Review

Standard and Non-Standard Aspects of Neutrino Physics

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Abstract: This review provides a succinct overview of the basic aspects of neutrino physics. The topics covered include neutrinos in the standard model and the three-neutrino mixing scheme; the current status of neutrino oscillation measurements and what remains to be determined; the seesaw mechanisms for neutrino mass generation and the associated phenomenology, including the leptogenesis mechanism to explain the observed matter–antimatter asymmetry of the Universe; and models for the origin of the pattern of neutrino mixing and lepton masses based on discrete flavour symmetries and modular invariance.

1. Introduction

Since W. Pauli's original proposal of the neutrino idea [1], E. Fermi's first description of the weak force [2], and the pioneering antineutrino detection in the Cowan–Reines experiment [3,4], our understanding of neutrino properties and their interactions has continuously expanded. Considerable progress has been made from the experimental side, enabling the detection of neutrinos from multiple sources: the Sun, Supernovae, the interior of the Earth, radioactive nuclei and reactor plants, cosmic rays interacting with the atmosphere, and, tentatively, extragalactic sources such as active galactic nuclei (see, e.g., ref. [5] for a complete description of neutrino fluxes across different energies). Such outstanding experimental advancements, along with the well-established electromagnetic and cosmic ray observation techniques and the advent of gravitational wave detectors [6], signal a promising era of multi-messenger astronomy.

In parallel, a series of theoretical developments ultimately led to the formulation of the *Standard Model* (SM) of particle physics [7–9]. The SM represents, at present, the most accurate description of the known fundamental particles, including neutrinos, and forces, with the exception of gravity. However, despite its great successes in predicting how Nature behaves at the sub-atomic scales, the SM remains incomplete. One motivation is that the SM assumes neutrinos are massless. However, the observed phenomenon of neutrino oscillations [10,11] can be explained by the standard paradigm of the *three-neutrino mixing scheme*, which considers at least two massive neutrinos and the mixing among three neutrino species. This scheme cannot be incorporated into the SM theory without interfering with some of its fundamental properties. Besides, the SM falls short in describing other challenging issues, including the long-standing problems of explaining the nature of dark matter [12] and the overabundance of matter over antimatter in the Universe [13]. These fundamental problems may be inter-related, with neutrinos potentially playing a primary role in establishing such connections.

The search for new physics beyond the SM is a central focus in high-energy (astro)particle physics. Numerous extensions of the SM have been proposed to explain the origin of neutrino masses while attempting to tackle other fundamental problems in modern physics. The phenomenology of many of these extensions can be tested through the

extensive experimental programs that are underway. Amidst a renewed excitement and recent advances in the field, this short review aims to gather concisely some of the standard and non-standard aspects of neutrino physics. Several reviews dedicated to specific topics treated in this work already exist in the literature, and we refer to these reviews throughout the text. This work is more oriented towards particle physics and the related phenomenology, although neutrinos are relevant also to cosmology and astrophysics (see, e.g., refs. [14,15] for monographs on the topic).

This manuscript is organised as follows: in Section 2, we briefly overview the physics of neutrinos in the SM and in the three-neutrino mixing scheme; in Section 3, we summarise our current knowledge of the neutrino oscillation parameters and what is left to determine; in Section 4, we describe possible mechanisms for neutrino mass generation, concentrating on the seesaw models; in Section 5, we briefly discuss models for the origin of the lepton mixing pattern; and we conclude in Section 6.

2. The Standard Neutrino Picture

2.1. Neutrinos in the Standard Model of Particle Physics

There are three kinds, or *flavours*, of (anti)neutrinos, namely, ν_e, ν_μ, and ν_τ ($\bar{\nu}_e$, $\bar{\nu}_\mu$, and $\bar{\nu}_\tau$). The flavour (anti)neutrino ν_α ($\bar{\nu}_\alpha$), $\alpha = e$, μ, τ, is produced with a charged antilepton α^+ (lepton α^-) or produces a charged (anti)lepton α^- (α^+) in Charged Current (CC) weak interaction processes. Hitherto, neutrinos (antineutrinos) have exhibited a polarisation in a direction that is opposite (parallel) to their motion or, equivalently, a negative (positive) helicity.To account for such experimentally observed neutrino properties, neutrinos ν_α and antineutrinos $\bar{\nu}_\alpha$ of the lepton flavour $\alpha = e$, μ, τ are described in the SM with a left-handed (LH) chiral field $\nu_{\alpha L}(x)$. In the absence of neutrino masses, this field destroys (creates) neutrinos (antineutrinos) with negative (positive) helicity. In the SM, neutrinos are considered massless, and no right-handed (RH) neutrino chiral field is included in its content as they would be singlet under the SM gauge group. As such, the corresponding RH neutrinos would be completely inert. Together with the LH charged lepton chiral fields $\alpha_L(x)$, the LH neutrino fields form SU(2) doublets $\psi_{\alpha L}(x) = (\nu_{\alpha L}(x), \alpha_L(x))$ with hypercharge $+1$.

The SM Lagrangian is symmetric under the action of global $U(1)_{L_\alpha}$, $\alpha = e$, μ, τ, and $U(1)_L$ transformation. In particular, the lepton flavour L_α assigned to (anti)leptons β^- (β^+), $L_\alpha(\beta^-) = L_\alpha(\nu_\beta) = \delta_{\alpha\beta}$, and $L_\alpha(\beta^+) = L_\alpha(\bar{\nu}_\beta) = -\delta_{\alpha\beta}$ and the total lepton number $L = L_e + L_\mu + L_\tau$ are conserved by the electroweak interactions. However, these symmetries are accidental and arise solely because of the particle content and renormalisability of the theory.

The CC and Neutral Current (NC) electroweak interactions between leptons are described by the following terms in the SM Lagrangian:

$$\mathcal{L}_{CC}^{SM}(x) = -\frac{g_w}{2\sqrt{2}} j_W^\mu(x) W_\mu(x) + \text{h. c.}, \quad \mathcal{L}_{NC}^{SM}(x) = -\frac{g_w}{2c_W} j_Z^\mu(x) Z_\mu(x), \quad (1)$$

where $g_w \simeq 0.65$ is the $SU(2)_L$ weak coupling constant; $c_W \equiv \cos\theta_W \simeq 0.88$ ($s_W \equiv \sin\theta_W \simeq 0.48$), and θ_W is the Weinberg angle; and $W_\mu(x)$ and $Z_\mu(x)$ are the 4-component vector fields describing the $W^\pm$ and Z^0 weak massive bosons with masses of $M_W \simeq 80.4\,\text{GeV}$ and $M_Z \simeq 91.2\,\text{GeV}$, respectively. The weak currents are defined as

$$j_W^\mu(x) = 2 \sum_{\alpha = e, \mu, \tau} \overline{\nu_{\alpha L}}(x) \gamma^\mu \alpha_L(x), \quad (2)$$

$$j_Z^\mu(x) = 2 \sum_{\alpha = e, \mu, \tau} [g_L^\nu \overline{\nu_{\alpha L}}(x) \gamma^\mu \nu_{\alpha L}(x) + g_L^\alpha \overline{\alpha_L}(x) \gamma^\mu \alpha_L(x) + g_R^\alpha \overline{\alpha_R}(x) \gamma^\mu \alpha_R(x)], \quad (3)$$

where γ^μ, $\mu = 0$, 1, 2, 3, are the four Dirac gamma matrices; $\alpha_R(x)$ is the RH chiral field for the charged lepton α; and the couplings take the values $g_L^\nu = +1/2$, $g_L^e = g_L^\mu = g_L^\tau = -1/2 + s_W^2$, and $g_R^e = g_R^\mu = g_R^\tau = s_W^2$. Quarks also participate in the CC and

NC electroweak interactions, albeit with different NC coefficients. The parameters of the electroweak interaction are determined at present with great accuracy thanks to precision electroweak measurements and global fits. A complete list with the values of the relevant constants, couplings, and masses can be found, e.g., in [16].

Neutrinos interact with ordinary matter via CC and NC electroweak currents. These interactions regulate both the production and the detection of neutrinos in all the different sources and experimental facilities. Consequently, determining the scattering cross sections of the different neutrino scattering interactions at relevant energies is a crucial task in neutrino physics (see, e.g., ref. [17] for a comprehensive review of neutrino cross sections across multiple energy scales).

2.2. The Three-Neutrino Mixing Scheme and Neutrino Oscillations

It has been widely established by experiments investigating solar, atmospheric, reactor, and accelerator neutrinos that flavour neutrinos undergo oscillations: an initial flavour neutrino ν_α (antineutrino $\bar{\nu}_\alpha$), after travelling a certain distance for a period of time t, can be detected as a flavour neutrino ν_β (antineutrino $\bar{\nu}_\beta$) of different flavour $\beta \neq \alpha$ with a non-zero probability $P(\nu_\alpha \to \nu_\beta, t) \neq 0$ ($P(\bar{\nu}_\alpha \to \bar{\nu}_\beta, t) \neq 0$). The three-neutrino mixing scheme explains the origin of such a phenomenon by assuming that (at least two) neutrinos are massive and are a mixture of the three lepton flavour states, namely, [16] $\nu_{\alpha L}(x) = \sum_{a=1}^3 U_{\alpha a} \nu_{aL}(x)$, where $\nu_{aL}(x)$, $a = 1, 2, 3$, is the LH component of the field of a light neutrino ν_a with mass m_a and U is the 3×3 unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino (lepton) mixing matrix [18–20]. By adopting the standard parametrisation and numbering of the massive neutrinos, the PMNS matrix can be written as [21]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{i\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{\frac{i\alpha_{31}}{2}} \end{pmatrix}, \quad (4)$$

with $c_{ab} \equiv \cos\theta_{ab}$, $s_{ab} \equiv \sin\theta_{ab}$, and the angles $\theta_{ab} \in [0, \pi/2]$, with a, $b = 1$, 2, 3; $\delta \in [0, 2\pi]$ is the Dirac phase; α_{21} and α_{31} are the two Majorana phases [22], $\alpha_{21(31)} \in [0, 2\pi]$.

The phenomenon of flavour neutrino oscillations is a fascinating manifestation of quantum mechanics on macroscopic scales. Derivations of the probabilities $P(\nu_\alpha \to \nu_\beta, t)$ can be found in textbooks (see, e.g., ref. [14]). Here, we provide a concise description. A flavour state $|\nu_\alpha\rangle$ is a coherent superposition of massive states, such that, at time $t = 0$, $|\nu_\alpha(t = 0)\rangle = \sum_a U_{\alpha a}^* |\nu_a(t = 0)\rangle$. Under the plane-wave approximation, the initial massive states have definite spatial momentum $p_\nu \equiv |\vec{p}_\nu|$ and energy $E_\nu \simeq p_\nu$, with $E_a - E_b \simeq (m_a^2 - m_b^2)/2E_\nu$, a, $b = 1$, 2, 3. The initial massive state is evolved unitarily in time through the Schrödinger equation and later sandwiched with a flavour neutrino state $|\nu_\beta\rangle$. The final result of the calculation in the vacuum is

$$P(\nu_\alpha \to \nu_\beta, t) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 \simeq \left| \sum_{a=1}^3 U_{\alpha a}^* U_{\beta a} \exp\left(-i \frac{\Delta m_{a1}^2 L}{2E_\nu} \right) \right|^2, \quad (5)$$

where $L \simeq t$ is the distance travelled by the initial relativistic neutrino, while $\Delta m_{ab}^2 \equiv m_a^2 - m_b^2$, a, $b = 1$, 2, 3. The expression for antineutrinos can be obtained by taking the conjugate of the PMNS matrix entries. Approaches with different levels of refinement, considering a superposition of wave-packets or in the quantum field theory formalism, yield the same results in all experimentally relevant situations [23–26]. More importantly, interactions with the constituents of the medium in which neutrinos propagate affect the neutrino oscillation probability through *matter effects* [27]. The Hamiltonian of the system is modified by the presence of an effective matter potential associated with CC and NC weak interactions. Matter effects differ between constant and varying density conditions. The constant density case can be studied analytically and is relevant for accelerator experiments with long-baseline facilities [28–30]. The varying density case, instead, is more subtle but crucial in

modelling, e.g., the propagation of solar neutrinos with energies of a few MeV. An adiabatic change in density in the Sun's core induces the Mikheyev–Smirnov–Wolfenstein (MSW) effect [27,31], leading to a ν_e survival probability of $P(\nu_e \rightarrow \nu_e, t) \simeq \sin^2 \theta_{12}$.

3. Current Measurements and Unknowns in the Standard Neutrino Mixing Paradigm

The parameters that enter the expressions of the neutrino oscillation probabilities are as follows: the three mixing angles θ_{12}, θ_{23}, and θ_{13}; the Dirac phase δ; and two independent squared mass differences denoted as $\Delta m_{\odot}^2 \equiv \Delta m_{21}^2$ and $\Delta m_{\mathrm{atm}}^2$, which, together with θ_{12} and θ_{23}, drive the oscillations of solar and atmospheric neutrinos, respectively. The parameters θ_{12}, θ_{23}, θ_{13}, Δm_{21}^2, and $|\Delta m_{\mathrm{atm}}^2|$ have been determined with remarkably high precision by solar neutrino experiments [32–36], atmospheric neutrino experiments [37,38], reactor neutrino experiments with either short-baseline (SBL) [39–41] and long-baseline (LBL) [42] facilities, and LBL accelerator-based neutrino experiments [43–49]. Measurements of δ have also been reported for LBL accelerator experiments, although the uncertainty in this determination is still relatively large (see Section 3.2).

Depending on the experimental setup, each neutrino oscillation experiment is most sensitive to specific sets of neutrino mixing parameters. Global fit analyses conducted by various collaborations [50–52] combine data from the different experiments on neutrino oscillations and provide an overall picture of the neutrino mixing pattern. For instance, the most recent `NuFit 5.2` global analysis [51,53] reported the following best-fit $\pm 1\sigma$ values and 3σ ranges of the mixing parameters in the case of a neutrino mass spectrum with normal (inverted) ordering (see further in Section 3.1):

$$
\begin{aligned}
\theta_{12} &= 33.41^{\circ}{}^{+0.75^{\circ}}_{-0.72^{\circ}}, [31.31^{\circ}, 35.74^{\circ}]; \\
\theta_{13} &= 8.58 \pm 0.11^{\circ} (8.57^{\circ} \pm 0.11^{\circ}), [39.7^{\circ}, 51.0^{\circ}]([39.9^{\circ}, 51.5^{\circ}]); \\
\theta_{23} &= 42.2^{\circ}{}^{+1.1^{\circ}}_{-0.9^{\circ}} (49.0^{\circ}{}^{+1.0^{\circ}}_{-1.2^{\circ}}), [8.23^{\circ}, 8.91^{\circ}]([8.23^{\circ}, 8.94^{\circ}]); \\
\Delta m_{\odot}^2/(10^{-5}\,\mathrm{eV}^2) &= 7.41^{+0.21}_{-0.20}, [6.82, 8.03]; \\
\Delta m_{\mathrm{atm}}^2/(10^{-3}\,\mathrm{eV}^2) &= 2.507^{+0.026}_{-0.027} (-2.486^{+0.025}_{-0.028}), [2.427, 2.590]([-2.570, -2.406]).
\end{aligned}
$$

Despite the significant progress we have made in determining the neutrino mixing parameters, much remains to be understood. While future experiments will undoubtedly seek greater precision, efforts will be dedicated to addressing the open problems that we outline in the subsequent subsections.

3.1. The Ordering of the Neutrino Mass Spectrum and the Absolute Mass Scale

None of the existing neutrino datasets collected so far permits the conclusive inference of the sign of $\Delta m_{\mathrm{atm}}^2$. Correspondingly, there are two possible orderings for the neutrino mass spectrum that are consistent with current neutrino mixing data:

- Normal Ordering (NO): $m_1 < m_2 < m_3$, $\Delta m_{31}^2 \equiv \Delta m_{\mathrm{atm}}^2 > 0$;
- Inverted Ordering (IO): $m_3 < m_1 < m_2$, $\Delta m_{32}^2 \equiv \Delta m_{\mathrm{atm}}^2 < 0$.

Depending on the value of the lightest neutrino mass, the neutrino mass spectrum can also be hierarchical, that is $0 \simeq m_1 \ll m_2 < m_3$, with $m_2 \simeq (\Delta m_{21}^2)^{1/2}$ and $m_3 \simeq (\Delta m_{31}^2)^{1/2}$ for NO, or $0 \simeq m_3 \ll m_1 < m_2$, with $m_1 \simeq (|\Delta m_{32}^2| - \Delta m_{21}^2)^{1/2}$ and $m_2 \simeq |\Delta m_{32}^2|^{1/2}$ for IO. The possibility of a quasi-degenerate spectrum with $m_1 \simeq m_2 \simeq m_3$, with $m_{1,2,3}^2 \gg |\Delta m_{31(32)}^2|$ is also not excluded by current data. The global fit analyses for oscillation data only [50–52], as well as with the inclusion of data on β-decay and from cosmological probes [54], indicate a preference of NO over IO up to only an $\sim 2.7\sigma$ confidence level (C. L.).

Since θ_{13} is relatively large, more information on the sign of $\Delta m_{\mathrm{atm}}^2$ can be obtained in the future from LBL accelerator facilities (NOνA [48,49,55], DUNE [56]) and atmospheric neutrino experiments (INO [57], PINGU [58,59], ORCA [60], DUNE [56], and Hyper-Kamiokande (HK) [61]) exploiting matter effects. Furthermore, based on a different method [62], the reactor experiment JUNO [63,64] is also designed to determine with

precision the neutrino mass ordering. If neutrinos are of the Majorana type, information on the mass ordering can also be attained from experiments on neutrinoless double beta $((\beta\beta)_{0\nu}$-)decay (see Section 3.3 for more details).

The absolute mass scale of neutrinos, i.e., the value of $m_{1(3)}$ in the NO (IO) case, is currently not known, and only upper bounds are available. The ongoing KATRIN experiment [65] is aimed at measuring the neutrino mass by examining the tail of the electron spectrum in the final state of β-decay of tritium ${}^{3}_{1}\mathrm{H}$ and provides the following limit [66,67]: $m_{1,2,3} < 0.8\,\mathrm{eV}$ (90% C.L.). From the best existing lower bounds on the $(\beta\beta)_{0\nu}$-decay lifetimes of ${}^{136}\mathrm{Xe}$ [68] and ${}^{76}\mathrm{Ge}$ [69], one can obtain a bound on neutrino masses, which, however, applies only if neutrinos have a Majorana nature [70]: $m_{1,2,3} \lesssim 0.58\,\mathrm{eV}$. The most stringent upper bound on the sum of neutrino masses comes from observations of the Cosmic Microwave Background by PLANCK [71], combined with supernovae and other cosmological and astrophysical data, yielding $\sum_a m_a < (0.12 - 0.69)\,\mathrm{eV}$ (95% C.L.) depending on the adopted cosmological model and level of statistical complexity [72,73] (see also [74–78]).

3.2. CP-Violation in the PMNS Matrix

Whether there is CP-violation in the lepton sector originating from the Dirac phase and/or the Majorana phases in the PMNS matrix is still not confidently known. The requisite for the CC interaction term of being symmetric under CP-transformation restricts the PMNS matrix entries to be either purely real or imaginary, $U_{\alpha a} = \pm U_{\alpha a}^*$, a condition that is satisfied when the Dirac and Majorana phases take the CP-conserving values $\delta = k\pi$, $\alpha_{21} = k_{21}\pi$, $\alpha_{31} = k_{31}\pi$, with $k, k_{21}, k_{31} = 0, 1, 2$ (see, e.g., ref. [79]). If $\delta \neq k\pi$ and/or, provided that neutrinos are of the Majorana type (see Section 3.3), $\alpha_{21} \neq k_{21}\pi$ and/or $\alpha_{31} \neq k_{31}\pi$, there is CP-violation in the lepton sector from the phases of the PMNS matrix.

The Dirac phase δ is responsible for CP-violating effects in neutrino oscillations. Such effects can be computed as the differences between the oscillation probabilities $P(\nu_\alpha \to \nu_\beta, t) - P(\bar\nu_\alpha \to \bar\nu_\beta, t)$, which results to be proportional to the invariant $J_{\mathrm{CP}} = s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$ [80]. There have been some indications of CP-violation in neutrino oscillations from the LBL experiments Tokai-to-Kamioka (T2K) and NOνA, although with some tensions (a review of their results and discrepancies is given in, e.g., ref. [81]). However, global fit analyses cannot confidently exclude CP-conserving values yet. E.g., the `Nufit 5.2` analyses determined $\delta = 232°{}^{+36°}_{-26°}(276°{}^{+22°}_{-29°})$, $[144°, 350°]([194°, 344°])$, with a relatively large uncertainty. Future determinations are planned for the upcoming LBL accelerator experiments T2K [46,47,82], NOνA [48,49,55], Tokai-to-HK (T2HK) [61], and DUNE [56] (see also the Tokai-to-HK-to-Korea (T2HKK) proposal [83,84]).

In contrast, the Majorana phases do not enter the flavour neutrino oscillation probabilities [22,85] but contribute to the $(\beta\beta)_{0\nu}$-decay rate (see Section 3.3).

3.3. Neutrinos Can Be Either Dirac or Majorana Fermions

A massive spin-1/2 particle ψ with mass m_ψ can either be of the Dirac or Majorana type if it is, respectively, distinguishable or indistinguishable from its corresponding antiparticle $\bar\psi$. Since all the known fundamental fermions are electrically charged, except neutrinos, they are intrinsically of the Dirac type. In quantum field theory terms, a Dirac particle ψ can be described by a 4-component spinor $\psi_D(x)$ that satisfies the Dirac equation. After quantisation, the field $\psi_D(x)$ annihilates (creates) particles ψ (antiparticles $\bar\psi$) having helicities with either sign, thus describing in total four degrees of freedom. The field $\psi_D(x)$ can be decomposed in terms of the LH- and RH-independent chiral fields $\psi_L(x)$ and $\psi_R(x)$, such that $\psi_D(x) = \psi_L(x) + \psi_R(x)$. The mass term for a Dirac field is as follows:

$$-\mathcal{L}_{\mathrm{Dirac}}^m(x) = \overline{\psi_L}(x)m_\psi\psi_R(x) + \mathrm{h.\,c.} = \overline{\psi_D}(x)m_\psi\psi_D(x). \tag{6}$$

Any U(1) symmetry acting on the Dirac field as $\psi_D(x) \to e^{iq}\psi_D(x)$, $q > 0$, leaves the Dirac mass term invariant.

A Majorana particle $\psi = \overline{\psi}$ can be described with a 4-component spinor $\psi_M(x)$ that satisfies both the Dirac equation and the Majorana condition $\psi_M^c(x) = C(\overline{\psi_M}(x))^T$, where C is the charge conjugation matrix [1]. The Majorana condition is invariant under a U(1) transformation acting on the Majorana field as $\psi_M(x) \rightarrow e^{iq}\psi_M(x)$ only if $q = 0$. In this sense, Majorana fields can only describe neutral particles. The Majorana condition restricts the number of degrees of freedom to two, and the quantised Majorana field $\psi_M(x)$ creates and annihilates particles ψ with either positive or negative helicity. As such, a Majorana field $\psi_M(x)$ can be decomposed in terms of its LH (or the RH) component $\psi_L(x)$ only, namely, $\psi_M(x) = \psi_L(x) + \psi_R^c(x)$, with $\psi_R^c = C(\overline{\psi_L}(x))^T$. The mass term of a Majorana field takes the form

$$-\mathcal{L}_{\text{Majorana}}^m(x) = \frac{1}{2}\overline{\psi_R^c}(x)m_\psi\psi_L(x) + \text{h. c.} = \frac{1}{2}\overline{\psi_M}(x)m_\psi\psi_M(x). \tag{7}$$

and breaks any U(1) transformation acting on $\psi_L(x)$ by two charge units [2].

The evidence of non-vanishing neutrino masses gives rise to the question of whether neutrinos are of the Dirac or Majorana type. The nature of neutrinos, either Dirac or Majorana, is intimately related to the conservation/breaking of the total lepton number. Dirac neutrinos would conserve $U(1)_L$ and the associated total lepton charge, while Majorana neutrinos would break it by two units. One of the consequences of this is that the Majorana phases α_{21} and α_{31} in the PMNS matrix of Equation (4) (can) cannot be reabsorbed into a redefinition of the charged lepton fields and the Majorana (Dirac) neutrino fields. This means that the Majorana phases α_{21} and α_{31} can only play a role in physical processes in which the Majorana nature of neutrinos is manifest, while they do not contribute to any physical observable if neutrinos are of the Dirac type. In both cases, the flavour lepton number is inevitably broken by neutrino oscillations.

Whether neutrinos are of the Dirac or Majorana type can be established experimentally by searching for lepton number violating processes. The most promising searches are those looking for the $(\beta\beta)_{0\nu}$-decay, that is the double β-decay of a nucleus without the emission of neutrinos. This process would be a direct manifestation of the Majorana nature of neutrinos (see, e.g., ref. [88] for a recent review). The decay rate of the $(\beta\beta)_{0\nu}$-decay, in the minimal case, is proportional to a nuclear matrix element $|\mathcal{M}_N|^2$, which contains most of the uncertainty, and the effective Majorana mass parameter

$$|m_{\beta\beta}|^2 = \left| m_1|U_{e1}|^2 + m_2|U_{e2}|^2 e^{i\alpha_{21}} + m_3|U_{e3}|^2 e^{i(\alpha_{31}-2\delta)} \right|^2, \tag{8}$$

which depends on the neutrino masses, the PMNS mixing angles, and the Dirac and Majorana phases. Through the mass dependence, the predictions of the effective Majorana mass parameter vary depending on the neutrino mass spectrum. The experiments that are currently searching for the $(\beta\beta)_{0\nu}$-decay, as well as proposals for the future, are several (see, e.g., an extended list in [88]), but, at present, no evidence has been found. The null detection allows putting constraints on the effective mass and the absolute mass scale, depending on the ordering and precision with which the nuclear matrix element is determined. The effective Majorana mass parameter depends on the Majorana phases, even though it remains challenging to obtain valuable information in the near future [88].

3.4. Deviations from the Standard Three-Neutrino Mixing Scheme

A pertinent aspect to understand is whether the three-neutrino mixing scheme concludes the narrative on neutrino oscillations or deviations from the standard picture should be considered [89]. Below, we briefly discuss a few examples of such deviations.

3.4.1. Sterile Neutrinos

There could be (at least) one additional neutrino, ν_s, that mixes with ordinary neutrinos. If the corresponding massive state, ν_4, is sufficiently light, neutrinos of this kind can be produced through coherent oscillations with ordinary neutrinos. However, the number of neutrino species,

N_ν, participating in the electroweak interaction is constrained by precision measurements of the rate of the invisible decay $Z^0 \to \nu_\alpha \bar{\nu}_\alpha$, yielding $N_\nu = 2.9963 \pm 0.0074$ [16,90,91]. Furthermore, any additional species oscillating with the ordinary neutrinos should exhibit a suppressed mixing to avoid compromising current neutrino oscillation data. In practice, extra neutrinos ν_s should be almost entirely *sterile*.

A $(3+1)$-neutrino mixing scheme including a sterile neutrino ν_4 with mass $m_4 \gtrsim$ eV and a squared mass difference $\Delta m^2_{\text{SBL}} \equiv m^2_4 - m^2_1 \gtrsim 1\,\text{eV}$ has been proposed as an explanation for the Liquid Scintillator Neutrino Detector (LSND) anomaly, the MiniBooNE low-energy excess, the Gallium Anomaly (GA), and the Reactor Antineutrino Anomaly (RAA), despite some tensions. In light of recent analyses, the RAA seems to be diminished, and the GA is being investigated with more care (see, e.g., refs. [92–94] for recent reviews), while the SBL Neutrino programme [95,96] is aimed at shedding more light on the LSND and MiniBooNE anomalies. Nevertheless, the issue is still unresolved, and it would be remarkable if the idea of a $\sim$ eV sterile neutrino is proven to be correct in the future. In turn, a sterile neutrino at the keV scale would be a good dark matter candidate, and active searches are ongoing [97].

3.4.2. Non-Unitary Neutrino Mixing Matrix

One major effect of the mixing with additional neutrino species is that the PMNS matrix becomes non-unitary. Essentially, the full $N_\nu \times N_\nu$ matrix that regulates the mixing between all the neutrino species is unitary, but when restricted to the 3×3 block corresponding to the PMNS matrix, unitarity is lost. A non-unitary PMNS matrix can affect CC and NC interactions, neutrino oscillations, electroweak precision data, and flavour observables as well as the history of the Universe, resulting in bounds on such deviations [98–100].

3.4.3. Non-Standard Neutrino Interactions

Neutrinos can also interact with matter through non-standard interactions (NSIs) [27,101], which can be of the CC or NC type and parameterised by the following 6-dimensional effective operators [102]:

$$\mathcal{L}^{\text{NSI}}_{\text{CC}}(x) = -2\sqrt{2}G_F \epsilon^{ff'L/R}_{\alpha\beta} (\overline{\nu_{\alpha L}}(x)\gamma_\mu \beta_L(x))(\overline{f'_{L/R}}(x)\gamma^\mu f_{L/R}(x)), \tag{9}$$

$$\mathcal{L}^{\text{NSI}}_{\text{NC}}(x) = -2\sqrt{2}G_F \epsilon^{fL/R}_{\alpha\beta} \;(\overline{\nu_{\alpha L}}(x)\gamma_\mu \nu_{\beta L}(x))(\overline{f_{L/R}}(x)\gamma^\mu f_{L/R}(x)), \tag{10}$$

α, $\beta = e$, μ, τ, where $G_F = g^2_w/(4\sqrt{2}M^2_W)$ is the Fermi coupling constant; $\epsilon^{ff'L/R}_{\alpha\beta}$ and $\epsilon^{fL/R}_{\alpha\beta}$ are the dimensionless quantities that set the strength of the NSIs; and $f_{L/R}(x) \neq f'_{L/R}(x)$ are the LH/RH chiral fields for quarks (or leptons) for the CC NSIs (NC NSIs). Neutrino NSIs can arise in a variety of SM extensions [103], with the form given being generated after integrating out some heavy mediator fields with a mass that is much heavier than the momentum transferred in neutrino interaction processes. The presence of neutrino NSIs can interfere with the picture of neutrino oscillations in various ways, by affecting detection/production and through variations in the matter effects when propagating in a medium. NSIs can also induce a degeneracy in the determination of the neutrino mixing parameters [102]. Current oscillation data, electroweak precision data, and data on coherent elastic neutrino–nucleus scattering impose constraints on their couplings [102,104,105].

4. Possible Origins of Neutrino Masses beyond the Standard Model

4.1. Dirac Mass Term from the Usual Higgs Mechanism

A minimal extension of the SM that explains the origin of neutrino masses consists of adding $n_R = 2$, 3 RH neutrinos (fields) ν_{jR} ($\nu_{Rj}(x)$), $j = 1, \cdots, n_R$ that couple via a Yukawa interaction with the charged leptons and Higgs doublets. Such coupling is allowed by the symmetry of the SM and reads:

$$-\mathcal{L}^\nu_Y(x) = \lambda^\nu_{\alpha j}\overline{\psi_{\alpha L}}(x)i\sigma_2\Phi^*(x)\nu_{jR}(x) + \text{h. c.} \tag{11}$$

where σ_2 is the second Pauli matrix and $\Phi(x) = (\Phi^+(x), \Phi^{(0)}(x))$ is the Higgs doublet field. After the electroweak symmetry is spontaneously broken and the neutral component of the Higgs doublet acquires a non-vanishing vacuum expectation value (VEV) $v/\sqrt{2} \simeq 174\,\mathrm{GeV}$, the Yukawa term above generates Dirac mass terms for the Dirac fields $\nu_a(x) = (U_L)^*_{\alpha a}\nu_{\alpha L}(x) + (U_R)^*_{aj}\nu_{jR}$, $a = 1, \cdots, n_R$, with U_L and U_R being, respectively, 3×3 and $n_R \times n_R$ unitary matrices that factorise λ^ν according to the singular value decomposition. The field $\nu_a(x)$ describes the massive Dirac neutrino ν_a with mass $m_a = (v/\sqrt{2})\lambda^\nu_{\alpha a}(U^*_L)_{\alpha a}(U_R)_{aj}$, $a = 1, \cdots, n_R$. On the basis of diagonal charged lepton Yukawa couplings, U_L coincides with the PMNS matrix, $U_L = U$, and regulates the mixing of the LH components of the Dirac neutrinos ν_a as in the three-neutrino mixing scheme.[3]

There are several reasons why this mechanism is not particularly appealing. Firstly, from a phenomenological perspective, it leads to a dead-end. The RH neutrinos are singlets under the SM gauge group and do not mix with the LH ones, with which they constitute Dirac pairs. As such, they do not participate in the SM interactions and are completely sterile. The only observable phenomenon beyond the SM in this extension is that of ordinary neutrino oscillations. From the theoretical standpoint, it seems artificial to justify tiny Yukawa couplings $\lambda^\nu \sim 10^{-12}$ to explain the smallness of the neutrino masses. Additionally, imposing a global $U(1)_L$ symmetry by hand to avoid a Majorana mass term for the RH neutrinos seems contrived. All these factors motivate the exploration of alternative mechanisms for neutrino mass generation.

4.2. Majorana Mass Term from the Weinberg Operator and Seesaw Mechanisms

The SM can be augmented with the effective Weinberg operator [106]

$$-\mathcal{L}_{\mathrm{Weinberg}}(x) = \frac{c_{\alpha\beta}}{\Lambda}\left(\overline{\psi^c_{\alpha R}}(x)\sigma_2\Phi(x)\right)\left(\Phi^T(x)\sigma_2\psi_{\beta L}(x)\right) + \mathrm{h.\,c.} \tag{12}$$

where $\psi^c_{\alpha R}(x) = C(\overline{\psi_{\alpha L}}(x))^T$, $c_{\alpha\beta} = c_{\beta\alpha}$ are symmetric coefficients and Λ has the dimension of a mass. The Weinberg operator is the only 5-dimensional operator that can be constructed out of the field content of the SM theory [107]. With such a term added, the SM loses its renormalisabilty. This means, in particular, that such an extension will not be valid at any scale. The limit of validity is given by Λ, setting the scale at which new physics beyond the SM manifests itself. After the breaking of the electroweak symmetry, the Weinberg operator leads to a Majorana mass term for the Majorana fields $\nu_a(x) = U^*_{\alpha a}\nu_{\alpha L}(x) + U^*_{\alpha a}\nu^c_{\alpha R}(x)$, $\nu^c_a(x) = \nu_a(x)$, where $\nu^c_{\alpha R} = C(\overline{\nu_{\alpha L}}(x))^T$, $a = 1, 2, 3$. The field $\nu_a(x)$ describes a Majorana neutrino ν_a with mass $m_a = v^2 c_{\alpha\beta}U^*_{\alpha a}U^*_{\beta a}/(2\Lambda)$, $a = 1, 2, 3$, suppressed by the scale $\Lambda \gg v$.

The Weinberg operator can be interpreted as the low-energy limit of a renormalisable interaction. This is in analogy with Fermi's theory being the low-energy limit of the CC weak interaction after integrating out the massive bosons $W^\pm$. There are only three possible 4-dimensional operators allowed by the SM symmetry that can generate the Weinberg operator at low energy. These are classified on the basis of the type of mediator, which can be either one (or more) singlet fermion(s) N with mass M_N [108–112], or SU(2)-triplet scalar(s) Δ with mass M_Δ [113–115], or SU(2)-triplet fermion(s) Σ with mass M_Σ [116]. A specific mechanism of neutrino mass generation is associated with each of these operators, all having in common that the masses of the neutrinos are suppressed by the mass of the heavy mediator, metaphorically as sitting on a see-saw. For this reason, such mechanisms (models) for neutrino mass generation are referred to as *type-I seesaw* for the singlet fermion, *type-II seesaw* for the triplet scalar, and *type-III seesaw* for the triplet fermion (see, e.g., ref. [117] for a review). The contributions to neutrino masses in the different seesaw mechanisms are schematised diagrammatically in Figure 1. In what follows, we give more details on each specific seesaw mechanism and related phenomenology [4].
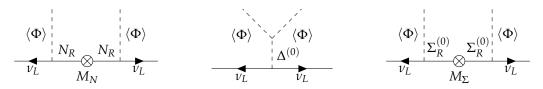
Figure 1. Diagrammatic representations of neutrino mass contributions in type-I (left), type-II (center), and type-III (right) seesaw mechanisms. Here, $\langle \Phi \rangle$ stands for the VEV of the Higgs boson, ν_L is the LH component of a massive neutrino, N_R is the RH component of a singlet fermion, $\Delta^{(0)}$ is the neutral component of the scalar triplet, and $\Sigma_R^{(0)}$ is the RH projection of the neutral component of a fermion triplet. The crosses in the fermion propagators signify mass insertions.

4.2.1. Type-I Seesaw

In the context of type-I seesaw mechanisms, the SM is augmented with n_R RH neutrinos (fields) N_{jR} ($N_{jR}(x)$), $j = 1, ..., n_R$, taken to be singlet under the SM gauge group. In what follows, we concentrate on the case of $n_R = 3$, even though the case with $n_R = 2$ is the minimal scenario compatible with the existing neutrino oscillation data. The RH neutrinos enter in the SM Lagrangian with a Majorana mass term and a Yukawa coupling to the Higgs and LH charged lepton doublets:

$$-\mathcal{L}_{\text{Type-I}}(x) = \left(Y_{\alpha j}\overline{\psi_{\alpha L}}(x)\, i\sigma_2\, \Phi^*(x)\, N_{jR}(x) + \text{h.c.} \right) + \frac{1}{2}M_j\overline{N_j}(x)N_j(x)\,, \qquad (13)$$

written in the basis for which both the charged lepton Yukawa and the Majorana mass matrix for the right-handed neutrinos are diagonal. The Majorana field $N_j(x) = N_{jR}(x) + N_{jL}^c(x)$, $N_{jL}^c(x) = C(\overline{N_{jR}}(x))^T$ describes a massive Majorana neutrino N_j with mass $M_j > 0$, $j = 1, 2, 3$ [5]. With the Higgs field acquiring a non-vanishing VEV, diagonalisation of the seesaw Lagrangian in Equation (13) leads to the type-I seesaw formula for the light neutrino masses: $m_a \simeq -v^2 U_{\alpha a}^* U_{\beta a}^* Y_{\alpha j} Y_{\beta j}/(2M_j)$, which is valid at tree level and up to second-order corrections in the quantity $\Theta_{\alpha j} \equiv (v/\sqrt{2})Y_{\alpha j}/M_j$, $|\Theta_{\alpha j}| \ll 1$ [6,7].

The typical seesaw mechanism consists of setting M_j at scales that are high enough to suppress the light neutrino masses. However, the smallness of the light neutrino masses can also be accounted for by tiny Yukawa couplings or cancellations occurring between the various terms (note that the seesaw formula is a sum of multiple terms). Consequently, the masses of the heavy neutrinos are basically unconstrained down to the $\sim$ eV scale when considered as an explanation for the light neutrino masses. The latter situation can be justified when, e.g., two heavy neutrinos form a pseudo-Dirac pair [128] and there is an approximately broken lepton number symmetry, as in *inverse*, *linear*, and *extended* seesaw scenarios (see, for instance, ref. [129] for a related discussion).

At the same order in $|\Theta_{\alpha j}|$, the heavy neutrinos N_j keep their masses M_j and mix with the light neutrinos [8]:

$$\nu_{\alpha L}(x) \simeq U_{\alpha a}\nu_{aL}(x) + \Theta_{\alpha j}N_{jL}^c(x)\,. \qquad (14)$$

Through mixing with the light neutrinos, the heavy Majorana neutrinos inherit a coupling to the SM particles and participate in the CC and NC interactions. If their masses lie at the sub-TeV scale and their mixing is relatively large, they can be directly produced or detected at accelerator-based experiments and leave a variety of indirect low-energy signatures [117,130,131]. An extensive experimental programme has been dedicated to the searches of heavy Majorana neutrinos. The absence of evidence thus far imposes constraints on the mixing for masses ranging from the eV to the PeV scale, with additional bounds from cosmology and astrophysics [132–134]. Further experiments are planned and proposed for the future with the aim to probe the relevant parameter space more deeply (see, e.g., the reports [135–137]). Also, future discussed colliders [138–141] have the potential to discover heavy Majorana neutrinos in the mass range from a few GeV to tens of TeV.

A common practice in phenomenological analyses is to express the Yukawa matrix in terms of low-energy observables via the Casas–Ibarra (CI) parameterisation [142]:

$$Y_{\alpha j} = \pm i \frac{\sqrt{2}}{v} U_{\alpha a} \sqrt{m_a} O_{ja} \sqrt{M_j}, \qquad (15)$$

where the *CI matrix O* is an arbitrary 3×3 complex orthogonal matrix that can be parameterised in terms of three complex angles (in the scenario of two heavy Majorana neutrinos, the number of complex angles reduces to one) and a phase that allows $\det(O) = \pm 1$. In general, the CI matrix contains CP-violating phases in addition to those of the PMNS matrix.

4.2.2. Type-II Seesaw

The minimal realisation of the type-II seesaw mechanism consists of extending the SM with a scalar $SU(2)_L$-triplet field $\Delta(x)$ given by

$$\Delta(x) = \begin{pmatrix} \Delta^+(x)/\sqrt{2} & \Delta^{++}(x) \\ \Delta^{(0)}(x) & -\Delta^+(x)/\sqrt{2} \end{pmatrix}. \qquad (16)$$

This field caries hypercharge $+1$ and enters the SM Lagrangian in the following lepton number violating terms:

$$-\mathcal{L}_{\text{Type-II}}(x) = \frac{1}{2} Y_{\alpha\beta}^\Delta \overline{\psi_{\alpha R}^c}(x) \Delta(x) \psi_{\beta L}(x) + \mu_\Delta \Phi^T(x) i\sigma_2 \Delta^*(x) \Phi(x) + \text{h. c.} \qquad (17)$$

Here, M_Δ is the mass of the scalar triplet, $Y_{\alpha\beta}^\Delta$ are dimensionless couplings, and μ_Δ has the dimension of a mass. The scalar triplet Δ interacts with gauge bosons and, along with the Higgs doublet, contributes to the full scalar potential. We omit such terms for brevity.

The masses of the light neutrinos are generated after both the Higgs and scalar triplets acquire non-vanishing VEVs, the latter being $v_\Delta \sim v^2 \mu_\Delta / (2M_\Delta^2)$. The masses of the light neutrinos are then given by $m_a = v_\Delta Y_{\alpha\beta}^\Delta U_{\alpha a}^* U_{\beta a}^*$. The smallness of the light neutrino masses is controlled by the smallness of the VEV v_Δ. Its value is constrained from above by electroweak precision data $v_\Delta \leq 4.8\,\text{GeV}$ at 95% C.L. (see, e.g., ref. [143]) and from below by the perturbativity of Yukawa couplings in the generation of light neutrino masses $v_\Delta \gtrsim 0.1\,\text{eV}$ [117].

The phenomenology of the type-II seesaw model is rich due to the presence of seven massive degrees of freedom: three neutral scalars (including the standard Higgs), two singly charged scalars and two doubly charged scalars. If the mass of the triplet is at the $\sim$ TeV scale or below, the new scalars could leave a plethora of signatures at colliders and other low-energy experiments [130]. Recent studies on type-II seesaw phenomenology and constraints can be found in, e.g., refs. [143,144] (see references therein for previous works).

4.2.3. Type-III Seesaw

In the type-III seesaw scenario, the SM is augmented with $n_T \geq 2$ hyperchargeless $SU(2)_L$-triplet fermion fields $\Sigma_{\kappa R}(x)$

$$\Sigma_{\kappa R}(x) = \begin{pmatrix} \Sigma_{\kappa R}^{(0)}(x)/\sqrt{2} & \Sigma_{\kappa R}^+(x) \\ \Sigma_{\kappa R}^-(x) & -\Sigma_{\kappa R}^{(0)}(x)/\sqrt{2}, \end{pmatrix} \qquad (18)$$

$\kappa = 1, \ldots, n_T$. These fields possess Majorana mass terms and couple to the Higgs and charged lepton doublets through a Yukawa interaction:

$$-\mathcal{L}_{\text{Type-III}}(x) = \left(Y_{\alpha\kappa}^\Sigma \overline{\psi_{\alpha L}}(x) \Sigma_{\kappa R}(x) i\sigma_2 \Phi^*(x) + \text{h. c.} \right) + \frac{1}{2} M_{\Sigma_\kappa} \text{Tr} \left[\overline{\Sigma_k^c}(x) \Sigma_\kappa(x) \right], \qquad (19)$$

with $\Sigma_\kappa(x) = \Sigma_{\kappa R} + \Sigma_{\kappa L}^c(x)$ and $\Sigma_{\kappa L}^c(x) = i\sigma_2 C(\overline{\Sigma_R}(x))^T i\sigma_2$. The fermion triplets have also gauge interactions, which we do not report here for brevity.

After the breaking of the electroweak symmetry, the generation of the non-zero neutrino masses proceeds exactly as in the type-I seesaw mechanism via the Yukawa term, with the neutral components $\Sigma_{\kappa R}^{(0)}$ playing the roles of RH neutrinos and a seesaw relation for the light neutrino masses.

The phenomenology of the type-III seesaw is similar to that in the type-I framework but richer due to the triplet nature. The neutral components of the triplets mix with the light neutrinos, akin to the RH neutrinos in the type-I seesaw scenario, through which they gain couplings to the SM particles. In addition, the fermionic triplets couple to the massive gauge bosons directly and their charged components mix with the other charged leptons. If the masses of the triplets are sufficiently small, there is the possibility to produce them copiously at colliders and observe their indirect signatures [130] (see, e.g., refs. [145,146] for recent discussions).

4.2.4. Leptogenesis within the Seesaw Extensions of the Standard Model

A remarkable feature of the seesaw extensions of the SM is the possibility to explain the present *baryon asymmetry of the Universe* (BAU) through the mechanism of *leptogenesis* (LG) [147]. In the presence of *L*-, C- and CP-violating processes (e.g., decays or scatterings) allowed by the seesaw models, when these occur out-of-equilibrium in the expanding Universe, all three of Sakharov's conditions for a dynamical generation of a lepton asymmetry are fulfilled [148]. Once a lepton asymmetry is generated in the early Universe, it gets converted into the present BAU by the non-perturbative sphaleron processes [149] predicted by the SM to be in thermal equilibrium above the electroweak scale [150].

The scenario of LG within the minimal type-I seesaw model is by far its most well-studied realisation. It can proceed thanks to decays, inverse decays, scatterings, oscillations involving the RH neutrinos, the Higgs, and the LH leptons (and sphalerons) when these happen out-of-equilibrium in the expanding Universe. LG within the type-I seesaw model has been shown to work over wide ranges of masses, with different contributions that dominate the BAU generation depending on the scale and heavy neutrino mass spectrum (see, e.g., ref. [151] for a recent review on the topic). We depict in Figure 2 a schematic summary of the various LG versions, see the caption for details and references.

The low-energy LG realisations with quasi-degenerate-in-mass heavy Majorana neutrinos at the sub-TeV scales are compatible with heavy neutrino masses and mixing in the sensitivity ranges of planned and proposed experiments looking for heavy neutrino signatures [152] (see also [153]). A relatively large part of the parameter space associated with successful LG will be scrutinised in the future by collider searches, beam-dump, and kaon decay experiments, possibly at future discussed colliders [154,155], and by experiments looking at charged lepton flavour violating processes [134,156]. However, it remains challenging to test the high-scale LG scenarios in which the heavy Majorana neutrinos have masses above the TeV scale. Nonetheless, an intriguing possibility arises when the requisite CP-violation in LG is uniquely provided by the Dirac and/or Majorana phases of the PMNS matrix. If this is the case, there is a direct link between the BAU and CP-violating phenomena in low-energy neutrino oscillations and $(\beta\beta)_{0\nu}$-decay. The possibility of having CP-violation only from the PMNS matrix can be achieved if the CI matrix is CP-conserving, as, e.g., in flavour models based on sequential dominance [157] or with residual CP-symmetries [158,159]. LG with CP-violation solely from the PMNS phases has been shown to work both in high-scale and low-scale scenarios [79,160–166]. If, in the future, LBL neutrino experiments and, possibly, experiments on $(\beta\beta)_{0\nu}$-decay, determine with confidence that the Dirac and/or the Majorana phases of the PMNS matrix have CP-violating values, that would be an indication in favour of LG as the origin of the predominance of matter over antimatter in our Universe.
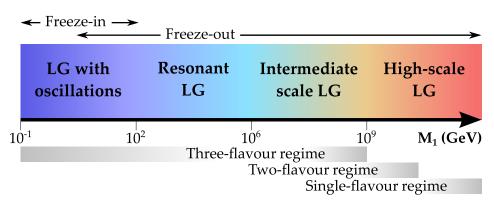
Figure 2. Scheme of LG scenarios within the type-I seesaw model. LG operates through either a *freeze-in* or *freeze-out* mechanism, if the BAU is generated when heavy Majorana neutrinos are being produced or depart from thermal equilibrium, respectively. The single-, two-, and three-flavour regimes are based on how many lepton flavours are distinguishable in the primordial plasma [167,168]. High-scale LG [147,169] is typically explored in the strong hierarchical limit $M_1 \ll M_2 \ll M_3$ and suffers from the Davidson–Ibarra bound $M_1 \gtrsim 10^9$ GeV [170], reducible to $M_1 \gtrsim 10^6$ GeV when milder hierarchies and flavour effects are considered [171,172]. Resonant enhancement of the CP-asymmetry in the decay of two nearly degenerate heavy Majorana neutrinos can reduce the LG scale down to the electroweak range [173]. In freeze-in LG taking place at sub-TeV scales, CP-violating RH neutrino oscillations can contribute substantially to the BAU generation [174].

Finally, we briefly comment on the LG realisation within the type-II and type-III seesaw models. Unlike the type-I seesaw scenario, LG within the minimal realisation of the type-II seesaw mechanism is unable to reproduce the present BAU [175]. To render LG successful, it takes either additional scalar triplets [176], support from RH neutrinos [177], or incorporation of the Affleck–Dine mechanism [178] with the neutral component $\Delta^{(0)}$ playing the role of the inflaton [179,180]. LG within the type-III seesaw works qualitatively as in the type-I scenario, with more complications caused by the gauge interactions of the charged fermionic triplets, which tend to bring them into thermal equilibrium faster [181,182].

5. The Origin of the Lepton Mixing Pattern

Understanding the origin of the neutrino mixing pattern and of the light neutrino masses, along with the origin and magnitude of the CP-violation in the leptonic sector, represents a fundamental task in neutrino physics. The structure of the PMNS matrix, derived from the neutrino oscillation data, and the spectrum of the lepton masses may be completely random [183–185] or hiding an underlying (approximate) symmetry. The latter idea is based, essentially, on the following logic. Fits to oscillation data give values of θ_{12} and θ_{23}, which are relatively large compared to θ_{13}, with θ_{23} being quasi-maximal (i.e., $\sin 2\theta_{23} \sim 1$), θ_{13} somewhat close to zero and θ_{12} not distant from special values (e.g., $\sin^2 \theta_{12} \sim 1/2, 1/3$). This structure closely resembles specific patterns like the tri-bimaximal with $\sin^2 \theta_{12} = 1/3$, $\sin 2\theta_{23} = 1$, and $\sin \theta_{13} = 0$ [186,187], the bi-maximal with $\sin 2\theta_{12} = \sin 2\theta_{23} = 1$ and $\sin \theta_{13} = 0$ [188,189], or other alternatives involving the golden ratio [190–194] or hexagonal mixing [195,196]. Deviations from such specific patterns can arise after the breaking of certain symmetries in the flavour sector, like those based on the action of discrete non-Abelian groups such as S_N, A_N, D_N, $\Sigma(2N^2)$, $\Sigma(3N^3)$, $\Delta(3N^2)$, and $\Delta(6N^2)$ (see, e.g., ref. [197] for the definitions and properties of these groups and, e.g., refs. [198–201] for reviews). Models based on such discrete flavour symmetries to describe the lepton mixing pattern are highly predictive and establish correlations between the values of the PMNS mixing angles and the CP-violating phases. It is also possible to combine the discrete flavour symmetry approach with a generalised CP-symmetry [202,203] to further reduce the number of free parameters and increase the correlations with the PMNS phases. Future high-precision determination of the mixing parameters will be crucial for discriminating between the models considered in the literature [204–206].

A possible drawback of such models is that the correct breaking of the discrete flavour symmetry generically implies the existence of many scalar fields, called *flavons* [207], entering an elaborate scalar potential with complicated symmetric shaping. Recently, the idea of *modular invariance* has been proposed to provide substantial simplification to this problem [208]. In this approach, the flavour (modular) symmetry is broken by the non-zero vacuum expectation value of a single scalar field, the *modulus*, the vacuum expectation value of which can also be the only source of violation of the CP-symmetry. The Yukawa couplings and the fermion mass terms in the theory are represented by special functions of the modulus called *modular forms* having particular properties under the modular group transformations. Over less than a decade, the number of works on the modular invariance solution to the flavour problem has grown enormously (see, for instance, ref. [209] and extensive lists of references therein), and the current hope is to explain under this same formalism the masses and mixing patterns of both leptonic and quark sectors.

6. Outlook and Conclusions

The parameters of the electroweak theory and most of the ones of the three-neutrino mixing paradigm are known today with exceptional precision. Nevertheless, the following fundamental tasks persist in neutrino physics.

- What are the ordering and the absolute mass scale of neutrino masses?
- What is the amount of CP-violation in the lepton sector?
- Are neutrinos Dirac or Majorana particles?
- What are the precise values of the mixing angles?
- Is the three-neutrino mixing accurate or are there deviations, such as sterile neutrinos, non-unitary PMNS matrix, and/or non-standard interactions?
- What mechanism is responsible for the generation of light neutrino masses?
- What is the origin of the lepton masses and mixing pattern, and why is it substantially different from that in the quark sector?

The above questions form the basis of an ambitious research programme and motivate extensions of the Standard Model that address the origin of neutrino masses and mixing. Ongoing and upcoming neutrino experiments on neutrino oscillations aim to refine the neutrino mixing picture, determining with more precision the parameters that have already been measured, unveiling those that are currently unknown, revealing potential correlations among them, and identifying/constraining deviations from the three-neutrino mixing paradigm. Additionally, planned and proposed accelerator-based and low-energy experiments will explore more in depth the parameter spaces of low-energy realisations of seesaw models, which constitute attractive mechanisms of light neutrino mass generations.

Resolving these central issues in neutrino physics may have profound implications also for other fundamental problems in modern physics, such as the matter-antimatter asymmetry of the Universe and the nature of dark matter. Moreover, the nascent field of multi-messenger astronomy involving neutrinos offers further avenues for exploration. Hopefully, in the foreseeable future, some, if not all, of these lingering questions in neutrino physics will be clarified.

Conflicts of Interest: The author declares no conflicts of interest.

Notes

[1]　The Majorana condition can be defined up to an overall unphysical phase which we have taken equal to unity.

[2]　More details on the basic properties of Dirac and Majorana fermions in quantum field theory can be found, e.g., in [86] (see also [87]).

[3]　We note that, in case $n_R = 2$, one neutrino remains massless. This latter situation is viable as neutrino oscillations dictates the existence of at least two non-zero neutrino masses.

[4]　Majorana mass terms can also be generated radiatively at the one or more loop levels [118–122].

[5]　Typically, the values of $M_{1,2,3}$ are larger than the sub-eV scale of the light neutrino masses, and so we will refer to $N_{1,2,3}$ further on as *heavy Majorana neutrinos*, or simply heavy neutrinos.

[6]　See, e.g., ref. [123] for a derivation of the seesaw relations at arbitrary order in $|\Theta_{\alpha j}|$.

[7]　One-loop corrections to light neutrino masses in the type-I seesaw scenario are discussed in, e.g., refs. [124–127].

[8]　Higher-order corrections lead, in particular, to deviations from unitarity of the PMNS matrix [100].

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