

Quantum Field Theory

Ralf Hofmann

Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany; hofmann@thphys.uni-heidelberg.de

This Special Issue on quantum field theory presents work covering a wide and topical range of subjects mainly within the area of interacting 4D quantum field theories subject to certain backgrounds. Investigations by Antonov, D. [1–3] concern the confining properties of SU(3) Yang–Mills vacuum models (instantons in confining monopole backgrounds and dual superconductor made of condensed monopoles) at zero temperature. The author also discusses string-breaking effects in the thermalised dual superconductor model where the distribution of the confining string is assumed to obey certain power laws of transverse distance in the exponent.

Dantas, R., Mota, H., de Mello, B., and Eugenio, R. [4] consider constrained vacuum fluctuations in an explicitly Lorentz symmetry, breaking the quantum theory of a linear scalar field, which is subject to specific derivative interactions invoking a constant four-vector as well as boundary conditions on parallel plates. In such a setting, the dependence of the Casimir effect (non-vanishing stress and energy per unit plate surface) on the direction of the Lorentz-symmetry-breaking vector on boundary conditions and on the interaction strengths, determined by the ratio of a length scale and the plate separation, is inferred through physically subtracted mode sums.

The deconfinement of SU(2) Yang–Mills thermodynamics is considered by Grandou, T. and Hofmann, R. [5] to generate a spherical zero-pressure plasma blob whose energy is determined by the product of the ground-state frequency of a vibrating, immersed magnetic monopole (with an electric-magnetically dual interpretation of its charge) and Planck’s quantum of action provided by a calorion center. To obtain the vanishing pressure of this blob, the plasma temperature T_0 in units of the critical temperature T_c for the deconfining–preconfining transition needs to be determined. A relevant mass formula, representing de Broglie’s ideas about the origin of particle–wave duality, when confronted with the mass of the electron, determines the blob radius, the monopole size, and T_0 . The authors also consider a zero-pressure plasma blob, which is void of the magnetic monopole, and compare, at the same blob radius, the lowest plasma frequency with the frequency of the perturbed magnetic monopole. The here-discussed model of the electron, which posits that the monopole-containing blob represents the self-intersection region of a figure-eight SU(2) center-vortex loop, is, however, incomplete: it neglects SU(2) gauge-theory mixing that associates with weak mixing in the Standard Model of Particle Physics (SMPP). An accordant extension, yielding a derivation of two dimensionless key parameter values of the electroweak part of the SMPP, is the subject of ongoing research [6].

Faber, M. [7] discusses the emergence of the Dirac equation for classical monopoles in his model of topological particles (MTPs). This model considers spacetime dependent-unit quaternions–elements of SO(3) as a fundamental field $Q(x)$ whose soft-core monopoles (spatial parts of $Q(x)$ are hedge-hogs in su(2)) come in four different species, each carrying two topological quantum numbers related to electric charge and spin. The affine connection in the tangential space of SU(2)—essentially the derivative of Q —corresponds to a non-abelian curvature. This model generalises the Skyrme model in allowing for solitons with long-range interactions. Faber argues that to the four soft-core monopoles at rest,



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unit vectors in a four-component Hilbert space can be assigned such that a zero-spatial-momentum wave equation arises. This equation can be generalised to finite momenta to represent the free Dirac equation. The electromagnetic field strength of a soliton can be defined in analogy to the 't Hooft tensor in the SU(2) adjoint Higgs model. This allows us to consider electromagnetic interactions, distinguishing internal and external contributions to the field strength, and therefore to recover the QED action. The approaches of M. Faber, on the one hand, and T. Grandou and R. Hofmann, on the other hand, to a resolution of the classical self-energy problem of the electron and its wavelike nature are conceptually quite different. While the former treats the electron as a topologically stabilised soliton at zero temperature and subject to an a posteriori quantum mechanical description in terms of a Dirac spinor, the latter proposal derives the pointlikeness of the electric charge on the scale of the transverse extent of the wavelike distribution of this charge in a boosted blob as a consequence of caloron topological winding in association with Planck's quantum of action in pressureless, deconfining SU(2) Yang–Mills *thermodynamics*. Conceptually, this is in agreement with [8].

't Hooft, G. [9] proposes an approach to the information-loss paradox of black-hole thermodynamics by deriving a unitary evolution law for a quantised black hole. The Penrose diagram for the eternal black hole as the background metric requires antipodal identification to guarantee that the two asymptotic domains of this metric associate with the same outside world. Antipodal identification is argued to invoke time reversal, in turn implying an 'anti-vacuum' state—a state where energy density reaches a maximal value. The re-arrangement of spacetime into two connected domains in the Penrose diagram is an example of how cut-and-paste procedures are dynamical features of space and time that are required to unravel particle quantum interactions at the Planck scale.

The black-hole information paradox is addressed in a different way by Ho, P.-M. and Yokokura, Y. [10]. These authors argue that a firewall in the near-horizon region is induced within the scrambling time due to higher dimensional operators of an effective matter-field theory. This happens because an uneventful horizon transmutes into an eventful horizon due to multi-particle states growing exponentially with time (due to a class of higher-derivative operators) after the collapsing matter enters the near-horizon region. This effect does not require large curvature invariants in the gravitational part of the effective action.

Calmet, X. and Sherrill, N. [11] make a case for atom interferometer experiments that can probe scalar and light dark matter but also quantum gravity if a simple coupling of the associated scalar field ϕ to Standard-Model matter and gauge fields is assumed. While a linear coupling is argued to be ruled out by torsion pendulum experiments, there is still room in the parameter space of quadratic or high-order couplings of ϕ . The authors discuss this in view of the possible signature detection of quantum gravity.

This Special Issue also contains two review-type contributions. The first one by Fried, H., Gabellini, I., Grandou, T., and Tsang, P. [12] discusses an interesting nonperturbative approach to QCD where gluon fields are integrated out in the partition function in favour of an antisymmetric Halpern field $\chi_{\mu\nu}^a$ which, in the eikonal, quenched approximation and in the strong coupling limit, mediates contact interactions between quarks (effective locality). This review discusses certain theoretical aspects (gauge fixing, etc.) of this venue and addresses phenomenological features (quark–quark binding potential, nucleon–nucleon binding potential, gluon bundles, elastic pp scattering, etc.). The second review by Floerchinger, S. [13] is a nice compendium of Clifford algebras and their spinor representations for various dimensions of flat spacetime as well as varying spacetime signatures. When generalising the physics of relativistic fermions away from the familiar Dirac, Majorana, or Weyl formulations, this review provides a condensed and highly useful presentation of the underlying mathematical structures. In particular, Clifford algebras are classified according to matrix realisations invoking elements in the reals, the complex numbers, and in the quaternions. Spinor spaces that represent Clifford algebras can be understood as elements of minimal or quasi-minimal left ideals of an algebra and as representations of the pin and spin groups. The author also distinguishes two types of Dirac adjoint spinors.

To summarise, this Special Issue on quantum field theory spans a wide scope of interesting ideas, and, as the Guest Editor, I value the high quality of all contributions, thank the authors for their efforts, and encourage reading their articles.

As an addition to this Special Issue topic introduction, I would like to share some personal thoughts about the present state of quantum field theory and what I believe are important developments. There is a broad consensus in the physics and mathematics community that gauge theories, defined on a four-dimensional spacetime manifold, are the basis for a description of fundamental reality. This goes for the theory of gravitation and for particle physics. In the realm of non-abelian gauge theory (or Yang–Mills theory), there is promise in understanding emergent phenomena (e.g., the dynamical breaking of chiral symmetries [14–18]), triggered by topologically stabilised, extended gauge-field configurations, to alleviate or even solve the mathematical problems of perturbatively spirited approaches with spontaneous gauge-symmetry breaking in spite of the latter’s many successes. These problems concern uncertainties in the convergence properties of small-coupling expansions [19], a shaky conceptual justification of the heuristically successful renormalisation programme [20], the probably oversimplistic, yet successful notion of elementary particles as pointlike quantum objects, and an embarrassingly poor modelling of the vacuum state [21,22]. Thus, a prime field of activity should be to improve our understanding of the emergence and implications of nonperturbative ground states in pure Yang–Mills theories, which differ strongly from their perturbative counterparts [23]. For example, given the experimentally determined values of Higgs and top-quark masses, radiative effects about the Higgs potential in the electroweak sector of the SMPP imply a metastability of the vacuum [24]! The potential vacuum decay is associated with a resolution of $\sim 10^{10}$ GeV; see [25] for a comprehensive review of the embedding of this result into expanding space, that is, cosmology. While this may indicate the limits of applicability of the SMPP as an effective theory, it also raises attention to an urgency to determine what exactly the proper theoretical framework, generalising the SMPP, is. It is likely that this framework requires a break with the perturbative paradigm. It should yield a predictive and thereby falsifiable unification of particle theory and the theory of gravitation in a setting where quantum mechanics and the Minkowskian signature of physical, asymptotically flat spacetime both emerge from a unified Euclidean and classical field theory [23] that is based on an appropriate gauge principle. In this context, I do share the opinion voiced in [26] that an understanding of the micro(quantum)structure of gravitational solitons, such as Schwarzschild black holes, will benefit from a deeper understanding of the nature of ordinary elementary particles such as the electron. Such a link could rest on the notion of spatially confined and quantum-stabilised phases of thermal gauge theory [5,8,27], and I do believe that fundamental problems such as the information paradox of black hole thermodynamics will find their resolution when faced from a vantage point within this more fundamental layer of reality, when compared to what the SMPP and our present theory of gravitation, together with its various strategies of quantisation, are able to provide: both, the SMPP and present approaches to quantum gravity are field or string theories quantised ‘after the fact’ when quantum behaviour is likely to emerge from classical Euclidean gauge theory (see above). Based on these deeper insights, it may well turn out, as Roger Penrose supposes [28], that the unitary evolution of a pure quantum state in interaction with a black hole is hampered by state reduction due to strong gravitational field interference in the vicinity of the black hole’s horizon or its singularity, and hence that information cannot be conserved as a matter of principle. Alternatively, if information *turns out to be conserved* [29], then, as a result of understanding gravitational and gauge-theoretic solitons in specific quantum environments, we will potentially be able to compute [30–32] the Page curve [33] and select how it is realised in detail, among the various presently pursued proposals (see e.g., [34,35]).

The idea that Yang–Mills theory and gravitation are interrelated is demonstrated on the level of scattering amplitudes. Namely, in [36], relations between closed- and open-string amplitudes were obtained at the tree level and do not depend on the particular string theory considered. Algebraically seen, these KLT relations allow us to express tree-level graviton amplitudes as squares of tree-level gauge-boson amplitudes (including sums over permutations

and kinematic prefactors) at infinite string tension. While this is not easily seen in terms of Feynman diagrams associated to the two field theories, a first step in demonstrating this was put forward in [37] for SU(N) Yang–Mills theories. Here, a simple expression was obtained for the square of (nontrivial) maximally helicity violating (MHV) amplitudes of n gluons scattering on tree-level ($n - 2$ gluons of the same helicity) to leading order in N . When two gluons are parallel, the square of the amplitude can be expressed by the square of the amplitude for $n - 1$ gluons. In a second step, using [36,37], it was shown in [38] that MHV amplitudes for the tree-level scattering of multi-gravitons can be written as a quadratic combination of sub-amplitudes of multi-gluons (squaring relation). As for one-loop generalisations of the tree-level relations between gauge-boson and graviton helicity amplitudes, gravity must be considered as being minimally coupled to massless matter [39] when applying the unitary method. In addition, squaring relations can be observed to occur here [40]; see also [40] for an insightful reorganisation of graviton tree amplitudes and [41] for a twistor-space representation of gauge-boson amplitudes being related to a D -instanton expansion of a certain string theory. It remains to be seen whether squaring relations for amplitudes in gravity and Yang–Mills theory are really the key to understanding quantum gravity or whether the relationship between these two theories needs to be explored nonperturbatively. In any case, these developments are good motivation for seeking nonperturbative correspondence of sufficient power to define a quantum theory of gravitation in terms of nonperturbative quantum gauge theory.

Nonperturbative results in gauge-field theory provide fundamental-physics ingredients to the cosmological model but also to the description of astrophysical systems. This is particularly true for the fuzzy-dark-matter paradigm, promisingly uniting the formation of cosmologically large-scale and small-scale structures (length scales of galaxy clusters and galaxies), in terms of ultralight, cold, and quantum-correlated axion-particle species [42–49] whose masses [14–17,50,51] may arise from a subtle interplay of Planck-scale physics with the ground states of pure Yang–Mills theories [52–55]. For example, pure SU(2) Yang–Mills theories would then associate with the leptonic mass spectrum, essentially represented by emergent blobs of deconfining phases [5,6], and may induce the electroweak interactions that are so successfully described by the SMPP [49]. It is possible that an analysis of blob embedding to the extent a_0 (Bohr radius) into condensates of extent ~ 1 kpc, formed from ultralight axions of mass $\sim 10^{-22}$ eV [47], will shed more light on the entanglement of electronic and photonic states in quantum mechanics, and on parity violation caused by the weak interactions.

The SMPP and Yang–Mills thermodynamics should thus describe complementary aspects of particle physics: the former the interactions of elementary particles (and at an impressively high precision over a vast range of energy-momentum transfers), and the latter the emergence of these particles together with derivations of (dimensionless) parameter values such as low-energy coupling constants and mixing angles [6] that the SMPP needs to extract from experimental data. The emergence of elementary particles as quantum-stabilised and phase-separated spatial regions in pure Yang–Mills theory (blobs) interrelates strongly with cosmological and astrophysical model building where the phases of pure Yang–Mills theory determine the right-hand sides of the respective global and local gravitational equations. As mentioned above, there is a considerable potential for such a theory of elementary particles and their quantum physics to also describe quantum gravitational solitons. Ultimately, this may yield hints towards a theory of quantum gravity tested through observables [56,57] in association with the supermassive black hole Sagittarius A* in the centre of the Milky Way.

To achieve an understanding of the cosmology beyond Λ CDM, the implications of Yang–Mills theories, spatially extended and evolving (thermal) ground states for early-time phase transitions, comparing, e.g., the discussions of Big-Bang-nucleosynthesis in [58] with that in [59], and late-time phase transitions, involving both dark matter/dark energy [43,49] and visible as well as dark radiation [60,61], should be fully explored. The accordingly improved cosmological model will modify the radiation sector and the dark sector at redshifts much higher than those accessible by direct cosmological or cosmographical observations of the

local universe (e.g., luminosity–distance redshift of standard(ised) candles [62] or time delays through strong gravitational lensing [63]) [64] to reconcile [65] them with global cosmology probes such as baryon-acoustic oscillations (BAO) [66], the CMB [67], as well as BAO anchored structure formation and weak lensing [68]. Depending on the results of terrestrial blackbody spectroscopy at low frequencies and temperatures ranging from 5 to 20 Kelvin [69], the thermodynamics of the CMB could fit better an SU(2) rather than a U(1) gauge principle [59,70], and the above-mentioned alterations in Λ CDM would then have a definite theoretical basis. This would also allow insights into the large-angle CMB anomalies [71?–73] and identify a late-time cause of extragalactic magnetic fields [75].

Concerning a stimulus of Conformal Quantum Field Theories (CFTs), and, in particular, of Yang–Mills thermodynamics on research in algebraic geometry, we believe that the classification of a manifold—embedded into the a priori phase space of a given, irreducible loop diagram and determined via constraints on physical momentum transfers through this diagram’s vertices—is interesting mathematically. In any case, it is useful physically in elucidating the convergence properties of the effective loop expansion in the deconfining phase [23]. In this phase, it is puzzling that the emergent mass scales $|\phi|$ (modulus of inert, adjoint Higgs field generated by (anti)caloron centers), $m_{1,2}$ (thermal quasiparticle masses of the off-Cartan vector modes determined using the Legendre consistency of thermal ground state and free thermal quasiparticle excitations), and ω^* (self-consistent one-loop radiatively generated gap in the blackbody spectrum of the Cartan gauge-boson mode) in the deconfining phase of SU(2) Yang–Mills thermodynamics all show a universal high-temperature power-law decay with exponent $-\frac{1}{2}$ [23]. Given certain integral representations of Riemann’s zeta function, an understanding of this meromorphic function’s structure within an environment of the critical line may or may not relate to the (emergent) quantum physics in deconfining Yang–Mills thermodynamics, which, in turn, generates universal exponents of objects with a mass dimension of one.

Finally, I would like to point out that there is a new and interesting research field in CFT, in part motivated by how fermion masses, generated by explicit chiral symmetry breaking (Yukawa couplings), depend on the resolution scale within the SMPP: symmetric mass generation (SMG). For a recent review, see [76]. This approach analyses emergent critical phenomena associated with the generation of fermion masses without breaking global symmetries. For low-dimensional examples in condensed-matter physics, see [77–80]. For lattice based investigations in 4D Yang–Mills theories with massless, fundamental fermions using gradient flow as a continuous renormalization-group method, see [81].

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