Review

The Nature of Dark Energy and Constraints on Its Hypothetical Constituents from Force Measurements

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Abstract: This review considers the theoretical approaches to the understanding of dark energy, which comprises approximately 68% of the energy of our Universe and explains the acceleration in its expansion. Following a discussion of the main approach based on Einstein's equations with the cosmological term, the explanations of dark energy using the concept of some kind of scalar field are elucidated. These include the concept of a quintessence and modifications of the general theory of relativity by means of the scalar–tensor gravity exploiting the chameleon, symmetron and environment-dependent dilaton fields and corresponding particles. After mentioning several laboratory experiments allowing us to constrain the hypothetical scalar fields modeling the dark energy, special attention is devoted to the possibility of constraining the parameters of chameleon, symmetron and environment-dependent dilaton fields from measuring the Casimir force. It is concluded that the parameters of each of these fields can be significantly strengthened in near future by using the next-generation setups in preparation suitable for measuring the Casimir force at larger separations.

Keywords: dark energy; cosmological constant; chameleon; symmetron; environment-dependent dilaton; Casimir force

1. Introduction

The concept of expanding Universe, which goes back to the Friedmann solutions of Einstein's equations published in 1922 [1] and 1924 [2], assumes that its expansion should decelerate with time due to the gravitational attraction of both visible and dark matter. It was a big surprise when, analyzing the redshift data of supernovae in binary systems, the two research teams independently found in 1998 that the Universe expansion is accelerating (see the pioneer Refs. [3,4] and reviews [5,6]).

If one wishes to explain the acceleration of the Universe expansion in the framework of the general theory of relativity, it is necessary to admit that there is an additional form of invisible matter with a positive energy density $\varepsilon > 0$, as it holds for both the usual and dark matter, but with a negative pressure, $P < 0$. Such a matter is characterized by the equation of state

$$P = -w\varepsilon, \tag{1}$$

where an acceleration in the expansion holds for $w > 1/3$. This kind of invisible matter violating the strong energy condition was called dark energy.

The advent of dark energy would not be so unusual if it constituted a small fraction of the total energy of the Universe. It turned out, however, that if one would like to preserve the standard cosmological scenario based on the general theory of relativity, the observational data demand that the dark energy constitutes about 68% of the Universe's energy [5,6]. When it is considered that the dark matter contributes approximately 27% of the Universe's energy, only 5% remains for the visible, baryonic, matter.

There are many theoretical approaches to the understanding of the nature of dark energy. These approaches can be grouped into four main divisions. The first of them describes dark energy by means of the cosmological constant Λ introduced into equations of the general theory of relativity by Einstein [7] for other purposes.

The second group of approaches to the description of dark energy considers it as some kind of classical time-varying scalar field called a quintessence. The cosmological applications of similar fields were considered in Refs. [8,9], whereas the term quintessence was introduced in Ref. [10].

The third group of approaches allows any change in the action and equations of the general theory of relativity by combining the metrical tensor with the classical scalar field within the formalism of scalar–tensor gravity in order to make the concept of dark energy unnecessary [11]. The chameleon field, symmetron field and the environment-dependent dilaton field were used in the literature for this purpose. Some of these approaches dispense with the need for either dark energy or dark matter (see, e.g., Ref. [12]). The modifications of the gravitational theory are also allowed in the unified models of dark matter and dark energy introducing the so-called dark fluid [13].

Note that the main ideas of the above three groups of approaches can be considered as based on the concepts of classical physics, although quantum physics was used in their further developments. As to the approaches of the fourth group aiming to understand the nature of dark energy, they consider it as composed of some hypothetical elementary particles with unusual physical properties that give rise to the negative pressure. The most popular particles of such kind are the chameleons, which possess a variable mass depending on the density of matter in the environment [14,15]. Another candidate for a dark energy particle is the symmetron whose interaction constant with the usual matter depends on the environmental density [16–18]. There are also other hypothetical particle candidates for the role of constituents of dark energy, e.g., the environment-dependent dilaton [19]. The classical fields with the variable masses and interaction constants were introduced in the third group of approaches mentioned above, whereas the unusual particles, such as chameleons, symmetrons, etc., are the result of their quantization.

In this review, we compare the approaches from the above four groups by the level of their credibility and discuss the main particle candidates for the role of dark energy constituents. Next, we pass to the constraints on the parameters of chameleon, symmetron and environment-dependent dilaton fields following from different laboratory experiments. The main attention is paid to the constraints that can be obtained from measuring the Casimir force arising between the closely spaced macroscopic bodies due to the zero-point and thermal fluctuations of the electromagnetic field.

There are also many alternative attempts to solve the dark energy issue, which are listed below for completeness. Thus, one can mention suggestions to consider modified gravity theories that introduce additional degrees of freedom in the gravitational and/or matter action [20,21]. It was also suggested to phenomenologically modify the Friedmann equation by additional terms that depend on the matter density in a nonlinear way [22–24]. Another option considered in the literature is to alter the mass–energy evolution equation with bulk viscosity terms [25–27].

Alternatively, some authors believe that dark energy may be only an apparent effect. They hypothesize that the supernovae data may be biased if the observer is located in a local underdense region (see, e.g., Ref. [28]) or suppose that the supernovae sources tend to be associated with overdensities (see, e.g., Ref. [29]). Finally, many papers focus on the role of matter inhomogeneities and anisotropies that may affect the cosmic expansion due to backreaction or statistical sampling effects (see, e.g., Refs. [30–34]).

The review is organized as follows. In Section 2, the theoretical approaches to understanding of the physical nature of dark energy based on classical physics are briefly considered and compared. Section 3 is devoted to a discussion of different particle candidates for the role of constituents of dark energy. The already obtained laboratory constraints on the parameters of chameleon, symmetron and environment-dependent dilaton fields,

as well as the prospective constraints obtainable from force measurements, including the Casimir force, are presented in Section 4. Section 5 contains the discussion and in Section 6 the reader will find our conclusions.

Below the relativistic units are used with $c = \hbar = 1$, where c is the speed of light and $\hbar$ is the reduced Planck constant.

2. Approaches to Theoretical Description of Dark Energy Based on Classical Physics

As discussed in Section 1, there are three groups of such kind approaches to understanding of what the dark energy is and none of them is either excluded or finally confirmed.

We begin with probably the most common approach describing the accelerations in the Universe expansion on the basis of classical Einstein equations with the cosmological term

$$R_{ik} - \frac{1}{2} R g_{ik} - \Lambda g_{ik} = 8\pi G T_{ik}, \tag{2}$$

where R_{ik} is the Ricci tensor, R is the scalar curvature of space-time, Λ is the cosmological constant, g_{ik} is the metrical tensor, G is the gravitational constant and T_{ik} is the stress–energy tensor of both visible and dark matter.

Equation (2) provides a very plausible explanation for the dark energy because in the homogenous isotropic 3-space of expanding Universe the metrical tensor is diagonal. Thus, raising the index k in Equation (2) and rearranging the cosmological term to the right-hand side of this equation, one obtains

$$R_i^k - \frac{1}{2} R \delta_i^k = 8\pi G \left(T_i^k + \frac{\Lambda}{8\pi G} \delta_i^k \right), \tag{3}$$

where δ_i^k is the Kronecker symbol. From this equation it is seen that the effective stress–energy tensor caused by the cosmological constant is

$$T_{(\Lambda)i}{}^k = \frac{\Lambda}{8\pi G} \delta_i^k. \tag{4}$$

Taking into account that in the homogeneous isotropic space for the stress–energy tensor of any kind of matter it holds [35]

$$T_0^0 = \varepsilon, \qquad T_1^1 = T_2^2 = T_3^3 = -P, \tag{5}$$

where ε is the energy density and P is the pressure, one obtains from Equation (4) the energy density, pressure and equation of state of the dark energy resulting from the cosmological constant

$$\varepsilon_\Lambda = \frac{\Lambda}{8\pi G}, \qquad P_\Lambda = -\frac{\Lambda}{8\pi G}, \qquad P_\Lambda = -\varepsilon_\Lambda. \tag{6}$$

Thus, in this case, Equation (1) is satisfied with $w = w_\Lambda = 1$ in violation of the second inequality of the strong energy condition

$$\varepsilon + P \geqslant 0, \qquad \varepsilon + 3P \geqslant 0 \tag{7}$$

valid for the usual and dark matter.

In spite of the fact that Equation (2) belongs to classical physics, it has long been understood [36] that the leading divergent term in the vacuum expectation values of the stress–energy tensor of quantized fields has the same geometric form as the cosmological term

$$\langle 0 | T_{ik} | 0 \rangle = I_\infty g_{ik}, \tag{8}$$

where I_∞ is an infinitely large constant. This is valid in both the Minkowski space–time and in the curved background of expanding Universe [37,38] as can be seen, for instance, by the method of dimensional regularization [39].

From Equations (5) and (8), it follows that

$$\langle 0|T_0^0|0\rangle = \varepsilon_{\text{vac}} = I_\infty,$$
$$\langle 0|T_1^1|0\rangle = \langle 0|T_2^2|0\rangle = \langle 0|T_3^3|0\rangle = -P_{\text{vac}} = I_\infty, \tag{9}$$

i.e., the equation of state of the quantum vacuum

$$P_{\text{vac}} = -\varepsilon_{\text{vac}} \tag{10}$$

is the same as due to the cosmological constant in Equation (6).

Thus, the vacuum stress–energy tensor of quantized fields could offer a plausible explanation for a generation of the cosmological constant. However, the great difficulty, called the vacuum catastrophe [40], arises from the infinitely large values of I_∞, ε_{vac} and P_{vac}. Even if one makes a cutoff in the expression for I_∞ at the Planck momentum, the obtained energy density is of the order

$$\varepsilon_{\text{vac}} \sim 10^{111} \text{ J/m}^3. \tag{11}$$

At the same time, the observed acceleration in the Universe expansion demands the value of the cosmological constant in Equation (2)

$$\Lambda \approx 10^{-52} \text{ m}^{-2}. \tag{12}$$

This results in the corresponding value of the vacuum energy density

$$\varepsilon_\Lambda = \frac{\Lambda}{8\pi G} \sim 10^{-9} \text{ J/m}^3, \tag{13}$$

which is different by the factor of 10^{120} from the estimation of ε_{vac} in Equation (11) obtained from quantum field theory [6,41]. In Ref. [39], it was suggested to consider the value of Λ from Equation (12) as a renormalized value of the cosmological constant as opposed to the enormously large bare value

$$\Lambda_{\text{vac}} = 8\pi G\varepsilon_{\text{vac}} \sim 10^{68} \text{ m}^{-2}. \tag{14}$$

Some grounds for such an approach are given by the quantum field theory in curved space-time [37,38], but the rigorous justification could be reached only in the framework of quantum theory of gravitation which is not yet available.

In spite of this problem, the cosmological constant, whose value is determined experimentally like the values of all other fundamental constants, provides a pretty convincing explanation for the acceleration in the Universe expansion. In fact, Equation (2) including the cosmological term can be considered as entirely classical with no connection with the problem of quantum vacuum. As a result, the cosmological constant is commonly considered as one of the main elements of the standard cosmological model Lambda-CDM along with the cold dark matter formed by the nonrelativistic particles (axions, weakly interacting massive particles) and the usual barionic matter.

The second group of approaches to an explanation of the acceleration in the Universe expansion considers dark energy as a time-varying classical scalar field Φ called quintessence [8–10]. Unlike the dark energy described by the cosmological constant, where the quantity w in Equation (1) is constant, $w = 1$, for the quintessence w depends on the form of the field potential $V(\Phi)$ and may vary with time.

There are many models of the quintessence dark energy proposed in the literature (see, for instance, Refs. [42–49] and review [50]) using different forms of the potential

$V(\Phi)$ [10,43,44,50–56]. Typically the sum of the actions of the general theory of relativity and the quintessence field is chosen in the form

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R - \frac{1}{2} g^{ik} \frac{\partial \Phi}{\partial x^i} \frac{\partial \Phi}{\partial x^k} - V(\Phi) \right], \tag{15}$$

where g is the determinant of the metrical tensor and the interaction with the usual baryonic matter ψ is lacking. Because of this, the total action is the sum of S and the action of the baryonic matter $S_m[\psi]$.

In the space-time of expanding Universe the quantity w takes the form [50]

$$w \equiv w_\Phi = \frac{2V(\Phi) - \left(\frac{\partial \Phi}{\partial t} \right)^2}{2V(\Phi) + \left(\frac{\partial \Phi}{\partial t} \right)^2}. \tag{16}$$

It was shown that with the exponential potential [42,43,50]

$$V(\Phi) = V_q(\Phi) = V_0 e^{-\lambda \sqrt{8\pi G} \Phi}, \tag{17}$$

where $\lambda = $ const, the equation of state of the quintessence dark energy approaches to Equation (1) with $w = w_q = 1 - \lambda^2/3$. As a result, the quintessence approach to the dark energy becomes capable to make approximately the same theoretical predictions for the accelerated expansion of the Universe as the standard model using the cosmological constant.

Note also that in some models of a quintessence the quantity w defined in Equation (1) satisfies the inequality $w > 1$. This means that the kinetic energy of a quintessence field is negative leading to a catastrophic acceleration of the Universe expansion without bounds. As a result, the distances between individual particles, even inside an atom, go to infinity. In the literature, this is called the Big Rip caused by the phantom energy [57]. There are also models of kinetic quintessence with a nonstandard form of negative kinetic energy but $0 < w < 1$ [58]. The fact is worth mentioning that the concept of a quintessence field is used for a solution of the so-called coincidence problem, i.e., why the energy densities of dark matter and dark energy are of the same order of magnitude in the present epoch of cosmic history [44] (see also Refs. [36,41]).

The third group of theoretical approaches essentially based on the classical physics admits modifications of the general theory of relativity in such a way that an introduction of the dark energy could be obviated. The most well-known modification of the general theory of relativity is the scalar–tensor theory, which assumes that the gravitational interaction is determined by the combined action of the metrical tensor and the scalar field Φ (see the pioneering paper [59], reviews [60,61] and the monograph [62]).

The typical action of the scalar–tensor theory is the sum of the action defined in Equation (15) and the action of usual matter, S_m, which is, however, coupled with the field Φ in this case

$$S_{\text{int}} = S_{\text{int}}[A^2(\Phi) g_{ik}, \psi], \tag{18}$$

where $A(\Phi)$ is some function describing the coupling to matter. Thus, in the Brans–Dicke theory [59]

$$A(\Phi) = A_{\text{BD}}(\Phi) = e^{-\frac{\sqrt{\pi G}}{C} \Phi}, \tag{19}$$

where $C = $ const.

Due to Equation (18) the effective potential depends on the usual matter. For example, for the dust-like matter with an energy density $T_0^0 = \varepsilon$ and $P = 0$, one has [11]

$$V_{\text{eff}}(\Phi) = V(\Phi) + \varepsilon, \qquad \Box \Phi = \frac{\partial V_{\text{eff}}(\Phi)}{\partial \Phi}. \tag{20}$$

Both the potential $V(\Phi)$ and the function $A(\Phi)$ take different forms in various models proposed in the literature [20]. Thus, the chameleon field with a choice [14,15,63]

$$V(\Phi) = V_{\text{ch}}(\Phi) = \frac{M^{4+n}}{\Phi^n}, \qquad A(\Phi) = A_{\text{ch}}(\Phi) \approx 1 + C\sqrt{8\pi G}\Phi,$$
$$V_{\text{eff}}(\Phi) = V_{\text{eff,ch}}(\Phi) = V_{\text{ch}}(\Phi) + C\sqrt{8\pi G}\varepsilon\Phi, \tag{21}$$

where M is a parameter with the dimension of mass, n is an integer number, C is a constant of the order of unity, is used in the models of dark energy. The effective mass of chameleon field is larger in the regions of larger density, $m_\Phi^2 \sim \varepsilon^{(n+2)/(n+1)}$.

Another choice used in the models of dark energy is the symmetron field for which [17,18]

$$V(\Phi) = V_{\text{s}}(\Phi) = -\frac{m^2}{2}\Phi^2 + \frac{\lambda}{4}\Phi^4, \qquad A(\Phi) = A_{\text{s}}(\Phi) \approx 1 + \frac{\Phi^2}{2M^2},$$
$$V_{\text{eff}}(\Phi) = V_{\text{eff,s}}(\Phi) = V_{\text{s}}(\Phi) + \varepsilon A(\Phi), \tag{22}$$

where λ is the dimensionless constant of self-interaction and m is one more parameter with the dimension of mass. The coupling strength of the symmetron field to the usual matter is of the order of Φ/M. It is perceptible in the regions of low density $\varepsilon/M^2 \ll m^2$ and goes to zero in the regions of sufficiently high density $\varepsilon/M^2 > m^2$ [64].

Another class of modifications of the general theory of relativity replaces the standard action of this theory linear in R with a nonlinear one [62,65]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,f(R) + \int d^4x\sqrt{-g}\mathcal{L}_M, \tag{23}$$

where $\mathcal{L}_M$ is the Lagrangian density of the usual matter, $f(R)$ can be presented as a series expansion

$$f(R) = \ldots + \frac{\beta_{-2}}{R^2} + \frac{\beta_{-1}}{R} + f(0) + R + \beta_2 R^2 + \ldots, \tag{24}$$

and $f(0) = 2\Lambda$ is expressed via the cosmological constant.

As shown in Ref. [65], the function of the form $f \sim 1/R^n$ with $n > 0$ in Equation (23) can explain the observed acceleration in the Universe expansion. It was shown, however, that the theories described by the action (23) are in fact the versions of the scalar–tensor theories of gravity considered above [66,67]. Thus, the dynamically equivalent action to (23) written in terms of an additional scalar field χ is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[f(\chi) + f'(\chi)(R - \chi)] + \int d^4x\sqrt{-g}\mathcal{L}_M. \tag{25}$$

Really, the variation of this action with respect to χ results in the equation of motion

$$f''(\chi)(R - \chi) = 0, \tag{26}$$

where $f'(\chi) = \partial f(\chi)/\partial\chi$. This means that $\chi = R$ if $f''(\chi) \neq 0$ and Equation (25) reduces to Equation (23).

Next, by introducing one more scalar field $\Phi = f'(\chi)$, one can transform the action (23) to the action of a Brans–Dicke theory with the potential [67]

$$V(\Phi) = \chi(\Phi)\Phi - f(\chi(\Phi)). \tag{27}$$

This means that any constraints obtained for a chameleon or symmetron fields from measuring the Casimir force (see Section 4) can be reformulated as the corresponding constraints on the function $f'(R)$ known as the scalaron field or, alternatively, as the

cosmological scalar field in theories of modified $f(R)$ gravity. The latter, however, is outside the scope of this review.

A comprehensive review of these and many others theories of modified gravity and their applications to cosmology is given in Ref. [20].

As is seen from the above, both the second and third groups of approaches to the theoretical description of an acceleration in the Universe expansion are heavily based on the consideration of some hypothetical scalar field whose form of potential, the function describing an interaction with matter and some parameters are not fixed uniquely. In this sense, the first approach exploiting the cosmological term in Einstein's equations seems preferable because it operates with only one parameter, the cosmological constant, which can be considered as a fundamental constant like the electric charge, speed of light, Planck constant etc. In the next section we discuss what could be added to this situation by the quantum theory, which brings an interpretation of the classical scalar fields used in the models considered above in terms of particles.

3. Particle Candidates for the Role of Constituents of Dark Energy

As discussed in previous section, the classical chameleon and symmetron fields were introduced in the context of modified gravity. This makes their immediate quantization problematic because the consistent quantum theory related to the standard part of gravitation described by the metrical tensor is not yet available. For this reason, the action of the form of Equation (15) or the sum of Equations (15) and (18) cannot be directly presented in the operator form.

It is possible, however, to consider the action of a scalar field Φ and its interaction with the matter fields separately of the gravitational action containing the scalar curvature. In so doing, the metrical tensor in the action (18), describing an interaction of the matter fields with Φ, is understood as the usual function in the spirit of quantum field theory in curved space-time [37,38].

Using this approach, the chameleon field can be quantized and the resulting particles are called the chameleons. Then it is possible to consider the interaction of chameleons with the curved gravitational background and with the elementary particles of the Standard Model. Thus, the quantum corrections to the chameleon potential were investigated in Ref. [68]. The effect of production of chameleons from vacuum in the early Universe was considered in the linear approximation in Ref. [69] by the method of Bogoliubov transformations. It was shown that in the radiation dominated Universe this effect makes a strong impact on the Universe evolution.

In addition to interaction with the baryon particles, chameleons can be coupled to photons via the additional term of the form $\Phi F_{ik} F^{ik}$, where F_{ik} is the tensor of the electromagnetic field. This term is in fact the linear approximation to the exact interaction which contains the chameleon field in the exponent [70]

$$S_{\text{int, ch}} = -\frac{1}{4} \int d^4 x \, e^{\frac{\Phi}{M}} F_{ik} F^{ik}, \tag{28}$$

where M is a fictitious mass controlling the coupling strength of chameleons to photons. Due to the interaction (28), chameleons can be turned to photons and vice versa in an external magnetic field.

Similar situation also holds as to the quantization of the symmetron field. If one considers its action separately from the action of gravitation, the symmetron field can be quantized with the metrical tensor g_{ik} being a classical function. The resulting quanta are called symmetrons. As discussed in Section 2, the coupling of symmetron field to the usual baryonic matter vanishes if the local energy density is large enough and is restored in the regions with sufficiently low energy density.

On the classical level, the symmetron field does not interact with the electromagnetic field. However, in the framework of quantum field theory, it was shown that quantum

corrections generate the interaction Lagrangian density between symmetrons and photons of the form [64,71]

$$\mathcal{L}_{\rm s} = \frac{\Phi^2}{M^2} \, A_{\rm s}^{-4} g^{ik} g^{ln} F_{il} F_{kn}, \tag{29}$$

where M is some new energy scale and $A_{\rm s} = A_{\rm s}(\Phi)$ is defined in Equation (22). This is the so-called axion-like coupling.

One more particle with unusual physical properties, which can be considered as a hypothetical constituent of dark energy, is the environment-dependent dilaton. The dilaton scalar field and its associated particles arise in many theoretical approaches beyond the Standard Model, e.g., in the extra-dimensional theories with a varied volume of compactified space, in the scalar–tensor theories of gravity, in string theory etc. (see, e.g., Refs. [62,72–74]).

Below we consider the model of an environment-dependent dilaton field which is formulated in the context of scalar–tensor gravity. In fact this field combines the properties of the quintessence, chameleon and symmetron fields. Thus, similar to the chameleon and symmetron fields, it is described by the sum of actions defined in Equations (15) and (18). The function A describing the coupling of an environment-dependent dilaton to matter is of the same form as was discussed for symmetrons in Equation (22) [75,76]

$$A_{\rm d}(\Phi) = 1 + \frac{A_0}{2M^2}(\Phi - \Phi_0)^2, \tag{30}$$

where Φ_0 is the current value of the dilaton field and A_0 is a constant.

As to the dilaton potential, it takes the exponential form [75,76] like for the quintessence field [see Equation (17)]

$$V_{\rm d}(\Phi) = V_0 e^{-\lambda\sqrt{8\pi G}\Phi}, \tag{31}$$

as opposed to the power-type potentials (21) and (22) for the chameleon and symmetron fields, respectively.

In the regions of space with sufficiently high density of matter, it holds $\Phi \approx \Phi_0$ and the coupling of the dilaton field to matter becomes negligibly small, although in the regions with low density the coupling of the dilaton field to matter is of the order of gravitational strength. In this regard the environment-dependent dilaton behaves in the same way as the symmetron. Similar to chameleons, however, the effective mass of a dilaton increases with increasing density of the environment.

The quantization of the environment-dependent dilaton field can be performed under the same conditions as discussed above for the chameleon and symmetron fields. In addition to coupling with baryons, the dilaton particles can be coupled to photons. This coupling has the form of Equation (28), the same as for chameleons [77].

4. Constraints on the Particle Constituents of Dark Energy from Force Measurements

The hypothetical scalar fields (the chameleon, symmetron and environment-dependent dilaton) discussed in Sections 2 and 3 interact with the usual matter and can be constrained in the laboratory experiments in a number of ways. Thus, it was shown [78] that individual atoms inserted into large high-vacuum chamber do not screen the chameleon field and the force acting on them from this field can be measured by means of atom interferometry.

One more method for searching chameleon particles uses their interaction with the electromagnetic field. For observation of oscillations between the chameleon and photon states, the vacuum chamber was used where the magnetic field of 5 T was initiated [70]. As a result, in the plane (effective chameleon mass) × (coupling to photon parameter), rather large region was excluded.

Strong limits on the parameters of chameleons were placed also by means of the gravity resonance spectroscopy used to measure the quantum states of ultracold neutrons confined near a mirror [79]. These limits are by the five orders of magnitude stronger than the previously known ones obtained from spectroscopic measurements [80].

The same methods can be used for searching and constraining the symmetrons and environment-dependent dilatons. For instance, in Refs. [81,82] it was shown that the parameters of symmetrons can be constrained by means of atom interferometry. As one more example, the possibility to constrain dilatons by measuring the dilaton–photon conversion in strong magnetic field was considered in Ref. [83] (see also the review [84] where several other possibilities are considered).

Constraints on the chameleon, symmetron and dilaton fields and respective particles can be obtained not only from the laboratory experiments mentioned above but from astrophysics and cosmology as well. One can mention constraints found from galaxy clusters' thermodynamic profiles, gravitational lensing and caustic techniques [85–88]. Specifically, the amplitude of the chameleon field and its coupling strength to matter were constrained by combining the gas and lensing measurements of the cluster [85]. The upper limits on the strength of chameleon force were placed by comparing X-ray and weak lensing profiles of the galaxy clusters [86]. It should be noted, however, that the constraints found from astrophysics and cosmology do not admit an immediate comparison with the laboratory constraints because the former unavoidably depend on some indefinite factors, whereas the latter are obtained in the fully controlled environments.

Below we concentrate our attention on constraining the parameters of chameleons, symmetrons and environment-dependent dilatons, which can be obtained from force measurements at short separations below a few micrometers. The point is that at such small distances between the material bodies the dominant force is not the gravitational one, but the Casimir force caused by the zero-point and thermal fluctuations of the electromagnetic field [89]. Precision measurements of the Casimir force have long been used for constraining the Yukawa-type corrections to Newton's law of gravitation and the interaction constant and mass of axions as the possible constituents of dark matter (see, e.g., Refs. [90–93] and reviews [94–96]).

The standard approach to obtaining constraints on some hypothetical force F_{hyp} from measuring the Casimir force is the following. According to the experimental data obtained over some separation interval, the theoretical expression for the Casimir force is confirmed within the total error ΔF, which includes the random and systematic experimental errors as well as possible theoretical uncertainties. The hypothetical force, e.g., from the Yukawa-type interaction or due to the axion exchange, is calculated in the experimental configuration as a function of separation and the parameters of this interaction. Since the hypothetical force was not observed, its magnitude is restricted by the inequality

$$|F_{\text{hyp}}(a)| < \Delta F(a), \tag{32}$$

where a is the value of separation. Then, by analyzing this inequality at different separations within the measurement interval, the strongest constraints on the parameters of hypothetical force are obtained [90–93,95].

This methodology can also be applied to the possible constituents of dark energy, such as chameleons, symmetrons and environment-dependent dilatons. The obtained results are considered in the following subsections.

4.1. Constraints on Chameleons from Measuring the Casimir Force

The possibility to constrain the chameleon parameters from measuring the Casimir force was proposed in Refs. [97,98] and further elaborated in Ref. [99]. Thus, in Ref. [99] the hypothetical force due to the presence of chameleons was calculated in the configurations of two parallel plates and a sphere above a plate. The latter configuration was used in all precise experiments on measuring the Casimir force [89].

As was noted in Section 2, different forms of the potential $V(\Phi)$ in Equation (15) have been proposed in the literature. The results of Ref. [99] are obtained with the most widely used potential of the form of Equation (21) and with the exponential potential

$$V_{\text{ch}}(\Phi) = \widetilde{\Lambda}_0^4 \, e^{\frac{\widetilde{\Lambda}^n}{\Phi^n}}. \tag{33}$$

The first term in the power expansion of Equation (33) corresponds to the vacuum energy density required for explanation of the accelerated expansion of the Universe and the second with $\widetilde{\Lambda} = \widetilde{\Lambda}_0 = M$ results in the potential (21).

Taking into account that the mass of the chameleon field strongly depends on the density of the environment, the macroscopic bodies are characterazed by the so-called thin shells regarding this field [14,15]. Let the body have the density ρ_b and outside the body the density of matter is ρ_m. Then, deep inside the body,

$$\Phi \approx \Phi_b \equiv \Phi_{\text{min}}(\rho_b), \tag{34}$$

where the effective potential $V_{\text{eff}}(\Phi)$ reaches its minimum value at Φ_{min}. As to the region outside the body, there it holds

$$\Phi \approx \Phi_m \equiv \Phi_{\text{min}}(\rho_m). \tag{35}$$

According to Ref. [99], for the thin-shelled bodies almost all of the change from Φ_m to Φ_b happens in the thin shell near a surface of the body. It turned out that the hypothetical force due to the presence of chameleon field between the thin-shelled bodies is much weaker than for sufficiently thin bodies where the thin shell near the surface is not formed [97–99].

According to the analysis performed in Ref. [99], the test bodies used in measurements of the Casimir force (see Refs. [89,100] for a review) have the thin shells for the most realistic models of the chameleon field used in the literature. It was noted also [99] that if the thin shells in the test bodies are absent, all the constraints on Yukawa interaction obtained from measuring the Casimir force remain valid for the chameleon theories.

In Ref. [99], the chameleon force was calculated between two parallel plates and between a sphere and a plate with account of the effect of thin shells for the potentials of the form (21) and (33). As a result, rather wide regions were excluded in the plane (chameleon-to-matter coupling) × (energy scale of chameleon potential) using the data of the most precise measurements of the Casimir force. For strengthening of the obtained constraints, it was suggested to perform measurements of the thermal Casimir force at larger separations and to use larger test bodies in order to avoid the effect of thin shells which decreases the magnitude of the chameleon force.

*4.2. Constraints on a Symmetron Field from Measuring the Casimir Force*

As discussed in Section 3, the coupling of the symmetron field to the barionic matter increases in the regions of low density and goes to zero with increasing density of matter. This field and the corresponding particles are described by the sum of actions (15) and (18), where the potential and the function $A(\Phi)$ describing the coupling to matter are given by Equation (22).

Constraints on the symmetron field following from measurements of the Casimir force can be obtained using the same methodology as described above in the case of a chameleon field. One should calculate the additional force caused by the symmetron field in the configuration of two plates or a sphere above a plate used in the Casimir experiments. If the theoretical expression for the Casimir force is confirmed by the measurement data in the limits of some error, then the magnitude of any additional force is restricted by this error.

In Ref. [101], the exact analytical solutions for the profiles of a symmetron field were found in the space near an infinite mirror occupying a semispace and between two such mirrors separated by a gap. The first of these solutions was applied for calculation of an additional frequency shift in the experiment measuring the reflection of ultracold neutrons

by a neutron mirror in the gravitational field of the Earth [102,103]. The second analytical solution concerning the case of two mirrors can be applied for calculation of the additional force induced by the symmetron field in the proposed CANNEX experiment on measuring the Casimir force between two parallel plates at separations up to 10 μm and more [104–106]. The principal scheme of the setup of this experiment, which is also discussed in Section 4.3, is shown in Figure 1 [104].

In the configuration of two parallel plates (like shown in Figure 1) and a sphere above a plate (up to now, the latter was used in the most precise measurements of the Casimir interaction) the additional force due to a symmetron field was calculated in Ref. [107]. For a sphere-plate geometry, these calculations were performed under the conditions $mR \ll 1$, $mR \sim 1$ and $mR \gg 1$, where m is the symmetron mass in the vacuum and R is the sphere radius, with account of the screening effects.
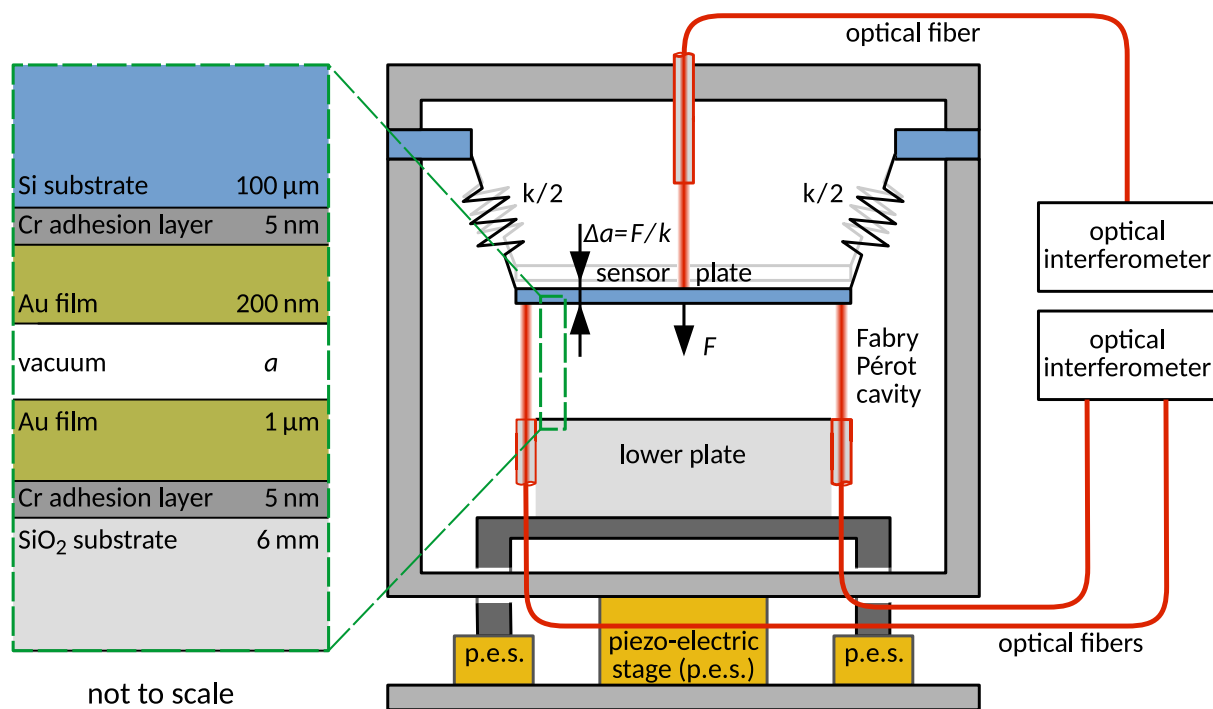
Figure 1. Schematic of the setup of CANNEX experiment for measuring the Casimir force between two parallel plates at large separations. The pressure between the fixed lower plate and the movable upper sensor plate separated by a distance a is measured by monitoring the extension Δa using the optical interferometer. The material structure of both the lower and upper plates is shown not to scale at the left of the figure.

In the case of two parallel plates of area S, it was found that at sufficiently small separations a between them satisfying the condition $am < \pi$ the additional symmetron force per plate area, i.e., the additional pressure, is given by [107]

$$P_s = \frac{F_s}{S} = -\frac{m^4}{4\lambda}. \tag{36}$$

At larger separations, the symmetron pressure goes to zero exponentially fast. These results are obtained for sufficiently dense plates with $\varepsilon \gg m^2 M^2$, where M is the mass scale entering the effective potential in Equation (22). This condition allows us to put $\Phi \approx 0$ inside the plates.

When considering the sphere–plate configuration, it was also assumed that these bodies are sufficiently dense. Under this condition, for the spheres of large radii, $R \gg m^{-1}$, the following approximate expressions for the additional symmetron force were obtained [107]

$$F_s = -\frac{m^4}{4\lambda}\pi R^2, \qquad a < \frac{\pi}{m} - R,$$

$$F_s = -\frac{m^4}{4\lambda}\pi\left(\frac{\pi}{m} - a\right)\left(2R + a - \frac{\pi}{m}\right), \qquad \frac{\pi}{m} - R < a < \frac{\pi}{m},$$

$$F_s = 0, \qquad a > \frac{\pi}{m}, \tag{37}$$

where a is the closest sphere–plate separation.

The approximate analytic calculation of F_s is also possible for the spheres of small radii $R \ll m^{-1}$ above a plate. The result is [107]

$$F_s = -\frac{4\pi m^3 R}{\sqrt{2}\lambda}\tanh\frac{m(a+R)}{\sqrt{2}}\operatorname{sech}^2\frac{m(a+R)}{\sqrt{2}}. \tag{38}$$

As is seen from Equation (38), in the limiting case $a \to 0$, i.e., when the sphere approaches the plate, the magnitude of the symmetron force decreases to

$$|F_s| = \frac{2\pi}{\lambda}m^2(mR)^2, \tag{39}$$

where $mR \ll 1$ in this case.

In the region of intermediate values of the sphere radius $mR \sim 1$, the additional force due to the symmetron field was computed numerically [107].

For obtaining constraints on the parameters of a symmetron field, it was suggested [107] to use a setup similar to that of Ref. [108]. In the proposed setup, a sphere of $R = 150$ μm radius is spaced at a distance $a = 15$ μm from a rotating disk covered with rectangular trenches of 50 μm depth in high vacuum. As a result, the distance between the sphere bottom and the disk surface varies between $a_{\min} = 15$ μm and $a_{\max} = 65$ μm. Taking into account that all the known forces at these separations are much smaller than the experimental error $\Delta F = 0.2$ fN, the constraints on the symmetron force can be obtained from the inequality [107]

$$F_s(a_{\min}) - F_s(a_{\max}) = \pm\Delta F, \tag{40}$$

using the expressions for F presented above. The expected constraints which can be obtained in this way are discussed in Ref. [107].

4.3. Constraints on the Environment-Dependent Dilaton from Measuring the Casimir Force

The parameters of the environment-dependent dilaton can be constrained from the same experiments as the parameters of chameleon and symmetron. Thus, in Refs. [19,109] the dilaton parameters were constrained using the experimental data of Ref. [79] on measuring the quantum states of ultracold neutrons near a mirror. As discussed in Section 3, these data have already been used for constraining the parameters of a chameleon model.

In Ref. [19] it was also suggested to constrain the parameters of an environment-dependent dilaton from the CANNEX experiment (see Figure 1) on measuring the Casimir force between two parallel plates at large separations [104–106]. For this purpose, using the potential (31) and the coupling function (30), the exact solutions for a dilaton field were obtained in the configurations with one and two mirrors.

The additional dilaton pressure arising between two parallel plates with an effective area of 1 cm^2 was computed in Ref. [109] in application to the CANNEX experiment. In this experiment, it was assumed that the separation distance between the plates can be varied from 1.5 to 15 μm. It was also suggested to vary the pressure around the plates by admitting Xe gas into the vacuum chamber. This option allows us to make the differential force measurements, which present many advantages in the case of an environment-dependent

force. As a result, the dilaton field between the plates and the corresponding additional pressure have been computed numerically under the condition

$$4\pi G A_0 \Phi^2 \ll 1. \tag{41}$$

This condition ensures that one can omit the coupling to matter of higher orders which is neglected in Equation (30).

Taking into account the planned sensitivity of the CANNEX experiment to force measurements equal to 0.1 nN/m^2, the prospective constraints on the dilaton parameters λ and A_0 were obtained in Ref. [109] from an assumption that no extra forces in addition to the Casimir one were registered.

5. Discussion

In the foregoing, we have considered different models of dark energy, which makes up to approximately 68% of the total energy of the Universe. These models differ significantly in their physical meaning and theoretical background. In some sense, the model of dark energy using Einstein's equations with the cosmological term provides the most economic description of the dark energy which does not require any changes in the mathematical formalism of fundamental physical theories and introduction of additional physical substances with unusual properties. It follows that this model can be considered as preferable.

All the other types of models considered above, using the concepts of the quintessence, modified gravity and hypothetical particles with unusual physical properties, in any event are based on an introduction of some additional scalar field with one or other type of the interaction potential and the function describing its interaction with the baryonic matter. There are many models specifying these functions in the one way or another, and in each case much work should be done to reconcile the model properties with all the available data from different experiments and astrophysical observations.

It should be emphasized that the chameleon, symmetron and environment-dependent dilaton fields and corresponding particles are radically different from the particles and fields used in the Standard Model of elementary particle physics. The particles and fields introduced for the understanding of dark energy are not similar to those introduced, for instance, in different approaches to the theoretical description of dark matter. In fact, the hypothetical particle constituents of dark matter, such as axions or weakly interacting massive particles, can be understood as some extensions of the Standard Model. Axions, for instance, were introduced [110,111] for a resolution of the problem of strong CP violation in quantum chromodynamics with no relation to the concept of dark matter.

It might be well to point out that in the framework of quantum theory the explanation of dark energy in terms of the cosmological constant is burdened by the problem called the vacuum catastrophe (see Section 2) and the alternative explanations using a variety of scalar fields imply a departure from the well approved general theory of relativity in favor of the scalar–tensor theory. Because of this, one may expect that the final resolution of the problem of dark energy will be found only in the context of quantum theory of gravitation. Meantime any experimental constraints on the proposed models of dark energy are of much importance by guiding the most prospective ways for further progress in cosmology.

6. Conclusions

To conclude, none of the model approaches to understanding of the dark energy discussed above can be considered as fully satisfactory. This increases the role of experiment, which may not only confirm the theoretical predictions, but to place so strong constraints on the parameters of some model that it will become completely unusable. In this regard, the laboratory experiments are the most promising because all their parameters are under the strict control which is often not the case for astrophysical observations.

In the above, we mentioned several laboratory experiments aimed to constrain the parameters of chameleon, symmetron and environment-dependent dilaton fields, such as using the atom interferometry, the interaction of the hypothetical scalar fields with

the electromagnetic field and scattering of ultracold neutrons (see Section 4). The main attention, however, was devoted to the possibility of constraining the parameters of these scalar fields from precise measurements of the Casimir force.

As is shown in the literature reviewed in Section 4, the parameters of chameleon, symmetron and environment-dependent dilaton fields can be constrained from the experiments on measuring the Casimir force. The prospective constraints, which can be obtained in this way, are quite competitive, as compared to the other laboratory experiments. For obtaining these constraints, it will be necessary, however, to create the next generation of setups which will allow measuring the Casimir interaction at large separations up to 10 micrometers and even more.

This work is currently in progress. Its successful completion will allow us to not only place new more strong constraints on the models of dark energy, but also solve the remaining problems of the Casimir physics.

Author Contributions: Conceptualization, G.L.K. and V.M.M.; investigation, G.L.K. and V.M.M.; writing—original draft, V.M.M.; writing—review and editing, G.L.K. All authors have read and agreed to the published version of the manuscript.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study. Date sharing is not applicable to this article.

Conflicts of Interest: The authors declare no conflicts of interest.

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