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Resource Allocation for Relay-Based OFDMA Power Line Communication System

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Abstract: In this paper, we investigate the resource allocation for multi-user orthogonal frequency division multiple access (OFDMA) power line communication (PLC) systems, which satisfy both the total average transmit power constrained by PLC networks and the required interference limitations. First, we propose an optimal resource allocation scheme, with respect to joint consideration of the subcarrier and power constraint to maximize the throughput of the PLC system. Then, we divide the intractable problem into two individual subproblems to decrease the computational burden, which can avoid proceeding the mixed integer nonlinear programming (MINLP) problem. The proposed resource allocation scheme can ensure the optimality and efficiency of the system with rigorous proofs. The simulation results show that our proposed resource allocation scheme for the PLC system outperforms other algorithms, and is efficient with acceptable complexity.

Keywords: joint optimization; orthogonal frequency division multiple access (OFDMA); power line communication (PLC); resource allocation

1. Introduction

The orthogonal frequency division multiplexing (OFDM) combined with dynamic resource allocation technology can dynamically allocate the subcarriers to every user and allocate the transmit power on each subcarrier according to the channel state. Applying OFDM into the power line communication (PLC) system can effectively eliminate the multi-path, attenuation, time-varying and frequency selectivity of the power line channel, therefore, effectively improving the resource utilization, quality of service and data transmission rate [1–5]. Motivated by the above technique, OFDM-based PLC systems have become increasingly attractive.

Due to the long transmission distance of the multi-conductor cable, it is necessary to adopt a relay for signal amplification and forwarding in the PLC system. The diagram of the relay-aided PLC system is shown in Figure 1, we can see that the base station (BS) of the communication network is connected to the Internet through the access point, and the BS is connected to the power grid by the coupling circuit on the low voltage side of the power transformer. The signal of the BS is transmitted to each PLC gateway through the multi-conductor cable, and the gateway is connected to the user terminal through the power line. Therefore, the network is composed of outdoor broadband access and indoor broadband interconnection. The resource allocation in the relay-aided OFDM PLC system is a joint optimization problem, including the power control, subcarrier assignment, relay selection, etc., therefore, when referring to the relay-aided OFDM PLC system, it is usually considered challenging due to the complexity [6]. The essence of the problem is how to optimally allocate subcarriers to users and allocate power to subcarriers [7–11]. Recently, a variety of multi-user dynamic resource allocation algorithms have been proposed with different optimization objectives or constraints for different communication systems. Ref. [12] proposed a cooperative wireless power communication network protocol, in which the time allocation of hybrid access point downlink transmission and user uplink transmission were jointly optimized for two different aspects. Ref. [13] proposed a multiuser OFDM system named simultaneous wireless information and power transfer, which can transfer information and power separately on different subcarriers. The optimal design for resource allocation is investigated to maximize the sum rate with a minimum transferred power constraint. In [14], for OFDM downlink distributed multiple input single output (MISO) systems in frequency selective fading channels, two power allocation strategies are proposed to solve the power constraint of a single antenna unit. In [15], the energy-saving user scheduling and power control in downlink multicell multiuser orthogonal frequency division multiple access (OFDMA) networks is investigated only with the channel distribution information, which can solve the resource allocation problem subject to the minimum throughput constraint of each user, even if the channel gain is the same for all the subcarriers. In [16], a stochastic optimization programming in dynamic wireless self-backhaul small cell networks is formulated to investigate the spectral efficiency maximization by both considering the resource reuse and allocation, additionally, the random and finite traffic loads are also considered to keep the system stable. The resource allocation problem in two-tier heterogenous networks with massive MIMO is investigated in [17], which used cellular frequency bands and millimeter wave frequency bands for wireless backhaul links. A block diagonalization based precoding scheme is proposed to eliminate the multi-user and inter-tier interference and a power allocation optimization problem is formulated to maximize the downlink sum rate of the system subject to per-small cells clusters power and users' quality of service requirements.

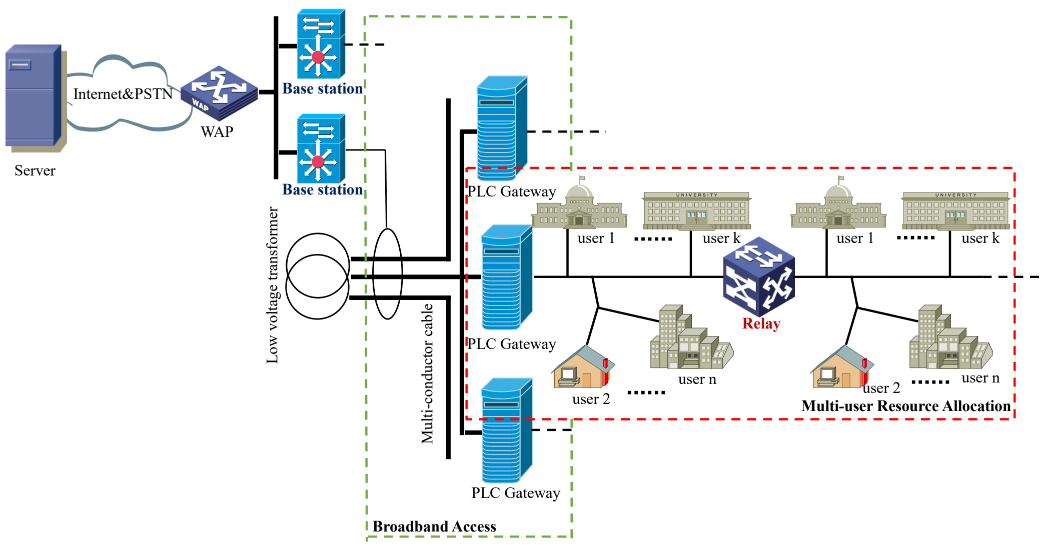

Figure 1. Diagram of the relay-aided power line communication (PLC) system.

However, the PLC networks have not been designed for communication purposes; the existing resource allocation strategies for wireless networks are not suitable [18–23]. The PLC systems usually use the phase line to neutral line to transmit informations. The imbalance of the two wire to ground results in common mode current which greatly enhances the electromagnetic radiation. Meanwhile, the impedance mismatch between the power line and the load causes the signal reflection and form standing wave, which also generates strong electromagnetic radiation. Therefore, the power line is not a good medium for communication. The power line channel also has defects, such as frequency selective multipath fading with time-varying characteristics and color background noise, etc., which are the main difference between PLC systems and wireless communication systems. In order to deal with the poor channel conditions, the PLC system must apply an efficient resource allocation technique. In [24], a sub-carrier and power allocation algorithm is proposed for power spectrum limitation of indoor power line communication OFDM systems and the performance of the algorithm under various variable channel conditions is simulated. For the Homeplug AV standard in power line communication, two bit allocation algorithms with the largest system capacity under the condition of guaranteed bit error rate (BER) and target average BER are proposed [25,26]. In [5], the power allocation problem of a hybrid system composed of a cascaded power-line communications (PLC)/visible light communications (VLC) link in parallel to an radio frequency (RF) wireless link is investigated. This formulated and analyzed the problem of minimizing the total transmission power consumption under a defined quality of service constraint. [27] proposed a dynamic load-based PLC system model for energy efficiency maximization. By optimizing the paremeters including the load impedance, transmission power and subchannel allocation, they investigated power allocation of the multi-receiver PLC OFDM downlink network with non-white Gaussian noise channel. The simulation results showed that compared with the baseline scheme, the proposed system is more energy-efficient, and can greatly improve the energy efficiency by optimizing the impedance and adopting the subchannel allocation strategy.

The resource allocation problem in a relay-aided OFDMA PLC system usually involves a mixed integer nonlinear programming (MINLP) problem [28,29], which leads to an algorithmic complexity of an order of magnitude. The subcarriers and power are jointly optimized, which makes it difficult to obtain an optimal resource allocation strategy. Based on the single-user system model, this paper discusses the optimal resource allocation scheme in multi-user environment, and proves that the system can maximize the system throughput by using the optimal resource allocation strategy proposed in the paper. The specific contents of this paper are as follows: Firstly, for the case of power constraints of the transmission source or relay, the problem description of maximizing the throughput of the system under the constraints of average power is given; secondly, for the case of multi-user OFDMA, The method of separating subcarrier and power allocation is used to give the optimal resource allocation strategy under the average power constraint and the interference power constraint.

The remainder of this paper is organized as follows. Section 2 presents a mathematical description of the relay-based multi-user OFDMA downlink system model. Section 3 discusses the optimal subcarrier allocation and power control schemes in a PLC environment; Section 4 gives the numerical simulations and analysis; Conclusions are given in Section 5.

Notations: $\mathbb{E}\{\cdot\}$ denotes the expectation w.r.t. all fading channels. $\mathcal{CN}(0,\sigma^2)$ denotes the complex Gaussian distribution with the mean being 0 and the variance being σ^2. $[\cdot]^{\mathrm{T}}$ denotes the matrix transpose. $[x]^+$ denotes $\max\{x,0\}$. $\mathrm{diag}\{z_1,z_2,\ldots,z_M\}$ denotes a diagonal matrix with the diagonal entries from $z_1,z_2,\ldots,z_M$. $|\cdot|$ denotes the absolute value of a real number or a modulus of a complex number. $\bigtriangledown^2 f(\cdot)$ denotes the Hessian matrix of $f(\cdot)$.

2. System Model and Problem Formulation

2.1. System Model

Consider a downlink multi-user OFDMA PLC system with one BS, one non-regenerative relay and K users. Since we mainly focus on the resourse allocation strategy for K users under one relay, this paper adopt a two-hop protocol for simplify. Assume that there are N subcarriers in each OFDM symbol and one subcarrier can only be used by one user. In the first hop, the source transmits OFDM symbols $s_{n,k}$ with transmit power $P_{s_{n,k}}$ through the backward channel h_1 to the relay, where $n \in \{1,\cdots,N\}$ is the subcarrier index and $k \in \{1,\cdots,K\}$ is the user index. Then, the received signal of relay can be expressed as

$$y_{n,k} = h_1\sqrt{P_{s_{n,k}}}s_{n,k} + n^r, \tag{1}$$

where n^r denotes the noise at the relay node. In the PLC system, the multi-source impulsive noise is considered as the main factor. According to the model of Middleton's Class A [21,30], we can formulate the total color noise as the sum of Gaussian noise and the impulsive noise, and the probability density function of the color noise that with amplitude c can be given by

$$p(c) = \sum_{c=0}^{\infty} \frac{e^{-A}A^c}{c!} \frac{1}{\sqrt{2\pi}\sigma_c} \exp(-\frac{z^2}{2\sigma_c^2}), \tag{2}$$

where σ_c^2 denotes the total color noise power, and

$$\sigma_c^2 = \sigma_g^2 + \sigma_I^2 \frac{\frac{c}{A} + \Gamma}{1 + \Gamma}, \quad \Gamma = \frac{\sigma_g^2}{\sigma_I^2}, \tag{3}$$

where σ_g^2, σ_I^2 denotes the Gaussian and impulsive noise power, respectively, A denotes the value of the detected impulsive noise. In the second hop, the relay amplifies the received signal with amplified factor α_n, and retransmits the amplified signal to the destination through the forward channel $h_{n,k}$, the forward signal is given by

$$g_{n,k} = \alpha_n h_1 \sqrt{P_{s_{n,k}}} s_{n,k} + \alpha_n n^r. \tag{4}$$

Let $P_{r_{n,k}}$ denotes the transmit power on the relay node, and $P_{r_{n,k}} = \mathbb{E}\left(|g_{n,k}|^2\right)$, the factor α_n can be expressed as

$$|\alpha_n|^2 = \frac{P_{r_{n,k}}}{|h_1|^2 P_{s_{n,k}} + \sigma^2}. \tag{5}$$

After two-hop transmission, we can obtain the k-th user's received signal as

$$d_{n,k} = h_1 h_{n,k} \alpha_n \sqrt{P_{s_{n,k}}} s_{n,k} + h_2 \alpha_n n^r + n_{n,k}^d, \tag{6}$$

where $n_{n,k}^d$ denotes the user node's noise, and $n_{n,k}^d \sim \mathcal{CN}\left(0, \sigma^2\right)$. Based on the above description, we can obtain the k-th user's information rate on the n-th subcarrier

$$R_{n,k} = \log_2\left(1 + \frac{P_{s_{n,k}} P_{r_{n,k}}}{\frac{P_{s_{n,k}}\sigma^2}{|h_{n,k}|^2} + \frac{P_{r_{n,k}}\sigma^2}{|h_1|^2} + \frac{\sigma^4}{|h_1|^2|h_{n,k}|^2}}\right). \tag{7}$$

Consider the background noise and radio interferences in the PLC channel, the subcarrier signal might give rise to interference with other subcarriers. We can model the interference as out-of-band emissions, the interference from the m-th subcarrier to the n-th subcarrier can be expressed as

$$I_{n,m} = P_{s_{n,k}} \eta_n, \tag{8}$$

where η_n denotes the interference coefficient [31,32], which is given by

$$\eta_n = \int_{f_{m,n} - \frac{\triangle f}{2}}^{f_{m,n} + \frac{\triangle f}{2}} T_s \left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2 df, \tag{9}$$

where $f_{n,m} = |f_m - f_n|$ denotes the spectral distance between the center frequency of subcarrier m and subcarrier n. T_s denotes the symbol interval of OFDM and $\triangle f$ denotes the channel bandwidth.

2.2. Problem Formulation

In this section, we derive the optimal resource allocation problem in the relay-based OFDM PLC system. The maximum information rate is regarded as the object function, and both the transmit power constraint and interference limitation between the subcarriers should be guaranteed. Then the problem can be formulated as

$$\begin{aligned}
\max_{P_{s_{n,k}},P_{r_{n,k}}} \quad & R = \sum_{k=1}^{K}\sum_{n=1}^{N} R_{n,k}, \\
\text{s.t.} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_{T_k}} P_{s_{n,k}} \le P_S, \\
& \sum_{k=1}^{K}\sum_{n,m\in\Omega_{T_k}} \eta_n P_{s_{n,k}} \le I_{tol}, \\
& P_{s_{n,k}} \ge 0, \qquad \forall\, k,\ n, \\
& \Omega_{T_1}\bigcup\cdots\bigcup\Omega_{T_K} \subseteq \Omega_T, \\
& \Omega_{T_i}\bigcup\Omega_{T_j} = \emptyset,\ \ \forall i\neq j,\ \ i,j\in\{1,\cdots,K\},
\end{aligned} \tag{10}$$

where P_s denotes the source node's average transmit power, I_{tol} denotes the average interference power, Ω_{T_k} represents the subcarrier set for the k-th user, Ω_T represents the set of all subcarriers. In (10), constraints 1 and 2 represent the transmission power constraint of the source of the PLC system and the interference power constraint between subcarriers, respectively, constraint 3 represents the non-negative property of the power value, constraints 4 and 5 indicate that the subcarrier sets are disjointed. It obvious that problem (10) belongs to mixed integer nonlinear programming (MINLP), which usually leads to a $\mathcal{NP}$-hard problem. Since the information rate $R_{n,k}$ can be explicitly expressed as in (7), a general mathematical form for simplicity can be expressed as

$$\begin{aligned}
\max_{x_n,\Omega_{T_k}} \quad & f(\mathbf{x}) = \sum_{k=1}^{K}\sum_{n=1}^{N} \log_2\left(1+\frac{x_n}{b_n x_n + c_n}\right), \\
\text{s.t.} \quad & \sum_{k=1}^{K}\sum_{n\in\Omega_{T_k}} x_n \le P_S, \\
& \sum_{k=1}^{K}\sum_{n,m\in\Omega_{T_k}} \eta_n x_n \le I_{tol}, \\
& x_n \ge 0, \qquad \forall\, k,\ n, \\
& \Omega_{T_1}\bigcup\cdots\bigcup\Omega_{T_K} \subseteq \Omega_T, \\
& \Omega_{T_i}\bigcup\Omega_{T_j} = \emptyset,\ \ \forall i\neq j,\ \ i,j\in\{1,\cdots,K\},
\end{aligned} \tag{11}$$

where x_n denotes the transmit power on the n-th subcarrier, b_n and c_n are constants determined by (7), which can be given by

$$b_n = \frac{\sigma^2}{h_{n,k}^2 P_{r_{n,k}}}, \qquad c_n = \frac{\sigma^2}{h_1^2}\left(1+\frac{\sigma^2}{h_{n,k}^2 P_{r_{n,k}}}\right). \tag{12}$$

3. The Optimal Resource Allocation Strategy for a PLC System

In this section, we will give the optimal resource allocation algorithm by considering both the transmit power and interference power constraints [33–36]. Since problem (11) is a joint optimization problem, we can divide the complex problem into two subproblems to obtain the global optimal solution, i.e., the subcarrier allocation strategy and power allocation.

3.1. The Subcarrier Allocation Strategy

Assume the resource allocation strategy is any given, the power on subcarrier n is $P_n^{'}$, and the information rate can be written as $R_n^{'}$ $(n \in \Omega_T)$. Let the n-th subcarrier licensed to the k-th PLC users that with the best forward channel, that is

$$k_n^* = \arg\max_k \{|h_{n,1}^2|^2, \cdots, |h_{n,K}^2|^2\}, \quad m \in \Omega_T, \tag{13}$$

then, the information rate can be given as $\widetilde{R}_n$ with the power $P_n^{'}$, submit $\widetilde{R}_n$ to (7), we can obtian

$$\widetilde{R}_n \geq R_n', \qquad n \in \Omega_T, \tag{14}$$

accordingly, the sum rate of the PLC system can be given by

$$\widetilde{R}_{sum} = \sum_{n \in \Omega_T} \widetilde{R}_n \geq \sum_{n \in \Omega_T} R_n'. \tag{15}$$

It can be seen that based on the subcarrier allocation scheme shown in (13), the maximum sum information rate can be achieved with the optimal power control strategy. In addition, by dividing the joint optimal problem into two subproblems, the complexity of the joint optimization problem can be descended. We can only consider the power allocation problem to obtain the global optimal solution.

3.2. The Optimal Power Control Scheme

Based on the above description, the optimization problem (11) can be degraded as

$$\begin{aligned} \max_{x_n} \; & f(\mathbf{x}) = \sum_{n \in \Omega_T} \log_2\left(1 + \frac{x_n}{b_n x_n + c_n}\right), \\ \text{s.t.} \; & \sum_{n \in \Omega_T} x_n \leq P_S, \\ & \sum_{n \in \Omega_T} \eta_n x_n \leq I_{tol}, \qquad x_n \geq 0, \quad \forall\, n, \end{aligned} \tag{16}$$

In order to obtain the optimal solution of (16), we investigate the property of the objective function. The second-order partial derivative of the objective function $f(x)$ with N independent variables can be given as

$$\frac{\partial^2 f(\mathbf{x})}{\partial x_n^2} = -c_n \frac{2x_n b_n(b_n+1) + 2b_n c_n + c_n}{(b_n x_n + c_n)^2 (b_n x_n + x_n + c_n)^2} < 0, \qquad \forall\; n \in \Omega_T,$$

and

$$\frac{\partial^2 f(\mathbf{x})}{\partial x_n \partial x_m} = 0, \qquad \forall\; n,\; m\; \in \Omega_T,\; \text{and}\; n \neq m,$$

we can see that the Hessian matrix $\nabla^2 f(\mathbf{x}) = \text{diag}\left\{\frac{\partial^2 \mathrm{f}(\mathbf{x})}{\partial \mathrm{x}_1^2}, \cdots, \frac{\partial^2 \mathrm{f}(\mathbf{x})}{\partial \mathrm{x}_\mathrm{N}^2}\right\}$ is a diagonal matrix, and the diagonal elements are all negative, therefore, the Hessian matrix of objective function is a negative matrix, the objective function of problem (16) is strictly convex on the space spanned by $\{x_1, \cdots, x_N\}$ [37–39]. Here we analyze the situation that the power control and interference constraint take the equal sign simultaneously, then, the problem can be formulated as

$$\begin{aligned} \max_{x_n} \quad & f(\mathbf{x}) = \sum_{n\in\Omega_T} \log_2\left(1+\frac{x_n}{b_n x_n + c_n}\right), \\ \text{s.t.} \quad & \sum_{n\in\Omega_T} x_n = P_S, \\ & \sum_{n\in\Omega_T} \theta_m x_n = I_{tol}, \qquad x_n \geq 0, \quad \forall\, n, \end{aligned} \tag{17}$$

where the constraint function $h_1(x_m) = \sum_{m\in\Omega_T} x_m - P_S$ and $h_2(x_m) = \sum_{m\in\Omega_T} \theta_m x_m - I_{th}$ is affine function, respectively. Therefore, the optimization problem (17) is convex optimization problem, which can be solved by the Karush-Kuhn-Tucher (KKT) conditions. In the following, based on the subcarrier allocation strategy, we discuss the optimal power control scheme with both the power and interference constraints by using the Lagrangian multiplier method.

The Lagrangian function of (17) is given by

$$L = f(\mathbf{x}) + \sum_{n\in\Omega_T} \lambda x_n - \nu_1\left(\sum_{n\in\Omega_T} x_n - P_s\right) - \nu_2\left(\sum_{n\in\Omega_T} \eta_n x_n - I_{tol}\right). \tag{18}$$

where λ is a constant. Let $\frac{\partial L}{\partial x_n} = 0$, the derivative of (18) with respect to x_n is given by

$$G_n(x_n) + \lambda - \nu_1 - \eta_{n,m}\nu_2 = 0, \tag{19}$$

where

$$G_n(x_n) = \frac{c_n}{(b_n x_n + c_n)(b_n x_n + x_n + c_n)}. \tag{20}$$

according to the KKT (i) condition [40–42], i.e., $\lambda \geq 0$, we can obtain

$$\nu_1 + \eta_n \nu_2 \geq G_n(x_n), \tag{21}$$

with KKT condition (ii), i.e., $\lambda x_n = 0$, substituting (19) into (21), we can obtain

$$\left[\nu_1 + \eta_{n,m}\nu_2 - G_n(x_n)\right] x_n = 0. \tag{22}$$

Since x_n denotes the power allocated on the n-th subcarrier, which is non-negative real number. Let $z_n \triangleq \frac{1}{c_n}$, we can obtain

$$\begin{cases} x_n > 0\,, \quad \nu_1 + \eta_n\nu_2 = G_n(x_n) < G_n(0) = z_n, & \forall\, n \in \Phi_1, \\ x_n = 0\,, \quad \nu_1 + \eta_n\nu_2 \geq G_n(0) = z_n, & \forall\, n \in \Phi_2. \end{cases} \tag{23}$$

where $\Phi_1 \bigcup \Phi_2 = \Omega_T$, $\Phi_1 \bigcap \Phi_2 = \emptyset$. When $x_m > 0$, substituting (23) into (19), we can obtain

$$x_n = \frac{c_n}{b_n(b_n+1)}\left[\sqrt{1+\frac{b_n(b_n+1)}{c_n(\nu_1+\eta_{n,m}\nu_2)}} - b_n - 1\right]^+, \tag{24}$$

where $[x]^+ = \max\{0, x_n\}$, parameters ν_1, ν_2 are determined by the two equality equations of power and interference constraints. (24) gives the expression of the optimal solution for (17) and in what follows, we will prove there exists a unique solution that satisfies (24).

Assume the subcarrier sets $\mathcal{N}_1,\ \mathcal{N}_2 \subseteq \Omega_T, \overline{\mathcal{N}}_1,\ \overline{\mathcal{N}}_2$ is the complement set, respectively. Let $\mathcal{N}_0 = \mathcal{N}_1 \bigcap \mathcal{N}_2,\ \mathcal{M}_0 = \mathcal{N}_1 \bigcup \mathcal{N}_2,\ \mathcal{C}_1 = \mathcal{N}_1 - \mathcal{N}_0,\ \mathcal{C}_2 = \mathcal{N}_2 - \mathcal{N}_0$, and suppose the power allocation solution of (24) is exist in subcarrier sets $\mathcal{N}_1$ and $\mathcal{N}_2$ simultaneously, we have

$$\left\{ \begin{array}{ll} x_{n\in\mathcal{N}_1}(\nu_1,\nu_2) > 0, & x_{n\in\overline{\mathcal{N}}_1}(\nu_1,\nu_2) \leq 0, \\ x_{n\in\mathcal{N}_2}(\nu_1',\nu_2') > 0, & x_{n\in\overline{\mathcal{N}}_2}(\nu_1',\nu_2') \leq 0. \end{array} \right\} \tag{25}$$

according to the assumption, there are two situations, i.e., $\nu_1 > \nu_1',\ \nu_2 < \nu_2'$ and $\nu_1 < \nu_1',\ \nu_2 > \nu_2'$, without loss of generality, we only consider the situation $\nu_1 > \nu_1',\ \nu_2 < \nu_2'$.

Calculate x_n by (24), if the value is negative, let $x_n = 0$, that is

$$x_n(\nu_1,\nu_2) = 0,\ \ n \in \overline{\mathcal{N}}_1, \qquad \text{and} \qquad x_n(\nu_1',\nu_2') = 0,\ \ n \in \overline{\mathcal{N}}_2. \tag{26}$$

and $\mathcal{C}_2 \subseteq \overline{\mathcal{N}}_1,\ \mathcal{C}_1 \subseteq \overline{\mathcal{N}}_2$, which leads to

$$x_n(\nu_1,\nu_2) = 0,\ \ n \in \mathcal{C}_2, \qquad \text{and} \qquad x_n(\nu_1',\nu_2') = 0,\ \ n \in \mathcal{C}_1.$$

Then, from $\sum_{n\in\mathcal{N}_1} x_n(\nu_1,\nu_2) = \sum_{n\in\mathcal{N}_2} x_n(\nu_1',\nu_2') = P_S$, we can derive that

$$P_S = \sum_{n\in\mathcal{N}_1} x_n(\nu_1,\nu_2) + \sum_{n\in\mathcal{C}_2} x_n(\nu_1,\nu_2) = \sum_{n\in\mathcal{N}_2} x_n(\nu_1',\nu_2') + \sum_{n\in\mathcal{C}_1} x_n(\nu_1',\nu_2'). \tag{27}$$

According to the definition of set $\mathcal{M}_0$, we can obtain

$$P_S = \sum_{n\in\mathcal{M}_0} x_n(\nu_1,\nu_2) + \sum_{n\in\mathcal{M}_0} x_n(\nu_1',\nu_2'). \tag{28}$$

Define two subcarrier sets $\mathcal{D}_1,\ \mathcal{D}_2$, which satisfy

$$\left\{ \begin{array}{ll} x_n(\nu_1,\nu_2) \geq x_n(\nu_1',\nu_2'), & n \in \mathcal{D}_1, \\ x_n(\nu_1,\nu_2) < x_n(\nu_1',\nu_2'), & n \in \mathcal{D}_2. \end{array} \right\}, \tag{29}$$

where $\mathcal{D}_1 \bigcup \mathcal{D}_2 = \mathcal{M}_0$, combined with (24), we can derived that

$$\eta_{n\in\mathcal{D}_1} > \eta_{n\in\mathcal{D}_2}. \tag{30}$$

Additionally, from (27) we can obtain

$$\sum_{n\in\mathcal{D}_1} \left(x_n(\nu_1,\nu_2) - x_n(\nu_1',\nu_2')\right) = \sum_{n\in\mathcal{D}_2} \left(x_n(\nu_1',\nu_2') - x_n(\nu_1,\nu_2)\right). \tag{31}$$

therefore, from (31) and (30) we can obtain the expression given by

$$\sum_{n\in\mathcal{D}_1} \eta_n \left(x_n(\nu_1,\nu_2) - x_n(\nu_1',\nu_2')\right) > \sum_{n\in\mathcal{D}_2} \eta_n \left(x_n(\nu_1',\nu_2') - x_n(\nu_1,\nu_2)\right), \tag{32}$$

similarly, when $\nu_1 < \nu_1',\ \nu_2 > \nu_2'$, we can obtain

$$\sum_{n\in\mathcal{N}_1} \eta_n x_n(\nu_1,\nu_2) < \sum_{n\in\mathcal{N}_2} \eta_n x_n(\nu_1',\nu_2'). \tag{33}$$

(32) and (33) imply that the following expression holds

$$\sum_{n\in\mathcal{N}_1} \eta_n x_n(\nu_1,\nu_2) \neq \sum_{n\in\mathcal{N}_2} \eta_n x_n(\nu_1',\nu_2'). \tag{34}$$

It can be seen that the above conclusion contradicts the previously held assumption. Therefore, there exists a unique solution for the optimization problem (24). Based on the above description, we can obtain the optimal power allocation scheme, however, the expression of the value L, ν_1 and ν_2 are not available. The details about the calculation of the parameters are given in Algorithm 1.

Algorithm 1 The optimal power control algorithm of problem (17).

Initialization: Let $\nu_1 \in [\nu_{1,\min}, \nu_{1,\max}]$, $\nu_2 \in [\nu_{2,\min}, \nu_{2,\max}]$, $\nu_{1,\max} = \min G_{(i)}(P_s)_{i \in \Omega_n}$, $\nu_{2,\max} = \min G_{(i)}(0)_{i \in \Omega_n}$, $\nu_{1,\min} = 0$, $\nu_{2,\min} = 0$.
Let $\nu_1 = \frac{\nu_{1,\min}+\nu_{1,\max}}{2}$, $\nu_2 = \frac{\nu_{2,\min}+\nu_{2,\max}}{2}$, calculate $x_n(\nu_1, \nu_2)$ by (24), and obtain the subcarrier set $\mathcal{N}_1 = \{n \in \Omega_T | x_n > 0\}$.
while $\frac{\left|\sum_{n \in \mathcal{N}_1} x_n(\nu_1,\nu_2) - P_S\right|}{P_S} > \epsilon$ or $\frac{\left|\sum_{n \in \mathcal{N}_1} \theta_n x_n(\nu_1,\nu_2) - I_{tol}\right|}{I_{tol}} > \epsilon$ **do**
  while $\left(\sum_{n \in \mathcal{N}_1} x_n(\nu_1, \nu_2) - P_S\right)\left(\sum_{n \in \mathcal{N}_1} \theta_n x_n(\nu_1, \nu_2) - I_{tol}\right) > 0$ **do**
    If $\sum_{n \in \mathcal{N}_1} x_n(\nu_1, \nu_2) - P_S > 0$, and $\sum_{n \in \mathcal{N}_1} \theta_n x_n(\nu_1, \nu_2) - I_{tol} > 0$, $\nu_{1,\min} = \nu_1$, otherwise, $\nu_{1,\max} = \nu_1$.
    Let $\nu_1 = \frac{\nu_{1,\min}+\nu_{1,\max}}{2}$, calculate x_n by (24), and obtain the set $\mathcal{N}_1$.
  end while
  If $\sum_{n \in \mathcal{N}_1} x_n(\nu_1, \nu_2) > P_S$ and $\sum_{n \in \mathcal{N}_1} \theta_n x_n(\nu_1, \nu_2) < I_{tol}$, $\nu_{1,\min} = \nu_1$, $\nu_{2,\max} = \nu_2$, otherwise, $\nu_{1,\max} = \nu_1$, $\nu_{2,\min} = \nu_2$.
  Let $\nu_1 = \frac{\nu_{1,\min}+\nu_{1,\max}}{2}$, $\nu_2 = \frac{\nu_{2,\min}+\nu_{2,\max}}{2}$, calculate $x_n(\nu_1, \nu_2)$ by (24), obtain the set $\mathcal{N}_1$.
end while
Output x_n.

4. Numerical Results

In this section, we show the numerical results of the proposed resource allocation strategy in the practical PLC environment. Assume the relay lies in the middle of the source and users, and we use the following parameters: The frequency band is $0 - 20$ MHz, the subcarrier band is $\Delta f = 62.5$ KHz. The total subcarrier number is $N = 128$. Substituting the OFDM symbol interval $\Delta T = \frac{1}{\Delta f}$ into (9) to obtain the normalized interference power η_n between channel n and channel m. With the proposed dynamic resource allocation algorithm, we can obtain the interference coefficient, interference power and optimal transmit power, as shown in Figure 2. It can be seen that the subcarrier $(1, 3, 6, 7)$ has a large interference coefficient, and no power allocated on these subcarriers. Meanwhile, the subcarrier $(9, 10, 11, 17)$ has small interference coefficient ($\eta_{17} = 0.011$) and more power is allocated for signal transmission, that is, as the interference increases, the power gradually decreases, and vice versa. In addition, the PLC channel status can also affect the power allocation result, we can see that the interference coefficient $\theta_{18} = 0.023 < \theta_{14} = 0.036$. However, the optimal transmit power allocated on 14 is larger than 18. Similary, due to the PLC channel status, the subcarrier $(12, 13, 15)$ did not assign power to transmit signals. Based on the above discussion, we can conclude that the optimal power allocation is affected by both the interference coefficient and the channel status.

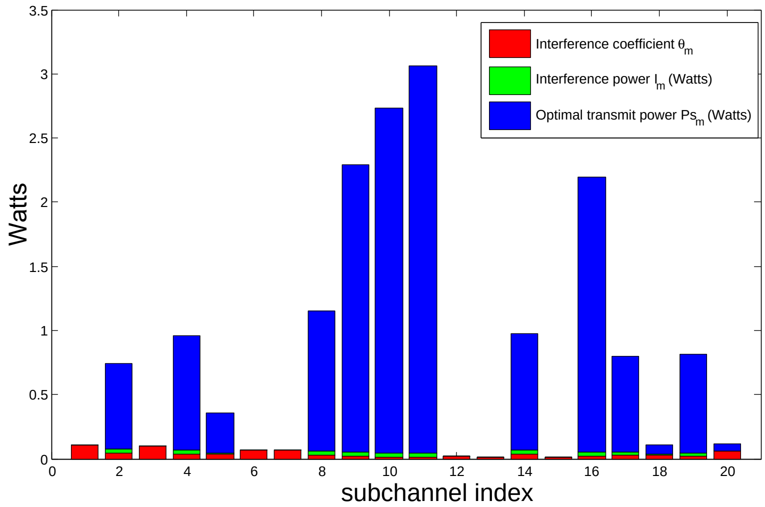

Figure 2. Schematic diagram of PLC system interference coefficient, optimal interference power value and optimal power allocation value.

Figure 3 depicts the PLC system throughput with the growth of the interference threshold I_{tol} under different transmit powers. It is obvious that when the interference threshold I_{tol} takes a smaller value, the system throughput increases with the growth of I_{tol} no matter the value of transmit power. With the further increase of I_{tol}, the system throughput is constrained by the transmit power and the larger power the higher throughput. In addition, the throughput under different transmit power P_s no longer increases till the accordingly points of I_{tol}, since with the growth of I_{tol}, the interference constraint ineffective to the resource allocation.

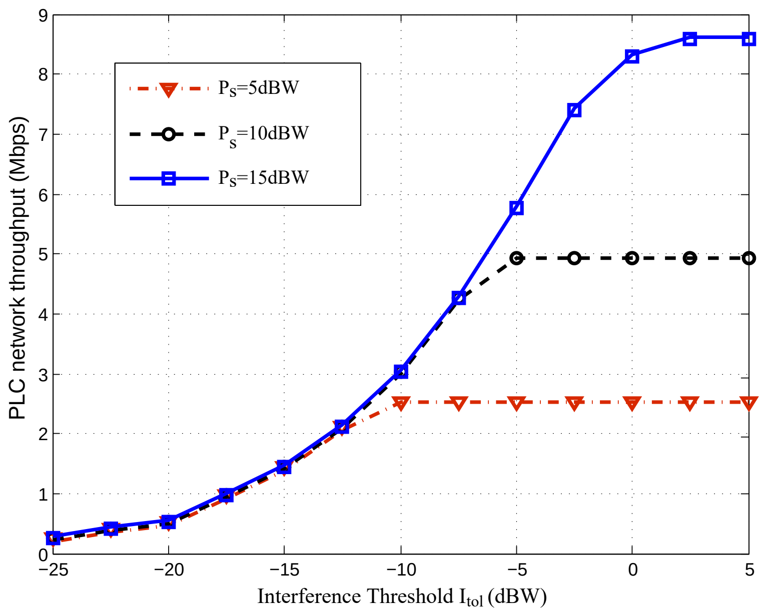

Figure 3. The PLC system throughput with the interference threshold I_{tol} under different transmit power.

In Figure 4, the system throughput is curved with the growth of interference threshold I_{tol}, and the transmit power is $P_s = 10$ dBW. Different resource allocation strategies have been shown in Figure 4, including the subcarrier pre-assignment method of FDMA and the optimal subcarrier allocation scheme (Opt. subcarrier), and the power control policy (Opt.PA and Equal PA). It can be seen that the proposed scheme achieves the maximum system throughput comparing with all the other schemes.

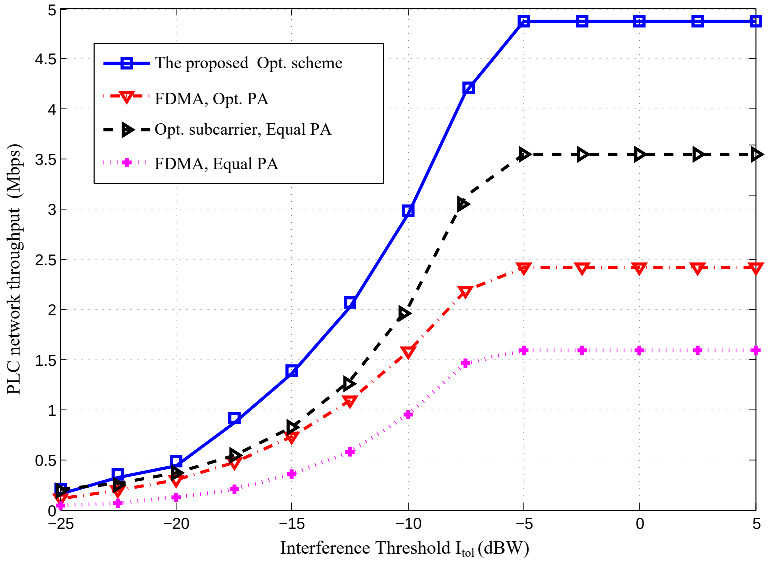

Figure 4. The PLC system throughput with the growth of the interference threshold under different resource schemes.

5. Conclusions

This paper focuses on the relay-based multi-user PLC downlink system, which can satisfy the total power constraint and the interference limitations required by the users. An optimal resource allocation scheme is proposed to maximize the system throughput with respect to the joint subcarrier and power constraints. In order to avoid the comuputational burden caused by the MINLP, we divide the joint optimization problem into two individual subproblems with rigorous proofs. The numerical results show that the global optimal solution can still obtained and the simulation results confirm that the proposed resource allocation scheme outperforms other schemes that mentioned in this paper.

Author Contributions: Q.Z. conceived and designed the algorithm; Z.C. performed the experiments; X.H. analyzed the data and contributed the analysis tools; Q.Z. wrote the paper.

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References

1. Do, L.P.; Lehnert, R. Dynamic resource allocation protocol for large PLC networks. In Proceedings of the 2012 IEEE International Symposium on Power Line Communications and Its Applications, Beijing, China, 27–30 March 2012; pp. 41–46. [CrossRef]
2. Zhou, N.; Zhu, X.; Huang, Y. Optimal Asymmetric Resource Allocation and Analysis for OFDM-Based Multidestination Relay Systems in the Downlink. *IEEE Trans. Veh. Technol.* **2011**, *60*, 1307–1312. [CrossRef]
3. Tsai, C.; Chang, C.; Ren, F.; Yen, C. Adaptive radio resource allocation for downlink OFDMA/SDMA systems with multimedia traffic. *IEEE Trans. Wirel. Commun.* **2008**, *7*, 1734–1743. [CrossRef]
4. Pavlidou, N.; Vinck, A.J.H.; Yazdani, J.; Honary, B. Power line communications: state of the art and future trends. *IEEE Commun. Mag.* **2003**, *41*, 34–40. [CrossRef]
5. Kashef, M.; Abdallah, M.; Al-Dhahir, N. Transmit Power Optimization for a Hybrid PLC/VLC/RF Communication System. *IEEE Trans. Green Commun. Netw.* **2018**, *2*, 234–245. [CrossRef]
6. Yuan, J.; Wang, Q. Delay quality-of-service driven resource allocation for relay-based OFDMA cognitive radio networks. In Proceedings of the 2012 IEEE Global Communications Conference (GLOBECOM), Anaheim, CA, USA, 3–7 December 2012; pp. 3994–3999. [CrossRef]

7. Wu, W.; Zhou, F.; Yang, Q. Adaptive Network Resource Optimization for Heterogeneous VLC/RF Wireless Networks. *IEEE Trans. Commun.* **2018**, *66*, 5568–5581. [CrossRef]
8. Kibria, M.G.; Hoque, R. Resource allocation for sum-power minimization in multiuser OFDM with user rate constraints. In Proceedings of the 2015 18th International Conference on Computer and Information Technology (ICCIT), Dhaka, Bangladesh, 21–23 December 2015; pp. 217–221. [CrossRef]
9. Ma, M.; Yang, Y. SenCar: An Energy-Efficient Data Gathering Mechanism for Large-Scale Multihop Sensor Networks. *IEEE Trans. Parallel Distrib. Syst.* **2007**, *18*, 1476–1488. [CrossRef]
10. Cano, C.; Pittolo, A.; Malone, D.; Lampe, L.; Tonello, A.M.; Dabak, A.G. State of the Art in Power Line Communications: From the Applications to the Medium. *IEEE J. Sel. Areas Commun.* **2016**, *34*, 1935–1952. [CrossRef]
11. Chen, Z.; Chen, P.; Li, J.; Miao, P. Non-orthogonal multi-carrier MIMO communication system using M-ary efficient modulation. *Digit. Signal Process.* **2018**, *76*, 14–21. [CrossRef]
12. Shin, W.; Vaezi, M.; Lee, J.; Poor, H.V. Cooperative Wireless Powered Communication Networks With Interference Harvesting. *IEEE Trans. Veh. Technol.* **2018**, *67*, 3701–3705. [CrossRef]
13. Yin, S.; Qu, Z. Resource Allocation in Multiuser OFDM Systems With Wireless Information and Power Transfer. *IEEE Commun. Lett.* **2016**, *20*, 594–597. [CrossRef]
14. Xu, W.; Yu, X.; Leung, S.; Chu, J. Energy-Efficient Power Allocation Scheme for Distributed MISO System With OFDM Over Frequency-Selective Fading Channels. *IEEE Access* **2018**, *6*, 51217–51226. [CrossRef]
15. Zhang, Y.; An, J.; Yang, K.; Gao, X.; Wu, J. Energy-Efficient User Scheduling and Power Control for Multi-Cell OFDMA Networks Based on Channel Distribution Information. *IEEE Trans. Signal Process.* **2018**, *66*, 5848–5861. [CrossRef]
16. Tang, L.; Wei, Y.; He, L.; Liao, H.; Chen, Q. Queue-Aware Dynamic Resource Reuse and Joint Allocation Algorithm in Self-Backhaul Small Cell Networks. *IEEE Access* **2018**, *6*, 61077–61090. [CrossRef]
17. Hao, W.; Yang, S. Small Cell Cluster-Based Resource Allocation for Wireless Backhaul in Two-Tier Heterogeneous Networks With Massive MIMO. *IEEE Trans. Veh. Technol.* **2018**, *67*, 509–523. [CrossRef]
18. Gassara, H.; Rouissi, F.; Ghazel, A. Statistical Characterization of the Indoor Low-Voltage Narrowband Power Line Communication Channel. *IEEE Trans. Electromagn. Compat.* **2014**, *56*, 123–131. [CrossRef]
19. Chen, P.; Qi, C.; Wu, L. Antenna placement optimisation for compressed sensing-based distributed MIMO radar. *IET Radar Sonar Navig.* **2017**, *11*, 285–293. [CrossRef]
20. Anatory, J.; Theethayi, N.; Thottappillil, R. Power-Line Communication Channel Model for Interconnected Networks-Part II: Multiconductor System. *IEEE Trans. Power Deliv.* **2009**, *24*, 124–128. [CrossRef]
21. Xu, Z.; Zhai, M.; Zhao, Y. Optimal Resource Allocation Based on Resource Factor for Power-Line Communication Systems. *IEEE Trans. Power Deliv.* **2010**, *25*, 657–666. [CrossRef]
22. Kuhn, M.; Berger, S.; Hammerstrom, I.; Wittneben, A. Power line enhanced cooperative wireless communications. *IEEE J. Sel. Areas Commun.* **2006**, *24*, 1401–1410. [CrossRef]
23. Chen, P.; Zheng, L.; Wang, X.; Li, H.; Wu, L. Moving Target Detection Using Colocated MIMO Radar on Multiple Distributed Moving Platforms. *IEEE Trans. Signal Process.* **2017**, *65*, 4670–4683. [CrossRef]
24. Newbury, J.; Miller, W. Potential communication services using power line carriers and broadband integrated services digital network. *IEEE Trans. Power Deliv.* **1999**, *14*, 1197–1201. [CrossRef]
25. Guerrini, E.; Dell'Amico, G.; Bisaglia, P.; Guerrieri, L. Bit-loading algorithms and SNR estimate for HomePlug AV. In Proceedings of the 2007 IEEE International Symposium on Power Line Communications and Its Applications, Pisa, Italy, 26–28 March 2007; pp. 77–82. [CrossRef]
26. Latchman, H.; Katar, S.; Yonge, L.; Amarsingh, A. High speed multimedia and smart energy PLC applications based on adaptations of HomePlug AV. In Proceedings of the 2013 IEEE 17th International Symposium on Power Line Communications and Its Applications, Johannesburg, South Africa, 24–27 March 2013; pp. 143–148. [CrossRef]
27. Hwang, Y.M.; Jung, J.H.; Seo, J.K.; Lee, J.; Kim, J.Y. Energy-Efficient Transmission Strategy With Dynamic Load for Power Line Communications. *IEEE Trans. Smart Grid* **2018**, *9*, 2382–2390. [CrossRef]
28. Singh, K.; Gupta, A.; Ratnarajah, T.; Ku, M. A General Approach Toward Green Resource Allocation in Relay-Assisted Multiuser Communication Networks. *IEEE Trans. Wirel. Commun.* **2018**, *17*, 848–862. [CrossRef]
29. Singh, K.; Gupta, A.; Ratnarajah, T. Energy Efficient Resource Allocation for Multiuser Relay Networks. *IEEE Trans. Wirel. Commun.* **2017**, *16*, 1218–1235. [CrossRef]

30. Kim, K.; Kim, S. Performance Analysis of LDPC Coded DMT Systems with Bit-loading Algorithms for Powerline Channel. In Proceedings of the 2007 IEEE International Symposium on Power Line Communications and Its Applications, Pisa, Italy, 26–28 March 2007; pp. 234–239.. [CrossRef]
31. Li, P.; Scalabrino, N.; Fang, Y.; Gregori, E.; Chlamtac, I. Channel Interference in IEEE 802.11b Systems. In Proceedings of the IEEE GLOBECOM 2007—IEEE Global Telecommunications Conference, Washington, DC, USA, 26–30 November 2007; pp. 887–891. [CrossRef]
32. Ding, Y.; Huang, Y.; Zeng, G.; Xiao, L. Using Partially Overlapping Channels to Improve Throughput in Wireless Mesh Networks. *IEEE Trans. Mob. Comput.* **2012**, *11*, 1720–1733. [CrossRef]
33. Zhang, G.; Yan, H.; Zeng, Y.; Cui, M.; Liu, Y. Trajectory Optimization and Power Allocation for Multi-Hop UAV Relaying Communications. *IEEE Access* **2018**, *6*, 48566–48576. [CrossRef]
34. Chen, P.; Wu, L.; Qi, C. Waveform Optimization for Target Scattering Coefficients Estimation Under Detection and Peak-to-Average Power Ratio Constraints in Cognitive Radar. *Circ. Syst. Signal Process.* **2016**, *35*, 163–184. [CrossRef]
35. Zhu, W.; Wang, W.; Li, X.; Gao, X. Power allocation for multiceli massive MIMO systems under Rician fading with statistical CSI. In Proceedings of the 2018 IEEE Wireless Communications and Networking Conference (WCNC), Barcelona, Spain, 15–18 April 2018; pp. 1–6. [CrossRef]
36. Yousaf, R.; Ahmad, R.; Ahmed, W.; Haseeb, A. Fuzzy Power Allocation for Opportunistic Relay in Energy Harvesting Wireless Sensor Networks. *IEEE Access* **2017**, *5*, 17165–17176. [CrossRef]
37. Chen, Z.; Wu, L.; Chen, P. Efficient modulation and demodulation methods for multi-carrier communication. *IET Commun.* **2016**, *10*, 567–576. [CrossRef]
38. Ezzine, S.; Abdelkefi, F.; Cances, J.P.; Meghdadi, V.; Bouallegue, A. On the evaluation and analysis of the data rate of an OFDM-based two-way relaying PLC system. In Proceedings of the 2017 13th International Wireless Communications and Mobile Computing Conference (IWCMC), Valencia, Spain, 26–30 June 2017; pp. 1610–1615. [CrossRef]
39. Chen, P.; Qi, C.; Wu, L.; Wang, X. Waveform Design for Kalman Filter-Based Target Scattering Coefficient Estimation in Adaptive Radar System. *IEEE Trans. Veh. Technol.* **2018**, *67*, 11805–11817. [CrossRef]
40. Chen, P.; Qi, C.; Wu, L.; Wang, X. Estimation of Extended Targets Based on Compressed Sensing in Cognitive Radar System. *IEEE Trans. Veh. Technol.* **2017**, *66*, 941–951. [CrossRef]
41. Ghamkhari, M.; Mohsenian-Rad, H. A Convex Optimization Framework for Service Rate Allocation in Finite Communications Buffers. *IEEE Commun. Lett.* **2016**, *20*, 69–72. [CrossRef]
42. Wadayama, T. Interior Point Decoding for Linear Vector Channels Based on Convex Optimization. *IEEE Trans. Inf. Theory* **2010**, *56*, 4905–4921. [CrossRef]