



Article

# Research on Harmonic Torque Reduction Strategy for Integrated Electric Drive System in Pure Electric Vehicle

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**Abstract:** In order to study the influence of harmonic torque on the performance of the integrated electric drive system (permanent magnet synchronous motor + reducer gear pair) in a pure electric vehicle (PEV), the electromechanical coupling dynamic model of a PEV was established by considering the dead-time effect and voltage drop effect of an inverter and the nonlinear characteristics of the transmission system. Based on the model, the dynamic characteristics of an integrated electric drive system (IEDS) are studied, and the interaction between the mechanical system and electrical system is analyzed. On this basis, a harmonic torque reduction strategy for an IEDS is proposed in this paper. The simulation results show that the proposed strategy can effectively reduce the harmonic torque of the motor and reduce the speed fluctuation and dynamic load of the system components, which can improve the stability of the IEDS and prolong the life of the mechanical components.

**Keywords:** integrated electric drive system; permanent magnet synchronous motor; electromechanical coupling; harmonic torque reduction strategy

## 1. Introduction

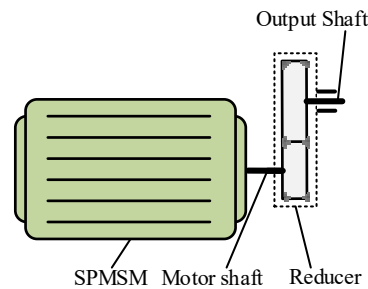
### 1.1. Motivation

Electric drive assembly is the key component of a pure electric vehicle (PEV), which plays a key role in the driving experience. Recent studies have shown that the integration of a drive motor and reducer can greatly improve the efficiency, power density, and reliability of the drive system, and it can also effectively reduce the volume, weight, and production cost of a PEV [1,2]. The research and development of high speed, high efficiency, high power density, lightweight integrated motor-reducer assembly has become a hotspot. The general structure of an integrated electric drive system (IEDS) is shown in Figure 1, which mainly includes a permanent magnet synchronous motor (PMSM), motor control system, helical gear pair reducer, and output shaft. The IEDS includes the driving motor part and the mechanical transmission part.

### 1.2. Literature Review

Due to the dead-time effect of an inverter, the voltage drop effect, and the structural factors of the motor itself, electromagnetic torque generated by the motor contains harmonic torque [3], which will cause adverse effects on the mechanical transmission system. At the same time, due to the nonlinear factors such as gear time-varying stiffness and meshing error, the mechanical transmission system will also affect the stability of the electrical system and finally significantly affect the performance of the

PEV. Therefore, scholars have paid much attention to the research of this problem, mainly including two categories: (1) motor and controller research and (2) gear transmission system research.



**Figure 1.** General structure diagram of an integrated electric drive system.

In order to reduce the motor torque ripple caused by the nonlinear characteristics of the inverter, scholars have done a lot of research on the optimization and improvement of the electrical system control strategy, mainly including the following two aspects: (1) inverter optimization research and (2) motor control strategy. From the aspect of inverter optimization, scholars have proposed various inverter topologies, which can greatly reduce the torque ripple [4–6], but this method has high requirements for development cost and it cannot be popularized in a short time. So, scholars have done a lot of research on the switching control strategy of an inverter such as hybrid space vector modulation (SVM) strategies in [7,8] to reduce torque ripple. In [9], a minimum root mean square (RMS) torque ripple-remote-state pulse-width modulation (MTR-RSPWM) technique was proposed for minimizing the RMS torque ripple under the reduced common-mode voltage condition of three-phase voltage source inverter-fed brushless alternating current motor drives. Besides, a new modulation method, modified trapezoidal modulation (MTM), was proposed for an inverter–PMSM drive in [10], which can increase torque and reduce torque ripple simultaneously. [11] studied the theoretical distortion index of a multilevel motor drive considering control sensitivity. By calculating the distortion index, the optimal equivalent carrier frequency to minimize the torque ripple was obtained. In addition to optimizing the inverter, some scholars studied the motor control strategy. In [12–14], the duty cycle of direct torque control (DTC) was optimized to reduce the torque and flux ripple at low switching frequency. Furthermore, in order to reduce the torque and flux ripple under all operating conditions, the three-level direct torque control (3L-DTC) based on constant switching frequency, which was suitable for low and constant switching frequency operation [15], and generalized direct torque control (GDTC) strategy, which was suitable for any voltage level inverter [16], were studied. Refs [17,18] proposed a seven-level torque comparator and a multi band torque hysteresis controller respectively, in which the voltage vectors were optimized to reduce the torque ripple at different speeds. Some scholars also adopted predictive torque control to reduce torque, flux ripple, and switching frequency [19]. In terms of motors for robots, the modified distributed control framework and on-line tuning fuzzy proportional-derivative (PD) controller of controlling of 5 degree-of-freedom (DOF) robot manipulators based on the equivalent errors method were used in [20,21] to improve the dynamic performance of robots under large disturbance and high frequency. In recent years, the harmonic injection method [22–26] has attracted the attention of many scholars. By adding a harmonic current feedback loop and harmonic voltage compensation loop to the traditional motor double closed-loop control system, and then using the method of injecting harmonic voltage [22] and harmonic current [23–26], the electromagnetic torque ripple of the motor was significantly reduced.

From single-stage gear transmission research to multi-stage gear transmission research, the field of gear system dynamics has formed a relatively mature theoretical system with the joint efforts of scholars [27]. Many scholars have studied the influence law of external load [28], time-varying meshing stiffness, backlash, tooth surface wear [29,30], meshing frequency, eccentricity [31], error, position error, bearing stiffness, and other internal and external factors on the gear transmission system,

and they have conducted in-depth research on the nonlinear dynamic characteristics of the gear system. Among them, gear noise and dynamic load caused by vibration are areas of major concern [32]. Due to the rise of IEDS, scholars have done some research on the electromechanical coupling characteristics of IEDS in recent years. A kind of trajectory-based stability preserving dimension reduction (TSPDR) methodology was proposed to investigate the nonlinear dynamic characteristics of the gear–motor system in the literature [33], which revealed the relationship between the stability and resonance of the gear motor system combined with modal analysis. Combining the nonlinear permeance network model of a squirrel cage induction motor (IM) and the bending torsion coupling dynamic model of a planetary gear rotor system and considering external excitation such as load mutation and voltage transient, [34] analyzed the dynamic characteristics of electromechanical coupling of a motor–gear system, and the author further provided an effective method for detecting the asymmetric voltage sag condition [35]. In [36], nonlinear damping characteristics, the time-varying meshing stiffness of gears, the wheel rail contact relationship, and other nonlinear factors were considered to reveal the dynamic performance of the motor car–track system model.

From the above analysis, it can be seen that scholars all over the world have conducted in-depth research on torque ripple within motor systems and the dynamic load of gear transmission systems, but little attention has been paid to the influence of coupling between a mechanical system and an electrical system on the dynamic performance of an IEDS. A small number of electromechanical coupling studies only focus on the electromechanical coupling characteristics of the IEDS, rarely conducting in-depth study on the overall torque ripple of the IEDS, and they do not propose methods to improve the system performance, which limits the ride comfort and NVH (noise, vibration, harshness) performance improvement of a PEV equipped with an IEDS.

### 1.3. Original Contributions of This Paper

In order to explore the influence of motor harmonic torque on the stability and dynamic load of an IEDS, the electromechanical coupling dynamic model of a PEV equipped with an IEDS is established in this paper. The simulation results show that the mechanical nonlinear factors such as time-varying meshing stiffness and meshing error will lead to the fluctuation of the motor shaft speed. The dead-time effect and voltage drop effect of the inverter will cause the harmonic torque of the motor and increase the dynamic load of the mechanical system. To reduce the influence of motor torque on the IEDS, a harmonic torque reduction strategy is proposed in this paper. Harmonic voltage is injected into the traditional field-oriented control (FOC) to reduce the harmonic torque of the IEDS, and it can ensure the system's stability and a response equal to the traditional FOC. Furthermore, the simulation results show that the harmonic torque reduction strategy proposed in this paper can effectively reduce the speed fluctuation and dynamic load of the system components and improve the stability of the IEDS.

## 2. Dynamics Modeling of PEV Equipped with IEDS

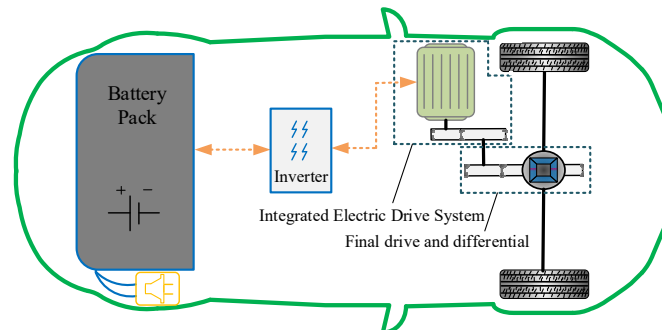
The structure of a PEV equipped with an IEDS is shown in Figure 2. Its power transmission system mainly includes an IEDS, final drive, differential, axle shaft, and wheel. In order to study the influence of harmonic torque on the working performance of an IEDS, the transient models of the electrical system and mechanical transmission system of an IEDS are established in this paper. On this basis, the dynamic model of the PEV equipped with an IEDS is established.

### 2.1. Harmonic Torque Mathematical Model of PMSM

Combined with Clarke transform and Park transform, the mathematical model of a three-phase PMSM in a d-q rotating coordinate system can be obtained. The stator voltage equation is as follows:

$$\begin{cases} u_d = Ri_d + L_d \frac{d}{dt} i_d - \omega_e L_s i_q \\ u_q = Ri_q + L_q \frac{d}{dt} i_q + \omega_e (L_s i_d + \psi_f) \end{cases} \quad (1)$$

where  $u_d$  and  $u_q$  are the voltages of the d-axis and q-axis respectively;  $i_d$  and  $i_q$  are the currents of the d-axis and q-axis respectively;  $L_d$  and  $L_q$  are the stator inductance of the d-axis and q-axis respectively;  $R$  is the stator resistance of the motor; and  $\omega_e$  is the rotor angular velocity.



**Figure 2.** Structure diagram of a pure electric vehicle (PEV) power transmission system equipped with an integrated electric drive system (IEDS).

The torque equation of a surface-mounted permanent magnet synchronous motor (SPMSM) is as follows:

$$T_m = \frac{3}{2} P_n i_q [i_d (L_d - L_q) + \psi_f]. \tag{2}$$

Stated thus, through the transformation from an a-b-c coordinate system to a d-q rotating coordinate system, the electromagnetic torque of PMSM is only related to its q-axis current. This means that by controlling the q-axis current of the motor, the electromagnetic torque control of the motor can be realized. The parameters of PMSM used in this paper are shown in Table 1.

**Table 1.** Parameters of the permanent magnet synchronous motor (PMSM) in this paper.

Parameter	Value	Unit
Peak power	60	kW
Peak torque	250	Nm
Maximum speed	7000	rpm
Base speed	2292	rpm
DC voltage	650	V
Stator resistance	0.153	$\Omega$
Stator inductance	1.8	mH
Permanent magnet flux linkage $\psi_f$	0.2778	Wb
Pole number $P_n$	4	-
Phase current limit	150	A

The inverter circuit of the PMSM for an electric vehicle is shown in Figure 3.  $S_a, S_b, S_c, S'a, S'b,$  and  $S'c$  are defined as the switch states of the inverter. The working principle of the inverter is as follows: when  $S_a/S_b/S_c$  is in 1 state,  $S'a/S'b/S'c$  is in 0 state. At this time, the three switching devices of the upper bridge arm of the inverter circuit are on, while the three switching devices of the lower bridge arm of the inverter circuit are turned off.

Figure 4 shows a structural diagram of the A-phase bridge arm of the inverter, in which the switch device is an insulated gate bipolar transistor (IGBT), and the motor stator winding is equivalent to a resistance inductance circuit, forming a series structure with the voltage source. As shown in Figure 4, when the inverter switching device  $S_a$  is on,  $S'a$  must be turned off. If the speed of  $S_a$  is turning on faster than that of  $S'a$  is turning off, the bridge arm will be short-circuited. Therefore, in order to ensure the safe operation of the inverter, dead time  $T_{dead}$  should be added to the process of switching between the two switches. During the dead time, it can be equivalent to the switch of the upper or lower leg of

the inverter being on. There is a deviation between the actual switch signal and the ideal switch driving signal, which leads to the deviation between the actual output voltage and the demand voltage.

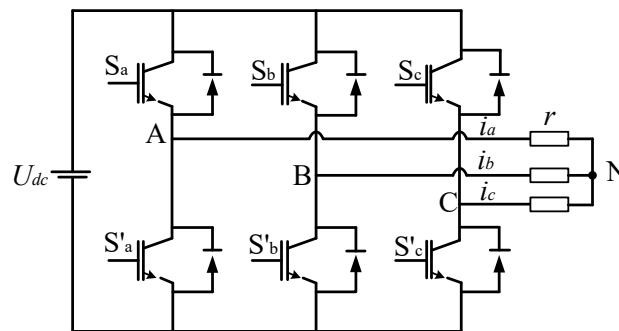


Figure 3. Inverter circuit of the PMSM.

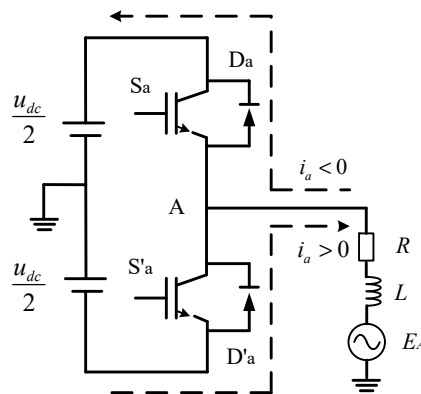


Figure 4. Structure diagram of the A-phase bridge arm of the inverter.

The dead time in the actual output voltage waveform of the inverter can be expressed as follows:

$$T_d = t_d + t_{on} - t_{off} \tag{3}$$

where  $t_{on}$  and  $t_{off}$  represent the on time and off time of the IGBT respectively; while  $t_d$  represents the preset dead time in the switch driving signal.

The above content is the error voltage brought by the dead time of the inverter to the system. The switch tube voltage drop and freewheeling diode conduction voltage drop will also bring an error voltage to the system. As shown in Figure 5, the current direction and path in the inverter bridge arm are different under different switching states. The error voltage  $U_{err}$  considering dead time and tube voltage drop is obtained by making a difference, as shown in Figure 5.  $v_t$  is the conduction voltage drop of the switch tube, and  $v_d$  is the conduction voltage drop of the freewheeling diode. Supposing that  $T_{s1}$  and  $T_{s2}$  are the real turn-on times of  $S_a$  and  $S'_a$  of the upper and lower switches in the A-phase of the inverter respectively during a pulse width modulation (PWM) cycle, the average error voltage  $\Delta u$  can be obtained by averaging the error voltage in a period of time according to the area equivalent principle. After averaging the error voltage over a period of time, a square wave signal can be obtained, as shown in Figure 6.

Through Fourier decomposition of the square wave signal, the mathematical expression of the average error voltage of the inverter can be obtained as follows:

$$u_{err} = \frac{4\Delta u}{\pi} \left( \sin \omega t + \frac{1}{3} \sin 3\omega t + \frac{1}{5} \sin 5\omega t + \frac{1}{7} \sin 7\omega t + \dots \right). \tag{4}$$

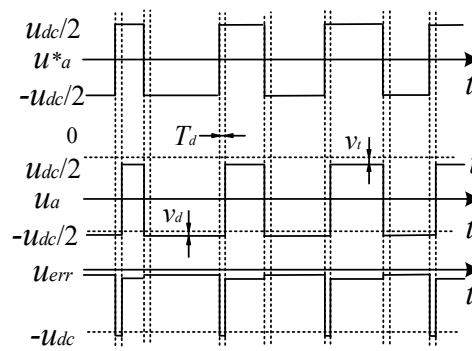


Figure 5. Voltage waveform of the A-phase bridge arm.

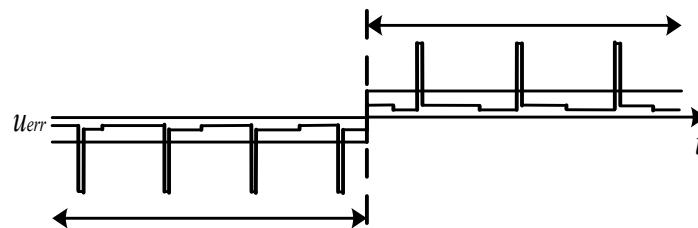


Figure 6. Error voltage waveform generated by inverter nonlinear characteristics.

From the results of Fourier decomposition, it can be seen that the nonlinear characteristics such as the dead-time effect and voltage drop of the inverter will introduce a large number of harmonics into the motor voltage, then causing the motor to produce a harmonic current. Since the stator winding of the PMSM mostly adopts star connection mode, the 3rd and integral multiple harmonic components of the stator current can not be circulated. Therefore, the actual system mainly includes 5th, 7th, 11th, and 13th harmonic components [37]. The specific parameters of the inverter used in this paper are shown in Table 2.

Table 2. Parameters of the inverter in this paper. IGBT: insulated gate bipolar transistor.

Parameter	Value	Unit
DC bus voltage $u_{dc}$	650	V
Modulation carrier period $T_{PWM}$	100	$\mu s$
IGBT turn on time $t_{on}$	1	$\mu s$
Voltage drop of IGBT switch tube $u_t$	3	V
Modulation carrier frequency $f_{PWM}$	10	kHz
Dead time $t_d$	4	$\mu s$
IGBT turn off time $t_{off}$	2	$\mu s$
Conduction voltage drop of freewheeling diode $u_d$	2	V

In the a-b-c coordinate system, the rotation speed of the 5th harmonic voltage vector in the motor stator winding is  $-5\omega$ , and the rotation speed of the 7th harmonic voltage vector in the motor stator winding is  $7\omega$  [38]. Therefore, the stator voltage equation in the three-phase static coordinate system can be expressed as follows:

$$\begin{cases} u_a = u_1 \sin(\omega t + \theta_1) + u_5 \sin(-5\omega t + \theta_5) + u_7 \sin(7\omega t + \theta_7) + \dots \\ u_b = u_1 \sin(\omega t + \theta_1 - \frac{2}{3}\pi) + u_5 \sin(-5\omega t + \theta_5 - \frac{2}{3}\pi) + u_7 \sin(7\omega t + \theta_7 - \frac{2}{3}\pi) + \dots \\ u_c = u_1 \sin(\omega t + \theta_1 + \frac{2}{3}\pi) + u_5 \sin(-5\omega t + \theta_5 + \frac{2}{3}\pi) + u_7 \sin(7\omega t + \theta_7 + \frac{2}{3}\pi) + \dots \end{cases} \quad (5)$$

where  $\theta_1$  is the initial phase angle of the stator fundamental voltage;  $\theta_5$  and  $\theta_7$  are the initial phase angles of the 5th and 7th harmonic voltages of the stator, respectively;  $u_1$  is the amplitude of the stator

fundamental voltage; and  $u_5$  and  $u_7$  are the amplitude of the 5th and 7th harmonic voltages of the stator, respectively.

Adopting the amplitude invariable transformation, the stator voltage equation expressed in Equation (5) is transformed into a d-q rotating coordinate system, and the converted stator voltage equation of the d-axis and q-axis is obtained as follows:

$$\begin{cases} u_d = u_{d1} + u_5 \cos(-6\omega t + \theta_5) + u_7 \cos(6\omega t + \theta_7) \\ u_q = u_{q1} + u_5 \sin(-6\omega t + \theta_5) + u_7 \sin(6\omega t + \theta_7) \end{cases} \quad (6)$$

where  $u_{d1}$  and  $u_{q1}$  are the d-axis and q-axis components of fundamental voltage in the d-q rotating coordinate system, respectively.

It can be seen from Equation (6) that in the d-q rotating coordinate system, the 5th and 7th harmonic components on the stator of the motor in the original three-phase static coordinate system are shown as the 6th harmonic component, with the rotation directions of them being opposite. If there is harmonic voltage in the stator voltage, there will be a corresponding harmonic current. In the same way, transforming the three-phase current of the motor into the d-q rotating coordinate system by means of amplitude invariance, the stator current equation in the d-q rotating coordinate system is obtained as follows:

$$\begin{cases} i_d = i_{d1} + i_5 \cos(-6\omega t + \theta'_5) + i_7 \cos(6\omega t + \theta'_7) \\ i_q = i_{q1} + i_5 \sin(-6\omega t + \theta'_5) + i_7 \sin(6\omega t + \theta'_7) \end{cases} \quad (7)$$

where  $i_{d1}$  and  $i_{q1}$  represent the d-axis and q-axis components of the stator fundamental current in the d-q rotating coordinate system;  $i_5$  and  $\theta_5$  represent the amplitude and initial phase angle of the 5th harmonic current in the d-q rotating coordinate system; and  $i_7$  and  $\theta_7$  represent the amplitude and initial phase angle of the 7th harmonic current of the motor in the d-q rotating coordinate system.

Ignoring the harmonic caused by the motor itself, there is no harmonic flux component in the permanent magnet flux linkage. By substituting Equation (7) into Equation (1), the stator voltage equation with harmonic component is obtained as follows:

$$\begin{cases} u_d = -\omega_e L_s i_{q1} + R i_{d1} + 5\omega_e L_s i_5 \sin(-6\omega t + \theta'_5) + R i_5 \cos(-6\omega t + \theta'_5) \\ \quad - 7\omega_e L_s i_7 \sin(6\omega t + \theta'_7) + R i_7 \cos(6\omega t + \theta'_7) \\ u_q = \omega_e \psi_f + \omega_e L_s i_{d1} + R i_{q1} - 5\omega_e L_s i_5 \cos(-6\omega t + \theta'_5) + R i_5 \sin(-6\omega t + \theta'_5) \\ \quad + 7\omega_e L_s i_7 \cos(6\omega t + \theta'_7) + R i_7 \sin(6\omega t + \theta'_7) \end{cases} \quad (8)$$

Projecting the fundamental current and harmonic current of the motor into the d-q rotating coordinate system, the current in the d-q axis is obtained as follows:

$$\begin{cases} i_d = i_{d1} + i'_{d5} + i'_{d7} + i'_{d11} + i'_{d13} \\ i_q = i_{q1} + i'_{q5} + i'_{q7} + i'_{q11} + i'_{q13} \end{cases} \quad (9)$$

where  $i_{d1}$  is the d-axis component of the stator fundamental current;  $i'_{d5}$ ,  $i'_{d7}$ ,  $i'_{d11}$ , and  $i'_{d13}$  are the d-axis components of the stator harmonic current of 5th, 7th, 11th, and 13th harmonic currents, respectively;  $i_{q1}$  is the q-axis component of the stator fundamental current; and  $i'_{q5}$ ,  $i'_{q7}$ ,  $i'_{q11}$ , and  $i'_{q13}$  are the q-axis components of the stator harmonic current of the 5th, 7th, 11th, and 13th harmonic currents, respectively.

By introducing Equation (9) into the electromagnetic torque equation of the motor, the harmonic equation of the torque is obtained as follows:

$$\begin{aligned} T_m &= \frac{3}{2} P_n \psi_f i_q \\ &= \frac{3}{2} P_n \psi_f \left[ i_{q1} + (i'_{q5} + i'_{q7}) + (i'_{q11} + i'_{q13}) \right] \\ &= T_{e0} + T_{e6} + T_{e12} \end{aligned} \quad (10)$$

where  $T_{e0}$  is the constant component of electromagnetic torque; and  $T_{e6}$  and  $T_{e12}$  are the 6th and 12th harmonics of the electromagnetic torque respectively.

It can be seen from Equation (10) that the constant component of electromagnetic torque is generated by the interaction of fundamental current and permanent magnet flux linkage under the premise of ignoring the harmonics introduced by the motor. The 6th harmonic of electromagnetic torque is mainly generated by the interaction of the rotor flux and stator 5th and 7th harmonic currents. Similarly, the 12th harmonic of electromagnetic torque is mainly generated by the interaction of the rotor flux and stator 11th and 13th harmonic currents. Generally, the larger the harmonic number, the smaller the corresponding harmonic torque amplitude becomes.

## 2.2. Transmission System Model of IEDS

The mechanical transmission system of an IEDS mainly includes a wheel gear reducer and output shaft. The dynamic model of a helical gear transmission system is established as below.

The meshing displacement of the gear tooth along the meshing line is:

$$\delta_y = R_1\theta_1 - R_2\theta_2 - e \quad (11)$$

where  $\theta_1$  is the angular displacement of gear 1;  $\theta_2$  is the angular displacement of driven gear 2;  $R_1$  is the radius of gear 1;  $R_2$  is the radius of gear 2; and  $e$  is the meshing error.

The normal elastic deformation of the gear tooth along the contact line is as follows:

$$\delta_i = \delta_y \cos \beta_b \quad (12)$$

where  $\beta_b$  is the helix angle of the base circle.

The meshing force of the gear pair can be expressed as follows:

$$F_m = c_m \dot{\delta}_i + k_m \delta_i = \left[ c_m (R_1 \dot{\theta}_1 - R_2 \dot{\theta}_2 - \dot{e}) + k_m (R_1 \theta_1 - R_2 \theta_2 - e) \right] \cos \beta_b \quad (13)$$

where  $c_m$  is the gear meshing damping and  $k_m$  is the gear meshing stiffness. The calculation methods of the two will be described in detail later.

The y-direction meshing force of the gear pair can be expressed as follows:

$$F_y = F_m \cos \beta_b = \left[ c_m (R_1 \dot{\theta}_1 - R_2 \dot{\theta}_2 - \dot{e}) + k_m (R_1 \theta_1 - R_2 \theta_2 - e) \right] \cos^2 \beta_b. \quad (14)$$

The torsional vibration model of a parallel shaft helical cylindrical gear pair is as follows:

$$\begin{cases} I_1 \ddot{\theta}_1 = T_1 - F_y R_1 \\ I_2 \ddot{\theta}_2 = -T_2 + F_y R_2 \end{cases} \quad (15)$$

where  $I_1$  is the moment of inertia of gear 1;  $I_2$  is the moment of inertia of gear 2;  $T_1$  is the active torque acting on gear 1; and  $T_2$  is the load torque acting on gear 2.

The specific parameters of helical gears involved in this paper are shown in Table A1 of Appendix A. By calculating the length change of the meshing line in the process of gear meshing and describing the time-varying meshing stiffness of the helical gear by the angular displacement of the driving gear, the relationship between the stiffness and angular displacement of the driving gear is obtained, as shown in Figure 7.



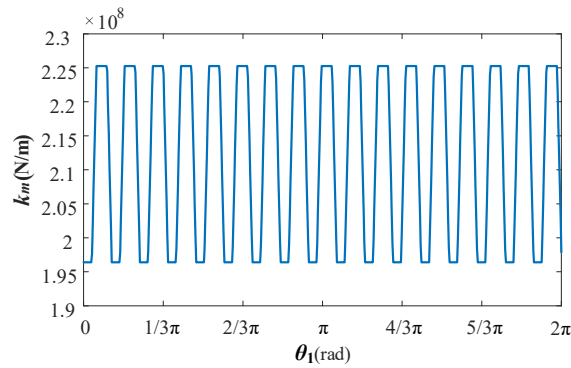


Figure 7. Time-varying meshing stiffness of gear pair.

The meshing damping of the gear pair can be expressed as follows:

$$c_m = 2\xi_m \sqrt{\bar{k}_{12} \frac{m_1 m_2}{m_1 + m_2}} \tag{16}$$

where  $\xi_m$  is the meshing damping ratio in the range of 0.03–0.17; and  $m_i$  is the mass of gear  $i$ .

The mathematical expression of the gear meshing error is as follows:

$$e = e_m + e_r \sin(Z_1\theta_1 + \delta) \tag{17}$$

where  $e_m$  is the constant value of the meshing error, and  $e_r$  is the amplitude of the meshing error, both of which are related to the manufacturing accuracy of the gear; and  $\delta$  is the initial phase of the meshing error.

### 2.3. Vehicle Powertrain Model

The moment of inertia of the whole vehicle is equivalent to the wheel in this paper, and the torsional vibration model of the whole vehicle powertrain is obtained, as shown in Figure 8. Ignoring the translational vibration of the system, a 6-DOF dynamic model of the power transmission system is established by using the lumped parameter method, as shown in Equation (18).

$$\begin{cases} I_m \ddot{\theta}_m + c_{s1}(\dot{\theta}_m - \dot{\theta}_1) + k_{s1}(\theta_m - \theta_1) = T_m \\ I_1 \ddot{\theta}_1 - c_{s1}(\dot{\theta}_m - \dot{\theta}_1) - k_{s1}(\theta_m - \theta_1) + R_1 F_y = 0 \\ I_2 \ddot{\theta}_2 + c_{s2}(\dot{\theta}_2 - \dot{\theta}_g) + k_{s2}(\theta_2 - \theta_g) - R_2 F_y = 0 \\ I_g \ddot{\theta}_g - c_{s2}(\dot{\theta}_2 - \dot{\theta}_g) - k_{s2}(\theta_2 - \theta_g) + 2/i_0 \cdot c_a(\dot{\theta}_g/i_0 - \dot{\theta}_w) + 2/i_0 \cdot k_a(\theta_g/i_0 - \theta_w) = 0 \\ I_w \ddot{\theta}_w - c_a(\dot{\theta}_g/i_0 - \dot{\theta}_w) - k_a(\theta_g/i_0 - \theta_w) + c_w(\dot{\theta}_w - \dot{\theta}_V) + k_w(\theta_w - \theta_V) = 0 \\ I_V \ddot{\theta}_V - 2c_w(\dot{\theta}_w - \dot{\theta}_V) - 2k_w(\theta_w - \theta_V) = -T_L \end{cases} \tag{18}$$

where  $T_m$  is the electromagnetic driving torque generated by PMSM; and  $T_L$  is the external resistance load when the vehicle is working. The parameters and their values represented by other symbols are shown in Table A2 of Appendix A.

Among them, the resistance load of the vehicle can be expressed as follows:

$$T_L = (mgf \cos \alpha + \frac{C_{DA}}{21.15} V^2 + mg \sin \alpha) \cdot r \tag{19}$$

where  $r$  is the wheel radius of the car.

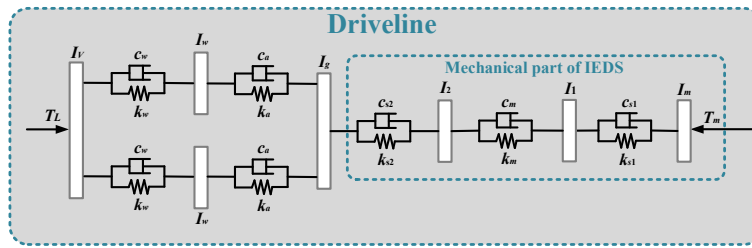


Figure 8. 6-DOF torsional vibration model of a power transmission system.

### 3. Research on Electromechanical Coupling Characteristics of IEDS

In order to study the electromechanical coupling effect of an IEDS, the transmission system dynamic model, PMSM, and PMSM control system model are built in this paper. The speed and torque of the motor shaft are used as common variables to transfer data between the electrical system and mechanical system in real time. Figure 9 shows the electromechanical coupling model of a PEV in motor torque mode. It should be noted that the simulation model of the IEDS is built on the MATLAB/Simulink 2018b simulation platform and the computer processor is Intel(R) Core(TM) i7-8700 CPU@ 3.20 Ghz.

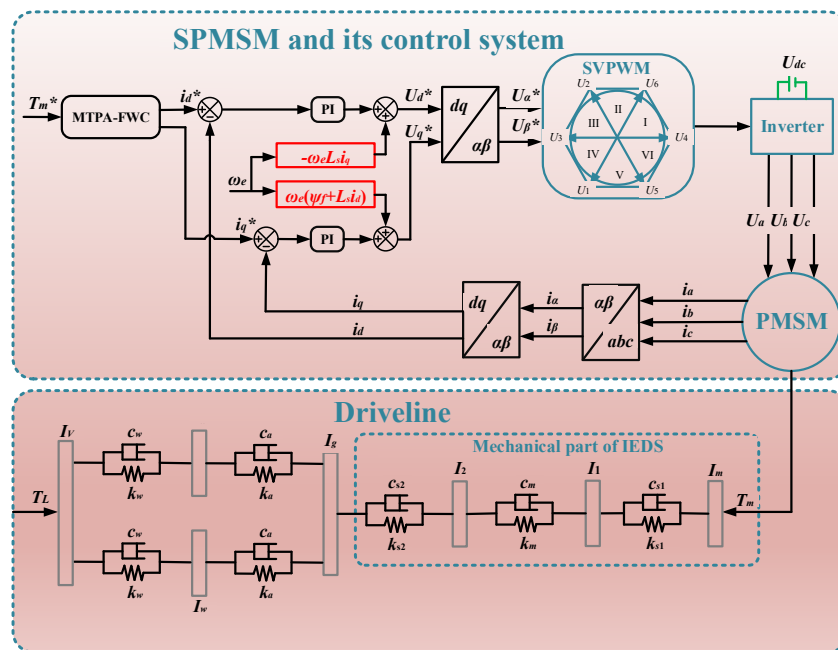
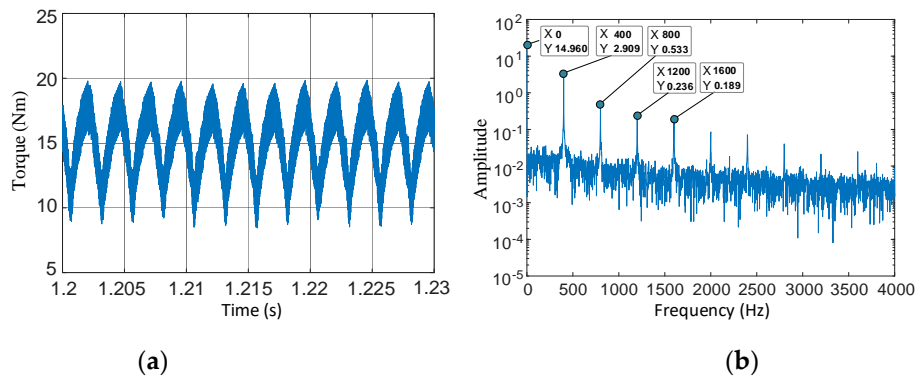


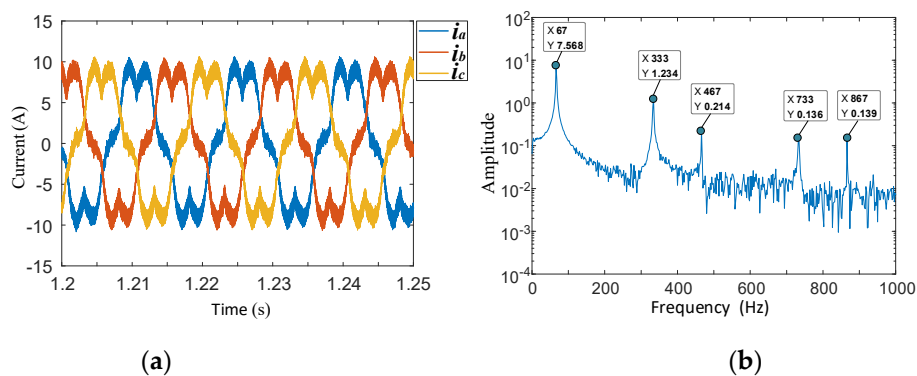
Figure 9. Electromechanical coupling dynamic model of a PEV.

In order to explore the electromechanical coupling characteristics of an IEDS under the condition of uniform speed, the simulation conditions are set as: the speed of PMSM is 1000 rpm; the vehicle load is 100 Nm; and the motor working mode is torque control mode. When the speed of the PMSM is 1000 rpm (the rotation frequency is 16.67 Hz), the electric angular frequency of the motor is 66.67 Hz, and the gear meshing frequency of the reducer of the IEDS is 283.33 Hz. The electromagnetic torque and its frequency spectrum of the PMSM are shown as Figure 10a,b, respectively. Under the influence of nonlinear factors such as inverter dead-time effect and voltage drop effect, the electromagnetic torque shows the characteristics of pulsation. Moreover, the fluctuation range of harmonic torque is about 11 Nm, and the fluctuation frequencies of harmonic torque are mainly 400 Hz, 800 Hz, and 1200 Hz, which are 6 times, 12 times, and 18 times the angular frequency, respectively. Figure 11a shows the three-phase current of the PMSM, and it can be seen that the current waveform in the time domain is not

completely sinusoidal. The frequency domain analysis of the A phase current in three-phase current by fast Fourier transform (FFT) shows that in addition to the fundamental frequency (67 Hz), the current also contains harmonic currents, among which the obvious components are 333 Hz, 467 Hz, 733 Hz, and 867 Hz, which are 5 times, 7 times, 11 times, and 13 times the angular frequency, respectively, as shown in Figure 11b.

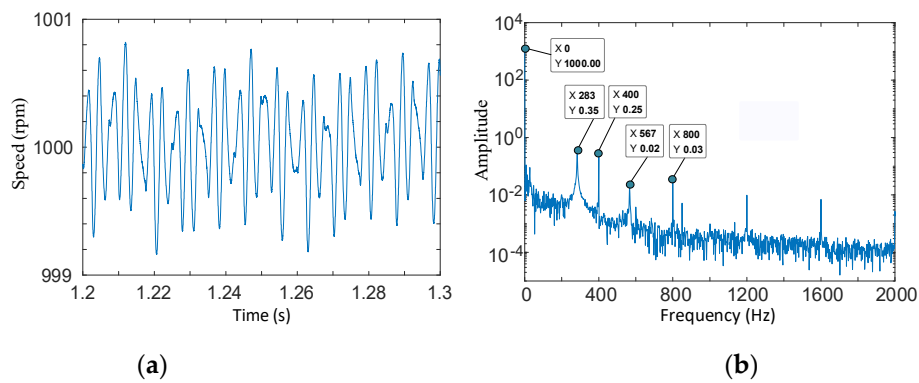


**Figure 10.** Electromagnetic torque of PMSM in an IEDS: (a) Electromagnetic torque in time domain; (b) Frequency spectrum of electromagnetic torque.

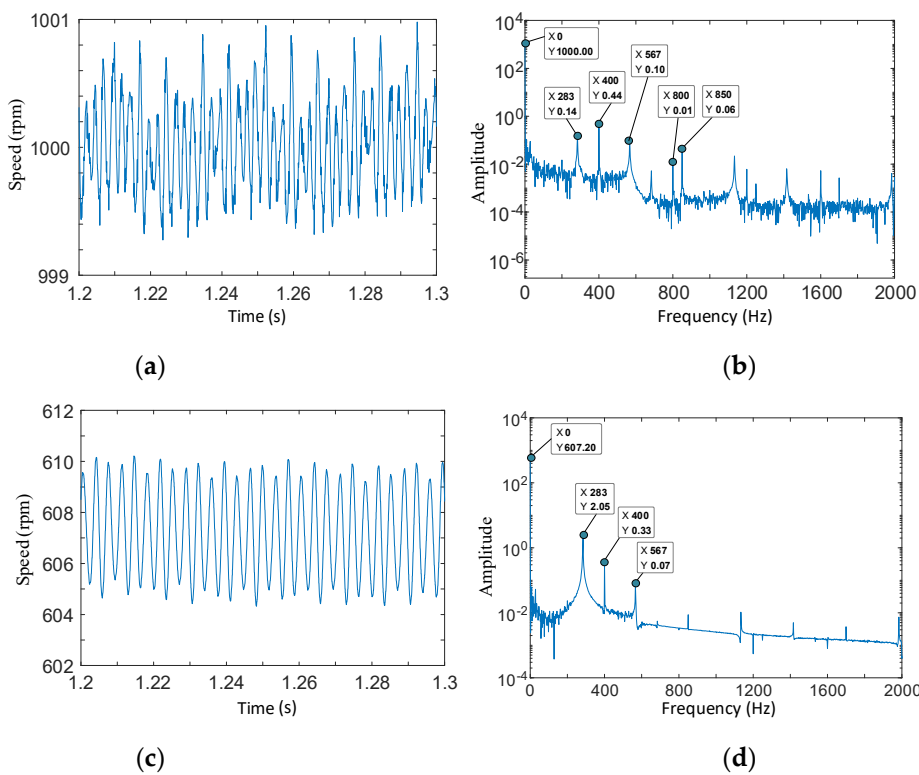


**Figure 11.** Three-phase current of PMSM in an IEDS: (a) Three-phase current in time domain; (b) Frequency spectrum of A phase current.

The shaft speed and the frequency spectrum of the PMSM are shown as Figure 12a,b, respectively. Under the influence of harmonic torque, gear time-varying stiffness, gear meshing error, and other factors, the motor shaft speed fluctuates continuously, and the fluctuation amplitude is about 2 rpm. The main components of the motor shaft speed fluctuation are gear meshing frequency (283 Hz), 2nd gear meshing frequency (567 Hz), motor 6th harmonic torque (400 Hz), and 12th harmonic torque (800 Hz). The gear pair speed and the frequency spectrum of the reducer in the IEDS are shown in Figure 13, which shows that the speed of the driving and driven gears fluctuates constantly, and the main frequency component of fluctuation is the same as that of the motor shaft. In conclusion, under the influence of mechanical transmission system factors such as gear time-varying stiffness and gear meshing error, the rotational speed of each component of the IEDS presents the characteristics of fluctuation. Meanwhile, the harmonic torque of the motor introduces more harmonics to the speed of each component, which intensifies the speed fluctuation of each component.

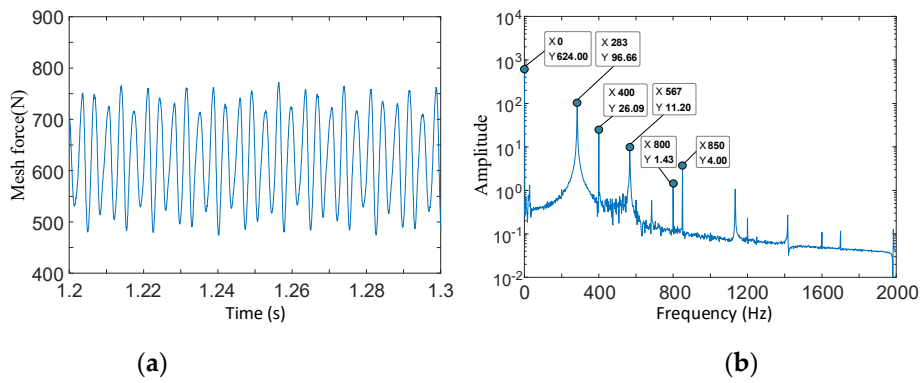


**Figure 12.** Shaft speed of the PMSM in the IEDS: (a) Shaft speed in time domain; (b) Frequency spectrum of shaft speed.

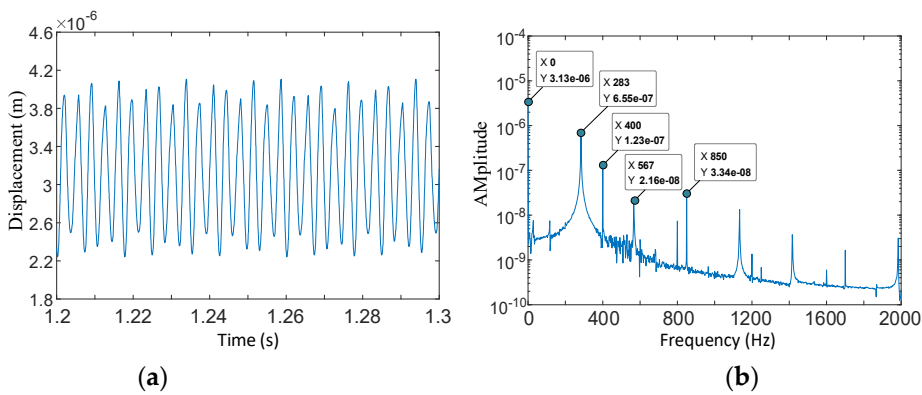


**Figure 13.** Gear pair speed of reducer in the IEDS: (a) Driving gear speed in time domain; (b) Frequency spectrum of driving gear speed; (c) Driven gear speed in time domain; (d) Frequency spectrum of driven gear speed.

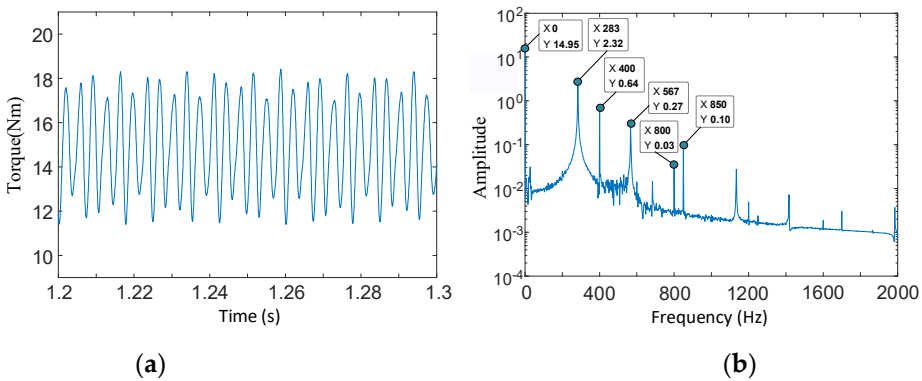
The meshing force and the frequency spectrum of the gear pair of the reducer in the IEDS is shown in Figure 14, and the meshing displacement and its frequency spectrum is shown in Figure 15. Under the influence of time-varying meshing stiffness and meshing error, the meshing force fluctuates around the theoretical meshing force. The frequency domain analysis of the gear meshing force and meshing displacement not only includes the gear meshing frequency (283 Hz) and its 2nd harmonic frequency (567 Hz), but it also includes the 6th harmonic torque (400 Hz) and the 12th harmonic torque (800 Hz). The dynamic transfer torque and the frequency spectrum of the motor shaft are shown in Figure 16, and that of the output shaft of the IEDS is shown in Figure 17. The dynamic torque transmitted by the two shafts also fluctuates continuously. It can be seen from the frequency domain analysis diagram that the main component of the fluctuation is the same as the meshing force of the gear.



**Figure 14.** Gear pair meshing force of the reducer in the IEDS: (a) Meshing force in the time domain; (b) Frequency spectrum of the meshing force.



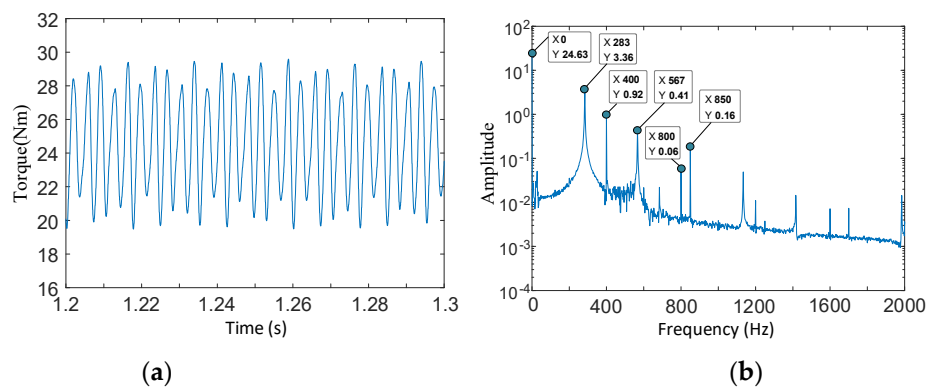
**Figure 15.** Gear pair meshing displacement of the reducer in the IEDS: (a) Meshing displacement in the time domain; (b) Frequency spectrum of the meshing displacement.



**Figure 16.** Dynamic transfer torque of the motor shaft in the IEDS: (a) Dynamic transfer torque in the time domain; (b) Frequency spectrum of dynamic transfer torque.

Through the above simulation and analysis of the electromechanical coupling characteristics of the IEDS under uniform speed conditions, it is found that the electrical system and mechanical system of the IEDS will affect each other. Gear meshing frequency is the main component of motor shaft speed fluctuation, which means that mechanical nonlinear factors such as time-varying meshing stiffness and meshing error are the main causes of motor shaft speed fluctuation. Besides, the electrical system will also affect the operation of mechanical system, because the dead-time effect and voltage drop effect of the inverter make the output electromagnetic torque of the PMSM contain 6th and 12th harmonics. The harmonic torque frequency of the motor is included in the meshing force of the gear and the

transmission torque of the shaft, which means that the harmonic torque of the motor will increase the dynamic load of the mechanical system.



**Figure 17.** Dynamic transfer torque of the output shaft in the IEDS: (a) Dynamic transfer torque in the time domain; (b) Frequency spectrum of the dynamic transfer torque.

It should be noted that when designing an IEDS, if the 6th harmonic torque of the motor is consistent with the gear meshing frequency, the amplitude of the motor speed fluctuation may be larger after superposition, which is not conducive to the smooth operation of the motor. Moreover, the dynamic load amplitude of the mechanical system is larger and the service life of the mechanical system is reduced. So, in order to avoid it, the gear meshing frequency of the reducer should not be equal to 6 times that of the electric angular frequency of the motor, namely  $Z_{p1} \neq 6P_n$  (where  $Z_{p1}$  is the number of driving gear teeth of the reducer gear pair; and  $P_n$  is the number of magnetic pole pairs).

#### 4. Harmonic Torque Reduction Strategy for IEDS

According to the analysis above, the harmonic torque will cause additional load fluctuation in an IEDS, which will aggravate the speed fluctuation of each component of the IEDS, and that is not conducive to the stable and efficient operation of the system. To solve the above problems, a harmonic torque reduction strategy to reduce the adverse effects of motor harmonic torque in an IEDS is proposed in this section.

##### 4.1. Design of Harmonic Torque Reduction Strategy

The stator of the motor mainly contains the 5th and 7th harmonic currents. In the three-phase static coordinate system, the rotation speed of the 5th harmonic current is  $-5\omega$ , and the rotation speed of the 7th harmonic current is  $7\omega$ . To better control the 5th and 7th harmonic currents of the motor, the 5th and 7th rotation coordinate systems are established in this section. Through coordinate transformation, the 5th harmonic voltage and current are converted into DC flow in the 5th rotation coordinate system, while the 7th harmonic voltage and current are converted into DC flow in the 7th rotation coordinate system. In a relative manner, other current components are converted into AC flow.

Consequently, at this time, a low-pass filter can be used to separate the 5th and 7th harmonics in the three-phase current of PMSM, and then the synchronous rotation a proportional-integral (PI) controller can be used to make the actual d-q axis current follow the reference current command to realize the injection of harmonic voltage to eliminate the torque harmonic component.

The 5th and 7th synchronous rotation coordinate systems used in Figure 18 are shown in Figure 19. It should be noted that the transformation matrix between the coordinate systems is shown in Appendix B.

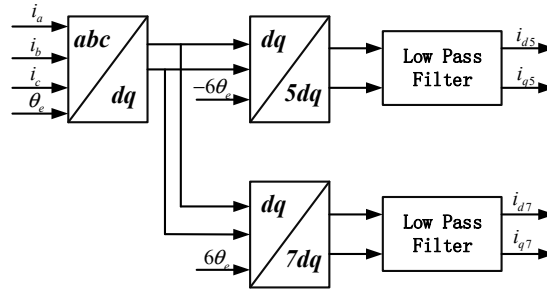


Figure 18. Separation strategy of 5th and 7th harmonic currents of a PMSM.

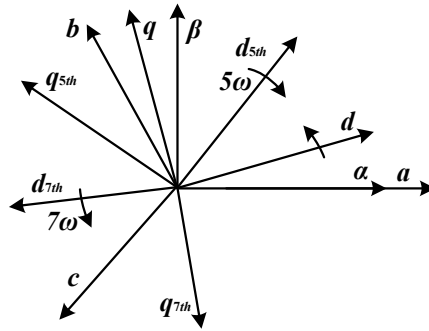


Figure 19. 5th and 7th synchronous rotation coordinate systems of a motor.

Based on the above coordinate transformation, Equation (8) is transformed into a 5th d-q rotating coordinate system:

$$\begin{cases} u_{d5} = \omega_e \psi_f \sin(-6\omega t + \theta_0) - \omega_e L_s i_1 \sin(6\omega t + \theta_1'') + Ri_1 \cos(6\omega t + \theta_1'') \\ \quad + 5\omega_e L_s i_{q5} + Ri_{d5} - 7\omega_e L_s i_7 \sin(12\omega t + \theta_7') + Ri_7 \cos(12\omega t + \theta_7') \\ u_{q5} = \omega_e \psi_f \cos(-6\omega t + \theta_0) + \omega_e L_s i_{d1} \cos(6\omega t + \theta_1'') + Ri_1 \sin(6\omega t + \theta_1'') \\ \quad - 5\omega_e L_s i_{d5} + Ri_{q5} + 7\omega_e L_s i_7 \cos(12\omega t + \theta_7') + Ri_7 \sin(12\omega t + \theta_7') \end{cases} \quad (20)$$

where  $i_{d5}$  and  $i_{q5}$  are the d-axis and q-axis DC current components in the 5th d-q rotation coordinate system.

The harmonic steady-state voltage equation in the 5th d-q rotating coordinate system is obtained by omitting the AC quantities contained in Equation (20), which is written as:

$$\begin{cases} u_{d5} = 5\omega L_q i_{q5} + Ri_{d5} \\ u_{q5} = -5\omega L_q i_{d5} + Ri_{q5} \end{cases} \quad (21)$$

Similarly, Equation (8) is transformed into the 7th d-q rotating coordinate system:

$$\begin{cases} u_{d7} = \omega_e \psi_f \sin(6\omega t + \theta_0) - \omega_e L_s i_1 \sin(-6\omega t + \theta_1'') + Ri_1 \cos(-6\omega t + \theta_1'') \\ \quad + 5\omega_e L_s i_5 \sin(-12\omega t + \theta_5') + Ri_5 \cos(-12\omega t + \theta_5') - 7\omega_e L_s i_{q7} + Ri_{d7} \\ u_{q7} = \omega_e \psi_f \cos(6\omega t + \theta_0) + \omega_e L_s i_1 \cos(-6\omega t + \theta_1'') + Ri_1 \sin(-6\omega t + \theta_1'') \\ \quad - 5\omega_e L_s i_5 \cos(-12\omega t + \theta_5') + Ri_5 \sin(-12\omega t + \theta_5'') + 7\omega_e L_s i_{d7} + Ri_{q7} \end{cases} \quad (22)$$

where  $i_{d7}$  and  $i_{q7}$  are the d-axis and q-axis DC current components in the 7th d-q rotation coordinate system.

The harmonic steady-state voltage equation in the 7th d-q rotating coordinate system is obtained by omitting the AC quantities contained in Equation (22), which is written as:

$$\begin{cases} u_{d7} = -7\omega L_q i_{q7} + Ri_{d7} \\ u_{q7} = 7\omega L_q i_{d7} + Ri_{q7} \end{cases} \quad (23)$$

Using a PI controller, combined with the 5th and 7th harmonic steady-state voltage equations, the harmonic current loop control strategy is obtained, as shown in Figure 20a,b. Among them, the 5th and 7th harmonic steady-state voltage and current are coupled with each other. In order to better control the harmonic current, this paper realizes the decoupling of stator harmonic voltage and current by adding compensation terms. Superimposing the voltage generated by the harmonic voltage steady-state equation and the harmonic current PI controller, the required injection voltage of each harmonic current in the rotating coordinate system can be obtained.

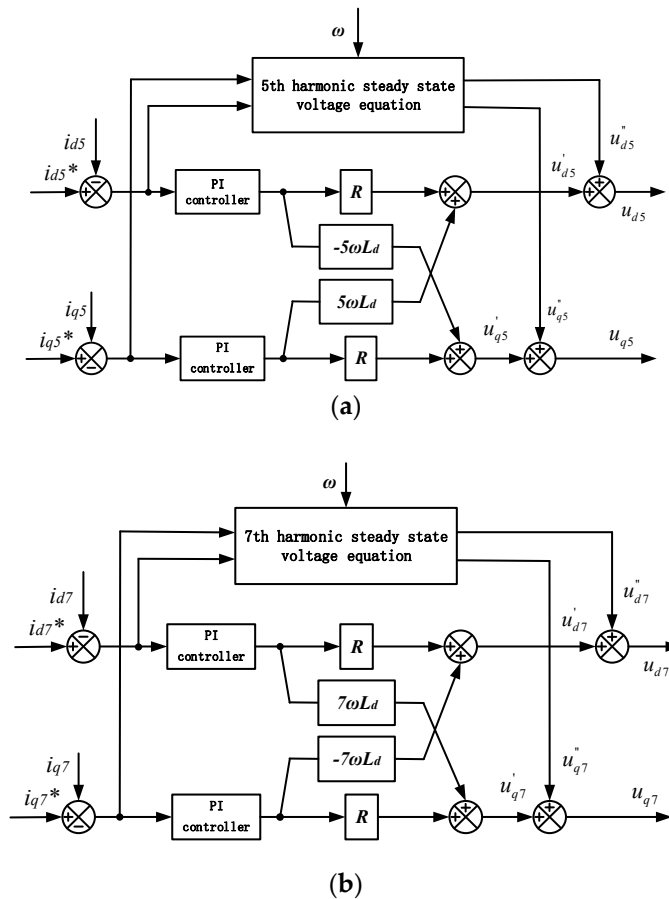


Figure 20. Control block diagram of motor harmonic current decoupling: (a) 5th harmonic current; (b) 7th harmonic current.

As shown in Figure 21, after coordinate transformation, injecting the harmonic output from the harmonic current controller into voltage signal  $U'_\alpha$  and  $U'_\beta$  and then adding them to the required voltage  $U^*_\alpha$  and  $U^*_\beta$  of the motor itself, harmonic voltage injection is realized, which constitutes the new reference voltage signals  $U_\alpha$  and  $U_\beta$  of the motor as the inverter demand voltages.

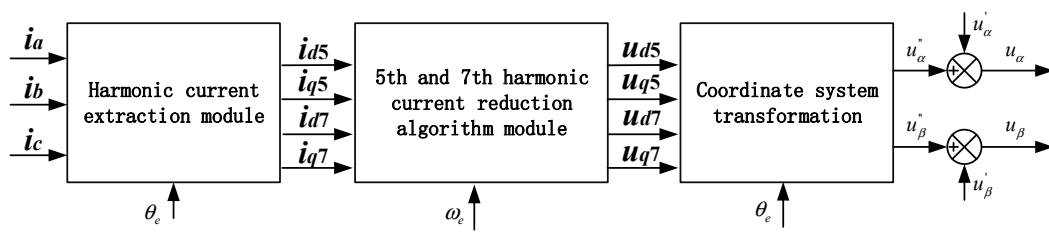


Figure 21. Harmonic torque reduction control strategy.



4.2. Simulation Analysis of Harmonic Torque Reduction Control Strategy Effectiveness

The electromechanical coupling dynamic model of a PEV with a harmonic torque reduction strategy is shown in Figure 22. In order to verify the effect of the harmonic torque reduction strategy, the simulation conditions are set as: the speed of the PMSM is 1000 rpm; the load of the vehicle is 100 Nm; and the motor is in torque control mode.

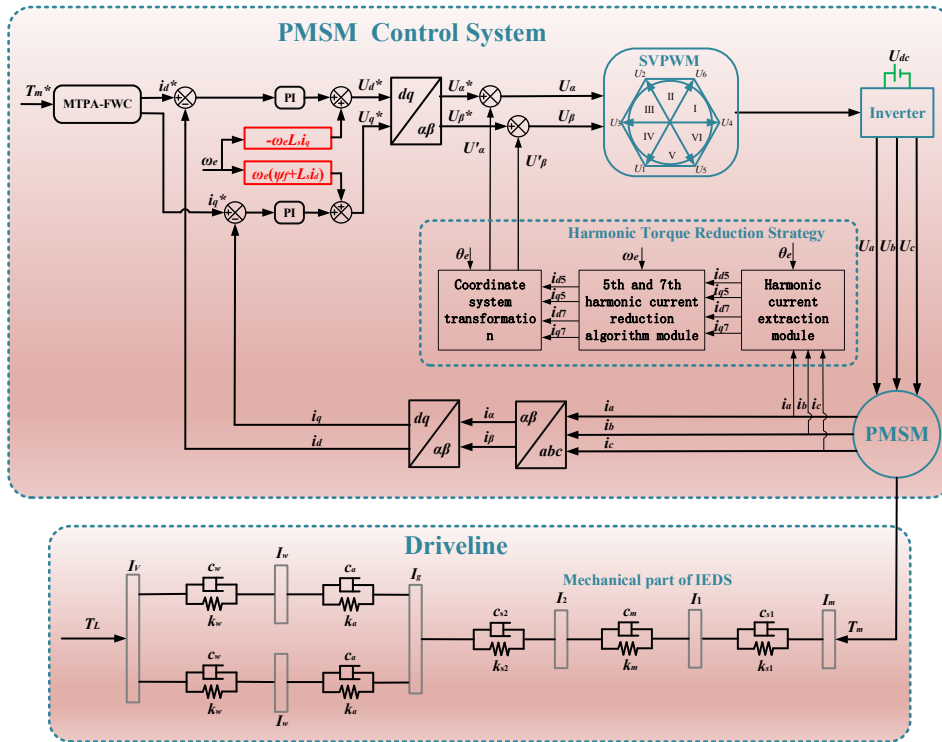
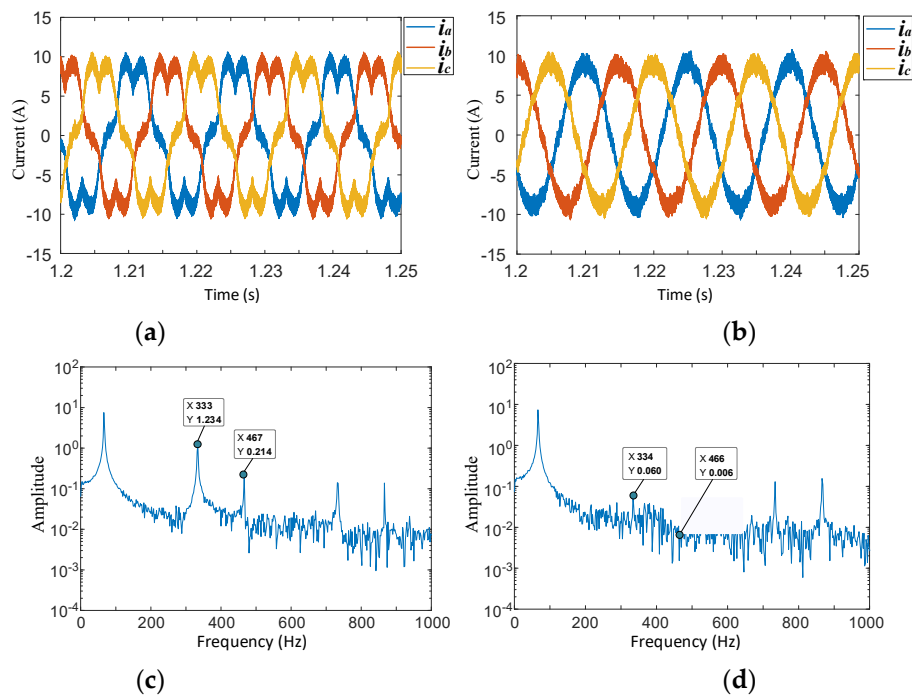


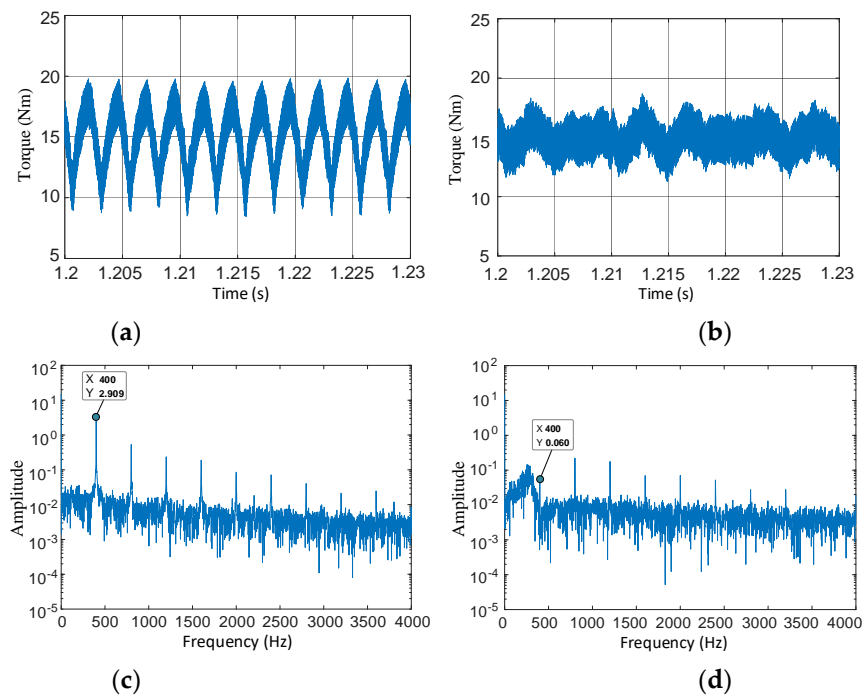
Figure 22. Electromechanical coupling dynamic model of a PEV considering harmonic torque.

Figure 23a,b show the three-phase current before and after adding the harmonic reduction strategy, respectively, while Figure 23c,d show the frequency domain analysis of the three-phase current before and after adding the harmonic reduction strategy, respectively. Moreover, Figure 24a,b show the electromagnetic torque of the motor before and after adding the harmonic reduction strategy, respectively, while Figure 24c,d show the frequency domain analysis of the motor electromagnetic torque before and after adding the harmonic reduction strategy, respectively. Under the effect of the harmonic reduction strategy, the 5th and 7th harmonics in the current are significantly reduced. Furthermore, the sinusoidal degree of the current is significantly improved, and the 6th harmonic torque of the motor is effectively reduced. It can be found that the amplitude of the 5th harmonic current is reduced from 1.234 A to 0.06 A, and that of the 7th harmonic current is reduced from 0.214 A to 0.006 A. Meanwhile, the total fluctuation amplitude of electromagnetic torque is reduced by 50% from 10 Nm to 5 Nm. The 6th harmonic torque amplitude of the motor is reduced from 2.909 Nm to 0.060 Nm. From the analysis of the above results, it can be seen that the harmonic current content can be effectively reduced by harmonic voltage injection, and then the harmonic torque can be reduced.

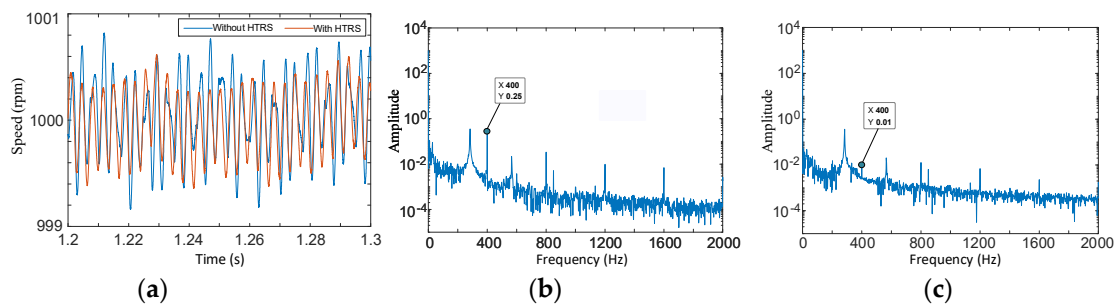
Figure 25a compares the motor shaft speed before and after adding the harmonic reduction strategy while Figure 25b,c show the frequency domain analysis of the motor shaft speed before and after adding the harmonic reduction strategy, respectively. The fluctuation amplitude of the motor shaft speed caused by the 6th harmonic torque of motor is reduced from 0.25 to 0.01 rpm, which means that the overall fluctuation amplitude of the motor speed is effectively reduced.



**Figure 23.** Three-phase current of a PMSM in an IEDS: (a) Three-phase current of the stator in the time domain before adding the harmonic reduction strategy; (b) Three-phase current of the stator in the time domain after adding the harmonic reduction strategy; (c) Frequency spectrum of the A-phase current before adding the harmonic reduction strategy; (d) Frequency spectrum of the A-phase current after adding the harmonic reduction strategy.

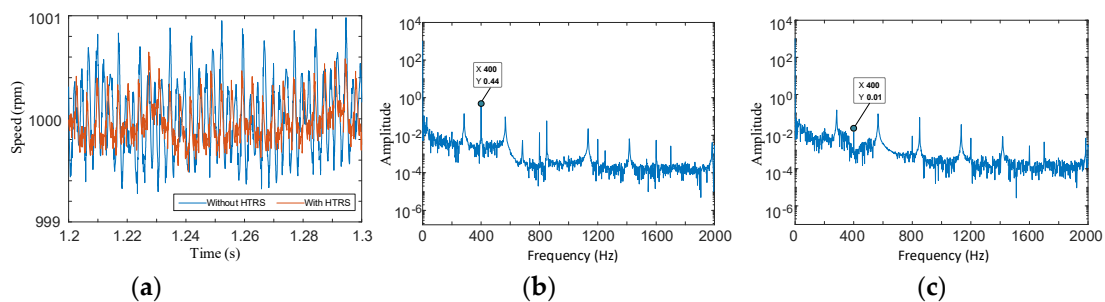


**Figure 24.** Electromagnetic torque of a PMSM in an IEDS: (a) Electromagnetic torque in the time domain before adding the harmonic reduction strategy; (b) Electromagnetic torque in the time domain after adding the harmonic reduction strategy; (c) Frequency spectrum of electromagnetic torque before adding the harmonic reduction strategy; (d) Frequency spectrum of electromagnetic torque after adding the harmonic reduction strategy.



**Figure 25.** Shaft speed of a PMSM in an IEDS: (a) Shaft speed in the time domain; (b) Frequency spectrum of the shaft speed before adding the harmonic reduction strategy; (c) Frequency spectrum of the shaft speed after adding the harmonic reduction strategy.

Figure 26a compares the driving gear speed of the reducer of the IEDS before and after adding the harmonic reduction strategy, while Figure 26b,c shows the frequency domain analysis of the reducer driving gear speed before and after adding the harmonic reduction strategy, respectively. The results show that the fluctuation amplitude of the driving gear speed caused by the 6th harmonic torque of the motor is reduced from 0.44 to 0.01 rpm.

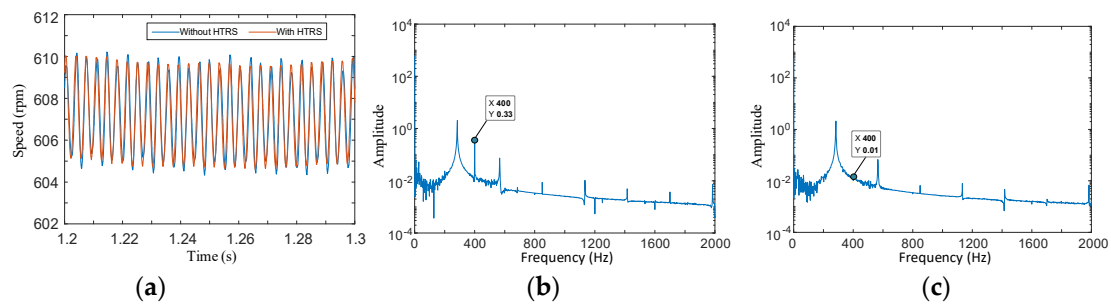


**Figure 26.** Driving gear speed of the reducer in an IEDS: (a) Driving gear speed in the time domain; (b) Frequency spectrum of the driving gear speed before adding the harmonic reduction strategy; (c) Frequency spectrum of the driving gear speed after adding the harmonic reduction strategy.

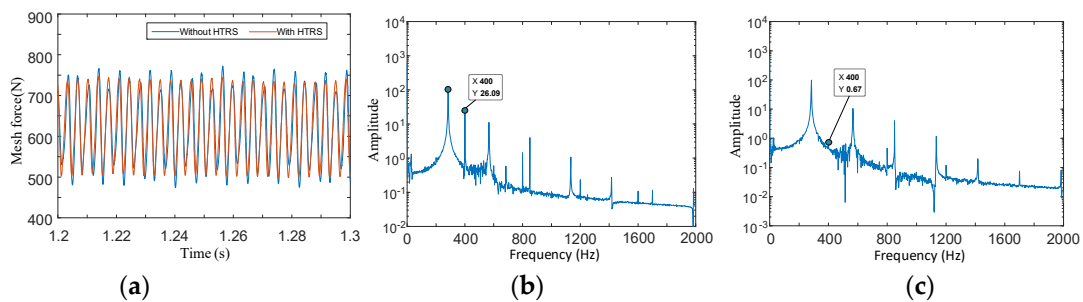
Moreover, Figure 27a compares the driving gear speed of the reducer of IEDS after adding the harmonic reduction strategy, while Figure 27b,c respectively show the frequency domain analysis of the driven gear speed of the reducer before and after adding the harmonic reduction strategy. It can be seen that the fluctuation amplitude of the driven gear speed caused by the 6th harmonic torque of the motor decreases from 0.33 to 0.01 rpm, and the overall fluctuation amplitude of the driven gear speed decreases slightly. Under the effect of the harmonic reduction strategy, the 6th harmonic torque component of the motor speed harmonics of each component of the IEDS is significantly reduced, and the fluctuation amplitudes of the speed of each component are reduced to a certain extent, which is conducive to the smooth operation of the system.

The meshing force of the reducer gear pair of the IEDS before and after adding the harmonic reduction strategy is shown in Figure 28a, while the frequency domain analysis of the meshing force of the gear pair before and after adding the harmonic reduction strategy is shown in Figure 28b,c respectively. It can be found that the amplitude of the gear pair meshing force caused by the 6th harmonic torque of motor is reduced from 26.09 to 0.67 Nm. Figure 29a compares the dynamic transmission torque of the motor shaft of the IEDS before and after adding the harmonic reduction strategy; then, Figure 29b,c show the frequency domain analysis of the motor shaft dynamic transmission torque before and after adding the harmonic reduction strategy, respectively. The amplitude of the motor shaft dynamic transmission torque caused by the 6th harmonic torque of the motor decreases from 0.64 to 0.02 Nm. Figure 30a compares the dynamic transmission torque of the output shaft of the

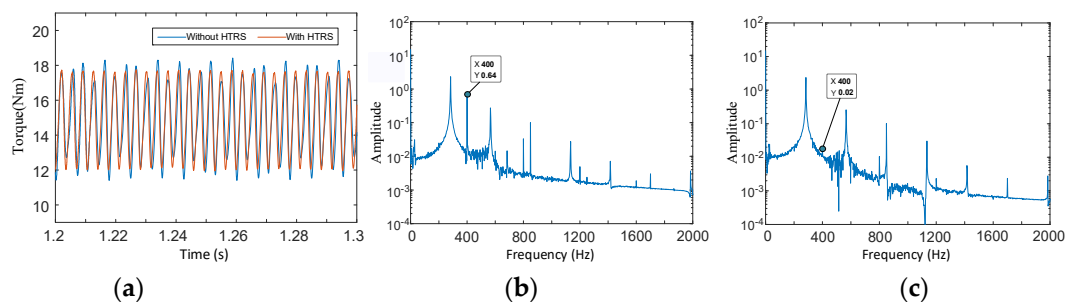
IEDS before and after adding the harmonic reduction strategy. Then, Figure 30b,c show the frequency domain analysis of the output shaft dynamic transmission torque before and after adding the harmonic reduction strategy. The dynamic transmission torque of the output shaft caused by the 6th harmonic torque of the motor is reduced from 0.92 to 0.02 Nm. That is to say, under the effect of the harmonic reduction strategy, the 6th harmonic torque component of the dynamic load of each component of the IEDS is significantly reduced, which is conducive to improving the reliability of the system and prolonging the service life of the mechanical system.



**Figure 27.** Driven gear speed of the reducer in an IEDS: (a) Driven gear speed in the time domain; (b) Frequency spectrum of the driven gear speed before adding the harmonic reduction strategy; (c) Frequency spectrum of the driven gear speed after adding the harmonic reduction strategy.



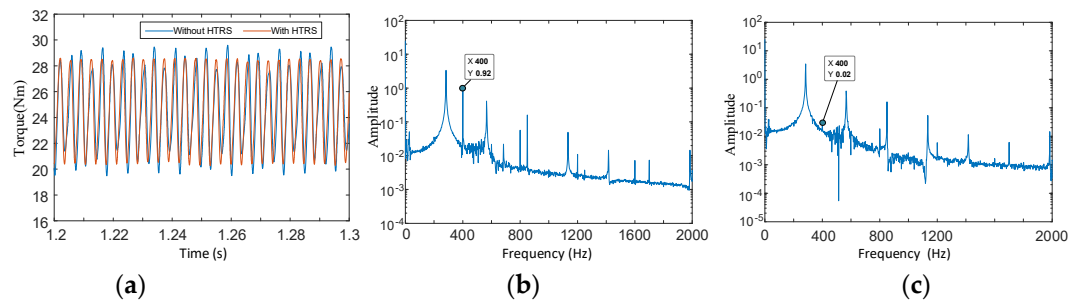
**Figure 28.** Driven meshing force of the reducer gear pair in an IEDS: (a) Reducer gear pair meshing force in the time domain; (b) Frequency spectrum of the meshing force before adding the harmonic reduction strategy; (c) Frequency spectrum of the meshing force after adding the harmonic reduction strategy.



**Figure 29.** Dynamic torque of the motor shaft in an IEDS: (a) Motor shaft dynamic torque in the time domain; (b) Frequency spectrum of dynamic torque before adding the harmonic reduction strategy; (c) Frequency spectrum of dynamic torque after adding the harmonic reduction strategy.

The detailed data comparison of IEDS before and after the harmonic torque reduction strategy is shown in Table 3. The simulation results show that the 5th and 7th harmonic currents of the PMSM are obviously reduced under the action of the harmonic torque reduction strategy, which makes the 6th harmonic torque successfully reduced, thus reducing the speed fluctuation of each component of the

IEDS and making the system run more stably. At the same time, the reduction of harmonic current also helps reduce the motor heating and improve the efficiency of the motor.



**Figure 30.** Dynamic torque of the output shaft in an IEDS: (a) Output shaft dynamic torque in the time domain; (b) Frequency spectrum of dynamic torque before adding the harmonic reduction strategy; (c) Frequency spectrum of dynamic torque after adding the harmonic reduction strategy.

**Table 3.** Data comparison of the IEDS before and after adding the harmonic torque reduction strategy.

Parameter	Before Adding Harmonic Torque Reduction Strategy	After Adding Harmonic Torque Reduction Strategy	Optimization Effect (%)
5th harmonic current amplitude (A)	1.234	0.06	95.1
7th harmonic current amplitude (A)	0.214	0.006	97.2
6th harmonic torque (Nm)	2.909	0.06	97.9
Overall fluctuation amplitude of motor torque (Nm)	10	5	50.0
Fluctuation amplitude of motor shaft speed (rpm)	0.25	0.01	96.0
Amplitude of driving gear speed fluctuation caused by 6th harmonic torque (rpm)	0.44	0.01	97.7
Fluctuation amplitude of driven gear speed caused by 6th harmonic torque (rpm)	0.33	0.01	97.0
Amplitude of meshing force fluctuation of gear pair caused by 6th harmonic torque (N)	26.09	0.67	97.4
Amplitude of shaft dynamic load fluctuation caused by 6th harmonic torque (Nm)	0.64	0.02	96.9
Amplitude of output shaft dynamic load fluctuation caused by 6th harmonic torque (Nm)	0.92	0.02	97.8

## 5. Conclusions

In this paper, the electromechanical coupling model of an electric vehicle equipped with an IEDS is established, and the electromechanical coupling characteristics of the IEDS are simulated and analyzed. On this basis, the method to suppress the harmonic torque of a PMSM is studied. There are two points that should be noted:

1. The electrical system and mechanical system of the IEDS will interact with each other. Mechanical nonlinear factors such as time-varying meshing stiffness and the meshing error

of the gears can lead to a speed fluctuation of the motor shaft. Meanwhile, the dead-time effect and voltage drop effect of the inverter will cause the 6th harmonic torque and 12th harmonic torque of the motor, which will increase the dynamic load of the mechanical system. When designing an IEDS, the gear meshing frequency of the reducer should not be equal to 6 times that of the electric angular frequency of the motor, namely  $Z_{p1} \neq 6P_n$ . In order to avoid that when the 6th harmonic torque of the motor is always consistent with the gear meshing frequency, the superposition may lead to a more serious speed fluctuation of the system, resulting in a greater dynamic load amplitude of the mechanical system and reducing the service life of the mechanical system.

- By injecting harmonic voltage, a harmonic torque reduction strategy is proposed for an IEDS in this paper. Under the effect of the harmonic torque reduction strategy, the 5th and 7th harmonic currents are effectively reduced, and the total fluctuation amplitude of the electromagnetic torque is reduced by 50%. The simulation results show that the harmonic torque reduction strategy proposed in this paper can effectively reduce the harmonic torque of the IEDS, thus reducing the speed fluctuation and dynamic load of each component of the system and improving the stability of the IEDS.

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**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A

**Table A1.** Parameters of the helical gear pair.

Parameter	Driving Gear	Driven Gear	Unit
Teeth number	$Z_1 = 17$	$Z_2 = 28$	-
Normal modulus $m_n$	3	3	mm
Modulus of end face $m_t$	3.19	3.19	mm
Tooth surface width $b$	23	23	mm
Normal pressure angle $\alpha_n$	20	20	deg
Transverse pressure angle $\alpha_t$	21.17	21.17	deg
Pitch circle helix angle $\beta$	20	20	deg

**Table A2.** Parameters of the transmission system.

Parameter	Value	Unit
Numerical motor rotor and motor shaft moment of inertia $I_m$	0.035	kg·m <sup>2</sup>
Rotational inertia of driving gear of reducer $I_1$	$1.67 \times 10^{-4}$	kg·m <sup>2</sup>
Rotational inertia of driven gear of reducer $I_2$	$1.2 \times 10^{-3}$	kg·m <sup>2</sup>
Equivalent moment of inertia of final drive and differential $I_g$	$8 \times 10^{-3}$	kg·m <sup>2</sup>
Wheel moment of inertia $I_w$	0.915	kg·m <sup>2</sup>
Body equivalent moment of inertia $I_V$	139.8	kg·m <sup>2</sup>
Torsional stiffness of motor shaft $k_{s1}$	$8 \times 10^4$	Nm/rad
Normal meshing stiffness per unit length of gear pair $k_u$	$6 \times 10^9$	Nm/mm
Torsional stiffness of output shaft $k_{s2}$	$2 \times 10^5$	Nm/rad
Half shaft torsional stiffness $k_a$	$8 \times 10^3$	Nm/rad
Wheel torsional stiffness $k_V$	$4.5 \times 10^3$	Nm/rad
Torsional damping ratio of motor shaft $c_{s1}$	2	Nm·s/rad
Gear meshing damping $c_m$	800	N/(m/s)
Torsional damping ratio of output shaft $c_{s2}$	2	Nm·s/rad

## Appendix B

$$C_{\frac{3s}{5r}} = \frac{2}{3} \begin{bmatrix} \cos(-5\theta_e) & \cos(-5\theta_e - \frac{2}{3}\pi) & \cos(-5\theta_e - \frac{4}{3}\pi) \\ -\sin(-5\theta_e) & -\sin(-5\theta_e - \frac{2}{3}\pi) & -\sin(-5\theta_e - \frac{4}{3}\pi) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix};$$

$$C_{\frac{5r}{3s}} = \frac{2}{3} \begin{bmatrix} \cos(-5\theta_e) & -\sin(-5\theta_e) & \sqrt{\frac{1}{2}} \\ \cos(-5\theta_e - \frac{2}{3}\pi) & -\sin(-5\theta_e - \frac{2}{3}\pi) & \sqrt{\frac{1}{2}} \\ \cos(-5\theta_e - \frac{4}{3}\pi) & -\sin(-5\theta_e - \frac{4}{3}\pi) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

$$C_{\frac{7s}{7r}} = \frac{2}{3} \begin{bmatrix} \cos(7\theta_e) & \cos(7\theta_e - \frac{2}{3}\pi) & \cos(7\theta_e - \frac{4}{3}\pi) \\ -\sin(7\theta_e) & -\sin(7\theta_e - \frac{2}{3}\pi) & -\sin(7\theta_e - \frac{4}{3}\pi) \\ \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} \end{bmatrix}; C_{\frac{7r}{3s}} = \frac{2}{3} \begin{bmatrix} \cos(7\theta_e) & -\sin(7\theta_e) & \sqrt{\frac{1}{2}} \\ \cos(7\theta_e - \frac{2}{3}\pi) & -\sin(7\theta_e - \frac{2}{3}\pi) & \sqrt{\frac{1}{2}} \\ \cos(7\theta_e - \frac{4}{3}\pi) & -\sin(7\theta_e - \frac{4}{3}\pi) & \sqrt{\frac{1}{2}} \end{bmatrix}$$

$$C_{\frac{2r}{5r}} = \begin{bmatrix} \cos(-6\theta_e) & \sin(-6\theta_e) \\ -\sin(-6\theta_e) & \cos(-6\theta_e) \end{bmatrix} C_{\frac{5r}{2r}} = \begin{bmatrix} \cos(-6\theta_e) & -\sin(-6\theta_e) \\ \sin(-6\theta_e) & \cos(-6\theta_e) \end{bmatrix}$$

$$C_{\frac{2r}{7r}} = \begin{bmatrix} \cos(6\theta_e) & \sin(6\theta_e) \\ -\sin(6\theta_e) & \cos(6\theta_e) \end{bmatrix} C_{\frac{5r}{2r}} = \begin{bmatrix} \cos(6\theta_e) & -\sin(6\theta_e) \\ \sin(6\theta_e) & \cos(6\theta_e) \end{bmatrix}$$

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