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Quantum Scalar Fields Interacting with Quantum Black Hole Asymptotic Regions

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Abstract: We continue our work on the study of spherically symmetric loop quantum gravity coupled to two spherically symmetric scalar fields, with one that acts as a clock. As a consequence of the presence of the latter, we can define a true Hamiltonian for the theory. In previous papers, we studied the theory for large values of the radial coordinate, i.e., far away from any black hole or star that may be present. This makes the calculations considerably more tractable. We have shown that in the asymptotic region, the theory admits a large family of quantum vacua for quantum matter fields coupled to quantum gravity, as is expected from the well-known results of quantum field theory on classical curved space-time. Here, we study perturbative corrections involving terms that we neglected in our previous work. Using the time-dependent perturbation theory, we show that the states that represent different possible vacua are essentially stable. This ensures that one recovers from a totally quantized gravitational theory coupled to matter the standard behavior of a matter quantum field theory plus low probability transitions due to gravity between particles that differ at most by a small amount of energy.

Keywords: loop quantum gravity; spherical symmetry; scalar field

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1. Introduction

Spherically symmetric loop quantum gravity is an effective symmetry-reduced laboratory for the study of black holes, singularity elimination by quantum theory, and other issues, and has been developing for over a decade now [1]. However, the introduction of matter has proven problematic. In the vacuum theory, one uses a redefinition of the constraints that allows one to turn them into a Lie algebra and complete the Dirac quantization, which at present is not known to exist in the case coupled to matter at the quantum level. The inclusion of massless scalar fields is a potentially attractive setting, as it is known to have rich dynamics that include black hole formation and the critical phenomena discovered by Choptuik [2].

Here, we would like to expand on our previous papers [3,4], which considered a spherically symmetric massless scalar field coupled to spherically symmetric gravity in the presence of a clock given by a second scalar field. The latter gives rise to a true Hamiltonian, so one quantizes a gauge-fixed theory and does not have to worry about constraints. This avenue of using matter clocks in quantum gravity has been considered by other authors as well (see [5–8] for references). In our approach, we exploit the advantages of the simplifications due to spherically symmetric gravity to make progress in defining the relevant quantum operators in a precise way. Our treatment allows us to study quantum field theory with a natural cutoff provided by the discreteness of quantum gravity. It makes contact with the expected results from quantum field theory in curved space-time. Our framework can, in principle, accommodate several space-time situations; here, we will concentrate on the one that yields quantum field theory on a black hole or other spherical backgrounds. We work in the far asymptotic region, keeping leading terms in the curvature in the calculations.

In this paper, we will consider using approximations to carry out concrete calculations of the space of states of the coupled theory. We will consider the theory for large values of the radial coordinate and expand it in powers of Newton's constant, as we are in spherical symmetry, which would mean far away from any black hole or star that may be present. This makes the calculations considerably more tractable. In contrast to our previous papers, we will consider terms in the sub-leading order in the expansion for large distances. This will allow us to study effects that may arise due to the presence of curvature and how they may modify the usual quantum field theory formulated in a Minkowskian background. We will see that quantum gravity effects add low probability transitions between physical states of the matter field. We concentrate on the low-energy eigenstates of the true Hamiltonian, which correspond to the small momentum of the clock and, therefore, lead to small interference of the clock with the system under study. When one applies gauge fixing using the second scalar field as the clock, the resulting total Hamiltonian is proportional to the momentum of the clock [4]. The solutions with low-energy eigenstates approximate those for the theory without being perturbed by the clock scalar field.

This article is organized as follows: in the next section, we set up the framework, in Section 3, we discuss perturbatively the effects of the terms we neglected in our previous papers. We end with a conclusion.

2. Classical Theory: Spherical Gravity with a Scalar Field and a Clock

We consider the Hamiltonian expanded in powers of G (strictly speaking, in powers of G/l_0) that we introduced in our previous paper [4],

$$\begin{aligned} H_{\rm true} &= H_{\rm grav} + H_{\rm matt} = \frac{|E^\varphi|\sqrt{-2C'}\sqrt{2}}{4\sqrt{\pi G}\, l_0^2} \\ &\quad - \frac{|E^\varphi|\sqrt{2\pi G}\sqrt{-2C'}\left(2(\phi')^2 x^4 - 8E^\varphi \phi' K_\varphi P_\phi x - 4(E^\varphi)^2 \rho_{\rm vac} + 2P_\phi^2\right)}{4(E^\varphi)^2(-C')l_0^2}. \end{aligned} \tag{1}$$

The phase space of the theory is that of spherically symmetric vacuum gravity, consisting of the radial triad E^φ, its conjugate momentum K_φ (the radial triad and its conjugate momentum have been gauge fixed), and the scalar field ϕ and its conjugate momentum. The scalar field of the clock and its conjugate momentum have also been gauge-fixed and, therefore, do not appear. As discussed in our previous paper [4], C is the Hamiltonian constraint of vacuum gravity, and $\rho_{\rm vac}$ is the expectation value of the scalar field energy in the vacuum. The subtraction of this term allows us to assume that the matter term does not perturb the gravitational one, which allows us to treat the matter term as a perturbation. l_0 is a spatial length that appears in gauge fixing and can be physically interpreted as characterizing the space-time domain within which the scalar field behaves effectively as a clock.

As is commonly conducted in spherically symmetric loop quantum gravity, one takes a kinematical basis of quantum eigenstates of the operators $\hat{E}^x$ and $\hat{E}^\varphi$. They are obtained by the direct product of a one-dimensional loop representation along a graph in the radial direction and a Bohr compactification in the transverse direction. That is,

$$\hat{E}^x_j|k_1,\ldots,k_N,\mu_1,\ldots,\mu_n\rangle = k_j\ell^2_{\rm Planck}|k_1,\ldots,k_n,\mu_1,\ldots,\mu_n\rangle, \tag{2}$$

$$\hat{E}^\varphi_j|k_1,\ldots,k_n,\mu_1,\ldots,\mu_n\rangle = \mu_j\ell_{\rm Planck}|k_1,\ldots,k_N,\mu_1,\ldots,\mu_n\rangle. \tag{3}$$

In terms of these, the discrete version of C, the vacuum Hamiltonian constraint is as follows:

$$\hat{C}_j = -j\Delta\left(1 - 2\Lambda + \frac{\sin\left(\rho\hat{K}_{\varphi,j}\right)^2}{\rho^2} - \frac{\left(\hat{E}^x_{j+1} - \hat{E}^x_j\right)^2}{4\Delta^2\left(\hat{E}^\varphi_j\right)^2}\right) + 2GM, \quad (4)$$

where ρ is the polymerization parameter of the Bohr compactification (not to be confused with the quantity ρ_{vac} that appears later on, which, as discussed in our previous paper [4], is the energy of the vacuum that leads in spherical symmetry to a solid angle defect $\Lambda = 2\pi G\rho_{\text{vac}}$). The operator $\hat{E}^x_j$ commutes with $\hat{H}_{\text{true}}$ and, therefore, is a constant of the motion that—in the spin network representation—has eigenvalues $k_j\ell^2_{\text{Planck}}$ with k_j integers. In order to simplify things, we choose an equally spaced lattice, where $E^x(x) = x^2$ and $x_j = j\Delta + x_0$, with $j > 0$. $\Delta = n\ell_{\text{Planck}}$ denotes the lattice spacing, and with this choice, $k_j = x_j{}^2/\ell^2_{\text{Planck}}$, where n denotes a small positive integer. As we mentioned, $x_0 \gg r_S = 2GM$, so we are in the asymptotic region.

At a quantum level, the purely gravitational part is given by the following:

$$\hat{H}_{\text{grav}}\Psi(l_1, \ldots, l_N) = \sum_j \frac{x_j F\sqrt{2\Delta}\sqrt{-\delta l_j}}{\sqrt{\frac{l_j - r_S}{x_j} + 1 - 2\Lambda}}\Psi(l_1, \ldots, l_N), \quad (5)$$

with $F = \left(2\sqrt{2\pi}\ell_{\text{planck}} l_0^2\right)^{-1}$ and $\delta l_j = l_{j+1} - l_j$. Here, l_j are the eigenvalues of $\hat{C}_j$.

In our previous paper, we studied the Hamiltonian and its properties in the region $x \gg r_S$ with r_S denoting the Schwarzschild radius at the zeroth-order [4]. For this, we considered normalizable states that effectively approximated the states of the continuous spectrum of the Hamiltonian. Here, we find it more convenient to work directly in the improper eigenstates of the Hamiltonian. We use the time-dependent perturbation theory to study the corrections to the Hamiltonian that we considered in our previous paper. We will use improper eigenstates because the spectrum of the Hamiltonian is continuous, and in perturbation theory, one usually uses the eigenstates.

We will divide the matter part into a zeroth-order term in the expansion in $1/x$, which corresponds to the H_{matt} from the previous paper [4], and a first-order term that corresponds to the asymptotic corrections that we ignored in that paper.

Let us consider the zeroth-order portion of the matter Hamiltonian,

$$\hat{H}^{(0)}_{\text{matt}} = \frac{\sqrt{2\pi}\sqrt{G}}{\sqrt{-2C'(x)^{(0)} l_0^2 E^\varphi(x)^{(0)}}}\left(\hat{\phi}'(x)^2 x^4 - 2\left(E^\varphi(x)^{(0)}\right)^2\rho_{\text{vac}} + \hat{P}(x)^2\right), \quad (6)$$

where, from now on, we call the momentum of the scalar field P instead of P_ϕ to simplify the notation. This Hamiltonian is obtained by taking expectation values on the gravitational variables with the normalizable gravitational state considered in our previous paper, leading to the following expectation values for the gravitational variables:

$$E^\varphi(x)^{(0)} = \frac{x}{\sqrt{1 - 2\Lambda}}, \quad (7)$$

$$K_\varphi(x)^{(0)} = 0, \quad (8)$$

$$C'(x)^{(0)} = \frac{x^2}{l_0^2\pi^2}. \quad (9)$$

The above Hamiltonian can be rewritten as follows:

$$\hat{H}^{(0)}_{\text{matt}}(x) = \frac{2\pi^{3/2}}{l_0}\sqrt{G}\sqrt{1 - 2\Lambda}\left(\frac{\hat{\phi}'(x)^2 x_k^2}{2} + \frac{\hat{P}(x)^2}{2x^2} - \frac{\rho_{\text{vac}}}{2(1 - 2\Lambda)}\right), \quad (10)$$

where—as we discussed in our previous papers—$\rho_{\rm vac}$ is a counterterm of the energy of the vacuum that we absorb in the solid deficit angle $\Lambda = 2\pi G\rho_{\rm vac}$. The quantity l_0 is used in the definition of the clock $\varphi = t/l_0^2$ with φ denoting the scalar field used as a clock (in our previous paper [4], we called ψ the scalar field and ϕ the clock one) and t denoting the asymptotic time. The physical interpretation of l_0 is the range of validity of the clock, which determines the size of the asymptotic region that we can analyze with it. By range of validity, we mean a region where there is a non-vanishing clock scalar field and its momentum is small.

We recognize the standard scalar field Hamiltonian on the lattice in the above expression (up to the constant term proportional to $\rho_{\rm vac}$, which will be evaluated later),

$$\hat{H}_{{\rm matt},j} = \sum_j \left(\frac{\hat{P}_j^2}{2x_j^2\Delta} + \frac{\left(\hat{\phi}_{j+1} - \hat{\phi}_j\right)^2 x_j^2}{2\Delta} \right). \tag{11}$$

and the total Hamiltonian (gravity plus matter), at zeroth-order, is as follows:

$$\hat{H}_j^{(0)} = \frac{x_j^{3/2}\sqrt{-\delta l_j \Delta}}{2\sqrt{\pi}\ell_{\rm Planck} l_0^2 \sqrt{l_j - r_S + (1-2\Lambda)x_j}} - \frac{2\pi^{3/2}\sqrt{1-2\Lambda}\sqrt{G}}{l_0}\hat{H}_{\rm matt}. \tag{12}$$

We consider the elements of a continuous basis for the gravitational part of the Hamiltonian:

$$\Psi\left(\vec{l}\right) = \prod_j \langle l_j | f_j^{(0)} + \epsilon f_j^{(1)} \rangle = \prod_j \delta(-l_j + f_j^{(0)} + \epsilon f_j^{(1)}), \tag{13}$$

with $f^{(1)} \ll x_0$ and $f^{(0)}$ chosen to recover the states that lead to the matter Hamiltonian discussed in the previous paper. ϵ is a small quantity to emphasize that the term in $f^{(1)}$ has a small contribution and the limit $\epsilon \to 0$ corresponds to the results of our previous paper. Here, δ is the Dirac delta. In order to compute the first-order correction to the Hamiltonian, we shall expand in ϵ and evaluate the first-order coefficient in ϵ. In basis (13), we choose $f^{(0)}$ to yield $\left(C^{(0)}(x)\right)'$, i.e., the Hamiltonian of vacuum gravity that makes $H_{\rm matter}$ take the Minkowskian form. In our previous paper [4], we carried out a similar construction for normalizable states. The choice that leads to this result is as follows:

$$f_{j+1}^{(0)} = -\frac{x_j^2\Delta}{l_0^2\pi^2} + f_j^{(0)}, \tag{14}$$

that is,

$$f_{j+1}^{(0)} = \frac{(\Delta j + x_0)^2\Delta}{l_0^2\pi^2} + f_j^{(0)}. \tag{15}$$

The recursion relation can be solved as follows:

$$f_{j+1}^{(0)} = -\frac{\left(\left(j^2 + \frac{3}{2}j + \frac{1}{2}\right)\Delta^2 + 3x_0\Delta(j+1) + 3{x_0}^2\right)j\Delta}{3{l_0}^2\pi^2}, \tag{16}$$

which satisfies the following:

$$\hat{C}'_{{\rm tot},j}\Psi\left(\vec{l}\right) \equiv \frac{\hat{C}_{j+1} - C_j}{\Delta}\Psi\left(\vec{l}\right) = \left[\frac{\epsilon}{\Delta}\left(f_j^{(1)} - f_{j+1}^{(1)}\right) + \frac{x_j^2}{l_0^2\pi^2}\right]\Psi\left(\vec{l}\right). \tag{17}$$

The solutions of the continuous spectrum of the gravitational part are for $f^{(1)} \ll f^{(0)}$ and take the following form:

$$\Psi_{\rm gr} = \prod_j \delta\left(l_{j+1} + \frac{\left(j^2 + \frac{3}{2}j + \frac{1}{2}\right)\Delta^2 + 3x_0\Delta(j+1) + 3{x_0}^2}{j\Delta} - \epsilon f^{(1)}_{j+1} \right). \tag{18}$$

The matrix elements of $\hat{E}^\varphi$ in the improper basis are as follows:

$$\langle \vec{l}^1 | \hat{E}^\varphi_j | \vec{l}^2 \rangle = \frac{x_j \delta\left(l^1_j - l^2_j\right)}{\sqrt{\frac{l^1_j - r_S}{x_j} + 1 - 2\Lambda}}, \tag{19}$$

from where we can read the form of the multiplicative operator to the first-order in $(r_s - l^1_j)/x_j$, and recalling that l_j are eigenvalues of $\hat{C}_j$:

$$\left(\hat{E}^\varphi_j\right)_{(1)} = \frac{r_S - \hat{C}_j}{2(1-2\Lambda)^{3/2}}, \tag{20}$$

and as before:

$$\left(\hat{E}^\varphi_j\right)_{(0)} = \frac{x_j}{\sqrt{1-2\Lambda}}. \tag{21}$$

For the inverses, we have the following:

$$\left(\hat{E}^\varphi_j\right)^{-1}_{(1)} = \frac{\hat{C}_j - r_S}{2\sqrt{1-2\Lambda}\, x_j^2}, \tag{22}$$

$$\left(\hat{E}^\varphi_j\right)^{-1}_{(0)} = \frac{\sqrt{1-2\Lambda}}{x_j}. \tag{23}$$

The above operators are diagonal on an improper basis. For the connection, it is a bit more complicated. We start by defining the basis of eigenstates of $\hat{E}^\varphi$:

$$\hat{E}^\varphi_j |\vec{\mu}\rangle = \mu_j \ell_{\rm Planck} |\vec{\mu}\rangle, \tag{24}$$

and given the eigenbasis for $\hat{C}$, we already considered $|\vec{l}\rangle$, and compute the following:

$$\langle \vec{l}' | \hat{K}_{\varphi,j} | \vec{l}\rangle = \int \langle \vec{l}' | \vec{\mu}\rangle \langle \vec{\mu} | \hat{K}_{\varphi,j} | \vec{l}\rangle d\vec{\mu} = \int \langle \vec{l}' | \vec{\mu}\rangle {\rm i} \frac{d}{d\mu_j} \langle \vec{\mu} | \vec{l}\rangle d\vec{\mu}. \tag{25}$$

The above eigenstates are direct products of the eigenstates at each site:

$$|\vec{l}\rangle = \prod_j |l_j\rangle, \tag{26}$$

and similarly for $|\vec{\mu}\rangle$. We also have the following (see our previous paper):

$$\langle \mu_j | l_k \rangle = \frac{\sqrt{2}}{2\sqrt{\ell_{\rm Planck}}} \delta\left(\mu_j - \frac{x_j}{\ell_{\rm Planck}\sqrt{\frac{l_j - r_S}{x_j} + 1 - 2\Lambda}} \right) \left(\frac{l_j - r_S}{x_j} + 1 - 2\Lambda \right)^{-3/4} \delta_{jk} \tag{27}$$

and, therefore,

$$\langle l_i|\hat{K}_\varphi|l'_j\rangle = \ell_{\text{Planck}}\left[2\left(\frac{l_j - r_S}{x_j} + 1 - 2\Lambda\right)\delta'\left(l_i - l'_j\right) + \frac{3}{2x_j}\delta(l_i - l'_j)\right]\sqrt{1 - 2\Lambda + \frac{l_j - r_S}{x_j}}. \tag{28}$$

3. Perturbative Analysis for the First-Order Correction to the Asymptotic Approximation

Taking into account that $\hat{E}^\varphi_j$ and $\hat{C}'_j$ have zeroth- and first-order terms, including $\hat{K}_{\varphi,j}$ as a first-order correction, and given that $\hat{C}'_j = \frac{\delta\hat{l}_j}{\Delta}$, the total Hamiltonian expanded to the second power in ϵ is as follows:

$$\hat{H}^{(0)} = \sum_j \left\{ \frac{x_j\sqrt{|\delta\hat{l}_j\Delta|}}{2\sqrt{\pi}\,\ell_{\text{Planck}}\,l_0{}^2\sqrt{\frac{\hat{l}_j - r_S}{x_j} + 1 - 2\Lambda}} - \left(\frac{x_j^2\sqrt{G}\,(\hat{\phi}_{j+1} - \hat{\phi}_j)^2}{\Delta l_0} + \frac{\sqrt{G}\,\hat{P}_j^2}{x_j^2\Delta l_0}\right)\pi^{3/2}\sqrt{1 - 2\Lambda}, \right\} \tag{29}$$

$$\begin{aligned}\hat{H}^{(1)} \quad = \quad \sum_j \Bigg\{ & \frac{\sqrt{1 - 2\Lambda}\,\pi^{\frac{7}{2}}\sqrt{G}\,(\hat{\phi}_{i+1} - \hat{\phi}_i)^2\hat{C}_j^{(1)'}l_0}{4\Delta} + \frac{\left(-\hat{l}_j + r_S\right)x_j\pi^{\frac{3}{2}}\sqrt{G}\,(\hat{\phi}_{i+1} - \hat{\phi}_i)^2}{2l_0\Delta\sqrt{1 - 2\Lambda}} \\ & + \frac{4\pi^{\frac{3}{2}}\sqrt{G}\,\hat{K}^{(1)}_{\varphi,j}\left[(\hat{\phi}_{j+1} - \hat{\phi}_j), \hat{P}_j\right]_+}{\Delta l_0 x_j} + \frac{\pi^{\frac{7}{2}}\sqrt{G}\,\hat{P}_j^2\sqrt{1 - 2\Lambda}\,\hat{C}_j^{(1)'}l_0}{4x_j^4\Delta} + \frac{\pi^{\frac{3}{2}}\sqrt{G}\,\hat{P}_j^2\left(-\hat{l}_j + r_S\right)}{2\sqrt{1 - 2\Lambda}\,x_j^3\Delta l_0}\Bigg\}\end{aligned} \tag{30}$$

where $[\ ,\ ]_+$ is the anti-commutator. The operator $\hat{C}^{(1)}$, when acting on $\psi\left(\vec{l}\right)$, is the term in (17), proportional to ϵ and $\hat{C}^{(0)}$, yielding the term independent of ϵ.

The energy at zeroth-order (of the gravitational field) per site for improper states $\delta\left(l_j - f_j^{(0)} - f_j^{(1)}\right)$ is as follows:

$$E_j^{(0)} = \frac{x_j\sqrt{\left|\Delta\left(\frac{\Delta x_j^2}{l_0^2\pi^2} - f_{j+1}^{(1)} + f_j^{(1)}\right)\right|}}{2\sqrt{\pi}\ell_{\text{Planck}}l_0^2\sqrt{1 - 2\Lambda - \frac{f_j^{(0)} + f_j^{(1)} - r_S}{x_j}}}. \tag{31}$$

Using the time-dependent perturbation theory, as discussed in [9], extended to the case of improper states, we schematically have the transition probabilities (densities) between (improper) eigenstates of the non-perturbed Hamiltonian $H^{(0)}$, which we call, for simplicity, a and b, given by the following:

$$W_{a\to b} = |H_{ab}^{(1)}|^2\left(2\frac{\sin^2\omega_{ab}t}{\omega_{ab}^2}\right), \tag{32}$$

with

$$H_{ba}^{(1)} = \langle b|H^{(1)}|a\rangle, \tag{33}$$

and

$$\omega_{ab} = E_a^0 - E_b^0, \tag{34}$$

and we recall that we are working with $\hbar = c = 1$.

The goal is to compute the probability densities for the situation we are considering and to analyze their consequences. To apply perturbation theory, we need the eigenstates of the zeroth-order Hamiltonian. In our previous paper, we analyzed the gravitational part of this Hamiltonian. We need to consider the matter part. Neglecting the point polymerization of the scalar field, it turns out that the resulting Hamiltonian on a spin network is the same as that of a scalar field on a lattice, as we discussed in (11).

We need to expand the scalar field present in that expression in terms of creation and annihilation operators:

$$\hat{\phi}_{v,j} = \left(\hat{a}_v \exp(-i\omega_v t) + \hat{a}_v^\dagger \exp(i\omega_v t)\right) \frac{\sin(k_v x_j)}{\sqrt{\pi v x_j}}, \tag{35}$$

where $k_v = 2\pi v/(j_N \Delta)$, with $j_N = (l_0 - x_0)/\Delta \sim l_0/\Delta$ is the number of nodes in the asymptotic region, where we study the field that ranges in x from x_0 to l_0. The integer v characterizes the different modes of the field. We recognize, at zeroth-order, the usual form for the Hamiltonian of a scalar field:

$$\hat{H}_{\rm matt} = \sum_v k_v \frac{\hat{a}_v \hat{a}_v^\dagger + \hat{a}_v^\dagger \hat{a}_v}{2} + O(\Delta). \tag{36}$$

As a consequence, the eigenstates of the complete Hamiltonian are given by the following:

$$\Psi^{(0)}_{\rm total} = \prod_j \delta\left(-l_j + f_j^{(0)} + f_j^{(1)}\right) \Phi_{k_{v_1},\ldots,k_{v_q}}, \tag{37}$$

with Φ denoting the eigenstate of the zeroth-order Hamiltonian of the scalar field, which has the modes from k_{v_1} to k_{v_q} excited. Since the spin network introduces a natural cut-off, the scalar field has a discrete spectrum. If the state of the matter part remains invariant, (34) takes the following form:

$$\omega_{ff'} = E_f^{(0)} - E_{f'}^{(0)} = \sqrt{\Delta} \sum_j x_j \frac{\sqrt{-2\delta {f^{(1)}}'_j l_0^2 \pi^2 + \Delta x_j^2} - \sqrt{-2\delta f_j^{(1)} l_0^2 \pi^2 + \Delta x_j^2}}{2\pi^{3/2} \ell_{\rm Planck} l_0^3 \sqrt{1 - 2\Lambda - \frac{f_j^{(0)}}{x_j}}}. \tag{38}$$

The expectation value in the gravitational part of the order one Hamiltonian, neglecting terms of order $\ell^2_{\rm Planck}$, between states like those in (37) with $f^{(1)}$ and ${f'}^{(1)}$, is given by the following:

$$\langle \hat{H}_j^{(1)} \rangle_{\rm grav} = \frac{\pi^{\frac{3}{2}} \left(2\left({f^{(1)}}_{j+1} - {f^{(1)}}_j\right)\pi^2\left(-\frac{1}{2} + \Lambda\right){l_0}^2 + \left(-{f^{(0)}}_j - {f^{(1)}}_j + r_S\right) x_j \ell_{\rm Planck}\right)}{\sqrt{1-2\Lambda}\, l_0 x_j^2} \delta\left(f_j^{(1)} - {f'}_j^{(1)}\right) \hat{H}_{{\rm matt},j}. \tag{39}$$

In order to manage this expression, it is good to provide a concrete form for the ${f^{(1)}}_j$ involved. We choose $f_j^{(1)} = f^{(1)}$, a constant.

We will compute transition probabilities for different values of the constant to obtain an idea about how they behave.

With this choice, since x_j will be typically large, the form of the expectation value in the gravitational part of the first-order Hamiltonian yields an operator acting on the matter variables,

$$\langle \hat{H}_j^{(1)} \rangle_{\rm grav} = \frac{\ell_{\rm Planck} x_j^2}{2\sqrt{\pi}\sqrt{1-2\Lambda} l_0^3} \delta\left(f_j^{(1)} - {f'}_j^{(1)}\right) \hat{H}_{{\rm matt},j}. \tag{40}$$

The presence of the x_j^2 factor on the right-hand side modifies the weights of the different terms of the zero-order Hamiltonian and will imply the existence of transitions between states of the scalar field due to coupling to gravity.

To simplify the calculations, we will go to the continuum limit, but we keep an ultraviolet cut-off in the momentum variable $2\pi/\Delta$. This is an excellent approximation, given that the spin network sites in spherical symmetry can be made as close as $\ell^2_{\rm Planck}/r_S$ due to the condition of quantization of the areas of symmetry. We choose it to be proportional to $\ell_{\rm Planck}$ to have a uniform lattice. In that limit, the equivalent expression to (35) is as follows:

$$\hat{\phi}(x,t) = \int dk \Big(\hat{a}_k e^{-ikt} + a^\dagger_k e^{ikt} \Big) \frac{\sin(kx)}{\sqrt{\pi k}}, \tag{41}$$

and similarly for the field momentum:

$$\hat{P}(x,t) = i \int dk \Big(-k\hat{a}_k e^{-ikt} + k a^\dagger_k e^{ikt} \Big) \frac{\sin(kx)}{\sqrt{\pi k}}. \tag{42}$$

Substituting these expressions into the first-order Hamiltonian, we obtain the following:

$$\begin{aligned} \langle \hat{H}^{(1)}(x) \rangle_{\rm grav} &= \Big(6\pi^{3/2}\sqrt{1-2\Lambda}\sqrt{kk'} l_0^3 \Big)^{-1} \Big(\hat{a}_k \hat{a}^\dagger_{k'} {\rm e}^{-it(k-k')} + \hat{a}^\dagger_k \hat{a}_{k'} {\rm e}^{it(k-k')} + \hat{a}_k \hat{a}_{k'} {\rm e}^{-it(k+k')} + \hat{a}^\dagger_k \hat{a}^\dagger_{k'} {\rm e}^{it(k+k')} \Big) \\ &\times \Big(\cos((k'-k)x) x^2 k' k - x \sin((k-k')x) k' + \sin(kx)\sin(k'x) \Big). \end{aligned} \tag{43}$$

Integrating in x, and recalling that $k_n = 2\pi n/(l_0 - x_0)$ with n denoting an integer, we have the following:

$$\begin{aligned} \langle \hat{H}^{(1)} \rangle^{k \neq k'}_{\rm grav} &= \frac{\ell_{\rm Planck}}{\pi^{\frac{3}{2}} \sqrt{1-2\Lambda}\, {l_0}^3} \left[\frac{2(l_0 - x_0)\sqrt{kk'}}{(k-k')^2} \right] \\ &\times \Big(\hat{a}_k \hat{a}^\dagger_{k'} {\rm e}^{-it(k-k')} + \hat{a}^\dagger_k \hat{a}_{k'} {\rm e}^{it(k-k')} + \hat{a}_k \hat{a}_{k'} {\rm e}^{-it(k+k')} + \hat{a}^\dagger_{k'} \hat{a}^\dagger_k {\rm e}^{it(k+k')} \Big), \end{aligned} \tag{44}$$

$$\begin{aligned} \langle \hat{H}^{(1)} \rangle^{k = k'}_{\rm grav} &= \frac{\ell_{\rm Planck}}{\pi^{\frac{3}{2}} \sqrt{1-2\Lambda}\, {l_0}^3} \left[\left(x_0 k\,(l_0 - x_0)^2 + {x_0}^2 k (l_0 - x_0) + \frac{k\,(l_0 - x_0)^3}{3} + \frac{(l_0 - x_0)}{k} \right) \right] \\ &\times \Big(\hat{a}_k \hat{a}^\dagger_{k'} {\rm e}^{-it(k-k')} + \hat{a}^\dagger_k \hat{a}_{k'} {\rm e}^{it(k-k')} + \hat{a}_k \hat{a}_{k'} {\rm e}^{-it(k+k')} + \hat{a}^\dagger_{k'} \hat{a}^\dagger_k {\rm e}^{it(k+k')} \Big) \delta_{k,k'}, \end{aligned} \tag{45}$$

and for the last integration, we recall the original form of the discrete k and k' in the spin network, which leads to the Kronecker delta instead of the Dirac delta.

Therefore, we see that the perturbative Hamiltonian can create and annihilate pairs of matter particles, such as the last terms in (44). However, such terms do not conserve energy (in the sense of the matter portion of $H^{(0)}$) and are, therefore, heavily suppressed in (32). There are energy-conserving contributions to the second order, but in the asymptotic regions, these terms are negligible. This ensures that in the asymptotically flat limit, one recovers the usual quantum field theory treatment, in which there is no particle production from the geometry. However, in other background geometries, this could lead to particle production, hinting at the emergence of Hawking radiation. This could lead to effects of interest, for instance, in cosmological backgrounds or closer to the horizon.

4. Conclusions

We studied spherically symmetric gravity coupled with a spherical scalar, field using a second spherical scalar field as a clock. We concentrated on the asymptotic region, using terms that we had neglected in previous publications. These terms induce transitions in the states of the scalar field due to interactions with gravity. Transitions are of low probability even when they do not induce changes in the energy of the scalar field. Therefore, we see the emergence of quantum gravity effects in the asymptotic region, but they are small, as expected. It would be more interesting to move closer to the horizon, where one could study quantum gravity corrections to Hawking radiation due to the back reaction. There are several methods proposed that would allow us to deal with that region perturbatively [10–14].

To summarize, we see the emergence of quantum field theory in a quantum space-time with spherical symmetry in the far asymptotic region. This includes gravitational effects that start to depart from the usual quantum field theory on curved spacetime.

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