universe

MDPI

Article

Quantum Tunneling Radiation from Loop Quantum Black Holes and the Information Loss Paradox

C. A. S. Silva [1,*] and Francisco A. Brito [2]

1 Instituto Federal de Educação Ciência e Tecnologia da Paraíba (IFPB), Campus Campina Grande-Rua Tranquilino Coelho Lemos, 671, Jardim Dinamérica I, Campina Grande 58432-300, Paraíba, Brazil
2 Departamento de Física, Universidade Federal de Campina Grande, Caixa Postal 10071, Campina Grande 58109-970, Paraíba, Brazil; fabrito@df.ufcg.edu.br
* Correspondence: carlos.souza@ifpb.edu.br

Academic Editor: Jaume Haro
Received: 8 December 2016; Accepted: 29 April 2017; Published: 8 May 2017

Abstract: In this work, we present some results relating to the issue of the Loop Quantum Black Holes (LQBH) thermodynamics by the use of the tunneling radiation formalism. The information loss paradox is also discussed in this context, and we have considered the influence of back reaction effects.

Keywords: loop quantum black hole; tunneling radiation; back-reaction; information recovery

1. Introduction

Starting from the Hawking demonstration, made in the 1970s, which shows that black holes can radiate thermally [1], some work has been undertaken in order to understand the black hole evaporation phenomenon. In this sense, several methods have been developed in order to calculate the temperature and entropy of black holes [2–5]. However, some questions about the black hole evaporation process remain open until now, such as the information loss paradox [6,7] and the issue about the origin of black hole entropy [8].

Among the methods that have been developed in order to understand black hole evaporation, more recently, a semiclassical method has been constructed upon the interpretation of Hawking radiation as a tunneling process across the black hole horizon [9–11]. The basic idea is that the Hawking flux, observed at infinity, has its origin in positive energy particles created just inside the horizon which could tunnel through it quantum mechanically. The tunneling approach is especially interesting in order to calculate black hole temperature since it provides a dynamic model for the black hole emission process. In this way, the tunneling approach turns out to be very useful when one wishes to incorporate back-reaction effects in order to describe black hole evaporation. In addition, even though calculations in the tunneling formalism are straightforward and relatively simple, they are robust in the sense that they can be applied to a wide variety of spacetimes [12–25].

Tunneling formalism has also contributed to the discussion of the black hole information loss paradox, even at the semiclassical level. In this way, Parikh [10] demonstrated, at first, that a nonthermal spectrum could be calculated when one interprets the black hole emission process as a tunneling phenomena. However, no information recovery was obtained from the Parikh analysis. Such treatment was used also by Arzano et al. [26] where quantum gravity effects was considered. However, Zhang et al. [27] have demonstrated that the Parikh argument needed to be rectified. In this way, by the use of a statistical argument, Zhang et al. have shown that in the view of the tunneling approach, some information could be recovered by black holes during their evaporation process.

On the other hand, in the framework of black hole evaporation, it is expected that quantum gravity effects must have a crucial role, especially in the last stages of black hole evaporation. In this way, additional investigations taking into account quantum gravity contributions to the black hole emission process have been done by considering noncommutative geometry, Generalized Uncertainty Principle (GUP), as well as Loop Quantum Gravity (LQG) and string theory scenarios [28–37], where the information loss issue has also been considered.

In this work, at first, we shall revisit the results of [38] where the Hamilton-Jacobi version of the tunneling formalism can be used to investigate how quantum gravity effects could have an influence in the emission process by means of a black hole. In order to do this, we shall investigate the thermodynamic properties of Loop Quantum Black Holes [39,40], which correspond to a quantum corrected black hole solution that appears in the context of LQG. The temperature and entropy of this kind of black hole are calculated by the use of the tunneling method. These first results presented in [38] replicate those found in the references [40–42], where other methods have been used. In this way, it can be demonstrated that the quantum tunneling formalism can be successfully applied to address the thermodynamics of LQBHs, opening a path for a whole range of applications. Among the possible applications, in the present paper, we shall investigate the possibility of information recovery through the calculation of the statistical correlations between consecutive modes emitted during the LQBH evaporation. In this case, the results of [38], which were based on the Parikh approach, have been improved by the use of the treatment introduced by Zhang et al. In order to perform the two last tasks, back reactions effects will be taken into account, since it is supposed that such effects could give us a mechanism for information recovery at the last stages of black hole evaporation.

This paper is organized as follows. In Section 2, we review the main features of the LQBHs scenario. In Section 3, we review the use of the tunneling formalism to calculate the temperature and entropy of LQBHs. In Section 4, we shall see how back reactions effects can be included. In Section 5, we shall address the information loss problem in the LQBH scenario by the use of the tunneling formalism. The last section is devoted to remarks and conclusions.

2. Loop Quantum Black Holes

Efforts in order to find out black holes solutions in the context of LQG have been made by several authors [43–56]. In this work, we shall investigate the thermodynamics of a particular solution called the self-dual solution which was obtained by the use of loop quantum cosmology quantization techniques to the Schwarzschild scenario [39].

The self-dual black hole solution is obtained in the semiclassical limit of LQG. Even though the semiclassical limit of the full theory of LQG is not completely understood yet, the situation is quite different in the case of symmetry-reduced models where the semiclassical limit can be performed in a trustable form [57].

In this way, the self-dual black hole scenario is obtained by the reduction from the full LQG theory to the minisuperspace model by spherical symmetric reduction of the Hamiltonian constraint [39]. The semiclassical approximation of the Hamiltonian is then performed and a quantum gravitationally corrected Schwarzschild solution, described by the following metric, is obtained

$$ds^2 = -G(r)c^2dt^2 + F(r)^{-1}dr^2 + H(r)d\Omega^2 \ , \tag{1}$$

with

$$d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2 \ . \tag{2}$$

In the Equation (1), the metric functions are given by

$$G(r) = \frac{(r-r_+)(r-r_-)(r-r_*)}{r^4+a_0^2} \ , \tag{3}$$

$$F(r) = \frac{(r-r_+)(r-r_-)r^4}{(r+r_*)^2(r^4+a_0^2)} \ , \tag{4}$$

and

$$H(r) = r^2 + \frac{a_0^2}{r^2} \,, \tag{5}$$

where

$$r_+ = \frac{2Gm}{c^2} \;\; ; \;\; r_- = \frac{2Gm}{c^2} P^2 \,.$$

In the loop black hole scenario, we have the presence of two horizons—an event horizon and a Cauchy horizon. Moreover, r_* is defined as

$$r_* = \sqrt{r_+ r_-} = \tfrac{2Gm}{c^2} P \;\; , \tag{6}$$

where P is the polymeric function [39].

Moreover, $a_0 = \frac{A_{\text{min}}}{8\pi}$, where A_{min} represents the minimal value of area in Loop Quantum Gravity, and the mass parameter m is related with the ADM mass M by

$$m = M(1+P) \;\; . \tag{7}$$

In this way, the LQBH scenario consist in a semiclassical scenario which differs from the classical Schwarzschild solution by the presence of quantum corrections in terms of the polymeric function P and the LQG minimal area. The classical Schwazschild scenario is restored when one takes the limit where $a_0 \to 0$ and $P \to 0$. The complete treatment regarding the way that the LQBH scenario is obtained from LQC is archived in [39].

In order to discuss the properties of LQBH solution, let us write the Kretschmann invariant for this solution which, for $r \approx 0$, is given by

$$K = R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} = \frac{3145728\pi^4 r^6}{a_0^4 \gamma^8 \delta^8 m^2} \;\; . \tag{8}$$

Consequently, in a different way from the classical Schwarzschild solution, the LQBH Kretschmann invariant does not diverge when $r \to 0$. This fact indicates that the resolution of the black hole singularity at $r = 0$. In fact, in the LQBH scenario, the black hole singularity is replaced by another asymptotic flat region, as can be seen in the LQBH Carter-Penrose diagram depicted in Figure 1.

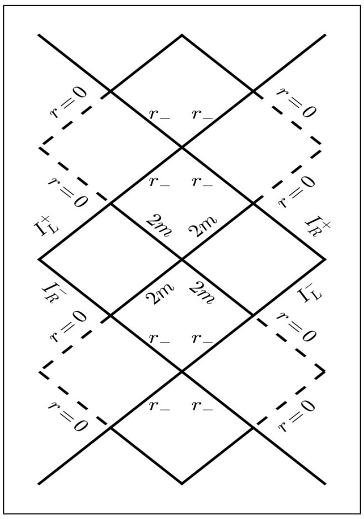

Figure 1. Carter-Penrose diagram for a self-dual black hole. The LQBH scenario possess two asymptotic regions, one at infinity and the other near the origin, which no observer can reach in a finite time.

The LQBH spacetime possess yet another interesting property, that of self-duality. This property is given by a symmetry present in the metric (1). In this way, the LQBH metric is invariant by the transformations $\tilde{r} = a_0/r$ and $\tilde{t} = tr_*^2/a_0$, with $\tilde{r}_\pm = a_0/r_\mp$. Such transformations connect the description of an outside to an inside observer, where the first sees a black hole with mass m described by the metric (1) while the inside observer sees a black hole with mass $1/m$ described by the dual metric. From the self-duality property, in a different way from the classical Schwarzschild solution, the LQBH scenario allows black holes to have a mass smaller than the Planck mass [39].

Another interesting feature of the LQBH scenario comes from the fact that, in the metric (1), r is only asymptotically the radial coordinate. This is because $g_{\theta\theta}$ is not given by r^2 but by $H(r)$. In this way, the physical radial coordinate, defined in order to measure the proper circumferential distance, is given by

$$R = \sqrt{r^2 + \frac{a_0^2}{r^2}} \ . \tag{9}$$

The Equation (9) reveals vital aspects of the LQBH's internal structure. From this expression, we obtain that, as r decreases from ∞ to 0, R initially decreases from ∞ to $\sqrt{2a_0}$, at $r = \sqrt{a_0}$, so it will increase once more to ∞. In this way, we have that, at the limit of $r \to 0$, we shall have another asymptotically flat Schwarzschild region rather than a singularity as has been depicted in the Carter-Penrose diagram (1). Such a new region corresponds to a wormhole whose dimensions are the order of the Planck length. The wormhole throat is described by the Kantowski-Sachs spacetime [58].

The thermodynamical properties of LQBH can be obtained from the metric (1). In fact, the Bekenstein-Hawking temperature T_{BH} is related with the surface gravity κ which is given by

$$\kappa^2 = -g^{\mu\nu} g_{\rho\sigma} \nabla_\mu \chi^\rho \nabla_\nu \chi^\sigma = -\frac{1}{2} g^{\mu\nu} g_{\rho\sigma} \Gamma^\rho_{\mu 0} \Gamma^\sigma_{\nu 0} \ , \tag{10}$$

where, in the expression above, $\chi^\mu = (1,0,0,0)$ is identified as a timelike Killing vector and $\Gamma^\mu_{\sigma\rho}$ are the connections coefficients.

In this way, from the metric (1), we obtain

$$T_H = \frac{\hbar}{2\pi c}\kappa = \frac{\hbar c^3}{4\pi G k_B} \frac{(2m)^3(1-P^2)}{[(2m)^4 + a_0^2]} \ . \tag{11}$$

As we can observe, the temperature above agrees with the classical Hawking temperature in the large mass limit. On the other hand, it goes to zero for $m \to 0$.

The entropy of LQBH is obtained from the usual thermodynamical relation $S_{BH} = \int c^2 dm/T(m)$, which gives us

$$S = \frac{4\pi k_B c^3}{\hbar G} \frac{(1+P)^2}{(1-P^2)} \left[\frac{16m^4 - a_0^2}{16m^2} \right] . \tag{12}$$

Further investigations about LQBH have been performed in order to calculate the gravitational wave spectrum and the gravitational lensing by this kind of black hole [59,60]. Moreover, the entropy-area relation that appears in the context of LQBH has been used in order to derive, based on a thermodynamical argument, quantum corrected bounce-type Friedmann equations [61], in agreement with the standard loop quantum cosmology [62].

As we can see, the LQBH metric brings quantum gravity corrections, expressed in terms of LQC parameters like P and a_0, to the black hole thermodynamical properties like temperature and entropy. Such corrections could induce modifications in the way how the black hole evaporates. In the following, we shall use the quantum tunneling formalism in order to address the thermodynamical properties of LQBHs. The information loss problem will also be addressed in this context.

3. Quantum Tunneling Radiation from Loop Quantum Black Holes

In 2000, Parikh and Wilczec [9], following previous discussions by Krauss and Wilczec [63–65], developed the first tunneling method in order to describe the black hole evaporation process, named the null geodesic method. Subsequently, in 2005, Angheben et al. [16] presented an alternative description to the black hole tunneling process based on a Hamiltonian-Jacobi ansatz, consisting of an extension of the complex path analysis developed by Padmanabham et al. [25,66–68].

By the use of the Hamilton-Jacobi method introduced by Angheben et al., the thermodynamical properties of LQBHs have been investigated by Silva and Brito [38], where the inclusion of back reaction effects and the information loss problem have been addressed. However, the discussion about the information loss problem undertaken in [38] has been based on an approach introduced by Parikh [10] which has been rectified by Zhang et al. [27].

In this way, we shall at first review the results of [38] related to the calculation of LQBH temperature and entropy and the inclusion of back reaction effects. In the Section 5, we shall improve the results of [38] related to the issue of information recovery, by the use of the approach introduced by Zhang et al. Regarding this point it is necessary to emphasize that the tunneling method, which works in the semiclassical level, will be applied here to a semiclassical black hole solution that appears in the context of LQG. In this way, even though the spacetime in LQG is discrete, the black hole geometry we consider here is an effective geometry that is obtained by taking the semiclassical limit of a symmetry reduced model of LQG [39].

In this way, we have that, near the event horizon, one could reduce the description of the particle emission by a black hole to a 2-dimensional theory [69,70], where the metric corresponds to the $(t-r)$ sector of the original metric since its angular part is red-shifted away in this limit. Therefore, the near-horizon metric becomes:

$$ds^2 = -G(r)c^2dt^2 + F(r)^{-1}dr^2 \; . \tag{13}$$

In addition, in the near-horizon limit, the effective potential vanishes and there are no grey-body factors.

Now, let us consider the Klein-Gordon equations

$$\hbar^2 g^{\mu\nu}\nabla_\mu \nabla_\nu \phi - m^2c^2\phi = 0 \;\; , \tag{14}$$

which, by the application of the metric (13) give us

$$-\frac{1}{c^2}\partial_t^2\phi + \Lambda\partial_r^2\phi + \frac{1}{2}\Lambda'\partial_r\phi - \frac{m^2c^2}{\hbar^2}G(r)\phi = 0 \, , \tag{15}$$

where $\Lambda = F(r)G(r)$.

At this point, we shall take the standard WKB ansatz:

$$\phi(r,t) = e^{-\frac{i}{\hbar}I(r,t)}. \tag{16}$$

Such WKB approximation is justified by the fact that, when the outgoing wave is traced back towards the horizon, its wavelength, as measured by a local fiducial observer, is ever-increasingly blue shifted and the point particle interpretation can be allowed [9].

In this way, by the use of the WKB ansatz (16), one obtains the relativistic Hamilton-Jacobi equation, in the limit of $\hbar \to 0$:

$$\frac{1}{c^2}(\partial_t I)^2 - \Lambda(\partial_r I)^2 - m^2c^2G(r) = 0 \, . \tag{17}$$

We shall seek a solution in the form:

$$I(r,t) = -\omega t + W(r) \; . \tag{18}$$

in a way that we obtain

$$W = \int \frac{dr}{\Xi}\sqrt{\frac{1}{c^2}\omega^2 - m^2c^2G} \; , \tag{19}$$

where $\Xi = \Lambda^{1/2}$.

In this point, we shall adopt the proper spatial distance

$$d\sigma = \frac{dr^2}{\Xi(r)} \quad , \tag{20}$$

where, by taking the near horizon approximation, we obtain

$$\Xi(r) = \Xi'(r_H)(r - r_H) + ... \quad . \tag{21}$$

In this way, we find that

$$\sigma = 2\frac{\sqrt{r - r_H}}{\Xi'(r_H)} \; , \tag{22}$$

where $0 < \sigma < \infty$.

In terms of the proper spatial distance, we obtain for the spatial part of the action I

$$\begin{aligned} W &= \frac{2}{\Xi'(r_H)}\int \frac{d\sigma}{\sigma}\sqrt{\frac{1}{c^2}\omega^2 - \frac{\sigma^2}{4}m^2c^2G'(r_H)\Xi'(r_H)} \\ &= \frac{2\pi i\omega}{\Xi'(r_H)c} + \text{real contribution} \; . \end{aligned} \tag{23}$$

In this way, the tunneling probability of the emission of a particle with energy ω will be given by

$$\Gamma \simeq Exp[-\frac{2}{\hbar}ImI] = Exp\Big\{ -\frac{\pi G}{c^3\hbar}\frac{[(2m)^4 + a_0^2]}{m^3(1-P^2)}\omega\Big\} \; . \tag{24}$$

Now, assuming a Boltzmann form, $\Gamma \sim e^{-\beta\omega}$, for the emission probability above, where β is the inverse temperature $\beta = 1/k_B T_H$, we obtain the LQBH temperature as:

$$T_H = \frac{\omega}{ImI} = \frac{\hbar c^3}{4\pi G k_B}\frac{(2m)^3(1-P^2)}{[(2m)^4 + a_0^2]} \quad , \tag{25}$$

which coincides with the former Expression (11) found out in the references [40–42].

From the expression for LQBH temperature, one obtains for the entropy:

$$S = \frac{4\pi k_B c^3}{\hbar G}\frac{(1+P)^2}{(1-P^2)}\Big[\frac{16m^4 - a_0^2}{16m^2}\Big] \; . \tag{26}$$

From the results above, we have that the tunneling formalism is straightforward in order to calculate the LQBH thermodynamical properties. Such results pave the way for many applications, some of which we shall address in the following sections.

4. Back Reaction Effects

Based on the results obtained in the last section, which demonstrated that the tunneling approach is appropriate to calculate the thermodynamical properties of LQBHs, following the results of [38], in this section we shall discuss how back reaction effects can be introduced in the description of its evaporation process. By taking into account such self-gravitational effects, in this way, one can refine the description of LQBH thermodynamics, mainly in the quantum gravitational regime. This is because back reaction effects must be taken into account in the late stages of black hole evaporation, where the usual framework for the emission process will loose its validity [9–11,23,25,64,66,67,71–77].

In this way, in the Hamilton-Jacobi formalism, back reaction effects can be introduced when one takes the action I to be given by the following relation [78]

$$I = -\frac{i}{2}\frac{\hbar}{k_B}[S(M-\omega) - S(M)] \,, \tag{27}$$

where M is identified as the black hole ADM mass.

In the case of LQBHs, we shall take, for practical purposes, the changing in the mass parameter m related with the black hole ADM mass through the Equation (7). Therefore, we shall consider that a reduction in the black hole ADM mass will correspond to a reduction of $\varepsilon = \omega(1+P)^2$ in the mass parameter m.

Let us consider the following relation:

$$\begin{aligned} I &= -\frac{i}{2}\frac{\hbar}{k_B}[S(m-\varepsilon) - S(m)] \\ &= -\frac{4\pi c^3}{G}\frac{i(1+P)^2}{(1-P^2)}\varepsilon(\varepsilon - 2m)\left[1 - \frac{a_0^2}{16m^2(m-\varepsilon)^2}\right] \,. \end{aligned}$$

Consequently, we shall have, for the probability of the black hole emit a quanta with energy ϵ, when back-reaction effects are taken into account:

$$\Gamma(\varepsilon) = Exp\left\{\frac{4\pi c^3}{G\hbar}\frac{(1+P)^2}{(1-P^2)}\varepsilon(\varepsilon - 2m)\left[1 - \frac{a_0^2}{16m^2(m-\varepsilon)^2}\right]\right\} \,. \tag{28}$$

In the next section, we shall apply these results in order to investigate the possibility to have some correlation between the quanta emitted by a LQBH due to quantum gravity corrections present in this scenario.

5. Information Recovery from LQBHs

In order to solve the black hole information loss paradox, many proposals have been made [79–85] (For a recent review, see [86]). Among these proposals, we have the idea that information lost behind the black hole event horizon could re-emerge via Hawking radiation by some process. The late stages of black hole evaporation consist of an interesting scenario in order to have some mechanism of information recovering through Hawking radiation. This is because, as has been emphasized in the last section, the usual description for the emission process will loose its validity at these stages, and gravitational back-reaction effects must be taken into account [9–11,23,25,64,66,67,71–77].

In order to investigate if some information can be recovered during the black hole evaporation, one can investigate if there exist statistical correlations between the Hawking modes emitted. This method was applied, at first by Parikh [10] and Arzano et al. [26] and consists of considering two emissions with energies ε_1 and ε_2, or one emission with energy $\varepsilon_1 + \varepsilon_2$. The statistical correlation between two Hawking modes emitted by a black hole can be measured by the function:

$$C(\varepsilon_1 + \varepsilon_2; \varepsilon_1, \varepsilon_2) = ln[\Gamma(\varepsilon_1 + \varepsilon_2)] - ln[\Gamma(\varepsilon_1)\Gamma(\varepsilon_2)] \,. \tag{29}$$

It has been initially demonstrated by Parikh and Wilczek [9] that non-thermal corrections to the black hole radiation spectrum can be obtained when one takes into account back reaction effects. However, Parikh at first demonstrated that, in the classical treatment by the aforecited authors, no statistical correlation between the quanta emitted by a black hole has been found [10]. Such treatment was later followed up by Arzano et al. [26]. On the other hand, based upon standard statistical methods, by distinguishing statistical dependence or independence of sequential emissions, Zhang et al. demonstrated that a statistical correlation can be established between the quanta emitted by a black hole [27]. In this way, by the use of the approach introduced by Zhang et al., we shall refine the results of [38] which was constructed upon the Parikh and Wilczek argument.

In order to do this, we have that the probability distribution for the simultaneous emission of two modes with energies ε_1 and ε_2 is given by

$$\begin{aligned} \Gamma(\varepsilon_1,\varepsilon_2) &= \Gamma(\varepsilon_1+\varepsilon_2) \\ &= Exp\left\{\frac{4\pi c^3}{G\hbar}\frac{(1+P)^2}{(1-P^2)}(\varepsilon_1+\varepsilon_2)[(\varepsilon_1+\varepsilon_2)-2m]\left[1-\frac{a_0^2}{16m^2(m-(\varepsilon_1+\varepsilon_2))^2}\right]\right\} \ , \end{aligned} \tag{30}$$

which is subject to a normalization factor Λ which is fixed by

$$\Lambda\int_0^M \Gamma(\varepsilon)d\varepsilon = 1 \ . \tag{31}$$

On the other hand, the probability distributions for two independent emissions ε_1 and ε_2 are given by

$$\Gamma(\varepsilon_1) = \Lambda\int_0^{M-\varepsilon_1}\Gamma(\varepsilon_1,\varepsilon_2)d\varepsilon_2 = Exp\left\{\frac{4\pi c^3}{G\hbar}\frac{(1+P)^2}{(1-P^2)}\varepsilon_1(\varepsilon_1-2m)\left[1-\frac{a_0^2}{16m^2(m-\varepsilon_1)^2}\right]\right\} \tag{32}$$

and

$$\Gamma(\varepsilon_2) = \Lambda\int_0^{M-\varepsilon_2}\Gamma(\varepsilon_1,\varepsilon_2)d\varepsilon_1 = Exp\left\{\frac{4\pi c^3}{G\hbar}\frac{(1+P)^2}{(1-P^2)}\varepsilon_2(\varepsilon_2-2m)\left[1-\frac{a_0^2}{16m^2(m-\varepsilon_2)^2}\right]\right\} \ . \tag{33}$$

Therefore, considering the quantum gravity corrections from LQBHs, using the Equation (28), the correlation function between two consecutive modes with energies ε_1 and ε_2 will be given by:

$$\begin{aligned} C(\varepsilon_1+\varepsilon_2;\varepsilon_1,\varepsilon_2) = &\frac{8\pi(1+P)^2}{(1-P)^2}\frac{c^3}{G\hbar}\varepsilon_1\varepsilon_2 - \frac{\pi a_0^2(1+P)^2}{4(1-P^2)m^2}\frac{G}{c^3\hbar}\left\{\frac{(\varepsilon_1+\varepsilon_2)(\varepsilon_1+\varepsilon_2-2m)}{(m-\varepsilon_1-\varepsilon_2)^2}\right. \\ &- \left.\left[\frac{\varepsilon_1(\varepsilon_1-2m)}{(m-\varepsilon_1)^2}+\frac{\varepsilon_2(\varepsilon_2-2m)}{(m-\varepsilon_2)^2}\right]\right\} \ , \end{aligned} \tag{34}$$

where the semiclassical term found out by [27] appears, unless the polymeric function, with a quantum gravity correction which comes from the LQBH metric. In the Figure 2, the correlation functions for a classical and for a LQBH are compared.

As we can observe, the quantum gravity contributions to the black hole evaporation process from the LQBH can, when compered with a classical black hole, relieve in a more substantial way the information loss problem. Such effects become more evident when the black hole approaches the Planck scale, in the final stages of the emission process, where both back-reaction and quantum gravity effects become more important.

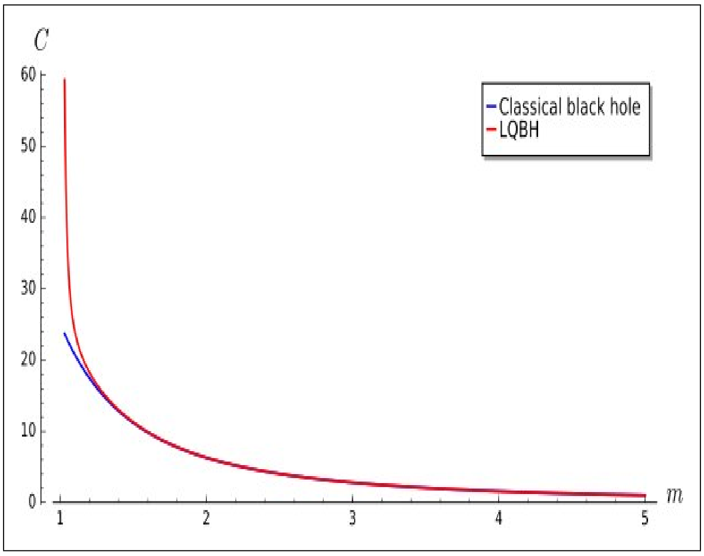

Figure 2. The correlation functions for a classical and for a LQBH. The results point to a substantial contribution to information recovery from LQBH's front classical black holes. We have considered $\omega m = 0.2$ (the peak of the emission spectrum [87–89]).

6. Conclusions and Remarks

In this work, at first, we have revisited the results of [33] relating to the investigation of the LQBH thermodynamics by the use of the Hamilton-Jacobi version of the tunneling formalism. We have shown that the results obtained in the references [40–42] for the LQBH temperature and entropy can be reproduced by the use of the tunneling method, in a way that such a method can be, in fact, reliably applied in order to address LQBH thermodynamics. We have also presented the results related with the inclusion of back reaction effects in the description of the LQBH evaporation process. Such effects are important in order to understand the thermodynamical dynamics of black holes during the late stages of their evaporation, where quantum gravity becomes important and the usual thermodynamical approach to such phenomena fails.

Finally, we have addressed the possibility of recovery of some information during the LQBH evaporation process, mainly during the its late stages. The results of the present work have revealed that the modes emitted by a LQBH are related by a non-thermal correlation function with a quantum gravity contribution. Consequently, some information can be recovered during the LQBH evaporation process. The results presented in this paper, related with correlation functions, calculated upon the argument by Zhang et al. [27], improve the results of [33], based on the Parikh argument [10].

Acknowledgments: The authors acknowledge the anonymous referees for the useful comments. F. A. Brito would like to acknowledge The Brazilian National Council for Scientific and Technological Development for the financial support.

Author Contributions: All the authors contributed equally to this work.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Hawking, S.W. Particle Creation by Black Holes. *Commun. Math. Phys.* **1975**, *43*, 199–220; Erratum in **1976**, *46*, 206.
2. Hooft, G. On the Quantum Structure of a Black Hole. *Nucl. Phys. B* **1985**, *256*, 727–745.
3. Robinson, S.P.; Wilczek, F. Relationship between Hawking Radiation and Gravitational Anomalies. *Phys. Rev. Lett.* **2005**, *95*, 011303.
4. Vagenas, E.C.; Das, S. Gravitational anomalies, Hawking radiation, and spherically symmetric black holes. *J. High Energy Phys.* **2006**, *2006*, 025.

5. Gibbons, G.W.; Hawking, S.W. Action Integrals and Partition Functions in Quantum Gravity. *Phys. Rev. D* **1977**, *15*, 2752–2756.
6. Hawking, S.W. Breakdown of Predictability in Gravitational Collapse. *Phys. Rev. D* **1976**, *14*, 2460–2473.
7. Preskill, J. Do black holes destroy information? In Proceedings of the International Symposium on Black Holes, Membranes, Wormholes, and Superstrings, The Woodlands, TX, USA, 16–18 January 1992.
8. Sorkin, R.D. Ten theses on black hole entropy. *Stud. Hist. Philos. Sci. B Stud. Hist. Philos. Mod. Phys.* **2005**, *36*, 291–301.
9. Parikh, M.K.; Wilczek, F. Hawking radiation as tunneling. *Phys. Rev. Lett.* **2000**, *85*, 5042–5045.
10. Parikh, M.K. Energy conservation and Hawking radiation. In Proceedings of the MG10 Meeting held at Brazilian Center for Research in Physics (CBPF), Rio de Janeiro, Brazil, 20–26 July 2003; pp. 1585–1590.
11. Vagenas, E.C. Generalization of the KKW analysis for black hole radiation. *Phys. Lett. B* **2003**, *559*, 65–73.
12. Jiang, Q.Q.; Wu, S.-Q.; Cai, X. Hawking radiation as tunneling from the Kerr and Kerr-Newman black holes. *Phys. Rev. D* **2006**, *73*, 064003; Erratum in **2006**, *73*, 069902.
13. Zhang, J.; Zhao, Z. Charged particles' tunnelling from the Kerr-Newman black hole. *Phys. Lett. B* **2006**, *638*, 110–113.
14. Kerner, R.; Mann, R.B. Tunnelling, temperature and Taub-NUT black holes. *Phys. Rev. D* **2006**, *73*, 104010.
15. Zhao, L. Tunnelling through black rings. *Commun. Theor. Phys.* **2007**, *47*, 835–842.
16. Angheben, M.; Nadalini, M.; Vanzo, L.; Zerbini, S. Hawking radiation as tunneling for extremal and rotating black holes. *J. High Energy Phys.* **2005**, *2005*, 014.
17. Wu, S.Q.; Jiang, Q.Q. Remarks on Hawking radiation as tunneling from the BTZ black holes. *J. High Energy Phys.* **2006**, *2006*, 079.
18. Ren, J.; Zhang, J.; Zhao, Z. Tunnelling Effect and Hawking Radiation from a Vaidya Black Hole. *Chin. Phys. Lett.* **2006**, *23*, 2019–2022.
19. Di Criscienzo, R.; Nadalini, M.; Vanzo, L.; Zerbini, S.; Zoccatelli, G. On the Hawking radiation as tunneling for a class of dynamical black holes. *Phys. Lett. B* **2007**, *657*, 107–111.
20. Kerner, R.; Mann, R.B. Tunnelling from Gödel black holes. *Phys. Rev. D* **2007**, *75*, 084022.
21. Unruh, W.G. Notes on black hole evaporation. *Phys. Rev. D* **1976**, *14*, 870–892.
22. Parikh, M.K. New coordinates for de Sitter space and de Sitter radiation. *Phys. Lett. B* **2002**, *546*, 189–195.
23. Medved, A.J.M. Radiation via tunneling from a de Sitter cosmological horizon. *Phys. Rev. D* **2002**, *66*, 124009.
24. Shankaranarayanan, S. Temperature and entropy of Schwarzschild–de Sitter space-time. *Phys. Rev. D* **2003**, *67*, 084026.
25. Srinivasan, K.; Padmanabhan, T. Particle production and complex path analysis. *Phys. Rev. D* **1999**, *60*, 024007.
26. Arzano, M.; Medved, A.J.M.; Vagenas, E.C. Hawking radiation as tunneling through the quantum horizon. *J. High Energy Phys.* **2005**, *2005*, 037.
27. Zhang, B.; Cai, Q.Y.; You, L.; Zhan, M.S. Hidden Messenger Revealed in Hawking Radiation: A Resolution to the Paradox of Black Hole Information Loss. *Phys. Lett. B* **2009**, *675*, 98–101.
28. Nozari, K.; Hamid Mehdipour, S. Quantum Gravity and Recovery of Information in Black Hole Evaporation. *Europhys. Lett.* **2008**, *84*, 20008.
29. Mehdipour, S.H. Generalized uncertainty principle and Parikh-Wilczek tunneling. *Int. J. Mod. Phys. A* **2009**, *24*, 5669–5680.
30. Fazeli, R.; Mehdipour, S.H.; Sayyadzad, S. Generalized Uncertainty Principle in Hawking Radiation of Non-Commutative Schwarzschild Black Hole. *Acta Phys. Polon. B* **2010**, *41*, 2365–2376.
31. Nozari, K.; Saghafi, S. Natural Cutoffs and Quantum Tunneling from Black Hole Horizon. *J. High Energy Phys.* **2012**, *2012*, 005.
32. Nozari, K.; Shahini, P. TeV scale black holes thermodynamics with extra dimensions and quantum gravity effects. *arXiv* **2012**, arXiv:1206.5624.
33. Silva, C.A.S.; Landim, R.R. Influences of a Generalized Uncertainty Principle on the black-hole area spectrum in the tunneling formalism. *Europhys. Lett.* **2012**, *100*, 10002.
34. Anacleto, M.A.; Brito, F.A.; Passos, E. Quantum-corrected self-dual black hole entropy in tunneling formalism with GUP. *Phys. Lett. B* **2015**, *749*, 181–186.
35. Kim, H. Hawking radiation as tunneling from charged black holes in 0A string theory. *Phys. Lett. B* **2011**, *703*, 94–99.

36. Nozari, K.; Mehdipour, S.H. Hawking Radiation as Quantum Tunneling from Noncommutative Schwarzschild Black Hole. *Class. Quantum Gravity* **2008**, *25*, 175015.
37. Miao, Y.G.; Xue, Z.; Zhang, S.J. Tunneling of massive particles from noncommutative inspired Schwarzschild black hole. *Gen. Relativ. Gravit.* **2012**, *44*, 555–566.
38. Silva, C.A.S.; Brito, F.A. Quantum tunneling radiation from self-dual black holes. *Phys. Lett. B* **2013**, *725*, 456–462.
39. Modesto, L. Semiclassical loop quantum black hole. *Int. J. Theor. Phys.* **2010**, *49*, 1649–1683
40. Modesto, L.; Premont-Schwarz, I. Self-dual black holes in loop quantum gravity: Theory and phenomenology. *Phys. Rev. D* **2009**, *80*, 064041.
41. Alesci, E.; Modesto, L. Particle Creation by Loop Black Holes. *Gen. Relativ. Gravit.* **2014**, *46*, 1656.
42. Hossenfelder, S.; Modesto, L.; Premont-Schwarz, I. Emission spectra of self-dual black holes. *arXiv* **2012**, arXiv:1202.0412.
43. Modesto, L. Disappearance of the black hole singularity in loop quantum gravity. *Phys. Rev. D* **2004**, *70*, 124009.
44. Modesto, L. The Kantowski-Sachs space-time in loop quantum gravity. *Int. J. Theor. Phys.* **2006**, *45*, 2235–2246.
45. Modesto, L. Loop quantum gravity and black hole singularity. *arXiv* **2007**, arXiv:hep-th/0701239.
46. Modesto, L. Gravitational collapse in loop quantum gravity. *Int. J. Theor. Phys.* **2008**, *47*, 357–373
47. Modesto, L. Quantum gravitational collapse. *arXiv* **2005**, arXiv:gr-qc/0504043.
48. Ashtekar, A.; Bojowald, M. Quantum geometry and the Schwarzschild singularity. *Class. Quantum Gravity* **2006**, *23*, 391–441.
49. Modesto, L. Loop quantum black hole. *Class. Quantum Gravity* **2006**, *23*, 5587–5602.
50. Gambini, R.; Pullin, J. Black holes in loop quantum gravity: The Complete space-time. *Phys. Rev. Lett.* **2008**, *101*, 161301.
51. Campiglia, M.; Gambini, R.; Pullin, J. Loop quantization of spherically symmetric midi-superspaces: The Interior problem. *AIP Conf. Proc.* **2008**, *977*, 52–63.
52. Campiglia, M.; Gambini, R.; Pullin, J. Loop quantization of spherically symmetric midi-superspaces. *Class. Quantum Gravity* **2007**, *24*, 3649–3672.
53. Modesto, L. Black hole interior from loop quantum gravity. *Adv. High Energy Phys.* **2008**, *2008*, 459290.
54. Boehmer, C.G.; Vandersloot, K. Loop Quantum Dynamics of the Schwarzschild Interior. *Phys. Rev. D* **2007**, *76*, 104030.
55. Chiou, D.W. Phenomenological loop quantum geometry of the Schwarzschild black hole. *Phys. Rev. D* **2008**, *78*, 064040.
56. Gambini, R.; Olmedo, J.; Pullin, J. Quantum black holes in Loop Quantum Gravity. *Class. Quantum Gravity* **2014**, *31*, 095009.
57. Alesci, E.; Cianfrani, F. Quantum Reduced Loop Gravity: Semiclassical limit. *Phys. Rev. D* **2014**, *90*, 024006.
58. Kantowski, R.; Sachs, R.K. Some spatially homogeneous anisotropic relativistic cosmological models. *J. Math. Phys.* **1966**, *7*, 443–446.
59. Cruz, M.B.; Silva, C.A.S.; Brito, F.A. Gravitational axial perturbations and quasinormal modes of loop quantum black holes. *arXiv* **2015**, arXiv:1511.08263.
60. Sahu, S.; Lochan, K.; Narasimha, D. Gravitational lensing by self-dual black holes in loop quantum gravity. *Phys. Rev. D* **2015**, *91*, 063001.
61. Silva, C.A.S. On the holographic basis of Quantum Cosmology. *arXiv* **2015**, arXiv:1503.00559.
62. Bojowald, M. Loop Quantum Cosmology. *Living Rev. Rel.* **2005**, *8*, 11.
63. Kraus, P.; Wilczek, F. A Simple Stationary Line Element for the Schwarzschild Geometry, and Some Applications. *arXiv* **1994**, arXiv:gr-qc/9406042.
64. Kraus, P.; Wilczek, F. Selfinteraction correction to black hole radiance. *Nucl. Phys. B* **1995**, *433*, 403–420.
65. Kraus, P.; Wilczek, F. Effect of self-interaction on charged black hole radiance. *Nucl. Phys. B* **1995**, *437*, 231–242.
66. Shankaranarayanan, S.; Srinivasan, K.; Padmanabhan, T. Method of complex paths and general covariance of Hawking radiation. *Mod. Phys. Lett. A* **2001**, *16*, 571–578.
67. Shankaranarayanan, S.; Padmanabhan, T.; Srinivasan, K. Hawking radiation in different coordinate settings: complex paths approach. *Class. Quantum Gravity* **2002**, *19*, 2671–2688.
68. Padmanabhan, T. Entropy of horizons, complex paths and quantum tunnelling. *Mod. Phys. Lett. A* **2004**, *19*, 2637–2643.

69. Iso, S.; Umetsu, H.; Wilczek, F. Anomalies, Hawking radiations, and regularity in rotating black holes. *Phys. Rev. D* **2006**, *74*, 044017.
70. Umetsu, K. Hawking Radiation from Kerr-Newman Black Hole and Tunneling Mechanism. *Int. J. Mod. Phys. A* **2010**, *25*, 4123–4140.
71. Keski-Vakkuri, E.; Kraus, P. Microcanonical D-branes and back reaction. *Nucl. Phys. B* **1997**, *491*, 249–262.
72. Parikh, M.K. A Secret tunnel through the horizon. *Gen. Relativ. Gravit.* **2004**, *36*, 2419–2422.
73. Vagenas, E.C. Are extremal 2D black holes really frozen? *Phys. Lett. B* **2001**, *503*, 399–403.
74. Vagenas, E.C. Two-dimensional dilatonic black holes and Hawking radiation. *Mod. Phys. Lett. A* **2002**, *17*, 609–618.
75. Vagenas, E.C. Semiclassical corrections to the Bekenstein-Hawking entropy of the BTZ black hole via selfgravitation. *Phys. Lett. B* **2002**, *533*, 302–306.
76. Setare, M.R.; Vagenas, E.C. Self-gravitational corrections to the Cardy—Verlinde formula of the Achúcarro–Ortiz black hole. *Phys. Lett. B* **2004**, *584*, 127–132.
77. Medved, A.J.M. Radiation via tunneling in the charged BTZ black hole. *Class. Quantum Gravity* **2002**, *19*, 589–598.
78. Medved, A.J.M.; Vagenas, E.C. On Hawking radiation as tunneling with back-reaction. *Mod. Phys. Lett. A* **2005**, *20*, 2449–2453.
79. Aharonov, Y.; Casher, A.; Nussinov, S. The Unitarity Puzzle and Planck Mass Stable Particles. *Phys. Lett. B* **1987**, *191*, 51–55.
80. Mathur, S.D. The Fuzzball proposal for black holes: An Elementary review. *Prog. Phys.* **2005**, *53*, 793–827.
81. Silva, C.A.S. Fuzzy spaces topology change as a possible solution to the black hole information loss paradox. *Phys. Lett. B* **2009**, *677*, 318–321.
82. Silva, C.A.S.; Landim, R.R. A note on black hole entropy, area spectrum, and evaporation. *Europhys. Lett.* **2011**, *96*, 10007.
83. Almheiri, A.; Marolf, D.; Polchinski, J.; Sully, J. Black Holes: Complementarity or Firewalls? *J. High Energy Phys.* **2013**, *2013*, 062.
84. Silva, C.A.S.; Landim, R.R. Fuzzy spaces topology change and BH thermodynamics. *J. Phys. Conf. Ser.* **2014**, *490*, 012012.
85. Hawking, S.W. The Information Paradox for Black Holes. *arXiv* **2015**, arXiv:1509.01147.
86. Unruh, W.G.; Wald, R.M. Information Loss. *arXiv* **2017**, arXiv:1703.02140.
87. Page, D.N. Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole. *Phys. Rev. D* **1976**, *13*, 198–206.
88. Page, D.N. Particle emission rates from a black hole. II. Massless particles from a rotating hole. *Phys. Rev. D* **1976**, *14*, 3260–3273.
89. Page, D.N. Particle emission rates from a black hole. III. Charged leptons from a nonrotating hole. *Phys. Rev. D* **1977**, *16*, 2402–2411.