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Fully Distributed Economic Dispatch with Random Wind Power Using Parallel and Finite-Step Consensus-Based ADMM

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Abstract: In this paper, a fully distributed strategy for the economic dispatch problem (EDP) in the smart grid is proposed. The economic dispatch model considers both traditional thermal generators and wind turbines (WTs), integrating generation costs, carbon trading expenses, and the expected costs associated with the unpredictability of wind power. The EDP is transformed into an equivalent optimization problem with only an equality constraint and thus can be solved by an alternating-direction method of multipliers (ADMM). Then, to tackle this problem in a distributed manner, the outer-layer framework of the proposed strategy adopts a parallel ADMM, where different variables can be calculated simultaneously. And the inner-layer framework adopts a finite-step consensus algorithm. Convergence to the optimal solution is achieved within a finite number of communication iterations, which depends on the scale of the communication network. In addition, leveraging local and neighbor information, a distributed algorithm is designed to compute the eigenvalues of the Laplacian matrix essential for the finite-step algorithm. Finally, several numerical examples are presented to verify the correctness and effectiveness of the proposed strategy.

Keywords: distributed economic dispatch; parallel ADMM; finite-step consensus; random wind power

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1. Introduction

Economic dispatch (ED) is a well-studied and critical problem in power system research, involving the efficient allocation of power among generators while satisfying total load demand and generator constraints. Various algorithms have been proposed to address the economic dispatch problem (EDP), including dynamic programming methods [1–3], heuristic algorithms such as particle swarm optimization [4–6], genetic algorithms [7,8], and deep-learning-based algorithms [9–11]. However, these methods traditionally operate in a centralized manner, gathering global information from all generators for optimization at a central node. As noted in previous studies, this centralized approach can be computationally and communicatively intensive, particularly as power systems grow larger, and may not align with the dynamic needs of modern smart grid systems. The reconfiguration process for new or decommissioned generators under such centralized optimization may also necessitate redesign.

To address the mentioned drawbacks, recent research has introduced distributed algorithms [12–20]. The multi-agent consensus algorithm is a typical distributed algorithm. Many researchers apply it to the optimal scheduling of power systems. The basic idea is to relax the power balance constraints using the Lagrange multiplier method and treat the incremental cost as a consensus variable. By using the consensus algorithm, the incremental costs of each generator agent converge to a consensus value. Then, based on the incremental costs obtained, the output power of generators that satisfies operational constraints is determined. For example, in [21], a consensus-based algorithm is proposed to solve economic dispatch problems and calculate the supply-demand power deviation using virtual nodes. In [22], using point-to-point communication, a consensus-based algorithm is presented to solve economic dispatch problems involving flexible loads. In [23], a "consensus + innovations" method is proposed to handle supply-demand balance global equality constraints and solve the energy management problems with flexible loads and energy storage units. A double-layer distributed control strategy based on the consensus algorithm is proposed in [24], effectively addressing distributed energy optimal dispatch problems. These algorithms aim to partition the central optimization process into multiple localized optimizations. By enabling communication among neighboring agents, the global objective cost function can be minimized. Distributed algorithms offer distinct advantages over centralized approaches, including: (1) reduced computational and communication costs; (2) compatibility with the plug-and-play nature of smart grid systems for enhanced flexibility in algorithm design; (3) resilience against single-point failures.

Recently, the integration of renewable energy generators into power systems has emerged as a strategic response to energy and environmental challenges [25–28]. Among the various types of renewable energy sources, wind turbines (WTs) have gained prominence due to the abundant availability and eco-friendly nature of wind energy, coupled with the advanced maturity of turbine technologies. Consequently, the EDP necessitates a reformulation that encompasses not only traditional generators (TGs) but also includes renewable energy sources like WTs. However, the intermittent and stochastic nature of wind speeds introduces uncertainty into wind power output, thus posing a challenge in designing efficient distributed economic dispatch strategies. The integration of energy storage devices within wind farms enables the bridging of the gap between scheduled and actual wind power outputs, thereby allowing for the scheduling of stochastic wind power akin to TGs. Addressing this scenario, a deterministic model proposed in [29] was employed to define the cost function associated with wind power, incorporating both underestimation and overestimation costs related to the available wind power.

The alternating direction method of multipliers (ADMM) is an optimization algorithm that combines the advantages of dual decomposition and augmented Lagrangian methods. Many scholars apply the ADMM algorithm in power systems, especially in the area of distributed optimal dispatch. In [30], a distributed ADMM algorithm is proposed to solve the economic dispatch problem of isolated microgrids, taking into account non-quadratic cost convex functions. In [31], the environmental cost term in the objective function is considered in a non-quadratic form, and time-coupled ramp constraints are included in the constraints. The optimal scheduling problem is solved in a distributed manner using the ADMM algorithm. Flexible loads and line losses are considered in [32]. A strategy combining the consensus algorithm and the serial ADMM algorithm is proposed to achieve a distributed solution to dynamic economic dispatch problems. In [33], an ADMM-based distributed economic dispatch strategy considering time delays and packet drops is proposed. This approach can address the EDP with local constraints of generators and nonquadratic convex cost functions.

However, few studies consider the communication times, which are associated with the iterations of the algorithm. The above ADMM algorithms typically demand a considerable amount of time and communications to achieve convergence, so the compute cost and communication cost are relatively high. In this paper, for EDP with random wind power, a parallel and finite-step consensus-based ADMM algorithm is proposed. The main contributions of this paper are as follows:

(1) An economic dispatch model considering random wind power is established. A fully distributed strategy combining the parallel ADMM algorithm and a finite-step consensus approach is proposed.
(2) The outer layer framework of the strategy adopts a parallel ADMM algorithm, where different variables can be iteratively calculated simultaneously to improve computational efficiency.
(3) The inner layer of the strategy adopts a finite-step consensus algorithm, which does not require a central controller. Agents in the network communicate with each other and perform local computations. The algorithm converges to the optimal value after a

finite number of communication iterations. The number of communication iterations depends on the size of the network.

(4) The finite-step consensus algorithm in the inner layer requires agents to obtain information about the eigenvalues of the communication network Laplacian matrix. A distributed algorithm is designed to calculate the eigenvalues based on local and neighbor information.

The rest of this paper is organized as follows: Section 2 introduces the graph theory and describes the EDP model with random wind power. A fully distributed parallel and finite-step consensus-based algorithm is proposed in Section 3. Simulation cases are carried out in Section 4 to demonstrate the effectiveness of the proposed method. Finally, Section 5 concludes this paper.

2. Problem Formulation

2.1. Graph Theory

$G = (V, E)$ describes the network topology of a communication graph, where $V = \{1, 2, \ldots, n\}$ denotes the set of communication nodes and $E \subseteq V \times V$ denotes communication links. The graph G here refers to an undirected graph with no graph loops. The adjacency matrix $A = \{a_{ij}\}_{n \times n}$ describes the connectivity between communication nodes. The diagonal element a_{ii} = 0, and a_{ij} = 1 if nodes i and j are connected; otherwise, a_{ij} = 0. $\mathcal{N}_i = \{V_j \in V \mid (V_j, V_i) \in E\}$ denotes the neighbor nodes of node i. $|\mathcal{N}_i|$ is the number of neighbor nodes. $d_i = \sum_{j \in N_i} a_{ij}$ represents the degree of node i. The corresponding Laplacian matrix L is given as [34]

$$\begin{cases} l_{ii} = \sum_{i \neq j} a_{ij} & \text{for diagonal elements} \\ l_{ij} = -a_{ij} & \text{for off-diagonal elements} \end{cases} \tag{1}$$

where l_{ij} denotes the element of the ith row and jth column of L. The Laplancian matrix L of an undirected graph with n nodes has n eigenvalues, including one zero eigenvalue, and the others are all positive, i.e., $\lambda_1 = 0, 0 < \lambda_2 \leq \cdots \leq \lambda_n$.

2.2. Economic Dispatch Model with Random Wind Power

In this section, an economic dispatch model with random wind power is proposed. For traditional generators, the cost function is usually modeled as the following quadratic function:

$$C_i(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{2}$$

where P_i denotes the output power of the ith generator. a_i, b_i, and c_i are the corresponding cost coefficients.

In addition to considering the generation costs, it is also important to consider the environmental benefits. To account for the carbon trading costs. Traditional power plants need to pay a certain penalty for greenhouse gas emissions. The carbon trading cost function is defined as follows:

$$G_i(P_i) = U_c \times \left(G_{r,i}(P_i) - G_{q,i}(P_i)\right) \tag{3}$$

where U_c denotes the carbon trading price. $G_{r,i}(P_i)$ and $G_{q,i}(P_i)$ represent the actual carbon emissions and the carbon emission standard quota of the power plant, respectively. The expressions for $G_{r,i}(P_i)$ and $G_{q,i}(P_i)$ are defined as follows:

$$G_{r,i}(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \tag{4}$$

$$G_{q,i}(P_i) = \delta P_i \tag{5}$$

where α_i, β_i, and γ_i are the carbon emission coefficients of the generator i, respectively. δ represents the standard carbon emission coefficient.

Due to the randomness of wind power output, there is often a deviation between the planned and actual wind power output, resulting in underestimation and overestimation phenomena. When underestimation occurs, meaning that the planned wind power output is less than the actual available wind power, the system needs to call upon the lower reserve capacity or charge the energy storage devices to balance the power. The cost incurred due to the underestimation is referred to as the underestimation penalty cost. Similarly, when overestimation occurs, meaning that the planned wind power output is greater than the actual available wind power, the system needs to call upon the upper reserve capacity or have the energy storage devices compensate for some of the power, which is referred to as the overestimation penalty cost. Additionally, the direct operating costs should also be considered. Based on this, the expected cost model for stochastic wind power is established as follows:

$$H_j(W_j) = \eta_j^d W_j + \eta_j^{ue} \mathbf{E}\left(W_j^{ue}\right) + \eta_j^{oe} \mathbf{E}\left(W_j^{oe}\right) \tag{6}$$

where W_j denotes the scheduled output power of the jth WT unit. The first term on the right side of the equation is the direct operating cost. The last two terms denote the underestimation and overestimation penalty costs, respectively. η_j^d, η_j^{ue}, and η_j^{oe} are the corresponding cost coefficients. The expressions for $\mathbf{E}\left(W_j^{ue}\right)$ and $\mathbf{E}\left(W_j^{oe}\right)$ are defined as follows.

$$\mathbf{E}\left(W_j^{ue}\right) = \int_{W_j}^{W_j^r} (w_{av,j} - W_j) f_W(w_{av,j}) dw_{av,j} \tag{7}$$

$$\mathbf{E}\left(W_j^{oe}\right) = \int_0^{W_j} (W_j - w_{av,j}) f_W(w_{av,j}) dw_{av,j} \tag{8}$$

where W_j^r denotes the rated output power of the jth WT unit. $w_{av,j}$ is the random variable of the available output power of the jth WT unit. And $f_W(w_{av,j})$ denotes the probability distribution function of the available wind power $w_{av,j}$, which is described by the Weibull distribution in this paper. The closed forms of $\mathbf{E}(W_l^{ue})$ and $\mathbf{E}(W_l^{oe})$ are given as follows:

$$\begin{aligned}\mathbf{E}\left(W_j^{ue}\right) =& \left(W_j^r - W_j\right)\left(\exp\left(-\frac{v_r^\kappa}{c^\kappa}\right) - \exp\left(-\frac{v_{\text{out}}^\kappa}{c^\kappa}\right)\right) \\ &+ \left(\frac{W_j^r v_{\text{in}}}{v_r - v_{\text{in}}} + W_j\right)\left(\exp\left(-\frac{v_r^\kappa}{c^\kappa}\right) - \exp\left(-\frac{v_j^\kappa}{c^\kappa}\right)\right) \\ &+ \frac{W_j^r c}{v_r - v_{\text{in}}}\left(\Gamma\left(1 + \frac{1}{\kappa}, \left(\frac{v_j}{c}\right)^\kappa\right) - \Gamma\left(1 + \frac{1}{\kappa}, \left(\frac{v_r}{c}\right)^\kappa\right)\right)\end{aligned} \tag{9}$$

$$\begin{aligned}\mathbf{E}\left(W_j^{oe}\right) =& W_j\left(1 - \exp\left(-\frac{v_{in}^\kappa}{c^\kappa}\right) + \exp\left(-\frac{v_{\text{out}}^\kappa}{c^\kappa}\right)\right) \\ &+ \left(\frac{W_j^r v_{in}}{v_r - v_{in}} + W_j\right)\left(\exp\left(-\frac{v_{in}^\kappa}{c^\kappa}\right) - \exp\left(-\frac{v_j^\kappa}{c^\kappa}\right)\right) \\ &+ \frac{W_j^r c}{v_r - v_{in}}\left(\Gamma\left(1 + \frac{1}{\kappa}, \left(\frac{v_j}{c}\right)^\kappa\right) - \Gamma\left(1 + \frac{1}{\kappa}, \left(\frac{v_{in}}{c}\right)^\kappa\right)\right)\end{aligned} \tag{10}$$

The optimization objective of the EDP is to minimize the total cost, including the generation cost and the carbon trading cost of TG units, and the expected cost of WT units. The objective function can be written as follows:

$$\min \sum_{i=1}^{m} C_i(P_i) + \sum_{i=1}^{m} G_i(P_i) + \sum_{j=1}^{n} H_j(W_j) \tag{11}$$

where m and n denote the amount of the TG units and WT units in the power grid, respectively.

The operation of the power systems required to satisfy certain constraints, which are as follows:

(1) Supply-demand balance constraint

$$\sum_{i=1}^{m} P_{i,t} + \sum_{j=1}^{n} W_j = D \tag{12}$$

(2) Individual generator capacity constraint

$$P_{i,t}^{\min} \leq P_{i,t} \leq P_{i,t}^{\max}, \quad i = 1, 2, \cdots, m \tag{13}$$

$$0 \leq W_{j,t} \leq W_{j,t}^{r}, \quad j = 1, 2, \cdots, n \tag{14}$$

Represent the output power of each generator unit uniformly as a vector:

$$X = [P_1, \cdots, P_m, W_1, \cdots, W_n]^T \tag{15}$$

The cost of each item in the objective function is only related to the output power of the unit itself, so it can be decomposed into individual $m + n$ sub-functions. Define the united form of the objective function as:

$$J_i(X_i) = \begin{cases} C_i(P_i) + G_i(P_i), & i = 1, 2, \cdots, m \\ H_j(W_j), & i = m+1, \cdots m+j, \cdots, m+n \end{cases} \tag{16}$$

Therefore, the EDP is modeled as:

$$\begin{cases} \min_X J(X) = \sum_{i=1}^{m+n} J_i(X_i) \\ \text{s.t.} \\ \sum_{i=1}^{m} P_i + \sum_{j=1}^{n} W_j = D \\ P_i^{\min} \leq X_i \leq P_i^{\max}, \quad i = 1, 2, \cdots, m \\ 0 \leq X_{m+j} \leq W_j^r, \quad j = 1, 2, \cdots, n \end{cases} \tag{17}$$

3. Distributed Implementation by Parallel and Finite-Step Consensus-Based ADMM Algorithm

In this section, we will propose a distributed ADMM algorithm to solve the EDP with random wind power. Each generation unit achieves overall optimization through mutual communication and cooperation.

3.1. Parallel ADMM Algorithm

To facilitate the algorithm design, the EDP model (17) is transformed into a standard form for ADMM. Define two closed convex sets as:

$$\Omega_1 = \left\{ \begin{array}{c} X \in \mathbb{R}^{m+n} : P_i^{\min} \leq X_i \leq P_i^{\max}, i = 1, \cdots, m; \\ 0 \leq X_{m+j} \leq W_j^r, j = 1, \cdots, n; \end{array} \right\} \tag{18}$$

$$\Omega_2 = \left\{ Y \in \mathbb{R}^{m+n} : \sum_{i=1}^{m} P_i + \sum_{j=1}^{n} W_j = D \right\} \tag{19}$$

where Ω_1 and Ω_2 are the closed convex sets. Then, define the indicator functions of the two convex sets as:

$$\hbar_1(X) = \begin{cases} 0 & : X \in \Omega_1 \\ \infty & : X \notin \Omega_1 \end{cases} \tag{20}$$

$$\hbar_2(Y) = \begin{cases} 0 & : Y \in \Omega_2 \\ \infty & : Y \notin \Omega_2 \end{cases} \tag{21}$$

Therefore, the alternative formulation of the EDP model (17) can be written as

$$\begin{array}{ll} \min & J(X) + \hbar_1(X) + \hbar_2(Y) \\ \text{s.t.} & X - Y = 0 \end{array} \tag{22}$$

The cost function is strongly convex, and the defined indicator function is a non-empty closed convex function. Therefore, the EDP model is convex. The ADMM algorithm can be used to solve the EDP.

$$L_\theta(X, Y, \lambda) = J(X) + \hbar_1(X) + \hbar_2(Y) + \lambda^T(X - Y) + \frac{\theta}{2}\|X - Y\|_2^2 \tag{23}$$

where λ is the Lagrange multiplier and θ is the coefficient of the regularization term.

Let $\rho = \frac{\lambda}{\theta} \in \mathbb{R}^{m+n}$, which is called the scaling dual variable. Equation (23) can be rewritten as:

$$L_\theta(X, Y, \rho) = J(X) + \hbar_1(X) + \hbar_2(Y) + \frac{\theta}{2}\|X - Y + \rho\|_2^2 - \frac{\lambda^T\lambda}{2\theta} \tag{24}$$

Different from the traditional Gauss-Seidel ADMM algorithm, a parallel ADMM algorithm is proposed, which requires adding proximal terms to the Lagrangian function during solving. The update protocol is as follows:

$$X^{k+1} = \arg\min_X L_\theta\left(X, Y^k, \rho^k\right) + \frac{1}{2}\left(X - X^k\right)^T \Phi\left(X - X^k\right) \tag{25}$$

$$Y^{k+1} = \arg\min_Y L_\theta\left(X^k, Y, \rho^k\right) + \frac{1}{2}\left(Y - Y^k\right)^T \Psi\left(Y - Y^k\right) \tag{26}$$

$$\rho^{k+1} = \rho^k + \sigma\theta\left(X^k - Y^k\right) \tag{27}$$

where Φ and Ψ are the coefficient matrix of the proximal terms regarding X and Y. σ is the update coefficients for scaling dual variables. They should satisfy

$$\Phi \succ \theta\left(\frac{1}{\mu_1} - 1\right)I, \Psi \succ \theta\left(\frac{1}{\mu_2} - 1\right)I, \mu_1 + \mu_2 < 2 - \sigma \tag{28}$$

For convenience, let $\Phi = \phi I$ and $\Psi = \psi I$, which satisfy the following constraints:

$$\phi > \theta\left(\frac{1}{\mu_1} - 1\right), \psi > \theta\left(\frac{1}{\mu_2} - 1\right), \mu_1 + \mu_2 < 2 - \sigma \tag{29}$$

3.2. Distributed Implementation Based on Finite-Step Consensus Algorithm

From Equations (25)–(27), it can be seen that ρ^{k+1} can be directly distributed locally after X^k and Y^k are obtained; what needs to be solved next is how to calculate X^k and Y^k in a distribute way.

(1) X-update step

According to Equation (25), one has

$$\begin{aligned} X^{k+1} &= \arg\min_{X} L_\theta\left(X, Y^k, \rho^k\right) + \frac{1}{2}\left(X - X^k\right)^T \Phi\left(X - X^k\right) \\ &= \arg\min_{X} J(X) + \hbar_1(X) + \frac{\theta}{2}\left\|X - Y^k + \rho^k\right\|_2^2 + \frac{1}{2}\left(X - X^k\right)^T \Phi\left(X - X^k\right) \\ &= \arg\min_{X\in\Omega_1} J(X) + \frac{\theta}{2}\left\|X - Y^k + \rho^k\right\|_2^2 + \frac{1}{2}\left(X - X^k\right)^T \Phi\left(X - X^k\right) \\ &= \arg\min_{X\in\Omega_1} f(X) \end{aligned} \tag{30}$$

where $f(X)$ can be expressed as

$$f(X) = J(X) + \frac{\theta}{2}\left\|X - Y^k + \rho^k\right\|_2^2 + \frac{1}{2}\left(X - X^k\right)^T \Phi\left(X - X^k\right) \tag{31}$$

$f(X)$ can be decomposed into the summation form of $m+n$ local functions, which can be written as

$$f_i(X_i) = J_i(X_i) + \frac{\theta}{2}\left(X_i - Y_i^k + \rho_i^k\right)^2 + \frac{\phi}{2}\left(X_i - X_i^k\right)^2 \tag{32}$$

It is easily seen that $X^k + 1$ is actually the solution to the following equivalent optimization problem

$$\begin{array}{ll} \min & \sum_{i=1}^{m+n} f_i(X_i) \\ \text{s.t.} & P_i^{\min} \le X_i \le P_i^{\max}, i = 1, \cdots, m; \\ & 0 \le X_{m+j} \le W_j^r, j = 1, \cdots, n; \end{array} \tag{33}$$

whose analytical solution is easily found by

$$X_i^{k+1} = \begin{cases} P_i^{k+1} = \min\left\{\max\left\{\nabla f_i^{-1}(0), P_i^{\min}\right\}, P_i^{\max}\right\}, & i = 1, \cdots, m; \\ W_j^{k+1} = \min\left\{\max\left\{\nabla f_{m+j}^{-1}(0), 0\right\}, W_j^r\right\}, & j = 1, \cdots, n; \end{cases} \tag{34}$$

(2) Y-update step

According to Equation (26), one has

$$\begin{aligned} Y^{k+1} &= \arg\min_{Y} L_\theta\left(X^k, Y, \rho^k\right) + \frac{1}{2}\left(Y - Y^k\right)^T \Psi\left(Y - Y^k\right) \\ &= \arg\min_{Y} \hbar_2(Y) + \frac{\theta}{2}\left\|X^k - Y + \rho^k\right\|_2^2 + \frac{\psi}{2}\left\|Y - Y^k\right\|_2^2 \\ &= \arg\min_{Y\in\Omega_2} \frac{\theta}{2}\left\|X^k - Y + \rho^k\right\|_2^2 + \frac{\psi}{2}\left\|Y - Y^k\right\|_2^2 \\ &= \arg\min_{Y\in\Omega_2} \sum_{i=1}^{m+n} \frac{\theta}{2}\left(X_i^k - Y_i + \rho_i^k\right)^2 + \frac{\psi}{2}\left(Y_i - Y_i^k\right)^2 \\ &= \arg\min_{Y\in\Omega_2} \sum_{i=1}^{m+n} g_i(Y_i) \end{aligned} \tag{35}$$

where $g_i(Y_i)$ is expressed as

$$g_i(Y_i) = \frac{\theta}{2}\left(X_i^k - Y_i + \rho_i^k\right)^2 + \frac{\psi}{2}\left(Y_i - Y_i^k\right)^2 \tag{36}$$

Similarly, it can be seen that Y^{k+1} is actually the solution to the following equivalent optimization problem

$$\begin{array}{ll} \min & \sum_{i=1}^{m+n} g_i(Y_i) \\ \text{s.t.} & \sum_{i=1}^{m} P_i + \sum_{j=1}^{n} W_j = D \end{array} \tag{37}$$

For problem (37), the Lagrangian multiplier method is used to transform it into an unconstrained optimization problem, and the Lagrangian function is constructed as follows:

$$L_Y = \sum_{i=1}^{m+n} g_i(Y_i) + \lambda_Y \left(D - \sum_{i=1}^{m+n} Y_i \right) \tag{38}$$

where λ_Y is the Lagrangian multiplier of the optimization problem. According to the KKT (Karush-Kuhn-Tucker) condition, one has

$$\frac{\partial L_Y}{\partial Y_i} = \theta\left(Y_i - \left(X_i^k + \rho_i^k\right)\right) + \psi\left(Y_i - Y_i^k\right) - \lambda_Y = 0, \quad i = 1, \cdots m+n \tag{39}$$

$$D - \sum_{i=1}^{m+n} Y_i = 0 \tag{40}$$

By solving the above two equations, the optimal Lagrange multiplier can be obtained as

$$\lambda_Y^* = \frac{(\theta+\psi)D - \sum_{i=1}^{m+n}\left[\theta\left(X_i^k + \rho_i^k\right) + \psi Y_i^k\right]}{m+n} \tag{41}$$

Then, the optimal solution to the optimization problem (37) can be obtained, which is expressed as

$$Y_i^{k+1} = \frac{\lambda_Y^* + \theta\left(X_i^k + \rho_i^k\right) + \psi Y_i^k}{\theta + \psi} \tag{42}$$

Next, the finite-step consensus algorithm is utilized to calculate λ_Y^* so that Y_i^{k+1} can be updated locally. The iterative protocol of the finite-step consensus algorithm is as follows

$$\begin{cases} \tau_i^k(r+1) = w_{ii}(r)\tau_i^k(r) + \sum_{j\in\mathcal{N}_i} w_{ij}(r)\tau_j^k(r), & i = 1, \cdots m+n \\ \eta_i^k(r+1) = w_{ii}(r)\eta_i^k(r) + \sum_{j\in\mathcal{N}_i} w_{ij}(r)\eta_j^k(r), & i = 1, \cdots m+n \end{cases} \tag{43}$$

where $\tau_i^k(r)$ and $\eta_i^k(r)$ are auxiliary variables. $w_{ij}(r)$ is the weight coefficient at kth iteration, which is updated by the following protocol

$$w_{ij}(r) = \begin{cases} 1 - \frac{|\mathcal{N}_i|}{\lambda_{r+1}}, & j = i \\ 0, & j \notin \mathcal{N}_i \cup \{i\} \\ \frac{1}{\lambda_{r+1}}, & j \in \mathcal{N}_i, \quad r = 1, \cdots, K \end{cases} \tag{44}$$

where $K = m + n$. And the initial values of $\tau_i^k(r)$ and $\eta_i^k(r)$ are set as

$$\tau_i^k(0) = D_i^k, \quad \eta_i^k(0) = \theta\left(X_i^k + \rho_i^k\right) + \psi Y_i^k \tag{45}$$

After K iterations, both auxiliary variables converge to the consensus values as follows.

$$\begin{cases} \overline{\tau_i^k} = \tau_i^k(K) = \frac{\sum_{i=1}^{m+n} D_i}{m+n} \\ \overline{\eta_i^k} = \eta_i^k(K) = \frac{\sum_{i=1}^{m+n}\left[\theta\left(X_i^k+\rho_i^k\right)+\psi Y_i^k\right]}{m+n} \end{cases} \tag{46}$$

Therefore, the optimal value of the Lagrange multiplier $\lambda_{Y_t}^*$ is given by

$$\lambda_{Y_t}^* = (\theta + \psi)\overline{\tau_i^k} - \overline{\eta_i^k} \tag{47}$$

(3) ρ-update step

$$\rho_i^{k+1} = \rho_i^k + \sigma\theta\left(X_i^k - Y_i^k\right) \tag{48}$$

(4) Algorithm convergence

The optimal output power and optimal Lagrange multiplier of the optimization problem (17) can be obtained through the above distributed algorithm. When both the primal and dual residuals of the ADMM algorithm meet certain threshold conditions, the algorithm terminates the iterative process. Define the primal residual and the dual residual as

$$res_p^k \triangleq \left\|X^k - Y^k\right\|_2, \quad res_d^k \triangleq \left\|-\theta\left(Y^k - Y^{k-1}\right)\right\|_2 \tag{49}$$

The primal and dual residuals must meet the following threshold conditions:

$$res_p^k \le \varepsilon^{\mathrm{pri}}, \quad res_d^k \le \varepsilon^{\mathrm{dual}} \tag{50}$$

where $\varepsilon^{\mathrm{pri}}$ and $\varepsilon^{\mathrm{dual}}$ are the convergence thresholds for the primal residual and the dual residual, respectively.

3.3. Distributed Calculation for Eigenvalues of Laplacian Matrix

It can be seen that the iteration of the finite-step consensus algorithm requires knowledge of the eigenvalues of the Laplacian matrix of the communication network. In literature [35], a graph discovery algorithm is proposed to obtain network topology information. This algorithm identifies topology information by assigning device numbers to intelligent agents and using broadcast algorithms. However, the iteration process of this algorithm is cumbersome and computationally intensive, especially when the scale of the network is relatively large. In this section, a distributed consensus algorithm is proposed to calculate the eigenvalues of the Laplacian matrix.

Firstly, transform the Laplace matrix into a nonsingular matrix through an elementary transformation.

$$\bar{L} = L + \varepsilon I_n \tag{51}$$

where ε is a positive constant and I_n is the identity matrix.

Since $\bar{L}$ is nonsingular, we then have the following linear problem of calculating $\bar{L}^{-1}$

$$\bar{L}\bar{L}^{-1} = I_n \tag{52}$$

The eigenvalues and eigenvectors of matrices $\bar{L}$ and $\bar{L}^{-1}$ satisfy the following equation:

$$\lambda_i(L) = \frac{1}{\lambda_i(\bar{L}^{-1})} - \varepsilon, \quad v_i(L) = v_i\left(\bar{L}^{-1}\right) \tag{53}$$

Then, a distributed consensus algorithm is used to calculate $\bar{L}^{-1}$. The update protocol of the algorithm is as follows:

$$Z_i(s+1) = Z_i(s) - \frac{1}{|\mathcal{N}_i|}P_{\perp,i}\left(|\mathcal{N}_i|Z_i(s) - \sum_{j\in\mathcal{N}_i} Z_j(s)\right) \tag{54}$$

where $Z_i(s)$ is the state variable of the node i at sth iteration. $P_{\perp,i} = P^T_{\perp,i} \in \mathbb{R}^{n\times n}$ is the orthogonal projection matrix of the kernel of $[\bar{L}]_{i*}$. $P_{\perp,i}$ can be expressed as

$$P_{\perp,i} = I_n - \frac{1}{[\bar{L}]^T_{i*}[\bar{L}]_{i*}}[\bar{L}]_{i*}[\bar{L}]^T_{i*} \quad (55)$$

where $[\bar{L}]_{i*}$ denotes the ith row vector of $\bar{L}$.

The initial value $Z_i(0)$ satisfies

$$[\bar{L}]^T_{i*}Z_i(0) = [I_n]^T_{i*} \quad (56)$$

The state values converge when satisfying

$$\|Z_i(s+1) - Z_i(s)\| \le \varepsilon^L \quad (57)$$

where ε^L is the convergence threshold.

Then, we can obtain that

$$\lambda_i(L) = \frac{1}{\lambda_i(Z_i^*)} - \varepsilon \quad (58)$$

where Z_i^* is the final steady-state consensus value of Equation (54).

Therefore, the eigenvalues of Laplacian matrix of the communication network can be calculated in a distributed manner, only requiring the local information.

4. Simulation Results

In this section, several case studies are presented in order to illustrate and validate the proposed algorithm. The case studies are simulated in the MATLAB R2021a environment on a laptop with Intel Core i5-7300U CPU @2.60 GHz and 8 GB RAM. The test systems are based on the IEEE 39-bus system, which includes 10 generator units. Generators 1 to 9 are TG units, while Generator 10 is replaced with a WT unit. The communication topology is an undirected network. The entire system including the power network and communication network is shown in Figure 1. The orange nodes represent the communication nodes. The red dashed lines indicate the connectivity between communication nodes.

In this case study, the generation cost parameters, operating parameters, and carbon emission coefficients of TG units are shown in Table 1. The relevant cost and operating parameters of WT units, as well as the parameters of the Weibull distribution function, are shown in Table 2. The power unit is MW, the currency unit is \$, and the carbon emission unit is t. The unit of wind speed is m/s, the carbon emission standard coefficient is $\delta = 0.7\ \text{t}/(\text{MW}\cdot\text{h})$, and the carbon trading price is $U_c = 1\ \$/(\text{tCO}_2)$.

The local load values are set as 250 MW, 250 MW, 250 MW, 250 MW, 250 MW, 50 MW, 50 MW, 50 MW, 50 MW, 50 MW, and respectively. Thus, the total power demand is 1500 MW. The stepsizes of the parallel ADMM algorithm are set as $\theta = 0.06$, $\sigma = 0.5$. The coefficients of the proximal terms are set as $\phi = 0.06$, $\psi = 0.06$. The convergence threshold for the distributed consensus algorithm when calculating the eigenvalues of the Laplacian matrix is set as $\varepsilon^L = 0.001$. The convergence thresholds for primal and dual residuals are set as $\varepsilon^{\text{pri}} = 0.001$ and $\varepsilon^{\text{dual}} = 0.001$, respectively.

The topology of the communication network is shown in Figure 1. The corresponding Laplacian matrix L is:

$$\begin{bmatrix} 4 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 5 & -1 & -1 & -1 & 0 & 0 & 0 & 0 & -1 \\ -1 & -1 & 4 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 5 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 6 & -1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 4 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 5 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & -1 & 5 & -1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 & -1 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 4 \end{bmatrix}$$

The positive constant is set as $\varepsilon = 1$. The eigenvalues of the Laplacian matrix can be calculated by the proposed distributed consensus algorithm. From λ_1 to λ_{10}, the values are 0, 2.1085, 2.4775, 4.4875, 5.0000, 5.3666, 6.0000, 6.3258, 6.8359, and 7.3982. Based on the obtained eigenvalues, the finite-step consensus algorithm is used to calculate λ_{Υ}^*. The iteration result of τ_i^k is shown in Figure 2. After 10 iterations, all agents converge to the average value of total load demand, i.e., 150 MW. Similarly, $\overline{\eta_i^k}$ can be obtained by 10 iterations. Then, the optimal value of the Lagrange multiplier for each iteration can be obtained, allowing for the calculation of Υ_i^k.

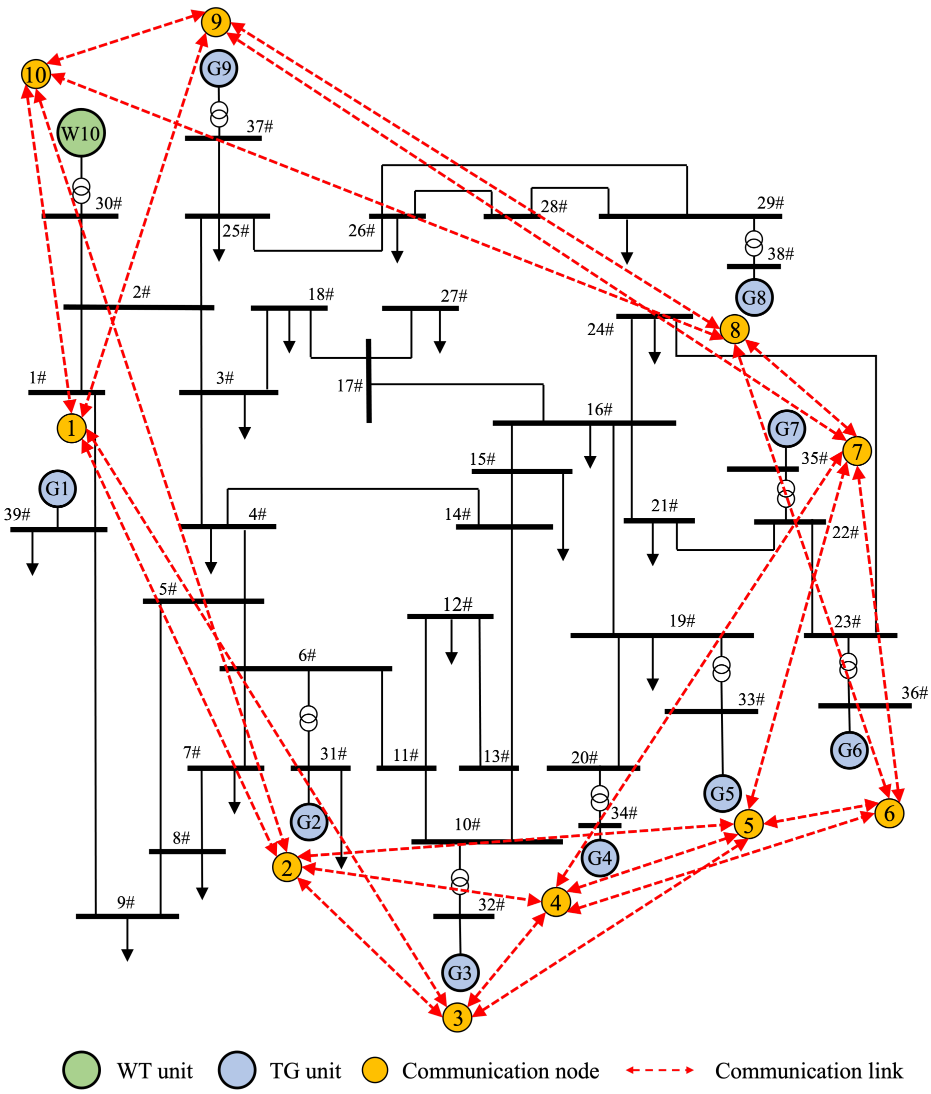

Figure 1. IEEE 39-bus system and undirected communication network.

Table 1. Parameters of TG units in IEEE 39-bus system.

Unit	a_i (\$/MW^{2}h)	b_i (\$/MWh)	c_i (\$/h)	α_i (t/MW^{2}h)	β_i (t/MWh)	γ_i (t/h)	P_i^{min} (MW)	P_i^{max} (MW)
G1	0.0024	5.56	30	0.00155	1.0347	0	60	339.69
G2	0.0056	4.32	25	0.00122	1.0314	0	25	479.10
G3	0.0072	6.60	25	0.00144	0.9836	0	28	290.40
G4	0.0047	3.14	16	0.00161	1.2423	0	40	306.34
G5	0.0091	7.54	6	0.00168	1.0836	0	35	593.80
G6	0.0018	3.28	54	0.00156	1.1727	0	29	137.19
G7	0.0053	7.31	23	0.00126	1.0878	0	45	595.40
G8	0.0063	2.45	15	0.00159	0.9115	0	56	162.17
G9	0.0028	7.63	20	0.00168	0.9162	0	12	165.10

Table 2. Parameters of WT units.

Wind Turbine	v_{in} 5	v_{out} 45	v_r 15	(c, κ) (8, 2)
Wind Power	d_j 5	η_j^{ue} 3.1	η_j^{oe} 3.1	W_j^r 50

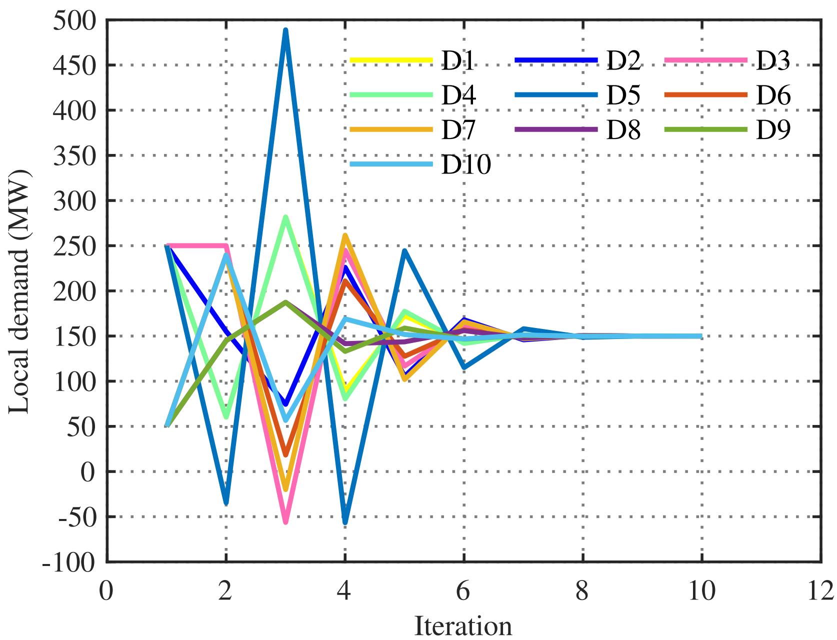

Figure 2. Local demand calculation based on finite-step consensus algorithm.

The simulation results of output power X and Y are shown in Figures 3 and 4. It can be seen that the iterative curve of X is smoother and has smaller fluctuations, which is because in the sub-optimization problem, the inequality constraints limit the range of variation. The power supply and demand relationship diagram is shown in Figure 5. It can be seen that after a certain number of iterations, the sum of X is equal to the total load demand value, while the sum of Y is always equal to the total load demand during the iteration calculation process. This is because in the sub-optimization problem, the equality constraint of supply and demand balance restricts the sum of Y. The residual results are shown in Figure 6. The dual residual first converges below the threshold, and after 508 iterations, the primal residual also converges below the threshold. Thus, the results converge to the optimal value.

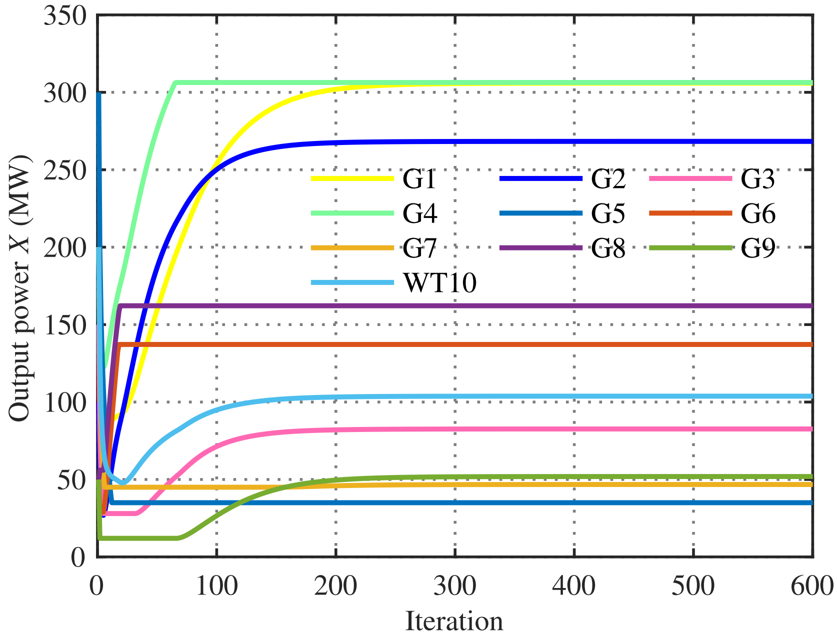

Figure 3. Simulation result of output power X.

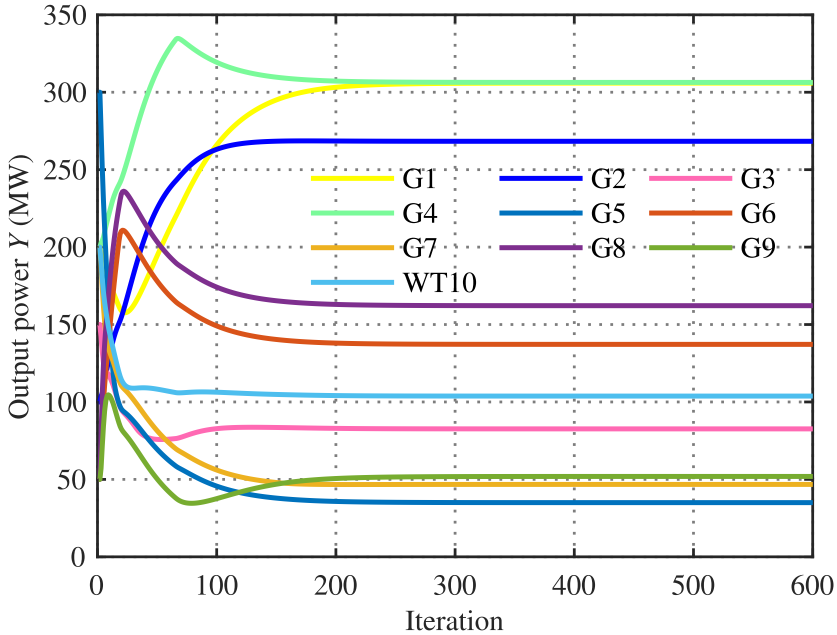

Figure 4. Simulation result of output power Y.

To verify the accuracy and effectiveness of the proposed PFC-ADMM algorithm, it is compared with the centralized algorithm, which is solved using the fmincon solver based on the interior point method. Furthermore, it is also compared with the consensus-based parallel ADMM (C-PADMM) algorithm proposed in reference [36] and the distributed gradient descent ADMM (DGD-ADMM) algorithm proposed in reference [37]. The comparison results are shown in Table 3.

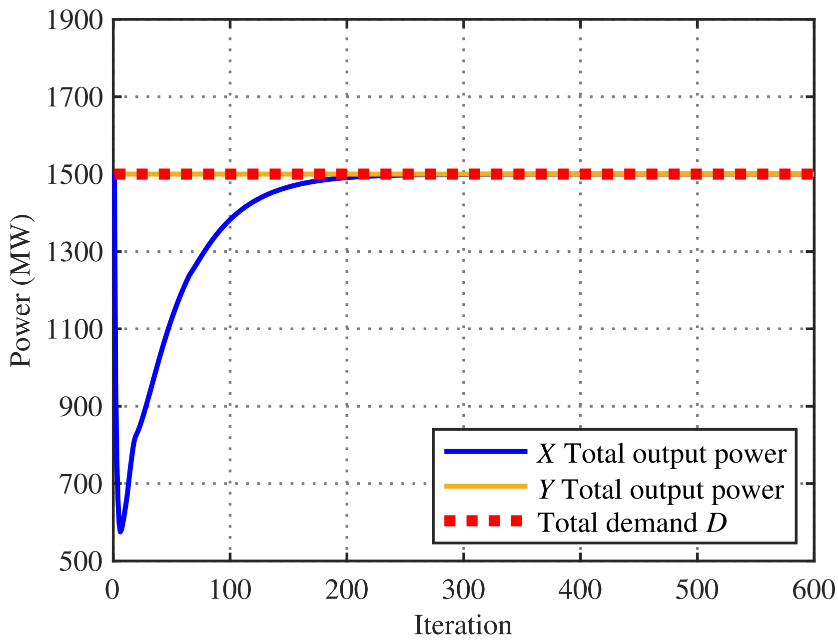

Figure 5. Supply-demand balance result.

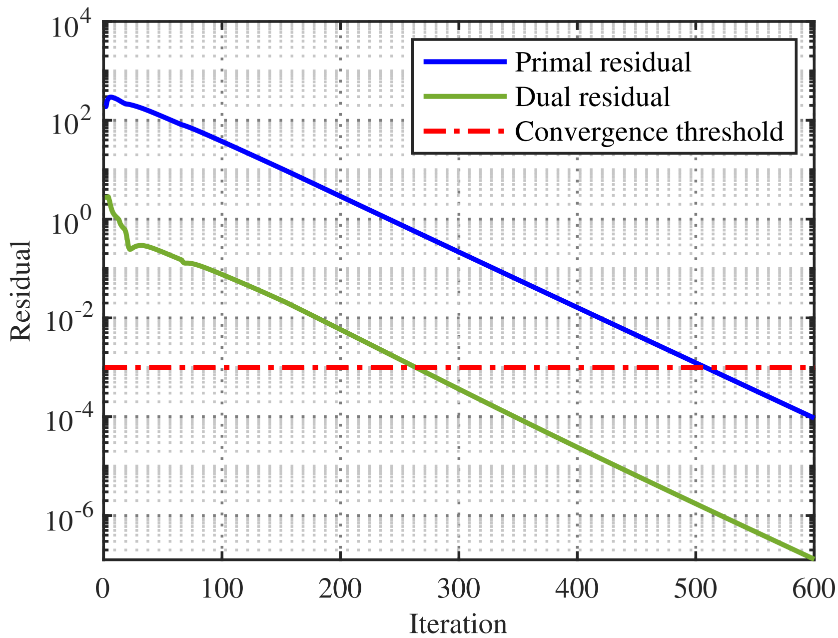

Figure 6. Primal and dual residuals.

It should be noted that the total number of communications here refers to the number of communications among agents when the algorithm converges to the optimal, rather than the cumulative number of communications between individual nodes and other nodes, nor the total number of communications that occurred in the communication network, because the latter two are closely related to the topology connectivity of the communication network. It can be seen that the simulation results of the three distributed algorithms, i.e., the optimal output power and the optimal incremental cost, both approach the centralized algorithm with very small errors. The proposed PFC-ADMM algorithm and the C-PADMM algorithm are both parallel ADMM algorithms, so they have the same number of outer-layer iterations. Due to using the finite-step consensus algorithm in this paper, the number of iterations is significantly reduced, so the total number of communications is correspondingly reduced. The DGD-ADMM algorithm uses a serial ADMM algorithm, so it has fewer outer-layer iterations. A distributed gradient descent algorithm is used in its inner-layer optimization, and the number of iterations gradually decreases as the outer-layer optimization approaches the optimal solution, decreasing from 144 times to 5 times in this case study. The total number of communications is slightly higher than the PFC-ADMM algorithm. As the network size expands and the number of nodes increases, the advantages of the algorithm proposed in this paper will become more significant. It is worth noting that the outer-layer framework of the algorithm proposed in this paper can also adopt the serial ADMM, further reducing the total number of communications. Although parallel ADMM algorithms are slower in convergence speed than serial ADMM, they also have their advantages. When computing variables X_i^{k+1}, Y_i^{k+1}, and ρ_i^{k+1} during a single-round calculation, there is no need to wait for the update of the previous variables. Instead, they are updated simultaneously in parallel, saving computation time. Therefore, the overall convergence time is a balanced issue concerning the convergence speed and computation time, which can be measured based on the computational complexity of the sub-optimization problem. Generally, the greater the number of nodes, the more obvious the advantages of adopting parallel ADMM algorithms.

Table 3. Algorithm comparison results.

	Our Algorithm	**C-PADMM [36]**	**DGD-ADMM [37]**	**Centralized Algorithm**
P_1	305.8952	305.8843	305.4740	305.8961
P_2	268.3193	268.3550	268.3043	268.3197
P_3	82.6201	82.6697	82.6160	82.6200
P_4	306.3400	306.3400	306.3400	306.3400
P_5	35.0000	35.0000	35.0000	35.0000
P_6	137.1900	137.1900	137.1899	137.1900
P_7	46.7588	46.7773	46.8605	46.7593
P_8	162.1700	162.1700	162.1699	162.1700
P_9	51.9055	51.8389	52.1444	51.9061
W	103.7987	103.7718	103.9011	103.7989
Incremental cost	8.3113	8.3115	8.3116	8.3113
Outer layer iterations	508	508	147	39
Inner layer iterations	10	26	[5,144]	
Total number of communications	5080	13,208	5404	—

5. Conclusions

In this paper, an economic dispatch model with random wind power is established. The generation cost, the carbon trading cost of TGs, and the stochastic expected cost of wind power are considered. A fully distributed strategy is proposed to deal with the EDP. The out-layer framework of the strategy is based on the parallel ADMM, where different variables can be calculated simultaneously. The inner-layer framework adopts a finite-step consensus algorithm. The algorithm converges to the optimal value after a finite number of communication iterations. In addition, based on local and neighbor information, a distributed algorithm is designed to calculate the eigenvalues of the Laplacian matrix. Thus, the algorithm has been fully distributed.

The optimal dispatch model studied in this paper only considered TGs and WTs. In future work, the coordinated optimization problem of distributed energy sources such as energy storage equipment and photovoltaic generation units needs further investigation. In addition, power flow constraints such as line power capacity limitations and node voltage limitations have not been considered. More security constraints during the actual operation can be further considered.

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References

1. Wang, R.; Ma, D.; Li, M.J.; Sun, Q.; Zhang, H.; Wang, P. Accurate current sharing and voltage regulation in hybrid wind/solar systems: An adaptive dynamic programming approach. *IEEE Trans. Consum. Electron.* **2022**, *68*, 261–272. [CrossRef]
2. Xiong, H.; Shi, Y.; Chen, Z.; Guo, C.; Ding, Y. Multi-stage robust dynamic unit commitment based on pre-extended-fast robust dual dynamic programming. *IEEE Trans. Power Syst.* **2022**, *38*, 2411–2422. [CrossRef]
3. Xue, X.; Ai, X.; Fang, J.; Yao, W.; Wen, J. Real-time schedule of integrated heat and power system: A multi-dimensional stochastic approximate dynamic programming approach. *Int. J. Electr. Power Energy Syst.* **2022**, *134*, 107427. [CrossRef]
4. Xiong, G.; Shuai, M.; Hu, X. Combined heat and power economic emission dispatch using improved bare-bone multi-objective particle swarm optimization. *Energy* **2022**, *244*, 123108. [CrossRef]
5. Aguila-Leon, J.; Vargas-Salgado, C.; Chiñas-Palacios, C.; Díaz-Bello, D. Energy management model for a standalone hybrid microgrid through a particle Swarm optimization and artificial neural networks approach. *Energy Convers. Manag.* **2022**, *267*, 115920. [CrossRef]
6. Zhang, X.; Wang, Z.; Lu, Z. Multi-objective load dispatch for microgrid with electric vehicles using modified gravitational search and particle swarm optimization algorithm. *Appl. Energy* **2022**, *306*, 118018. [CrossRef]
7. Liu, Y.; Ćetenović, D.; Li, H.; Gryazina, E.; Terzija, V. An optimized multi-objective reactive power dispatch strategy based on improved genetic algorithm for wind power integrated systems. *Int. J. Electr. Power Energy Syst.* **2022**, *136*, 107764. [CrossRef]
8. Lü, X.; Wu, Y.; Lian, J.; Zhang, Y.; Chen, C.; Wang, P.; Meng, L. Energy management of hybrid electric vehicles: A review of energy optimization of fuel cell hybrid power system based on genetic algorithm. *Energy Convers. Manag.* **2020**, *205*, 112474. [CrossRef]
9. Yang, Y.; Yang, Z.; Yu, J.; Xie, K.; Jin, L. Fast economic dispatch in smart grids using deep learning: An active constraint screening approach. *IEEE Internet Things J.* **2020**, *7*, 11030–11040. [CrossRef]
10. Wen, L.; Zhou, K.; Yang, S.; Lu, X. Optimal load dispatch of community microgrid with deep learning based solar power and load forecasting. *Energy* **2019**, *171*, 1053–1065. [CrossRef]
11. Yin, L.; Gao, Q.; Zhao, L.; Wang, T. Expandable deep learning for real-time economic generation dispatch and control of three-state energies based future smart grids. *Energy* **2020**, *191*, 116561. [CrossRef]
12. Zhao, C.; He, J.; Cheng, P.; Chen, J. Consensus-based energy management in smart grid with transmission losses and directed communication. *IEEE Trans. Smart Grid* **2016**, *8*, 2049–2061. [CrossRef]
13. Cherukuri, A.; Cortes, J. Initialization-free distributed coordination for economic dispatch under varying loads and generator commitment. *Automatica* **2016**, *74*, 183–193. [CrossRef]
14. Tang, Z.; Hill, D.J.; Liu, T. A novel consensus-based economic dispatch for microgrids. *IEEE Trans. Smart Grid* **2018**, *9*, 3920–3922. [CrossRef]
15. Wang, R.; Li, Q.; Zhang, B.; Wang, L. Distributed consensus based algorithm for economic dispatch in a microgrid. *IEEE Trans. Smart Grid* **2018**, *10*, 3630–3640. [CrossRef]
16. Chen, W.; Li, T. Distributed economic dispatch for energy internet based on multiagent consensus control. *IEEE Trans. Autom. Control* **2020**, *66*, 137–152. [CrossRef]
17. Yan, Y.; Chen, Z.; Varadharajan, V.; Hossain, M.J.; Town, G.E. Distributed consensus-based economic dispatch in power grids using the paillier cryptosystem. *IEEE Trans. Smart Grid* **2021**, *12*, 3493–3502. [CrossRef]
18. Zhang, Y.; Ni, M.; Sun, Y. Fully distributed economic dispatch for cyber-physical power system with time delays and channel noises. *J. Mod. Power Syst. Clean Energy* **2021**, *10*, 1472–1481. [CrossRef]
19. Huang, H.; Li, Z.; Sampath, L.M.I.; Yang, J.; Nguyen, H.D.; Gooi, H.B.; Liang, R.; Gong, D. Blockchain-enabled carbon and energy trading for network-constrained coal mines with uncertainties. *IEEE Trans. Sustain. Energy* **2023**, *14*, 1634–1647. [CrossRef]
20. Li, Z.; Xu, Y.; Wang, P.; Xiao, G. Restoration of multi energy distribution systems with joint district network recon figuration by a distributed stochastic programming approach. *IEEE Trans. Smart Grid* **2023**.
21. Yang, S.; Tan, S.; Xu, J.X. Consensus based approach for economic dispatch problem in a smart grid. *IEEE Trans. Power Syst.* **2013**, *28*, 4416–4426. [CrossRef]
22. Rahbari-Asr, N.; Ojha, U.; Zhang, Z.; Chow, M.Y. Incremental welfare consensus algorithm for cooperative distributed generation/demand response in smart grid. *IEEE Trans. Smart Grid* **2014**, *5*, 2836–2845. [CrossRef]

23. Hug, G.; Kar, S.; Wu, C. Consensus+ innovations approach for distributed multiagent coordination in a microgrid. *IEEE Trans. Smart Grid* **2015**, *6*, 1893–1903. [CrossRef]
24. Xu, Y.; Li, Z. Distributed optimal resource management based on the consensus algorithm in a microgrid. *IEEE Trans. Ind. Electron.* **2014**, *62*, 2584–2592. [CrossRef]
25. Negnevitsky, M.; Nguyen, D.H.; Piekutowski, M. Risk assessment for power system operation planning with high wind power penetration. *IEEE Trans. Power Syst.* **2014**, *30*, 1359–1368. [CrossRef]
26. Zhao, J.; Wang, H.; Wu, Q.; Hatziargyriou, N.D.; Shen, F. Distributed risk-limiting load restoration for wind power penetrated bulk system. *IEEE Trans. Power Syst.* **2020**, *35*, 3516–3528. [CrossRef]
27. Shang, H.; Liu, T.; Bu, T.; He, C.; Yin, Y.; Ding, L. Operational risk assessment of power system considering wind power and photovoltaic grid connection. *Mod. Electr. Power* **2020**, *37*, 358–367.
28. Gui, J.; Lei, H.; McJunkin, T.R.; Chen, B.; Johnson, B.K. Operational resilience metrics for power systems with penetration of renewable resources. *IET Gener. Transm. Distrib.* **2023**, *17*, 2344–2355. [CrossRef]
29. Hetzer, J.; David, C.Y.; Bhattarai, K. An economic dispatch model incorporating wind power. *IEEE Trans. Energy Convers.* **2008**, *23*, 603–611. [CrossRef]
30. Chen, G.; Yang, Q. An ADMM-based distributed algorithm for economic dispatch in islanded microgrids. *IEEE Trans. Ind. Inform.* **2017**, *14*, 3892–3903. [CrossRef]
31. He, X.; Zhao, Y.; Huang, T. Optimizing the dynamic economic dispatch problem by the distributed consensus-based ADMM approach. *IEEE Trans. Ind. Inform.* **2019**, *16*, 3210–3221. [CrossRef]
32. Nguyen, D.H.; Narikiyo, T.; Kawanishi, M. Optimal demand response and real-time pricing by a sequential distributed consensus-based ADMM approach. *IEEE Trans. Smart Grid* **2017**, *9*, 4964–4974. [CrossRef]
33. Yang, Q.; Chen, G.; Wang, T. ADMM-based distributed algorithm for economic dispatch in power systems with both packet drops and communication delays. *IEEE/CAA J. Autom. Sin.* **2020**, *7*, 842–852. [CrossRef]
34. Yang, Z.; Xiang, J.; Li, Y. Distributed consensus based supply–demand balance algorithm for economic dispatch problem in a smart grid with switching graph. *IEEE Trans. Ind. Electron.* **2016**, *64*, 1600–1610. [CrossRef]
35. Guo, F.; Wen, C.; Mao, J.; Song, Y.D. Distributed economic dispatch for smart grids with random wind power. *IEEE Trans. Smart Grid* **2015**, *7*, 1572–1583. [CrossRef]
36. Nguyen, D.H.; Azuma, S.i.; Sugie, T. Novel control approaches for demand response with real-time pricing using parallel and distributed consensus-based ADMM. *IEEE Trans. Ind. Electron.* **2018**, *66*, 7935–7945. [CrossRef]
37. Li, W.; Liu, Y.; Liang, H.; Shen, Y. A new distributed energy management strategy for smart grid with stochastic wind power. *IEEE Trans. Ind. Electron.* **2020**, *68*, 1311–1321. [CrossRef]