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Optimized Profit Allocation Model for Service Alliance Transactions Considering Risk

Wei Liu [1,2], Mengxing Huang [2,*] and Wenlong Feng [2,*]

[1] College of Biomedical Information and Engineering, Hainan Engineering Research Center for Health Big Data, Hainan Medical University, Haikou 571199, China; liuwei@hainmc.edu.cn
[2] School of Information and Communication Engineering, Hainan University, Haikou 570228, China
* Correspondence: huangmx09@163.com (M.H.); fwlfwl@163.com (W.F.)

Abstract: In service alliances, where multiple service providers collaborate to complete service transactions, the equitable allocation of profits based on their respective contributions and risk-bearing capacities is paramount. This paper introduces an optimized profit allocation game model that integrates risk considerations into the Nash bargaining framework. Initially, the study established a service alliance transaction model that considered the interactions among multiple participants, providing a robust theoretical foundation for cooperation. Subsequently, the concept of marginal risk was introduced, and a unique calculation method based on the Shapley value was devised to quantify risk contributions. Finally, an improved Nash bargaining model was proposed, which introduced a risk adjustment factor, explicitly addressing the impact of each participant's risk on profit allocation. Through computational cases and result analyses, this model demonstrated its ability to balance profit and risk and to optimize outcomes for all participants, and it validated the fairness and rationality of the proposed allocation method.

Keywords: profit allocation; service alliance; Nash bargaining; cooperative game

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1. Introduction

With the rapid development of globalization and informatization, service alliances have emerged as crucial forms of collaboration for enterprises [1,2]. A service alliance refers to a scenario in which multiple independent service providers collaborate to share resources and technologies, aiming to achieve common business objectives. An increasing number of enterprises have realized that relying solely on their own capabilities is insufficient to optimize service resources and maximize profit [3]. As service scales continue to expand, the models for service transactions have become increasingly complex. Single types of services are no longer adequate to meet the needs of service consumers. Instead, multiple service providers collaborate, each contributing value in a specific aspect of the service, thereby forming high-quality service alliances that fulfill user demands. In this collaborative model, numerous service providers join forces to collectively offer comprehensive service solutions to customers. Such alliances typically integrate resources, technologies, and expertise from all parties, thereby enhancing the quality and efficiency of services to meet a broader range of customer needs. Service alliances have been widely adopted across various industries, including technology, healthcare, and logistics, where they help address challenges such as resource integration, customer satisfaction, and competitive advantage. For example, in the healthcare industry, service alliances allow hospitals, pharmaceutical companies, and technology providers to collaborate on integrated patient care solutions, while in logistics they enable seamless supply chain management and last-mile delivery. An illustration of service alliance scenarios is provided in Figure 1:

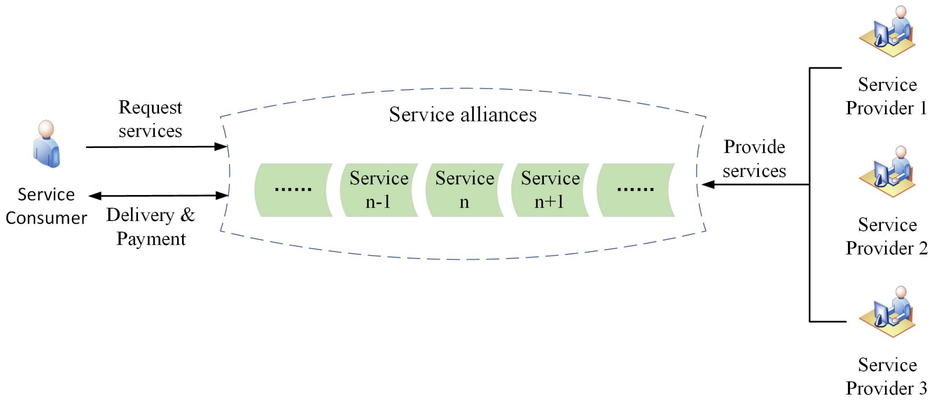

Figure 1. Service alliance transaction.

Collaboration among parties is essential to complete service transactions, with profit allocation being a crucial aspect of service alliance cooperation. Fairness and equity in profit allocation not only affect party cooperation and the stability of long-term relationships but also influence the overall profitability and sustainability of the service alliance. Without a clear profit-allocation mechanism, the contributions of some members may be overlooked, leading to a lack of deserved profits and resulting in a lack of enthusiasm from certain members for participating and providing services. Consequently, this can impact the overall efficiency and effectiveness of the service alliance [4]. If some members bear excessive risks without corresponding returns, doubts regarding the cooperative relationship may arise, which can affect the stability and sustainability of the service alliance [5]. Traditional profit allocation methods often rely on the proportion of contributions from each party or adhere to fixed allocation rules, neglecting the actual risk-bearing situations of each party. This results in unfair and opaque profit allocation, potentially leading to disputes and conflicts among partners. Therefore, addressing how to achieve fair and equitable profit allocation in service alliance transactions has become an urgent issue [6].

Although significant progress has been made in the study of service alliances, there remains a considerable research gap regarding fair and effective profit allocation, especially when considering the risks borne by each party. Many existing models focus on contribution proportions but fail to adequately address the complexity of risk distribution. Furthermore, while cooperative game theory has been extensively studied, there is limited research on its practical applications in specific industries. Addressing these gaps could provide actionable insights for industries such as technology, healthcare, and logistics, where service alliances play a critical role in fostering collaboration and improving service outcomes. Another approach is to apply cooperative game theory. In service alliance transactions, the challenge of profit allocation, which involves cooperation among multiple service providers, represents a typical cooperative game [7]. Cooperative game theory can help determine a fair method for profit allocation, thereby encouraging better participation and promoting the stability and sustainability of the alliance. However, cooperative game theory encompasses multiple solution concepts, and different solutions may lead to different profit allocation outcomes, potentially causing subjectivity and controversy [8]. Therefore, further research is needed to identify the appropriate solution concept that ensures the fairness and acceptability of the final scheme. These research gaps indicate the need for more refined models that not only consider marginal profit but also take into account the risk-bearing capacities of each party, thereby ensuring fairness and sustainability within service alliances.

A commonly used theoretical approach to modeling fair profit allocation is Nash bargaining theory, introduced by John Nash in 1950 [9]. Nash bargaining theory provides a framework for allocating resources or profits in a way that maximizes collective benefits while ensuring fairness among participants. In the context of service alliances, Nash bargaining theory focuses on how multiple service providers can negotiate profit sharing based on their profits and the risks they bear, ensuring an equitable distribution of benefits.

The Nash bargaining solution is particularly suitable for situations where participants must cooperate to achieve mutual gains but must also balance the distribution of those gains based on the bargaining power of each participant. Several studies have applied Nash bargaining theory to analyzing cooperation and profit allocation in service alliances. For example, Chen et al. [10] applied Nash bargaining theory to studying profit allocation in multi-party cooperation, emphasizing the balance between risk and contribution proportions. Li et al. [11] used Nash bargaining theory to address profit distribution in supply chains, considering the risk and contribution proportions of different suppliers, thus optimizing resource allocation. The Shapley value, introduced by Lloyd Shapley in 1953 [12], is another widely used solution concept in cooperative game theory. The Shapley value provides a fair method for distributing the total profit among participants based on their individual contributions to the overall outcome. It calculates each player's marginal contribution in every possible coalition and then averages it to determine their fair share of the total profit. The Shapley value has been applied in various fields, including supply chain management, to allocate profits fairly among participants. In the context of service alliances, the Shapley value can be particularly useful for quantifying the contributions of each service provider and ensuring that profits are allocated proportionally to these contributions. For instance, Liu et al. [13] proposed a Shapley value-based profit allocation model to optimize resource sharing and cooperation in supply chains. Wang et al. [14] studied risk-bearing among service alliance members and proposed a profit allocation method integrating the Shapley value to address fairness concerns when members face different levels of risk. In the context of service alliance profit allocation, the Nash bargaining theory and the Shapley value can complement each other. Nash bargaining provides a framework for fair negotiation, while the Shapley value allows for precise quantification of each participant's profit to the alliance's success.

The main objective of this paper was to propose an improved Nash bargaining profit allocation model that would consider marginal profit and risk, aiming to provide a reasonable allocation scheme for alliance services jointly completed by multiple service providers. Our model seeks to identify an optimal method for profit allocation that maximizes the overall profit of the cooperative alliance while ensuring the profits of individual participants, thereby achieving the goal of win–win cooperation. The main contributions of this paper are summarized as follows:

(1) A service alliance transaction model based on Nash bargaining theory is established, which takes into account the interactions among multiple participants in the service alliance, providing an effective transaction framework for alliance members.
(2) The concept of marginal risk is introduced, and a calculation method for marginal risk is designed, using the Shapley value approach. By considering risk and a risk adjustment factor, an improved Nash bargaining model is proposed, addressing the shortcomings of conventional methods that fail to incorporate dynamic risk analysis into profit allocation.
(3) Through numerical calculations and result analyses, this model combines Nash bargaining theory with the Shapley value, demonstrating its ability to optimize the profits of each service provider while maximizing overall profit, thereby achieving fair profit allocation.

The remainder of the paper is organized as follows: Section 2 outlines related works; Section 3 describes how our study was performed, including a description of the problem, the definition of the model, and the Nash bargaining model; Section 4 introduces the concept of marginal risk and the method for designing the improved Nash bargaining model, focusing on enhancing the model with considerations for marginal risk; Section 5 discusses the numerical calculations and results in detail and provides a high-level summary of our findings; finally, Section 6 presents our conclusions and explores possible directions for future work.

2. Related Works

Profit allocation in collaborative ventures has become a significant focus of research across various disciplines, with models evolving to address the increasing complexity of economic interactions. From traditional methods that rely solely on fixed contributions to more advanced approaches incorporating game theory and risk factors, scholars have proposed numerous frameworks to ensure equitable and efficient allocation of profits.

2.1. Traditional Models of Profit Allocation

Early studies introduced traditional methods of profit allocation that are primarily based on the contributions or production inputs of each party, often employing fixed proportions to allocate profits. These methods are well-suited for relatively straightforward cooperative relationships, emphasizing profit allocation according to participants' contributions, such as sales or resource inputs. Within this collaborative framework, researchers have proposed various forms of profit allocation, including the equal allocation model [15], quantity discounts [16,17], repurchase agreements [18], two-part tariffs [19], revenue-sharing contracts [20], and mail-in rebates [21], all of which have been widely applied.

Govindan et al. [22] implemented profit-sharing contracts considering decentralized reverse setting, defining customer preferences as a function of discounts offered by the seller, and distributing profit proportionally based on participants' contributions or sales volume. Noori-Daryan et al. [23] introduced quantity and freight discounts, providing discounts or preferential prices based on participants' contributions or purchase quantities, optimizing the overall profit of sales prices and order volumes under composite incentive contracts, studying the impact of optimal decision-making strategies. Taleizadeh et al. [24] proposed two-part tariff contracts, dividing profit into two parts, one allocated based on participants' contributions and the other evenly distributed among all participants.

However, these allocation forms often rely on simplistic rules or fixed proportions, overlooking the actual risk-bearing situations of parties during the collaboration process. Such an approach can lead to unfair and opaque allocations, especially in more complex cooperative settings where participants assume different levels of risk. For example, parties who assume higher risk exposure, such as those investing in new technology or bearing operational risks, may receive the same share of profit as those with lower risk exposure, leading to dissatisfaction and undermining the willingness to cooperate. This lack of consideration for risk factors can destabilize cooperative relationships, especially in contexts like service alliances where risk is often a significant component. Incorporating fairness theory and game theory principles, such as Nash bargaining or the Shapley value, is essential for ensuring a more equitable and flexible profit allocation that better reflects the risks and profits of all parties involved.

2.2. Profit Allocation Based on Cooperative Game Theory

As economic cooperation become more complex, cooperative game theory [25,26] has emerged as a vital theoretical foundation for addressing profit allocation challenges. This theory primarily focuses on allocating profits generated from the collaboration of multiple participants, employing analytical approaches to develop mathematical models for quantitative research. Classic profit allocation methods, such as the Shapley value model [12] and the Nash bargaining model [9], provide solutions for fair and stable profit allocation.

The Shapley value offers a clear allocation equation that divides total profits among all collaborators forming an alliance, satisfying properties such as set monotonicity and coalition monotonicity. The Nash bargaining model focuses on how two participants share surplus value in cooperation. These methods have been widely applied in various fields, including transportation energy [27–30], supply chain management [31–33], and agricultural-product-sharing transactions [34]. Eissa et al. [35] developed a conceptual framework using the Shapley value as an alternative to the traditional investment-based approach, proposing an axiomatically fair methodology for profit-sharing negotiations. This profit-allocation scheme was based on the marginal profits of each participant. Jiang et al. [7] proposed a profit allocation mechanism based on the Shapley value method, to maintain stable cooperative relationships. Various factors, including contribution, synergistic effect, and cost coefficient, were considered. However, the risks borne by enterprises were not assessed, and the incorporation of multiple factors increased the complexity. Wang et al. [27] proposed an improved Shapley value model that addresses existing deficiencies by identifying key factors such as risk, input, and service quality. This model combines these factors with the modified Shapley value, to determine the allocation for each participant. Wang et al. [36] established a multi-weight interval Shapley value method for the benefit allocation model. This model reflects the impact of key parameter variables, including resource input ratios, allocation operation scales, risk taking, and other factors. However, the added complexity may hinder practical implementation, suggesting the need for a more straightforward approach that still considers risk. Meng et al. [37] constructed a four-level supply chain model and proposed three new Shapley values to address participants' varying risk attitudes. However, the Shapley value lacks flexibility and cannot capture dynamic changes. Li et al. [38] developed a two-layer revenue allocation model for road data assets, using a modified Shapley value approach. It adjusts allocations for data risk factors and determines correction factors for a consolidated allocation. However, this model may still struggle with scalability and might not fully account for the complexities of interdependencies in larger networks, reinforcing the need for a robust bargaining method that incorporates risk considerations.

In contrast, the proposed Nash bargaining model in this study addresses these issues by incorporating both marginal profit and risk, providing a more flexible and equitable approach to profit allocation. This model resolves the existing gap by considering risk-sharing mechanisms alongside traditional profit-based models, offering a solution that better suits the complexities of service alliances.

2.3. Profit Allocation Models Considering Risk

In recent years, researchers have increasingly recognized the importance of risk factors in profit allocation. In real-world collaborations, parties contribute differently and face varying levels of risk. Recent studies have focused on optimizing both profit and risk allocation among collaborators in various contexts of uncertainty. Radwa et al. [39] explored a model that allocates both profits and risks among collaborators under uncertain conditions by introducing a risk-sharing mechanism, which optimizes overall profit allocation and enhances cooperation sustainability. Ding et al. [40] presented a quantitative approach that models alliance members' inequity aversion, to analyze risk-sharing arrangements in an alliance project. The derivation involves solving a constrained optimization problem using concepts and methods from Stackelberg game theory. Gao et al. [41] adopted a profit distribution method that combines improved Shapley values and independent risk contribution theory to allocate the total revenue, taking into account the risk levels and comprehensive marginal benefits of various entities. Feng et al. [42] presented several factors that affect profit allocation, including input level, effort level, innovation level, risk factor, and value-added factor. They constructed a profit allocation method for service-oriented manufacturing alliances, based on the modified Shapley value method, with risk being one of the key considerations. Yang et al. [43] employed a risk objective function, which aims to minimize risk while controlling operational costs, and they proposed a Shapley value cost-allocation mechanism.

Collectively, these studies highlight that the allocation of profit and risk is crucial for ensuring the success of collaborations within alliance services. The model proposed by Radwa et al. [39] emphasizes the contribution of optimizing the distribution of profits and risks under uncertain conditions to the sustainability of cooperation. Ding et al. [40] and Gao et al. [41] revealed the importance of risk-sharing arrangements for equitable profit distribution by analyzing the risk aversion of alliance members, and they offered a new perspective by integrating risk levels with the marginal benefits of various parties, providing a framework for revenue allocation. The research by Feng et al. [42] and Yang et al. [43] validated the importance of minimizing risk within the cost-allocation mechanism through their risk objective function model.

However, while these studies introduced risk considerations into profit allocation, they largely focused on static models that do not account for the varying risk of participants over time. In some cases, the proposed models still use simplified allocation methods that fail to capture the complexity of risk interactions in more dynamic collaborations.

In summary, while the literature on profit allocation has examined the topic from various perspectives, most existing models primarily emphasize tangible contributions, such as sales or effort, often neglecting the risk exposure faced by each participant. In the context of service resource transactions, overlooking the uncertainties encountered by service providers can adversely impact the successful completion of service alliances and affect the earnings of other members, ultimately resulting in imbalanced profit allocation. Therefore, this study proposes a profit allocation model that accounts for the risks undertaken by each party, offering a fairer and more efficient solution for service alliances. The proposed Nash bargaining model is specifically designed to address these gaps by incorporating both profit and risk factors, ensuring that profits are allocated more equitably and flexibly in environments characterized by uncertainty and dynamic risks.

3. Problem Description and Definition

In service alliances, profit allocation involves cooperation among multiple service providers and requires careful consideration of the key components involved in the problem and the model development process. In what follows, we describe the problem, define the service alliance transaction model, and examine the interactions between multiple participants, ultimately establishing a Nash bargaining-based model.

3.1. Problem Description

Service alliances are formed by groups of service providers that collectively address personalized user needs by integrating different services to achieve more complex functionalities. Within these alliances, transactions and collaborations among multiple participants are essential. Consumers seeking services can include individuals, businesses, institutions, and other users, while providers may be single entities or a consortium of institutions or enterprises that collaborate as service-provider alliances. These dynamic service alliances emerge in response to the needs of service consumers, facilitating the extensive combination of services tailored to customer requirements. Through the contributions and collaboration of participants, the personalized service demands of consumers are met within the service alliance, as illustrated in Figure 2.

After service consumers receive the services, they pay the corresponding fees, which the service alliance distributes as profit shares to the participating providers. The service alliance serves as the primary entity responsible for creating and delivering services, with each provider establishing relationships within the alliance. However, the formation of such alliances carries inherent risks, including the potential for incomplete service transactions following establishment. Therefore, it is essential for multiple entities within the alliance to reach a consensus on both profit and risk sharing.

3.2. Model Definition

In a cooperative game involving n service providers, each provider seeks to maximize their own interests by forming alliances with others and offering alliance services composed of multiple services. Let S represent the formed alliance, denoted as $S = \{S_1, S_2, \ldots, S_n\}$, where each service S_i $(1 < i < n)$ represents the service provided by the i-th provider. Then, the members not included in the alliance are represented as $N - S$. Profit allocation is carried out after obtaining the profit based on the cooperative game, which can be represented by a binary tuple, as shown in Equation (1):

$$G = \{N, v\}$$
$$v : 2^N \to R \tag{1}$$

where N represents the set of participants and v represents the profit function of all possible cooperative alliances in the participant set. The notations of the parameters used in this paper are given in Table 1, and the notations of the variables used in this paper are given in Table 2.

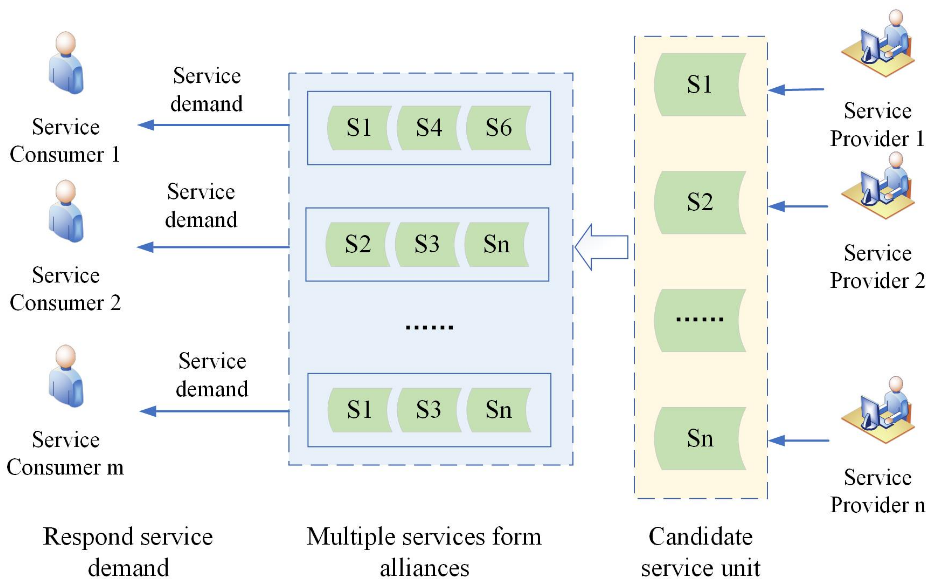

Figure 2. Multiple services form alliances.

The risk function $r(S)$ is a key element in ensuring that risk considerations are appropriately integrated into the profit allocation process. It quantifies the overall risk faced by the alliance, taking into account the varying levels of exposure each member faces based on their contribution to the alliance and their individual risk profiles.

Let $f(v_i)$ represent the risk exposure for provider i in the alliance. This function is typically modeled as a function of the profits generated by the alliance and the specific risks undertaken by each provider. The definition of $f(v_i)$ is as shown in Equation (2):

$$f(v_i) = \beta_i \cdot r(S) \tag{2}$$

where α_i is the proportion factor that quantifies how much risk member i bears relative to the entire alliance's risk, and where $r(S)$ is the total risk of the alliance, which is a function of the individual risks of all the members in the alliance.

The total risk function $r(S)$ is computed by aggregating the individual risks $f(v_i)$ across all the alliance members, possibly using a weighted sum depending on the level of risk exposure each provider faces. The model for $r(S)$ is as shown in Equation (3):

$$r(S) = \sum_{i=1}^{n} f(v_i) \tag{3}$$

where $f(v_i)$ is the individual risk function, reflecting the risk exposure of member i, and where n is the total number of members in the alliance.

Table 1. Notations of parameters.

Notation	Description
n	Assume there are n service providers in the cooperative game system.
N	The set of service providers, $N = \{1, 2, \ldots, n\}$.
S	The set of service alliances, where $S \subseteq N$, and where S represents a specific alliance of service providers.
$v(S)$	The profit function of alliance S. The total profit generated by the alliance S, which depends on the services provided by its members.
α_i	The weight factor of cooperative profit for provider i, reflecting how much each provider's profit to the alliance's total profit is weighted.
$f(v_i)$	The risk shared by member i in the alliance. This is a function of the profit generated by the alliance and the risk exposure faced by provider i. Specifically, it quantifies the share of the total risk borne by i in proportion to their profit.
$r(S)$	The risk function of alliance S, representing the total risk borne by the alliance. This function quantifies the uncertainty or potential loss that the entire alliance faces. It is a function of the risks taken by each individual provider and their combined interaction.
β_i	The proportion factor of risk for member i, representing how much of the total risk $r(S)$ is allocated to member i. This factor reflects the relative risk exposure of i in the context of the entire alliance.
λ	Risk adjustment factor, used to represent the impact of risk on profit.

Table 2. Notations of variables.

Notation	Description
$\varphi_(v_i)$	The profit obtained by service provider i in the alliance. It is calculated based on their profit to the total profit: $\varphi(v_i) = \alpha_i \cdot v(S)$, where α_i is the weight factor of service provider i's profit and $v(S)$ is the total profit of the alliance S.
$U_i(x_i)$	The reward function of service provider i. This function quantifies the net benefit that service provider i gains from the collaboration, considering both profit and risk. It is defined as: $U_i(x_i) = \varphi(v_i) - \beta_i \cdot f(v_i)$, where $f(v_i)$ represents the risk borne by provider i and β_i is the risk proportion coefficient for i.
u^*	The reward point (equilibrium point), which is the optimal solution or equilibrium point obtained by maximizing the reward function $U_i(x_i)$.
Y	A set of feasible solutions, representing all possible profit and risk allocation schemes that satisfy the constraints. These solutions can be found either analytically or iteratively.
d_i	The threat point, representing the potential negative impact on the alliance if service provider i were to exit. It can quantify the degree of threat or bargaining power each participant has in the collaboration.

The cooperative game is super-additive, which is defined as Definition 1:

Definition 1 (Super-Additivity)**.** *For a cooperative game $G = \{N, v\}$, for any alliances A and B formed by service providers $A \subseteq S$, $B \subseteq S$, if A and B have no intersection, i.e., $A \cap B = \emptyset$ then the profit of the new alliance formed by A and B, $v(A \cup B)$, is greater than or equal to the sum of the profit of A and B, as shown in Equation (4):*

$$v(A \cup B) \geq v(A) + v(B) \tag{4}$$

3.3. Nash Bargaining Model

In profit allocation for service alliance transactions, multiple service providers collaborate to form an alliance and deliver services to consumers. The Nash bargaining model is employed, to seek a fair allocation of profit based on both marginal profit and risk within the alliance.

The Nash bargaining model offers a solution to allocating profit among participants based on a utility function that incorporates influencing factors, such as risk, marginal profit, and the bargaining positions of each party. The Nash bargaining solution must satisfy four key axioms: symmetry, linearity preservation, Pareto efficiency, and the independence of irrelevant alternatives. These axioms ensure that the solution is fair, stable, and consistent across different bargaining scenarios. In service alliances, ensuring fairness and stability in profit allocation is crucial for maintaining long-term cooperation among participants. The Nash bargaining model addresses these issues by providing a mathematically rigorous framework that guarantees each participant's utility is maximized within the bounds of fairness and stability.

We assume that the reward function $U_i(x_i)$ maps from $[0,1]$ to the real number space, which is a strictly increasing concave function. Let (d_1, d_2) be the bargaining breakdown point, and the lower bound is $d_1 \geq U(0), d_2 \geq U(0)$. If the four axioms are satisfied then the balanced reward point (equilibrium point) $u^* = ({u_1}^*, {u_2}^*)$ has only one unique and valid solution. These four axioms are discussed below.

3.3.1. Symmetry

If service A and service B provided by two service providers are exactly the same then their bargaining breakdown points are the same, as well as the return function, i.e., ($d_1 = d_2$, $U_1 = U_2$); then, the final return is also the same, as shown in Equation (5):

$$ {u_1}^* = {u_2}^* \quad (5) $$

Both parties have equal bargaining power. Symmetry ensures that if two participants are identical in their roles and contributions then they will receive equal shares of the profit. This ensures fairness, as no participant will feel disadvantaged or unfairly treated, thus promoting cooperation and minimizing the risk of alliance breakdown.

3.3.2. Linearity Preservation

If the service provider's reward function U or the bargaining rupture point is scaled and shifted to perform a linear transformation then the resulting u^* is also scaled and shifted in the same way; that is, the linear transformation invariance axiom is satisfied. Linearity preservation means that any scaling or shifting of reward functions or bargaining points will be consistently reflected in the resulting profit allocation. This property ensures that the solution remains consistent and scalable, regardless of any external changes to the underlying model, thus providing stability in the face of fluctuations in the external environment.

3.3.3. Pareto Efficiency

There is no other point of return that is better than u^*. Neither party can be made better-off without making the other party worse-off. Pareto efficiency guarantees that the allocation cannot be improved for one participant without making another worse-off. This is crucial for maintaining cooperation among service providers, as no party can increase their utility without reducing the others, which ensures a stable and non-exploitative distribution of profits.

3.3.4. Independence of Irrelevant Alternatives

We assume that service providers A and B reach a consensus on x^*, resulting in u^* in the value range U. Then, if in a new bargaining problem the effective reward point set is a strict subset of the value range U, and u^* is still in this subset, then the equilibrium point of this new problem is still u^*. The independence of irrelevant alternatives ensures that the equilibrium outcome remains unchanged even if irrelevant alternatives are introduced into the bargaining process. This guarantees the robustness of the Nash bargaining solution, ensuring that minor changes or the introduction of unrelated alternatives will not disturb the final equilibrium, which further strengthens the stability of the profit allocation.

Together, these axioms ensure that the Nash bargaining model yields a solution that is not only mathematically rigorous but also practical and equitable for the participants in the service alliance. Compared to other profit allocation models, the Nash bargaining model is preferable because it incorporates well-established game-theoretic principles to ensure fairness and stability, addressing both the marginal profit and the risk faced by participants. In service alliances, this model is particularly effective in maintaining long-term cooperation by ensuring that all parties are motivated to contribute without fear of exploitation or unfair treatment.

3.3.5. Objective Function

Based on the preceding analysis, the issue of profit allocation in service alliance transactions satisfies the four axioms of symmetry, linearity preservation, Pareto efficiency, and independence of irrelevant alternatives. Therefore, the Nash bargaining model is applicable for profit allocation. In the cooperative game $G = \{N, v\}$, we assume the profit allocation factors of service providers N in the alliance S are $\{x_1, x_2, \ldots, x_n\}$, where these factors represent the proportion of the alliance's total profit allocated to each service provider N. The objective function of the Nash bargaining for the profit allocation of service provider i after joining the alliance S is represented in Equation (6):

$$\begin{aligned} &MAX_{\varphi \in Y}\left(\prod_{i=1}^{n}(\varphi_i - d_i)\right) \\ &s.t. \ \ \varphi_i \geq d_i \end{aligned} \tag{6}$$

in which, n is the number of service providers participating in the bargaining, Y is a set of feasible solutions, φ_i is the allocation profit of service provider i participating in the alliance S, and d_i represents the threat points (i.e., the reservation payoffs of each participant).

In order to motivate mutual coordination and cooperation among service providers, the solution of Nash bargaining can exclude the scenario of bargaining breakdown and only consider outcomes that are superior to the reservation point. This exclusion is crucial, because it focuses the negotiation on achievable, mutually beneficial outcomes, thus promoting cooperation rather than conflict. In a multi-participant Nash bargaining model, logarithmic transformation can be used to simplify the calculation process and ensure the solvability of the maximization problem of the product, transforming the product maximization problem into a sum maximization problem, as shown in Equation (7):

$$\ln\left(\prod_{i=1}^{n}(\varphi_i - d_i)\right) = \sum_{i=1}^{n}\ln(\varphi_i - d_i) \tag{7}$$

Therefore, the objective function of Nash bargaining can be equivalently transformed into Equation (8):

$$\begin{aligned} &MAX_{\varphi \in Y}\left(\sum_{i=1}^{n}\ln(\varphi_i - d_i)\right) \\ &s.t. \ \ \varphi_i \geq d_i \end{aligned} \tag{8}$$

Maximizing the utility product of each service provider corresponds to maximizing the utility of the alliance. Service providers engage in cooperative games, to ensure fair allocation of profits. By focusing on maximizing the utility of the collective group rather than individual gain, the Nash bargaining model aligns the interests of all participants, thus ensuring a stable and fair profit distribution. The Nash bargaining model, which is based on influencing factors and the bargaining equilibrium point, appropriately addresses the allocation of total profit among multiple service providers.

4. Proposed Improved Nash Bargaining Model

The valuation of each participant's contributions in a service alliance is crucial for achieving fair profit allocation, and the following two aspects should be taken into consideration: (1) the resources invested, the expertise provided, and the amount of work performed by each participant in the service delivery process; (2) the risks faced by each party in the service alliance, including project risks, financial risks, and market risks. To address these issues, we first conducted an impact factor analysis. Subsequently, an improved Nash bargaining model considering risk was designed, to achieve fair profit allocation.

4.1. Impact Factor Analysis

The allocation of profit in a service alliance transaction involves multiple service providers, with influencing factors determined by the necessary costs and value generated by the alliance services. According to the principle of fairness, the greater the value created, the larger the share of profit that should be allocated. There are two main factors that affect profit allocation.

4.1.1. Profit

Marginal profit generated by service providers after joining the alliance, which is defined as in Definition 2:

Definition 2 (Marginal profit)**.** *For a cooperative game $G = \{N, v\}$, considering the order in which service providers join the alliance, when a new service provider joins the alliance the overall profit of the alliance increases. The additional profit is called the marginal profit.*

Assuming service provider i joins service alliance S, its marginal profit can be given by Equation (9):

$$Marginal\ Profit = v(S \cup \{i\}) - v(S) \tag{9}$$

Here, S represents the coalition of service providers before i joins, $S \cup \{i\}$ is the new coalition including i, $v(S)$ is the profit generated by coalition S and $v(S \cup \{i\})$ is the profit of the coalition after adding i.

The Shapley value method can determine the weight factor α_i for the marginal profit of cooperative profit, as it calculates the allocation of the cooperative profit that satisfies the core allocation. The Shapley value was chosen for its well-established properties of fairness and stability in cooperative game theory. It ensures that each participant's share reflects their marginal profit across all potential coalition formations, thus making it suitable for scenarios where both value and shared responsibilities, such as risk, are considered. In a cooperative game, $G = \{N, v\}$, the equation for calculating the profit allocation of the Shapley value for service provider i after joining alliance S is presented in Equation (10):

$$\varphi_i[N, v] = \sum_{S \subset N, i \notin S} [v(S \cup \{i\}) - v(S)] \frac{(N - |S|)!(|S| - 1)!}{N!} \tag{10}$$

where $|S|$ represents the number of service providers included in coalition S, and where $(|S| - 1)!$ represents how many kinds of cooperation sequences there are after service provider I joins coalition S, where $S \subseteq N$. Then, $(N - |S|)!$ represents how many kinds of cooperation sequences the remaining coalition members have; $v(S \cup i) - v(S)$ represents the marginal profit of the coalition S after joining the service provider i. Service provider i's participation in different sequence combinations in alliance S is divided by the sequence possibilities of all n members; that is, the profit that service provider I should share for alliance S.

Based on the calculation by Equation (10) of the Shapley value allocation, the calculation equation for the weight factor α_i of the cooperative profit after service provider i joins the alliance S can be expressed as Equation (11):

$$\alpha_i = \frac{\varphi_i[N,\ v]}{\sum_{i=1}^{n} \varphi_i[N,\ v]} \tag{11}$$

where α_i represents the weight factor for the marginal profit of the cooperative profit.

4.1.2. Risk

The risk incurred by service providers after joining the alliance can be understood as the Operation and Maintenance (O&M) cost that service providers need to incur during the process of delivering services. This cost is typically a fixed value and can be quantified after a certain O&M period. When providing services individually, the independent risk only needs to consider the O&M cost incurred by the service providers. However, upon joining the alliance, the independent risk incurred by service providers in delivering services includes not only the O&M cost but also the shared risk of the alliance. During the service process, failure to complete the service or consumer dissatisfaction leading to non-payment can impose significant costs on service providers, impacting their profitability and reputation.

To allocate risk fairly, the Shapley value is employed, as it considers the marginal impact of each participant on the total risk, distributed across all possible coalition formations. This method ensures that each provider's risk allocation reflects the additional burden they contribute when joining or leaving the alliance, thereby promoting an equitable balance. By treating risk as a quantifiable cost shared among participants, the Shapley value provides a mechanism for determining each member's fair share of the total incurred risk. This ensures that risk allocation, similar to profit allocation, reflects the marginal change each participant brings to the alliance.

With reference to marginal profit, we introduced the concept of marginal risk, as defined in Definition 3, which represents the additional risk or loss incurred by adding or removing a service member in the alliance.

Definition 3 (Marginal risk)**.** *For a cooperative game $G = \{N, v\}$, taking into account the sequence in which service providers join the alliance, the additional risk incurred when a new service provider joins the alliance and the overall risk as the alliance increases is considered as the marginal risk.*

Assuming service provider i joins service alliance S, its marginal risk can be given by Equation (12):

$$Marginal\ Risk = r(S \cup \{i\}) - r(S) \tag{12}$$

Here, S represents the coalition of service providers before i joins, $S \cup \{i\}$ is the new coalition including i, $r(S)$ is the risk generated by coalition S, and $r(S \cup \{i\})$ is the risk of the coalition after adding i.

Utilizing the Shapley value model, the marginal risk of each service provider can be calculated. In the cooperative game $G = \{N, v\}$, the risk allocation equation for service provider i after joining the alliance S can be expressed as Equation (13):

$$f_i[N,\ v] = \sum_{S \subset N, i \notin S} [r(S \cup \{i\}) - r(S)] \frac{(N - |S|)!(|S| - 1)!}{N!} \tag{13}$$

where $f_i[N,\ v]$ represents the risk value, $|S|$ denotes the number of service providers in S, $(|S| - 1)!$ presents the number of cooperation sequences after service provider i joins S, $S \subseteq N$; $(N - |S|)!$ presents the number of cooperation sequences the remaining coalition members have; $r(S \cup \{i\}) - r(S)$ represents the increased marginal risk after the service provider i joins S. Service provider i participates in different sequence combinations in S, divided by the sequence possibilities of all n members, that is, the risk value of service provider I for S.

The proportion factor of risk β_i is the proportion factor of cooperative profit after service provider I joins alliance S, which can be calculated by the proportion of marginal risk in the alliance. Assuming that the marginal risk of each service provider I joining the alliance is expressed as f_i, the equation for calculating the proportion factor β_i is as shown in Equation (14):

$$\beta_i = \frac{f_i[N,\ v]}{\sum\limits_{i=1}^{n} f_i[N,\ v]} \tag{14}$$

where β_i represents the proportion factor of risk.

The interaction between profit and risk is critical in ensuring a balanced profit allocation. The improved Nash bargaining model considers both factors by integrating risk-adjusted profit measures. Specifically, the profit allocated to each service provider not only depends on the marginal profit they bring but also on the marginal risk they bear. This ensures that service providers are compensated in proportion to their profits and the associated risk, thereby fostering a sustainable and fair alliance.

4.2. Improved Nash Bargaining Model Considering Risk

Based on the above analysis, we propose an improved Nash bargaining model that considers risk. The architecture of the model is illustrated in Figure 3, which outlines the key components and their interactions. The model considers both marginal profit and risk-adjusted factors, allowing for a more balanced and equitable distribution of profits among service providers in a cooperative alliance.

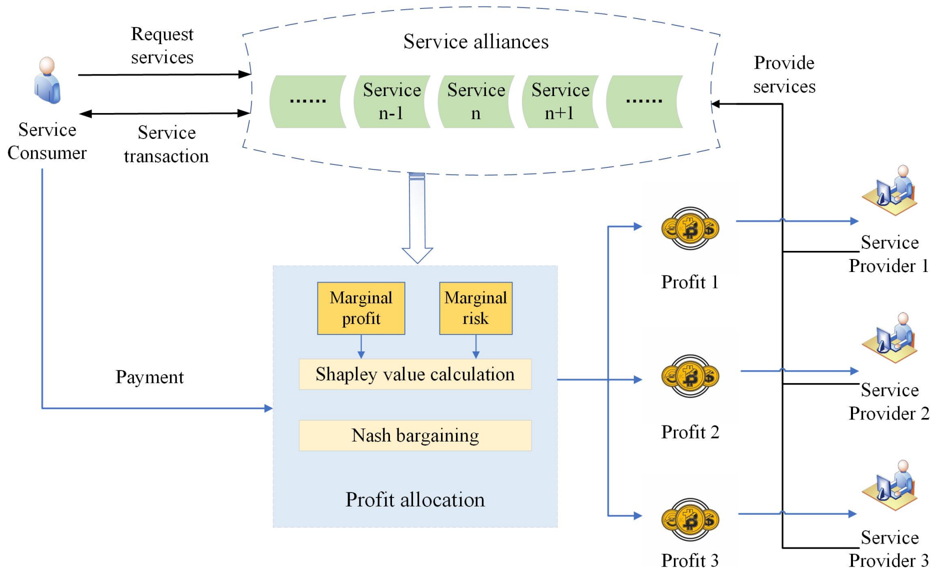

Figure 3. Architecture of the improved Nash bargaining model considering risk.

To better address the interaction between the profit and risk of each service provider in the alliance, the Nash bargaining objective function presented in Equation (8) is enhanced to derive a profit allocation model that incorporates these dual aspects. This refined model, which takes into account both marginal profit and marginal risk, is expressed as Equation (15):

$$MAX\sum_{i=1}^{n} ln[(\varphi_i - d_i) - \lambda \cdot (f_i - p_i)]$$
$$s.t. \begin{cases} \varphi_i \geq d_i + \lambda \cdot (f_i - p_i) \\ d_i \geq p_i \end{cases} \quad (15)$$

where φ_i denotes the profit allocated to service provider i upon joining the alliance S, d_i represents the profit of service provider i's independent decision making, f_i signifies the risk allocated to service provider i after joining the alliance S, p_i represents the risk of service provider i's independent decision making, and λ represents the risk adjustment factor, used to represent the impact of risk on profit.

5. Case Calculation and Result Analysis

5.1. A Case

Suppose there are three service providers, 1, 2, 3, denoted (n_1, n_2, n_3). When the three cooperate, they form an alliance, and they cooperate to complete a service alliance transaction. The individual profit of the three service providers is as shown in Equation (16):

$$\begin{aligned} v(n_1) &= 60 \\ v(n_2) &= 90 \\ v(n_3) &= 150 \end{aligned} \quad (16)$$

Among the three service providers, the profits of two cooperating are as shown in Equation (17):

$$\begin{aligned} v(n_1 \cup n_2) &= 360 \\ v(n_1 \cup n_3) &= 240 \\ v(n_2 \cup n_3) &= 300 \end{aligned} \quad (17)$$

The profits of three cooperating are as shown in Equation (18):

$$v(n_1 \cup n_2 \cup n_3) = 690 \quad (18)$$

Collaborating with another service provider brings greater profit than the sum of their individual profit, while collaborating with three service providers brings greater profit than collaborating with two service providers. Therefore, for each service provider, collaborating with others to provide services brings greater profit than providing services alone. The three service providers collaborate to provide services to consumers, and then they allocate the profit.

Assuming that the three service providers provide services independently, the risk is as shown as Equation (19):

$$\begin{aligned} r(n_1) &= 30 \\ r(n_2) &= 30 \\ r(n_3) &= 60 \end{aligned} \quad (19)$$

Assuming the risk of two cooperating is shown as Equation (20):

$$\begin{aligned} r(n_1 \cup n_2) &= 180 \\ r(n_1 \cup n_3) &= 120 \\ r(n_2 \cup n_3) &= 150 \end{aligned} \quad (20)$$

Assuming the risk of the three service providers cooperating is shown as Equation (21):

$$r(n_1 \cup n_2 \cup n_3) = 360 \quad (21)$$

5.2. Case Calculation

According to the above assumptions, the cases of the models were calculated, respectively.

5.2.1. Nash Bargaining Model

The Nash bargaining solution could obtain the maximum value when the distributed profit of the three service providers joining the alliance minus the profit of their independent services was equal. According to the objective function equation of Nash bargaining (8), this became Equation (22):

$$MAX\sum_{i=1}^{3}[\ln(\varphi_i - d_i)]$$
$$s.t.\ \begin{cases} \varphi_1 + \varphi_2 + \varphi_3 = v(S) \\ \varphi_1 \geq d_1, \varphi_2 \geq d_2, \varphi_3 \geq d_3 \\ v(S) = 690 \\ d_1 = 60, d_2 = 90, d_3 = 150 \end{cases} \tag{22}$$

The profit values for service provider n_1, n_2, n_3 were calculated to be $(190, 220, 280)$.

5.2.2. Improved Nash Bargaining Model

(1) Calculate profit value

For the three service providers, Equation (10) could be used to obtain the specific profit ability. The Shapley values of profit calculated for service provider n_1, n_2, and n_3 were calculated as follows: $(210, 255, 225)$. The sum of the profit of the three was 600. This was equal to the total profit of the three alliances, which met the requirements of the definition of the example.

(2) Calculate risk value

With the Shapley value model, the marginal risk of the three service providers could be calculated by Equation (13). The marginal risk calculation values of service provider n_1, n_2, and n_3 are shown as follows: $(115, 130, 115)$.

(3) Calculate multi-impact factor

The allocation method of the improved Nash bargaining model, which considers multiple influencing factors, as articulated in Equation (15), was utilized to compute and examine a case study. The Nash bargaining solution could attain the maximum value when the difference between the allocated profit of the three service providers who joined the alliance and their individual service profit, minus the difference between the risk, equaled the final profit obtained, as demonstrated in Equation (23):

$$MAX\sum_{i=1}^{3} ln[(\varphi_i - d_i) - \lambda \cdot (f_i - p_i)]$$
$$s.t.\ \begin{cases} \varphi_1 + \varphi_2 + \varphi_3 = v(S) \\ \varphi_i \geq d_i + \lambda \cdot (f_i - p_i) \\ v(S) = 690 \\ d_1 = 60, d_2 = 90, d_3 = 150 \\ f_1 = 115, f_2 = 130, f_3 = 115 \\ p_1 = 30, p_2 = 30, p_3 = 60 \end{cases} \tag{23}$$

The profit values of the three service providers, considering multi-impact factors ($\lambda = 1$), were calculated to be $(195, 240, 225)$.

5.3. Results Analysis

5.3.1. Comparison of Profit Allocation Results Across Models

Based on the aforementioned calculation results, the model profit allocation outcomes for service provider n_1, n_2, and n_3 are presented in Table 3. The detailed calculation process of Shapley can be found in Tables S1–S3. It can be observed that, with independent decision making, the independent service profits of service providers n_1, n_2, and n_3 were $V(60, 90, 150)$, respectively, indicating that their respective value creation capabilities followed the order $n_3 > n_2 > n_1$. However, in the profit allocation using the Shapley value $V(210, 255, 225)$, the profit allocated to n_2 was the largest, which did not match the expected order of $n_3 > n_2 > n_1$. In contrast, the profit allocation from the Nash bargaining model $V(190, 220, 280)$ followed the order $n_3 > n_2 > n_1$, which was consistent with the trend of the value creation abilities of the participating members. The adjusted profit allocations in the improved Nash bargaining model ($\lambda = 1$) were $V(195, 240, 255)$. This model took into account the risk influencing factor and still followed the order $n_3 > n_2 > n_1$, aligning with the value creation abilities of the participating members and meeting the expectation of fair allocation. These findings confirm the feasibility of the improved Nash bargaining model in achieving fair profit allocation. The comparison of the benefits allocation for service provider n_1, n_2, and n_3 is depicted in Figure 4:

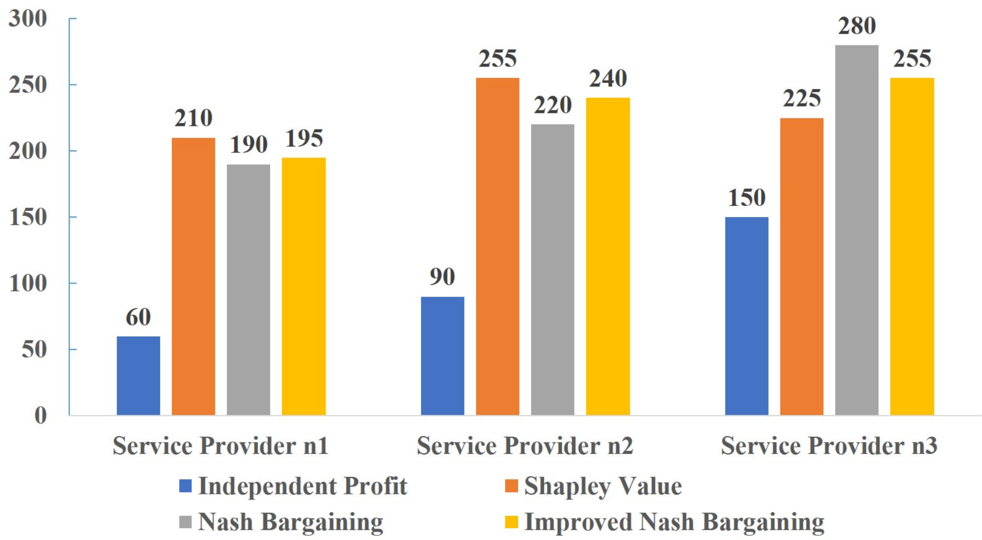

Figure 4. Comparison of benefits allocation results for service providers.

Table 3. Profit allocation of the three service providers (n_1, n_2, and n_3).

	Independent Profit	Shapley	Nash	Improved Nash ($\lambda = 1$)
n_1	60	210	190	195
n_2	90	255	220	240
n_3	150	225	280	255

5.3.2. Analysis of the Impact of Risk and Risk Adjustment Factors on Model Performance

(1) The influence of risk

Based on the aforementioned calculation results, the risk and profit allocation results of service provider (n_1, n_2, and n_3) are presented in Table 4. It can be observed that the Shapley value of risk for provider n_2 was 130. In the service alliance cooperation among the three service providers, n_2 bore the highest risk. Therefore, with the inclusion of the risk impact factor in the improved Nash bargaining model, compared to the original Nash bargaining model, the income allocated to n_2 also increased accordingly. This indicates that n_2 undertook more risk and, consequently, that it could be allocated more income.

Conversely, n_3 had a larger independent profit, so in the original Nash bargaining model it received the largest allocation of income. However, because its Shapley value of risk was relatively small, implying lower risk, its profit allocation value decreased after the inclusion of the risk factor. This demonstrates that the improved Nash bargaining model, considering risk, aligned better with the comprehensive considerations of value creation and risk assumption for all the participants. The comparison of the risk and benefits allocations of service providers n_1, n_2, and n_3 is depicted in Figure 5:

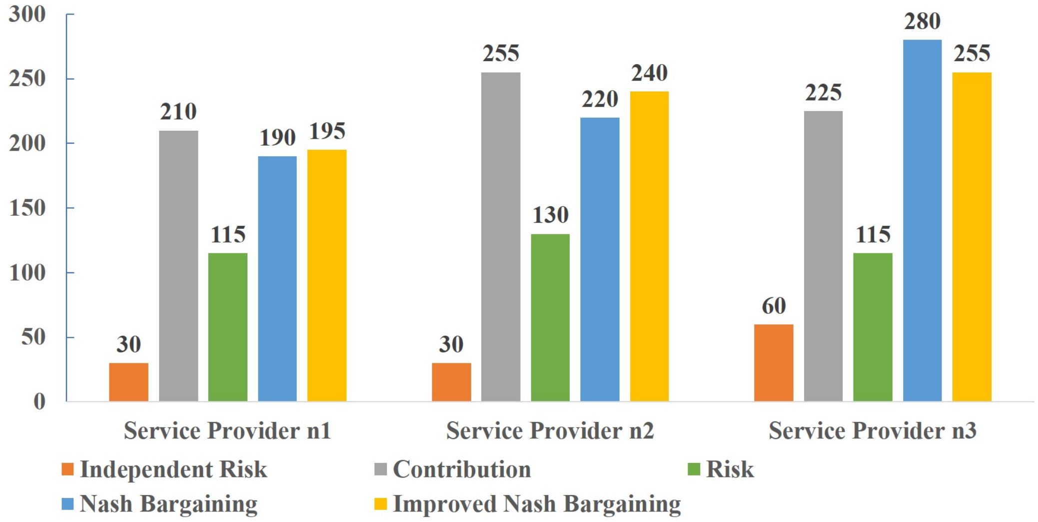

Figure 5. Comparison of benefits and risk impact results for service providers.

Table 4. Profit allocation of service providers (n_1, n_2, and n_3), considering risk.

	Independent Risk	**Profit**	**Risk**	**Nash**	**Improved Nash ($\lambda = 1$)**
n_1	30	210	115	190	195
n_2	30	255	130	220	240
n_3	60	225	115	280	255

(2) The risk adjustment factor λ changes

We assumed that the value of λ ranged from 0 to 1, changing in increments of 0.1. The optimal profit allocation scheme for the three service providers (n_1, n_2, and n_3) was calculated, and the results are shown in Figure 6. The analysis illustrates that changes in the risk adjustment factor λ significantly impacted the profit allocation among the three service providers, while the total allocated profit remained constant at 690. As λ increased from 0 to 1, $n1$'s profit gradually rose from 190 to 195, and $n2$'s profit increased from 220 to 240, indicating that both benefited from higher risk adjustments. In contrast, $n3$'s profit decreased from 280 to 255, suggesting it became less favored or assumed a higher share of the risk as λ rose. This redistribution pattern shows that higher λ values shifted the allocation toward $n1$ and $n2$ at the expense of $n3$. The results imply that the risk adjustment factor played a critical role in balancing profit distribution by reflecting the perceived risk or contribution of each provider. When $\lambda = 0$, the risk factor was not considered, and the allocation scheme was effectively equivalent to the traditional Nash bargaining solution. As λ gradually increased, the weight placed on risk in the profit allocation increased. Therefore, by carefully adjusting λ, stakeholders can manage the trade-off between risk and reward, enabling more equitable or strategically aligned allocations depending on the specific objectives of each service provider.

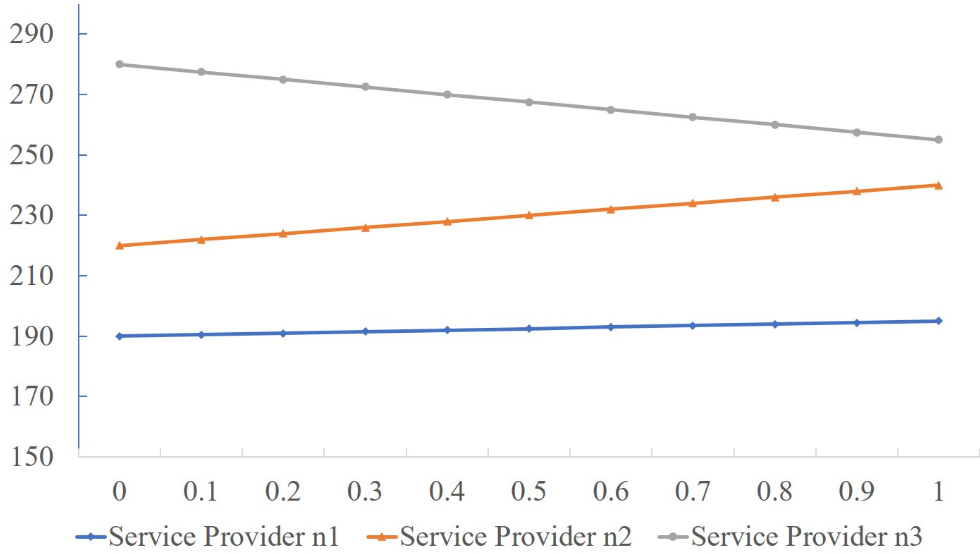

Figure 6. Impact of risk adjustment factor λ changes from 0 to 1.

(3) The independent risk changes

We let $\lambda = 1$, while keeping the independent risks of n_2 and n_3 unchanged. The independent risk of n_1 was gradually increased from 6 to 60, and the impact of this change on profit allocation was observed. The results are depicted in Figure 7. It can be observed that when the independent risk of n_1 exhibited a linear increasing trend, the Shapley value of risk also showed a linear increasing trend accordingly. However, since the total risk remained constant, the marginal risk actually decreased, consistent with the trend of the marginal risk curve in the figure. Consequently, due to the reduction in the marginal risk, the profit allocation decreased accordingly, aligning with the trend of the improved Nash bargaining model curve in the figure. This indicates that the profit allocation of the improved Nash bargaining model can accurately reflect the impact of changes in independent risk.

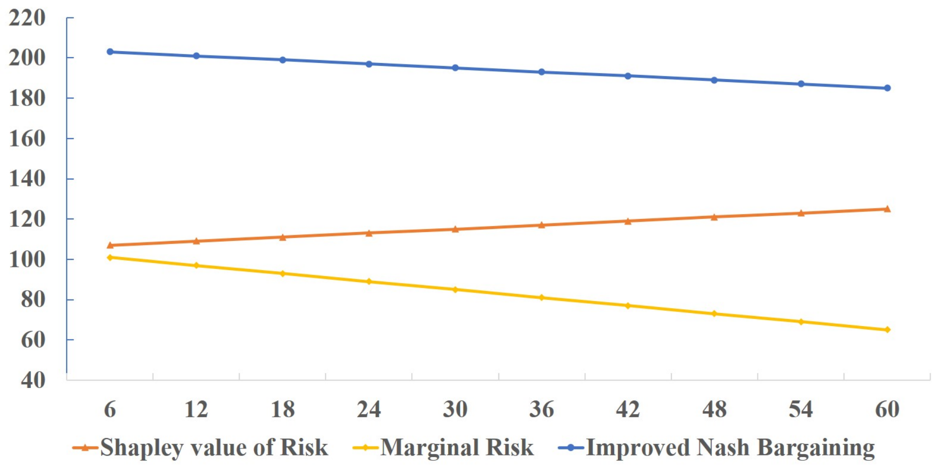

Figure 7. Impact of independent risk change of service provider n_1 on profit allocation.

(4) Comparison with other methods

We then compared the performance of our method with other approaches reported in the literature. The comparative study was conducted from three key perspectives: methodology and advantages, influencing factors, and limitations. Our literature review demonstrated that the proposed approach yielded better results than the previous methods, showing improved efficiency in profit allocation and enhanced fairness through the integration of risk adjustments. A summary of these methods is presented in Table 5:

Table 5. Comparative study of the proposed method with other methods.

Literature	Method and Advantage	Influencing Factors	Limitation
Govindan et al. (2013) [22]	Profit-sharing contracts	Contribution	No risk
Noori-Daryan et al. (2018) [23]	Quantity and freight discounts	Contribution	No risk
Taleizadeh et al. (2018) [24]	Two-part tariff contracts	Contribution	No risk
Jiang et al. (2021) [7]	Nash bargaining, Shapley	Contribution, synergic effect, cost coefficient	No risk, complexity
Wang et al. (2023) [27]	Shapley	Risk, input, and service quality	Performance, lacks flexibility
Meng et al. (2023) [37]	Shapley	Contribution, evaluation	No risk, performance
Wang et al. (2023) [36]	Shapley	Resource input, allocation operation, risk bearing	Performance, lacks flexibility
Eissa et al. (2021) [35]	Nash equilibrium, Shapley	Marginal contribution	No risk
Xu et al. (2022) [44]	Shapley, Stackelberg	Contribution	No risk, performance
Radwa et al. (2024) [39]	Shapely, Owen, and Myerson	Risks and rewards	Lacks flexibility
Gao et al. (2023) [41]	Shapley	Risk levels, marginal benefits	Lacks flexibility
Proposed method	Nash bargaining, Shapley	Contribution, risk	-

5.3.3. Managerial Insights and Practical Findings

From the analysis of the aforementioned results, we can draw the following findings:

Firstly, ensuring equitable profit allocation based on each party's profits promotes collaboration among service providers, enhances trust, and establishes long-term partnerships. Computational analysis indicates that this model not only optimizes profits for individual providers but also improves overall system efficiency, thereby enhancing service delivery. By explicitly considering profit contributions, managers can ensure that allocation mechanisms are both equitable and performance-driven, creating a strong foundation for sustainable cooperation.

Secondly, incorporating risk factors into the profit allocation model allows managers to better understand and mitigate operational risks, thus increasing the resilience of service transactions. Moreover, recognizing the different levels of risk faced by each service provider ensures that profit allocation reflects both profits and the inherent uncertainties of the business environment. This nuanced understanding of risk-adjusted profit allocation enables managers to design adaptive strategies for various industries, such as technology, logistics, and healthcare, where risk profiles can vary significantly. By doing so, organizations can align profit-sharing practices with real-world challenges, fostering an environment of fairness and innovation.

Thirdly, the proposed Nash bargaining model considers the risks faced by participants, providing managers an effective method for profit allocation that enhances satisfaction among all service providers. Additionally, this model provides a clear framework for decision makers, helping them assess the impact of various factors on profit allocation and facilitating more informed strategic decisions. Specifically, the inclusion of a risk adjustment factor within the model offers practical guidance for addressing discrepancies in risk exposure, enabling tailored solutions that balance financial returns with operational stability. This approach can be instrumental in industries where dynamic changes and risk-bearing capacities significantly influence collaborative outcomes.

6. Conclusions

This paper introduces an optimized profit allocation game model based on Nash bargaining theory, which accounts for both marginal profit and risk. The unique contribution of this study lies in integrating marginal risk into the traditional Nash bargaining framework, offering a more comprehensive approach to profit allocation in service alliances. Unlike many previous models that primarily focus on contribution proportions, our model ensures a fairer distribution by considering the risk-bearing capacities of each service provider. This enhancement improves both fairness and cooperative efficiency among alliance members. A contribution of this work is the introduction of the Shapley value method for calculating marginal risk, addressing a common issue in the literature where risk is often neglected or oversimplified. The addition of the risk adjustment factor further strengthens the model's ability to account for the varying risk-bearing capacities in profit allocation. Our computational analysis shows that this improved Nash bargaining model effectively balances profit and risk, maximizing overall profitability while ensuring equitable profit distribution. This represents a significant advancement in the field, offering a more balanced and fair solution for service alliance cooperation.

However, this study has certain limitations. The current model relies on assumptions regarding the interactions between service providers and does not fully account for dynamic changes in real-time transactions or fluctuating service conditions. Additionally, obtaining accurate risk data in practical settings may present challenges, potentially affecting the precision of the model. The model, as currently designed, is tailored to specific types of service alliances and may require modifications to be applicable to other business environments or industries.

Future work will focus on extending the model to scenarios involving multiple providers, various service alliances, and real-time business environments. This will involve adapting the model to different business models, market conditions, and service complexities. We also plan to explore applications beyond those covered in this study, such as logistics, healthcare, and technology services, where unique risk profiles and profit-sharing dynamics exist. These expansions will enhance the applicability and effectiveness of our approach, addressing challenges related to fairness, incentive mechanisms, trust, and transparency across a wider array of service alliance contexts.

Supplementary Materials: The following supporting information can be downloaded at https://www.mdpi.com/article/10.3390/electronics13234648/s1, Table S1: Shapley values calculated for service provider n_1. Table S2: Shapley values calculated for service provider n_2. Table S3: Shapley values calculated for service provider n_3.

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