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Signal Processing from the Radiation Detector of the Radiometric Density Meter Using the Low-Pass Infinite Impulse Response Filter in the Measurement Path in the Coal Enrichment Process Control System

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Abstract: One of the most common coal preparations is enrichment in a jig using a float regulation system. The latest solutions propose to compensate for significant measurement errors of the float by introducing a radiometric density meter operating on the principle of gamma radiation absorption into the bottom product discharge zone of the jig. The signal from the radiometric density meter detector is in the form of a sequence of pulses with a Poisson time distribution, which are counted by a counter, as a form of digital low-pass filter. The requirement to maintain accuracy at an appropriate level forces the measurement time to be extended, which worsens the dynamic properties. Changes in the density of the coal–water medium have an unsteady, cyclical course, resulting from the principle of operation of the jig. The research goal was to develop an algorithm for processing the signal from the radiation detector using an *IIR* filter in the measurement path in a way that ensures optimization of the dynamic properties of the radiometric density meter operating in the control system of the coal enrichment process in the jig. For this purpose, a low-pass *IIR* filter was introduced into the measurement path to process the signal from the pulse counter. The identified course of the medium density for one cycle (the first) served as a reference signal. A first-order *IIR* filter was proposed, with a constant parameter selected on the basis of the reference signal and a parameter depending on the time derivative of the identified density of the medium. The mean squared error *MSE* was adopted as an indicator for assessing the dynamic properties of the radiometric density meter. The results of simulation tests showed that introducing an *IIR* filter into the measurement path gives better results in terms of the adopted criterion than using a counter with a constant measurement time. The best results ($MSE = 2.05 \times 10^{-4}$) were obtained using an *IIR* filter with a parameter that is a linear function of the derivative of the medium density over time, determined for one air pulsation cycle and applied in four subsequent cycles. These results were obtained for an adaptive first-order filter with a variable parameter a from the designated range from 0.833 to 0.999, for a measurement time of 2 ms.

Keywords: adaptive digital filter; radiometric meter; coal preparation; jigging; separation density

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1. Introduction

Radiometric meters have been widely used in the processing of mineral raw materials. In the field of hard coal production, both energy and coke, they are used in the control systems for enriching this raw material, as well as to monitor the quality parameters of coal as a commercial product. In these systems, radiometric meters are used for measuring the quantitative and qualitative parameters of coal, such as density, ash content, concentration of solid parts in transmission for flotation, as well as the density of heavy liquid [1–3]. In connection with recent events taking place in Ukraine and Russia, the energy security of the European Union member states, and therefore also of Poland [4,5], was being endangered. Poland is in the privileged situation that it has natural coal resources of 5% of world resources. Coal from Polish deposits is the only one hundred percent guarantee of the country's energy security. The main assumption of the European Green Deal [6] is the de-carbonization of energy EU systems. However, as of today, this is directly related to the liquidation of the heating and coal-powered power plants [7]. It should be noted that there is a need to ensure a mild transformation of the energy mix, before a source of energy is created which is ready to take over the role of coal in the structure of production and energy consumption in Poland. The solution in this matter is clean coal technologies that will allow the supply of high-quality coal to the combustion process [8,9]. One of the most widespread technologies is enriching coal in a pulsating jig. In this process, the grains are distributed according to their density as a result of cyclical water movement up and down, causing adequate loosening of the material, and then stratification of grains due to differences in the speeds of their falling depending on the density and size. A properly conducted process ensures stratification of the material in such a way that the layers consisting of greater density grains are in the lower part, and the grains with lower density form the upper layers [10,11]. In the automatic control system of the coal enrichment process usually the basic sensor indicating the position of material layer with a selected density is a metal float. In this system, the location of the separation layer is identified by the position of the float and should be stabilized at the level of the overflow threshold using the jig flow rate control. Correct setting of the distribution layer ensures that its upper half together with the material of layers above the distribution layer moves to the upper product above the overflow threshold. In turn, the lower distribution layer along with layers below should go to the lower product [12].

Due to the occurrence of significant measurement errors of the float in automatic control systems, stabilizing the position of the separation layer at the overflow threshold level by the flow rate of the lower product of the jig is a difficult goal to achieve. In order to improve the operation of the system, the latest solutions propose replacing the float or compensating its measurement errors with a radiometric density meter placed at an appropriate height [13,14]. Thanks to this, the signal from the density meter can be used to achieve two main control purposes: stabilizing the course of changes in the density of the medium and stabilizing the separation density. This action will be all the more effective the smaller the deviation of the signal at the density meter output from the actual density value at each moment. In such a situation, a cascade control system can be used with a P controller in the loop with feedback from the float and PI in the main loop from the radiometric density meter, in which the set value is the separation density [14].

Due to the Poisson noise present in the signal from the radiometric density meter, the value of the measured quantity determined at a given moment is subject to error. Therefore, the issue of separating the useful component from the radiometric density meter signal is important when using a radiometric meter to compensate for float measurement errors in automatic control systems of the coal enrichment process in jigs. This shows that achieving the two main goals presented in the automatic control system of the coal enrichment process in the jig using a radiometric density meter requires optimization of the dynamic properties of this meter. In a radiometric density meter, the signal at the output of a gamma ray detector (often in the form of a scintillation counter) is a series of pulses. This sequence is stochastic with a Poisson time distribution, the average value of which is a measure of the analysed density.

One of the basic elements of the measurement path is a counter that counts pulses from the gamma radiation detector in a cyclical manner in a defined unit of time, called the measurement time τ_s. The value of the counted pulses is converted into the value of the measured quantity using the calibration characteristic. In the measurement path, the counter plays the role of a digital low-pass filter with a parameter in the form of measurement time. The traditional method of filtering the signal from the detector using a counter with a constant but correctly selected measurement time will always be a compromise solution between the measurement accuracy (in the steady state) and the dynamic properties of the radiometric meter. Another solution may be digital filtering of the signal from the gamma radiation detector of a radiometric density meter by supplementing the measurement path with an additional low-pass digital filter. In such a situation, it is possible to shape the dynamic properties of the meter using an appropriately selected low-pass filter connected to the measurement path with a pulse counter. The article describes the method of designing a low-pass *IIR* filter for use in processing the signal from the detector of a radiometric density meter operating in the control system of the coal enrichment process in a jig. An adaptive form of the *IIR* filter was also proposed, with the filter parameter selected on the basis of a model of changes in the density of the coal–water medium during the pulsation cycle in the jig, which is a new approach to signal processing from a gamma radiation detector. The results of simulation tests using an adaptive *IIR* filter in the measurement path for digital signal processing from a radiometric density meter installed at the overflow threshold in the jig are presented.

2. Changes in the Density of the Medium in the Jig during the Air Pulsation Cycle

In a pneumatic jig, medium pulsations are caused by the supplied air using appropriately controlled air valves—inlet and outlet. Automatic control in an open system is then implemented. In the first phase of the air pulsation cycle, when the air inlet valve is opened, the entire bed is lifted with no loosening of grains. A clear increase in the density of the medium is then observed as a result of rising layers of increasing density. After closing the inlet valve, there is a phase of relaxation, as a result of which the grains fall (the density decreases). As a result, they gradually reach a state of constriction, which is accelerated by opening the outlet valve, which makes the density increase visible. The exhaust valve then closes to ensure the same conditions for the next cycle. The cycle duration is from 1 to 1.3 s. The stratified material is separated into two products in the final part of the jig compartment, i.e., in the product discharge zone, which is closed by an overflow threshold.

The cyclical nature of changes in the density of the coal–water medium during a properly conducted coal enrichment process in the jig can be described by a mathematical model. Industrial research carried out at one of the Polish mines has shown that the time course of changes in the density of the medium during the air pulsation cycle can be described sufficiently precisely by a mathematical model [15,16]:

$$\rho(t) \approx A_j \cdot e^{-\beta_j \cdot t} \cdot \sin\left(\omega_j \cdot t - \psi_j\right) + \rho_{uj} \tag{1}$$

where A_j, β_j, ω_j, ψ_j, ρ_{uj}—equation parameters (1) for the pulsation cycle.

Values of model parameters (1), determined on the basis of industrial data recorded during five cycles of air pulsation, and corresponding to the times of each cycle separately, standardized from the value of 0 s (start of a given cycle) to the end of the pulsation period (ψ_j = 0), are listed in Table 1. The identification results are shown in Figure 1.

Table 1. Values of the model of the medium density changes parameters in the jig, determined on the basis of industrial data recorded during ten cycles of air pulsation [15].

Cycle	A	β	ω	ρ_u	t
I	0.1711	2.52	5.72	1.3830	from 0 to 1.30 s
II	0.2088	3.74	6.83	1.3978	from 0 to 1.25 s
III	0.1681	3.87	7.00	1.4004	from 0 to 1.20 s
IV	0.1476	2.05	5.64	1.4034	from 0 to 1.30 s
V	0.2016	4.21	6.66	1.4018	from 0 to 1.30 s
Mean value	0.1794	3.28	6.37	1.3973	-

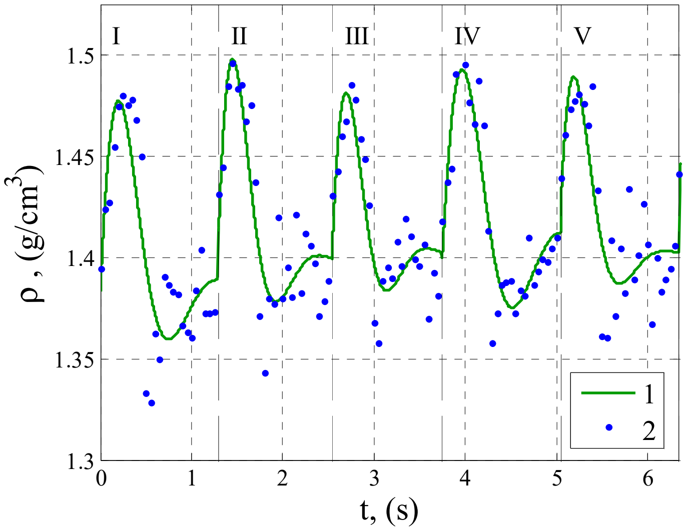


Figure 1. Changes in the medium density during five cycles of air pulsation in the jig. Key: 1—model; 2—measurement data; I–V—air pulsation cycles.

As can be seen, changes in the density of the coal–water medium are characterized by an unsteady cyclical pattern (except for very short moments of confinement at the end of each cycle), resulting from the principle of operation of the jig.

In the analysis presented below, a simulation model is presented of the measurement method (source, absorber, and scintillation counter) in the form of discrete Poisson noise and controlled average pulse intensity described in [17]. Due to the exponential distribution of random variables in the Poisson process, the time T_i, after which the next impulse appears is determined from the relationship:

$$T_i = -\frac{\ln(R)}{\lambda} \tag{2}$$

where:

R—random number from 0 to 1;

λ—pulse intensity corresponding to the measured density.

T_i values calculated according to Formula (2) are transformed into subsequent, narrow component time windows and are created on the timeline pulse series with Poisson distribution. This model, verified in laboratory conditions with high accuracy, represents the actual radiometric method of density measurement [15].

3. Signal Processing from the Gamma Radiation Detector of a Radiometric Density Meter

3.1. Radiometric Density Meter

The radiometric densimeter used to measure the density of the medium in the bottom product discharge zone of the jig uses the phenomenon of gamma radiation absorption. The construction of such a density meter is described in [18]. Radiation passing through a specific material (absorber) is weakened, which can be expressed by the formula:

$$I = I_0 \cdot e^{-\mu \cdot x \cdot \rho} \tag{3}$$

where:

I_0—radiation intensity without absorber;

μ—mass attenuation coefficient of gamma radiation;

x—absorbent thickness;

ρ —density of the absorbing layer.

The basic elements of an industrial radiometric density meter are a measuring head containing a gamma radiation source, a detector, and an electronics block, the main element of which is a pulse counter used to process the signal from the detector (Figure 2). In the radiometric meter, the radiation source is the ^{137}Cs isotope, and a scintillation counter is used as a gamma radiation detector [19–21]. The signal at the output of the radiation detector is a series of pulses $s(t)$. These pulses are counted using a counter (block 2, Figure 2) in a cyclical manner during a time interval called measurement time τ_s, which can be written:

$$k[i] = \sum_{l=1}^{\tau_s/\Delta t} S\left[\frac{\tau_s}{\Delta t} - l\right] \tag{4}$$

where:

S—counted impulse occurring in an elementary time interval;

Δt—elementary pulse counting time determined by the dead time of the radiation detector $\Delta t < \tau_s$.

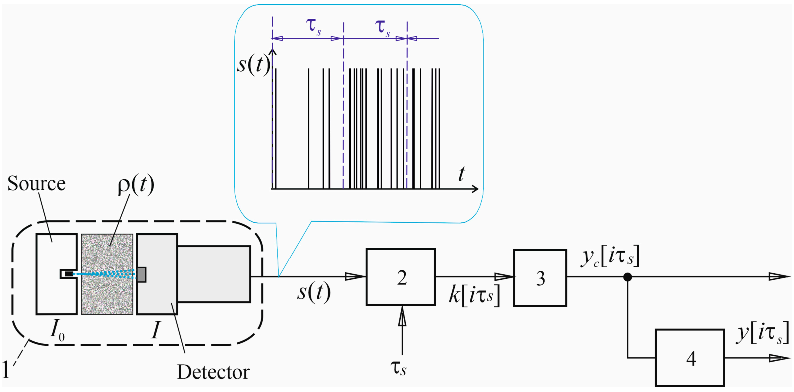


Figure 2. Schematic diagram of a density meter with a pulse counter and a constant measurement time, with the possibility of including an additional filter in the measurement path. Key: 1—measuring head; 2—pulse counter; 3—block for pulse counting to measured value recalculation; 4—additional low-pass digital filter.

The value of the counted pulses is converted into the value of the quantity measured using the calibration characteristic (block 3, Figure 2), which is the dependence of the measured density on the number of counted pulses during the measurement time, usually standardized to one second. The measurement path can be supplemented with an additional low-pass filter (block 4, Figure 2).

3.2. Radiometric Meter with a Pulse Counter and Constant Measurement Time

The traditional approach to processing the signal from the detector is to use a counter with a constant measurement time in the measurement path. In such a situation, the radiometric density meter can be rearranged schematically as in Figure 2, excluding block 4, i.e., the additional digital filter. With respect to a radiometric meter, the Poisson distribution describes the probability of k pulses appearing at the output of a radiation detector in the form of a scintillation counter in a time interval equal to the measurement time τ_s. The series of pulses from the radiation detector $s(t)$ is always a stochastic signal with a Poisson time distribution, regardless of the measured quantity and the nature of its changes. In the radiometric density meter system, the counter performs cyclical counting of pulses from the detector in a specific time interval, called the measurement time τ_s, and the obtained values are converted according to the calibration characteristics. In practice, this can be expressed as:

$$y_c[i] = a_0 - a_1 \cdot \ln\left(\frac{1}{\tau_s} \cdot k[i]\right) \tag{5}$$

where: a_0, a_1—calibration characteristic coefficients determining the relationship with the parameters of Equation (3), where a_0 is proportional to $(\mu x)^{-1}$ and $a_1 = (\mu x)^{-1}$; i—sampling step.

The value of the counted pulses k using the counter (block 3, Figure 2) is a measure of the measured density. The signal from the radiation detector is always burdened with stochastic noise. In the measurement path, the counter functions as a digital low-pass filter with the parameter τ_s, enabling a reduction in the share of noise in the measurement signal. In the case of a radiometric density meter, the selection of the measurement time comes down to determining a value that will allow obtaining good dynamic properties of the meter without significantly impairing its accuracy. The measurement time should have an optimal value that simultaneously minimizes the dynamic error and maximizes the suppression of fluctuations resulting from the stochastic nature of the signal from the density meter detector.

Analysing the results of identifying the medium density model during the pulsation cycle—presented in Figure 1 and Table 1—it can be seen that the model coefficients change slightly from cycle to cycle in quantitative terms. It can therefore be assumed that the measurement time selected using the identified coal–water density model for the first cycle can be used to filter the signal in subsequent cycles without significant deterioration of the dynamic properties and accuracy of the meter.

3.3. IIR Filter in the Measurement Path of a Radiometric Density Meter

In the case of signal processing only using a pulse counter, the measured values (y_c) are maintained for the next measurement time, during which subsequent pulses are counted. Therefore, a step reconstruction of the signal takes place, i.e., the sample value (converted to density) is maintained for a time corresponding to the measurement time τ_s. This is an analogy to the use of a zero-order extrapolator with a time to maintain the value obtained from the counter equal to the measurement time τ_s, whose frequency response includes side lobes next to the main lobe. In order to reduce their negative impact, an additional low-pass filter should be included in the measurement path with the pulse counter. One of the possible variants is the use of a low-pass *IIR* filter due to its satisfactory frequency response and, in particular, its speed.

To design an additional *IIR* (Infinite Impulse Response) filter included in the measurement path of a radiometric density meter, a method based on the prototype of an analogue filter can be used:

$$K(s) = \frac{\alpha^n}{(s+\alpha)^n} \tag{6}$$

where:

α—filter parameter;

n—filter order.

The prototype analogue filter with structure (6) was chosen due to its advantageous form, which requires only the selection of the α and the filter order.

A characteristic feature of these filters is that for the first-order filter limit pulsation $\omega = 1/T$, i.e., equal to the filter parameter α. The attenuation increases for this pulsation for higher-order filters. The attenuation in the stopband in the case of a first-order filter is 20 dB/decade. As the order of the filter in Equation (6) increases, the phase delay and attenuation in the stopband increase for the same value of α. This shows that for the same value of α the attenuation in the stopband will be the smallest, with the smallest phase delay for the first-order filter.

The discrete transmittance equation of the n-th order filter, corresponding to Equation (6) of the analogue filter, was determined using the impulse response invariance method. The transmittance function equation in such a situation takes the following form:

$$K(z) = \frac{\alpha \cdot \tau_s^{n-1} \cdot \left(e^{-\alpha \cdot \tau_s}\right)^{n-1} D_{n-1} \cdot z}{\left(z - e^{-\alpha \cdot \tau_s}\right)^n} \tag{7}$$

In expression (7), D_{n-1} is a determinant that is a function of the expression $(z \cdot e^{-\alpha \cdot \tau_s})$ and with dimensions depending on the filter order n [22]:

$$D_{n-1} = \begin{vmatrix} 1 & 1 - ze^{-\alpha\tau_s} & 0 & \vdots & 0 \\ \frac{1}{2!} & 1 & 1 - ze^{-\alpha\tau_s} & \vdots & 0 \\ \frac{1}{3!} & \frac{1}{2!} & 1 & \vdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{n!} & \frac{1}{(n-1)!} & \frac{1}{(n-2)!} & \vdots & 1 \end{vmatrix} \tag{8}$$

The filter parameters (7) obtained by applying the impulse response invariance method should be scaled and standardized. When using an additional low-pass digital filter to process the signal from the radiometric density meter (Figure 2), the output signal from the pulse counter (after conversion to the value of the measured density) becomes the input signal of the *IIR* filter, and then the sampling period is equal to the measurement time τ_s. To ensure good dynamic properties, the measurement time should be short, but its value should also be determined. In such a situation, the order and parameters of the *IIR* filter must be selected, as well as the value of the measurement time. Also, in this case, the selection of the filter order and parameters based on the medium density model for the first cycle of air pulsation can be used in practice to filter the signal in subsequent cycles.

In the case of a first-order filter, it is preferable to use an equation of the form:

$$y[i] = a \cdot y[i-1] + (1-a) \cdot u[i] \tag{9}$$

where a is the filter parameter.

Parameter a is selected, and in relation to the signal processing from the gamma radiation detector, the value of the measurement time should also be determined.

3.4. A First-Order Filter with a Parameter Depending on the Modulus of the Time Derivative of the Medium's Density

Analysing the shape of the medium density curve (Figure 1), it can be seen that the cut-off pulsation of the *IIR* filter should have a large value in moments of sudden, significant density changes, so that the dynamics of changes are not dampened by the filter, and a small value in time intervals characterized by small changes ρ, in order to increase the degree of noise reduction from the measurement signal. This shows that when a filter with Equation (9) is used, the parameter a can be related to the rate of change in the density of the medium. Due to the fact that the empirical model of the density of the medium is known from Equation (1), the derivative of the density over time for a given pulsation cycle is expressed by the equation:

$$\frac{d\rho(t)}{dt} = A_j \cdot e^{-\beta_j \cdot t} \cdot \left(\omega_j \cdot \cos\left(\omega_j \cdot t - \psi_j\right) - \beta_j \cdot \sin\left(\omega_j \cdot t - \psi_j\right)\right) \tag{10}$$

The use of Equation (10) allows one to avoid difficulties with numerically determining the derivative of a signal burdened with noise. This makes it possible to improve the dynamic properties of the radiometric density meter while maintaining its accuracy at the required level by making the parameter a of the filter Equation (9) dependent on the time derivative of the medium density in such a way that for a high rate of change of the medium density the coefficient a will have a small value, and for a small value of $d\rho/dt$ a large value. Knowing the parameters of the model (1) for one cycle, the derivative of density with respect to time (10) in this cycle can be determined. In order to use the derivative of the density of the medium, determined for one cycle, to filter the signal from the density meter in subsequent cycles, it is advantageous to standardize it in the range [0, 1].

$$\left(\frac{d\rho}{dt}\right)_N = \left(\frac{d\rho(t)}{dt} - \left(\frac{d\rho}{dt}\right)_{\min}\right)\cdot\left(\left(\frac{d\rho(t)}{dt}\right)_{\max} - \left(\frac{d\rho}{dt}\right)_{\min}\right)^{-1} \tag{11}$$

Due to the fact that the filter parameter (9) should depend on the value and not the sign of the medium density derivative with respect to time, this function should be expressed as follows:

$$a = f\left(\left|\frac{d\rho}{dt}\right|_N\right) \tag{12}$$

where a_N is the normalized parameter filter value.

With reference to the empirical model (1), the standardized course of the density derivative over time determined for the first cycle is shown in Figure 3.

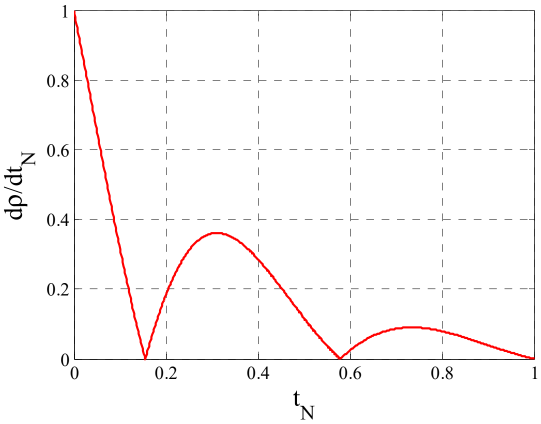


Figure 3. Derivative of the medium density over time determined on the basis of model (1) for the first pulsation cycle, standardized in the range [0, 1].

By identifying the model of the density course of the medium (1) for the first cycle, it is possible to determine the derivative of this quantity with respect to time in this cycle and use its standardized module to filter the signal in subsequent cycles. The proposed method of processing the signal from the detector using a first-order filter with Equation (9) and a parameter depending on the rate of change of medium density during the air pulsation cycle in the jig is shown in Figure 4.

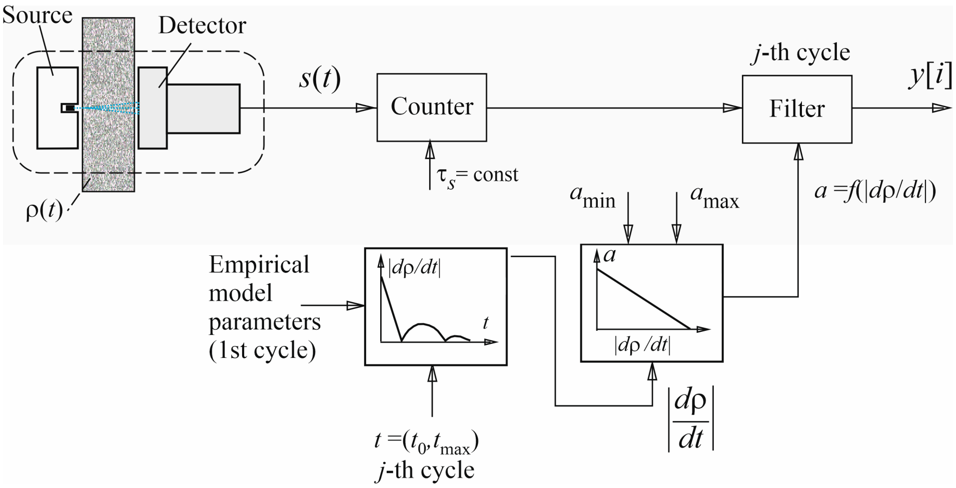


Figure 4. Processing the signal from the gamma radiation detector using a filter with a parameter depending on the modulus of the time derivative of the medium density identified for the first cycle and used in subsequent cycles.

In a properly operating coal enrichment system in the jig, the modulus of the standardized derivative of the medium density determined for the first cycle will be qualitatively similar to the rate of changes in subsequent cycles. As the starting time of each subsequent air pulsation cycle (t_0) is known and results from the operation of the control system, the process of changing the filter parameter a is carried out cyclically in the same way. In the proposed algorithm, function (12) was adopted as a linear dependence of the normalized value of the filter parameter with Equation (9) on the normalized modulus of the model of the derivative of the medium density over time, which can be expressed by the formula:

$$a_N = 1 - \left|\frac{d\rho}{dt}\right|_N \tag{13}$$

The normalized a_N value, determined on the basis of Equation (13), is converted into an absolute value (so-called denormalization) using the equation:

$$a = (a_{\max} - a_{\min}) \cdot a_N + a_{\min} \tag{14}$$

When using a filter (9) with a parameter changing according to the relationship (12), the measurement time τ_s should be as short as possible, and assuming that $a_{\max} \approx 1$ (e.g., 0.999), the value of $a_{\min}$ should be selected.

4. Simulation Studies of the Use of the *IIR* Filter to Process the Signal from a Radiometric Density Meter

4.1. Test Conditions

The research task included a comparative analysis of the results of using the classic form of signal processing from the detector, i.e., using a counter with a constant measurement time τ_s, with the results obtained when the measurement path was supplemented with a low-pass *IIR* filter.

The research was carried out on the basis of simulated waveforms representing the course of the density of the medium in the jig during five successive cycles of air pulsation. A series of pulses, representing the signal from the gamma radiation detector with changing density of the medium in the jig (Table 1), was generated using the simulation model described in Section 2. Taking into account the elementary pulse counting time Δt equal to 1 µs, a data sequence with a length of 6,350,000 points was obtained, corresponding to the duration of six pulsation cycles, equal to 6.35 s. The lengths of the data series for cycles I to V were, respectively: 1,300,000; 1,250,000; 1,200,000; 1,300,000; 1,300,000. The output signals from the counter y_c, the filter y, and the density waveform ρ were observed discretely with a sampling period of T_s = 1 ms. This ensured the possibility of conducting a comparative analysis of the obtained results, regardless of the method used to process the signal from the detector. To convert the number of pulses counted in time τ_s, Equation (5) was used, with the parameter values used in the industrial device being a_0 = 5.25010 and a_1 = 0.40206, respectively.

With regard to the processing of the signal from the detector using a counter with a constant measurement time, first, the optimal values of τ_s in terms of criterion (9) were determined for each pulsation cycle. This has practical justification due to the fact that, in industrial conditions, the starting time of each subsequent air pulsation cycle is known and taken into account in the control algorithm. Then, the value of the measurement time was determined, for which the minimum was reached by expression (9) for the entire data sequence. With respect to cycles I to V, for the shortest and longest measurement times, calculations were carried out to determine the value of the expression (9).

In the second stage of the research, the process of selecting the measurement time and parameters of *IIR* filters with n orders from 1 to 5 for the first air pulsation cycle was carried out. Also, in this case, a criterion value was used (9). For the case for which the minimum value of expression (9) was achieved, the order and values of the filter parameters were accepted as correct and used to process the signal from the detector in subsequent air pulsation cycles. In the last variant of the conducted simulation tests of signal processing from the gamma radiation detector into the measurement path with a pulse counter, a first-order filter was introduced with a parameter changed depending on the value $|d\rho/dt|_N$. In the proposed algorithm, the modulus of the standardized density time derivative identified for the first pulsation cycle was used in subsequent cycles. In such a case, the measurement time should be as short as possible so that the pulse counter does not introduce an additional phase shift. Therefore, τ_s = 2 ms was assumed. The largest value of the filter parameter (15) $a_{\max}$ close to 1, i.e., 0.999, was assumed, and the parameter $a_{\min}$ was determined by simulation, changing its value from 0.750 to 0.925. For the correct value of $a_{\min}$, the one for which the value of the indicator (15) was the smallest was adopted.

4.2. Criterion for Selecting the Measurement Time and IIR Filter Parameters

Knowing the mathematical model of changes in the density of the medium for one cycle, it is possible to select the measurement time by minimizing the value of the criterion in the form of a square error expressed by the formula:

$$MSE_j = \frac{1}{N_j}\sum_{i=1}^{N_j}\left(y_j[i] - \rho_j[i]\right)^2 \tag{15}$$

where:

$y_c(j)$—signal at the output of a density meter with a constant measurement time counter or with a constant measurement time counter and an additional *IIR* filter connected to the measurement path;

N_j—number of data during the j-th pulsation cycle;

j—cycle number (from I to V).

The indicator (15) can also be used to select the measurement time or assess its correctness for all pulsation cycles. In that case, j stands for cycles I to V.

To select the order and parameters of the *IIR* filter with Equation (4), a reference signal in the form of model (1) can also be used. In such a situation, the correct order and parameters of the filter can be considered those for which the value of the criterion in the form of the mean square error is minimized (15).

4.3. Results

The results of the first stage of simulation tests, in the form of measurement time values (τ_s) determined by minimizing criterion (15) in the situation of using a pulse counter with a constant measurement time to process the signal from the detector, are summarized in Table 2. The table also gives the optimal measurement time determined for all pulsation cycles, and the results of using the shortest and longest of the optimal (in the sense of *MSE*) measurement times. Treating the measurement time determined for the first cycle as the value used in subsequent air pulsation cycles, the values of the *MSE* index were determined for cycles II-V and for the entire course using this parameter, and the results are also summarized in Table 2.

Table 2. Values of optimal measurement times in terms of criterion (15) for individual pulsation cycles and the entire course of the medium density and *MSE* values, including the shortest and longest measurement times and the measurement time determined for cycle I.

Cycles	I	II	III	IV	V	from I to V
τ_s, (ms)	45	41	61	38	41	45
$MSE\ 10^{-4}$	3.18	3.33	2.24	2.75	3.49	3.92
τ_s, (ms)	45					
$MSE\ 10^{-4}$	3.18	4.53	3.22	3.31	4.81	3.92
τ_s, (ms)	38					
$MSE\ 10^{-4}$	4.04	4.61	3.15	2.75	6.02	4.36

Table 2. *Cont.*

Cycles	I	II	III	IV	V	from I to V
τ_s, (ms)			61			
$MSE\ 10^{-4}$	5.30	4.40	2.24	3.56	5.36	4.89

The course of the signal from the density meter using a constant measurement time of 45 ms is shown in Figure 5.

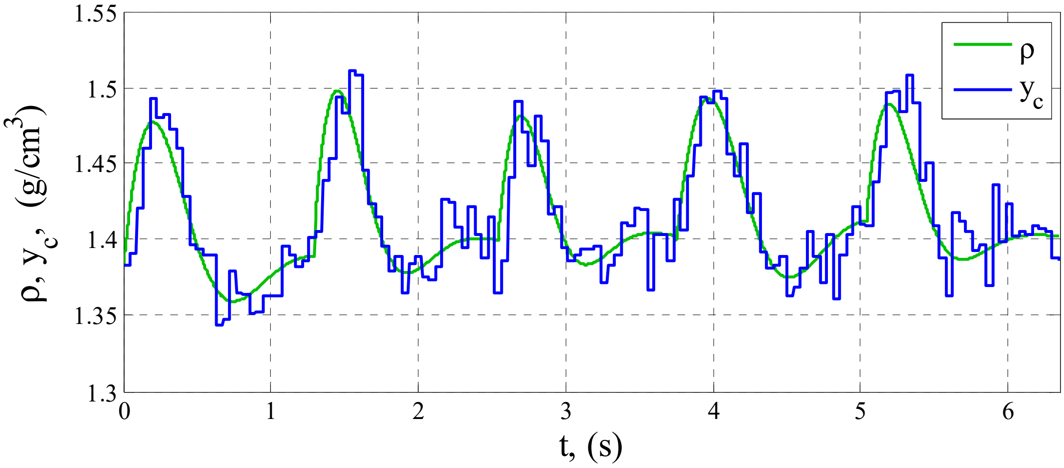


Figure 5. Processing of the signal from the gamma radiation detector using a filter with a parameter depending on the modulus of the time derivative of the medium density identified for the first cycle and used in subsequent cycles.

The assessment of the order of the *IIR* model with Equation (7) was carried out in relation to density changes during the first air pulsation cycle, and the results are summarized in Table 3. The values of filter parameter α are given, and measurement times for which the minimum value of the mean square error was obtained for a given filter order.

Table 3. *IIR* filter parameters of Equation (7) of orders one to five, determined by minimizing criterion (15) determined for the first air pulsation cycle.

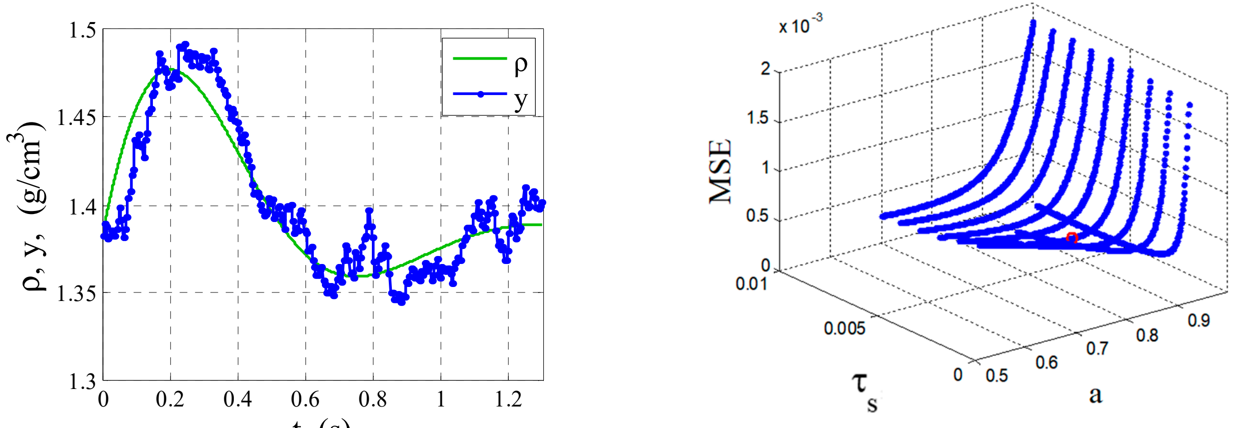
n	1	2	3	4	5
τ_s, (ms)	5	5	5	5	10
$1/\alpha$ (ms)	9.00	3.60	2.40	1.80	1.22
$MSE \times 10^{-4}$	2.47	2.85	3.64	3.86	4.54

Based on the results in Tables 2 and 3, the first-order filter was adopted as the most advantageous from the point of view of obtaining good dynamic properties of the radiometric meter while maintaining accuracy at an appropriate level. The values of the parameter a of the filter using Equation (9) determined to be optimal, in the sense of *MSE* for individual air pulsation cycles, are included in Table 4. Due to the proposed filter parameter selection algorithm, which involves determining the parameter a (as well as the measurement time τ_s) based on the identified course of the medium density during the first cycle and using this setting for signal processing during subsequent cycles, calculations were carried out aiming at determining the *MSE* value for signal filtering in pulsation cycles II to V for $a = 0.884$ and $\tau_s = 5$ ms. The results are also summarized in Table 4.

Table 4. The values of the filter parameter with Equation (9) determined by minimizing the *MSE* criterion for all air pulsation cycles and the value of the indicator (15) using the parameter *a* determined for the first cycle and used to process the signal for the remaining pulsation cycles.

Cycle	I	II	III	IV	V
τ_s, (ms)	5	8	9	4	10
a	0.884	0.798	0.828	0.902	0.780
$MSE \times 10^{-4}$	2.47	2.70	1.39	1.65	3.07
τ_s, (ms)	5				
a	0.884				
$MSE \times 10^{-4}$	2.47	2.82	1.46	1.67	3.41

The calculation results from Table 4 for the first air pulsation cycle are shown in Figure 6. The value of the mean square error for cycles II to V, using the *IIR* filter with the parameter calculated for the first air pulsation cycle, is 2.25×10^{-4}. The waveform of the signal from the first-order filter for all pulsation cycles is shown in Figure 7.



(**a**) (**b**)

Figure 6. Results of determining the optimal value of the $a_{\min}$ parameter of a first-order filter: (**a**) waveform of the signal at the filter output against the background of the medium density during the first cycle of air pulsation; (**b**) relationship between *MSE* and the *a* parameter and the τ_s measurement time.

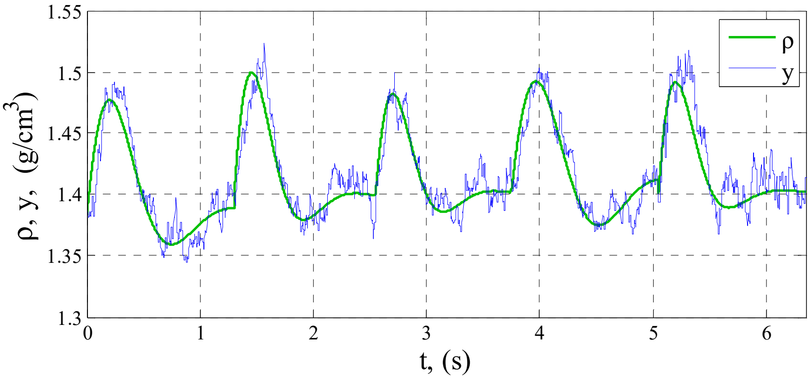
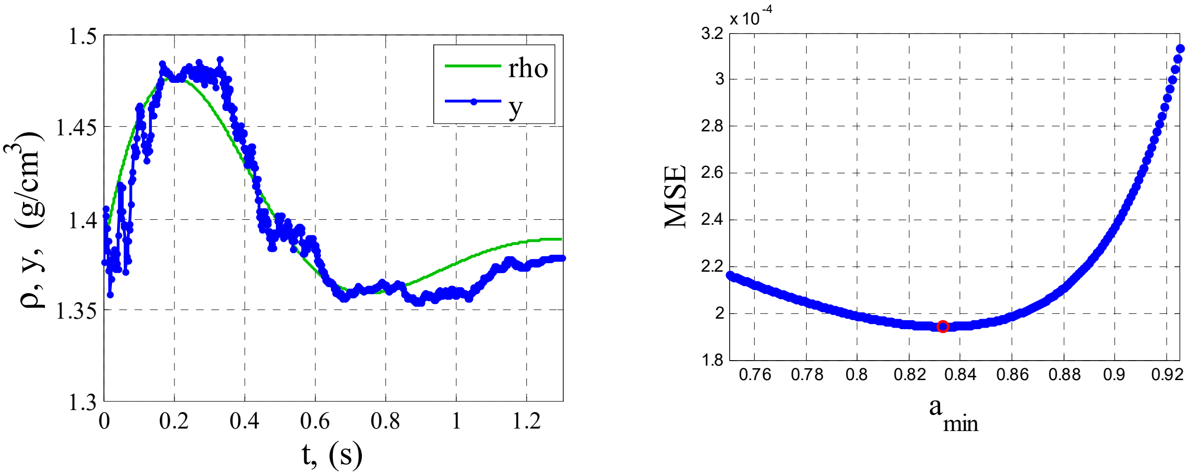


Figure 7. Signal waveforms from the *IIR* filter with $a = 0.884$ and a measurement time of 5 ms against the background of medium density waveforms for five air pulsation cycles.

In the last stage of the research, calculations were carried out to determine the range of variability of parameter *a* in accordance with the algorithm described in Section 3.4. The results obtained are summarized in Table 5 and presented graphically (Figure 8). However, the waveforms of the signal from the filter with parameter a depending on $|d\rho/dt|$ are shown in Figure 9. The value of criterion (15) determined for the course from cycle II to V is 2.05×10^{-4}.

Table 5. Values of the mean square error obtained using the changed parameter a of the filter in the range calculated for the first pulsation cycle $[a_{\min}, a_{\max}] = [0.833, 0.999]$ and used in processing the signal from the detector for the remaining cycles (τ_s = 2 ms).

Cycle	I	II	III	IV	V
$MSE \times 10^{-4}$	1.94	1.37	0.70	1.86	2.57



(**a**) (**b**)

Figure 8. Results of determining the optimal value of the $a_{\min}$ parameter of a first-order filter with a parameter changed as a function of the derivative of the medium density over time: (**a**) waveform of the signal at the filter output against the background of the medium density during the first cycle of air pulsation; (**b**) dependence of MSE on the $a_{\min}$ parameter.

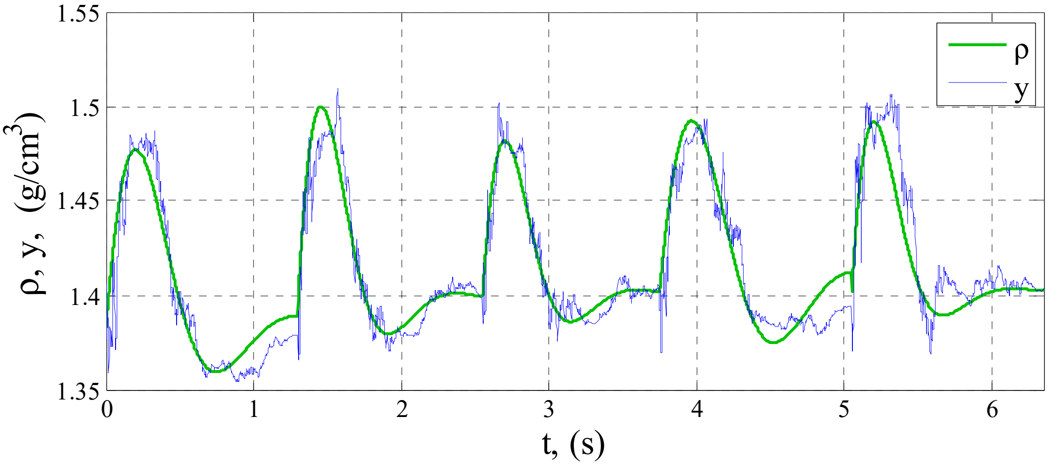


Figure 9. Signal processing from the gamma radiation detector using a filter with a parameter depending on the modulus of the time derivative of the medium density identified for the first cycle and used in subsequent cycles.

The mean square error values obtained for all applied methods of signal processing from the gamma radiation detector of the radiometric density meter are presented graphically in Figure 10.

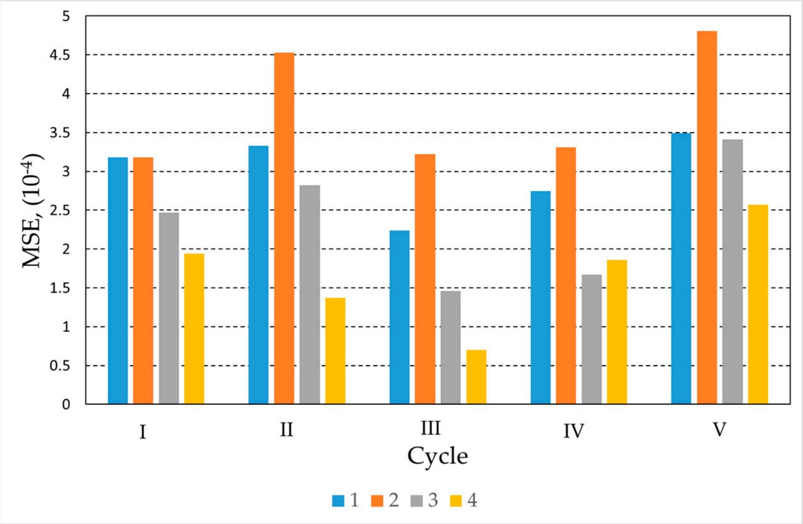


Figure 10. The results of signal processing from the gamma radiation detector of the radiometric density meter used in the enrichment process control system in the jig, obtained for various filtration methods. Key: 1—constant measurement time determined for each cycle of air pulsation separately—reference values; 2—constant measurement time determined on the basis of the first cycle; 3—first-order *IIR* filter with equation (9) with a parameter selected on the basis of $\rho(t)$ of the first cycle; 4—first-order *IIR* filter with a parameter value resulting from the relationship: $a = f(|d\rho/dt|)$ determined for the first air pulsation cycle.

5. Discussion

The comparative analysis of the results was carried out based on the quantitative assessment of the criterion in the form of the mean square error (Equation (15)). This was possible thanks to the use of the sampling period T_s with an identical value in the observation of the resulting signals (after processing) for each of the considered cases, regardless of the value of the measurement time τ_s.

The traditional method of processing the signal from a gamma radiation detector, which involves using a counter with a selected measurement time, requires determining its value. In a comparative analysis of the results, they can be treated as reference values. The results in the form of $MSE = f(\tau_s)$ waveforms presented in Table 2 show that the optimal values of measurement time in the sense of *MSE* for five air pulsation cycles range from 38 ms to 61 ms. It should be noted that as many as three values of τ_s obtained from the calculation results for five cycles amount to approximately 40 ms. This affects the measurement time determined on the basis of the density curves of the medium of all considered pulsation cycles. It is 45 ms with an *MSE* value of 3.92×10^{-4}. It is true that it is equal to the result obtained for the first cycle, but this similarity should be treated as accidental. As the results in Table 2 show, introducing a measurement time of 45 ms for processing the signal from the detector for all cycles obviously leads to the deterioration of the *MSE* results, increasing their values by 1.2 to 1.4 times.

Based on the results obtained in relation to the first pulsation cycle (Table 3), it can be concluded that supplementing the measurement path with an *IIR* filter with Equation (7) is justified only for first and second order filters, because for these algorithms the mean square error is, respectively, 2.47×10^{-4} and 2.85×10^{-4}, i.e., their values are lower than when using only a pulse counter with a constant measurement time. Due to the lowest *MSE* value obtained among the considered *IIR* filters with Equation (7), a first-order filter was adopted in the further part of the research, using its more convenient form—Equation (9). Comparing the results in Tables 3 and 4, it should be concluded that supplementing the measurement path with a first-order *IIR* filter with a parameter selected for each cycle separately leads to significantly lower *MSE* values than in the case of using a constant measurement time counter. The *MSE* values in this case range from 60% to 88% of the corresponding criterion values obtained for classical signal processing from the detector. The parameter a of the filter (9) has values ranging from 0.780 to 0.902.

Here again, the determined values of the measurement time (Table 4), as the second parameter in the situation of processing the signal from the detector using a first-order *IIR* filter, range from 4 to 10 ms and, unfortunately, have different values in each pulsation cycle.

In the case of the proposed algorithm for selecting the filter parameter (and possibly measurement time) only for the first cycle and applying it in subsequent air pulsation cycles, differences in the determined optimal values of parameters a and τ_s in terms of the mean square error have an unfavourable impact on the value of this criterion (15).

Based on the results from Table 4, it can be concluded that the filter with Equation (9) and parameter $a = 0.884$ and $\tau_s = 5$ ms, selected for the first cycle while minimizing *MSE* and used in subsequent pulsation cycles, gives criterion values from 1.46×10^{-4} (cycle III) to 3.41×10^{-4} (cycle V), which is smaller than when using the pulse counter alone. In this case, the *MSE* value determined for pulsation cycles II to V is 2.25×10^{-4}. This leads to the conclusion that supplementing the measurement path with an additional first-order *IIR* filter is justified. A further reduction in the *MSE* criterion was achieved when the value of the parameter a of the filter (9) was dependent on the rate of change in the density of the medium in the first pulsation cycle. By selecting the minimum value of the filter parameter 0.833 from the range of its changes $[a_{\min}, a_{\max}]$ using the mean square error criterion (as shown in Figure 8b), assuming $a_{\max} = 0.999$, the *MSE* index values were lower than in the case of classic signal processing with a detector (counter). In this case, the indicator values range from 31% to 74% of the *MSE* of the reference value of each cycle (Table 2, row 3). Comparing the results of using the *IIR* filter with Equation (9) with a constant value of parameter a with the algorithm in which $a = f\,|d\rho/dt|$, it should be stated that lower *MSE* values were obtained in four out of five cycles when a changed form of parameter a was used. Only a slightly more favourable result was obtained with a constant value of parameter a for cycle IV. Calculated for cycles II to V, i.e., during the period of use of the range of variability of the filter parameter a (9) determined in cycle I, the value of index (15) is 2.05×10^{-4} and is the smallest value. From this point of view, supplementing the measurement path of the radiometric density meter with a first-order *IIR* filter with parameter a, the value of which depends on the time derivative of the density of the medium, is the best solution.

6. Conclusions

In traditional industrial signal processing systems from a gamma ray detector with a pulse counter (with a constant measurement time), due to the fast-changing component of Poisson noise in the measurement signal of a radiometric density meter, the density values determined in subsequent moments corresponding to the measurement time are burdened with a measurement error, as the greater the value, the shorter the measurement time. Therefore, the issue of separating the useful component from the radiometric density meter signal in the coal enrichment process control system in the jig is important, where changes in the density of the coal–water medium are unsteady and cyclical (except for very short moments of confinement at the end of each cycle), resulting from the principle of operation of this coal enrichment process. In such a case, the radiometric density meter should have good dynamic properties and sufficient accuracy. Based on the research carried out, it can be concluded that the use of the measurement time selected based on the density course of the medium of the first cycle to process the signal from the detector in subsequent cycles is possible, but it only provides intermediate features of a radiometric density meter between the appropriate measurement accuracy and its dynamic properties. The advantage of this solution is its simplicity. The disadvantages include fluctuations in the signal and a phase shift in relation to the density of the medium.

The introduction of an *IIR* filter designed based on a prototype analogue filter (Equation (6)) into the measurement path of a radiometric density meter using the impulse response invariance method is justified only in relation to the first-order filter. Filters with higher orders ($n > 2$), despite better attenuation in the stopband, introduce a larger phase shift, resulting in an increase in the dynamic error. This is especially important in the first phase of the pulsation cycle, when a significant increase in the density of the medium is observed. A practical algorithm consisting in identifying the course of the density of the coal–water medium during the first cycle of air pulsation and using this model to select the parameter a of the filter with Equation (9) used in the subsequent four cycles is a better solution than using only a counter with a constant measurement time. Further improvement of the dynamic properties (without deteriorating the accuracy) of the radiometric density meter was achieved by changing the parameter a of the *IIR* filter from the model of the derivative of the medium density over time in the form of Equation (10), considering the identified model parameters (Table 1) for cycle I. Using the relationship $a_N = f(|d\rho/dt|_N)$ standardized in the range [0, 1] in the form of a linear function, the filter parameter a is related to the rate of change of the medium density at a given moment of the pulsation cycle in the jig. The method of using the model of the derivative of the medium density over time, determined for cycle I and used in the subsequent four cycles to change the parameter a of the *IIR* filter, turned out to be the best method among those considered.

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