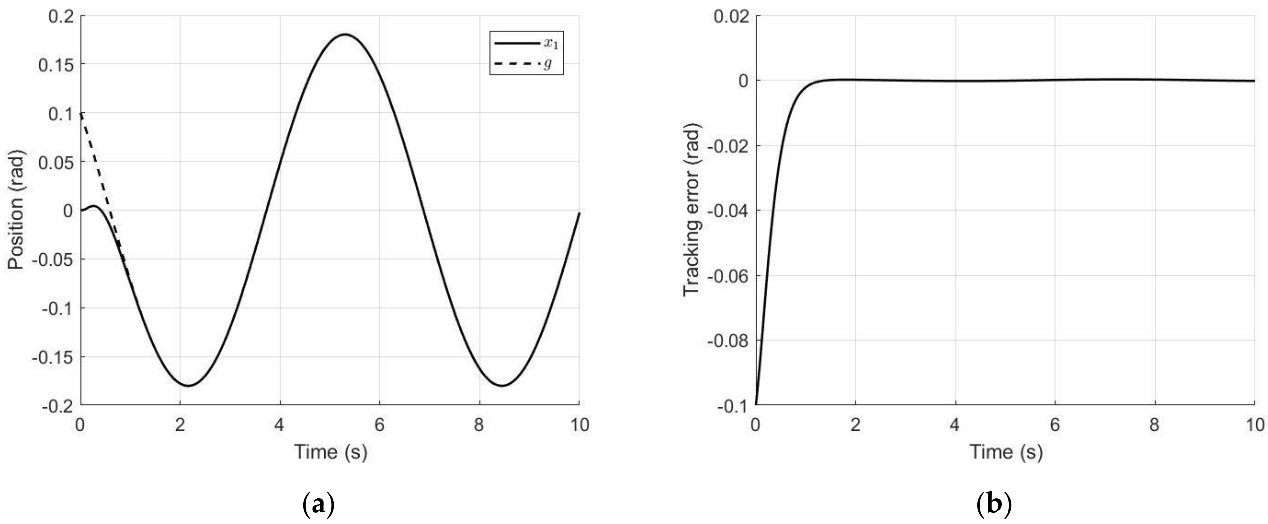


Figure 15. Experiment 4. In (**a**), plots of the reference signal $g(t)$ and the output $x_1(t)$; in (**b**), plot of the tracking error $e_1 = x_1(t) - g(t)$.

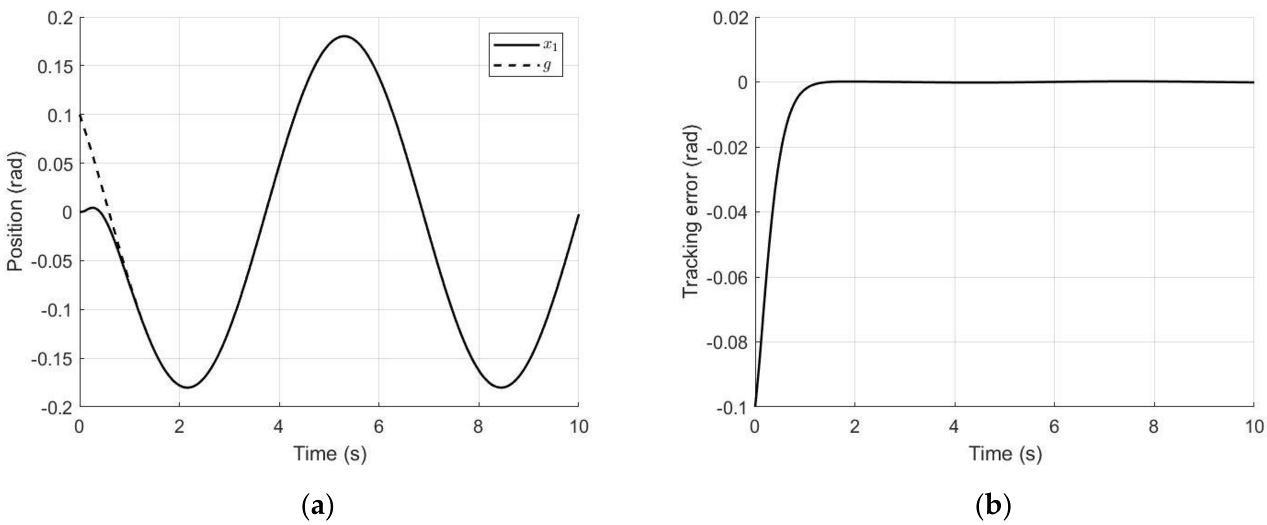


Figure 16. Experiment 5. In (**a**), plots of the reference signal $g(t)$ and the output $x_1(t)$; in (**b**), plot of the tracking error $e_1 = x_1(t) - g(t)$.

Table 3. Values of performance indicators of closed-loop systems.

Indicator	Experiment's Number	
	4	5
t_1, s	0.8403	0.8430
$e_{1,\max}$, rad	0.1000	0.1000
Δ_1, rad	$2.49 \cdot 10^{-4}$	$1.73 \cdot 10^{-4}$
$u_{\max}$, A	0.4394	0.4321

It follows from Figures 15 and 16, and Table 3 that the algorithms remain robust to the allowable changes in parametric and external disturbances.

Thus, the results of experiments 2–5 confirmed the effectiveness of the developed algorithms.

5. Discussion

The paper aims to develop a tracking system for a full-actuated manipulator with flexible joints without considering the dynamics of the current loop. The problem was complicated by the action of parametric and external disturbances on the system, both matched and unmatched. We assumed that matched disturbances could be non-smooth. In addition, we considered different options for installing sensors: when only the generalized coordinates of the manipulator were measured, and when only the positions and velocities of the motors were measured. The aim was achieved by the system transformation to the block form "input–output", with respect to tracking errors. In this form, all uncertainties were in the last equation and became matched. Based on this block form, we have synthesized a combined control law with compensation for uncertainties. For its information support, observers of mixed variables with piecewise linear corrective actions have been developed. These corrective actions allow us to limit the peaks of evaluation signals in contrast to corrective actions of the high-gain observers. In addition, the piecewise linear corrective actions provide estimates of mixed variables with a given accuracy. Hence, the tracking of a given signal also occurs with a given accuracy.

In the proposed block form, we used linear local feedbacks due to the choice of coefficients, of which one can directly influence the stabilization rate of mixed variables and tracking errors. Therefore, the "input–output" block form is a more convenient tool for stability analysis compared to the standard canonical system. In addition, the developed observers of mixed variables have the smallest possible order. They improve the performance of a closed-loop system. In particular, they allow us to avoid cumbersome off-line calculations typical for known solutions with an uncertain control matrix under control, such as the hierarchical control method. The simulation results confirmed the effectiveness of the developed approach. We demonstrated the possibility of its application for real electromechanical plants with the provision of specified technological requirements.

A further direction in the development of this paper is studying the performance of observers of mixed variables in the presence of noise in the measurements, as well as considering the dynamics of the current circuit in the plant model.

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