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Dark Energy, QCD Axion, and Trans-Planckian-Inflaton Decay Constant

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Abstract: Pseudoscalars appear frequently in particle spectra. They can be light if they appear as pseudo-Goldstone bosons from some spontaneously broken global symmetries with the decay constant f. Since any global symmetry is broken at least by quantum gravitational effects, all pseudoscalars are massive. The mass scale of a pseudoscalar is determined by the spontaneous symmetry breaking scale f of the corresponding global symmetry and the explicit breaking terms in the effective potential. The explicit breaking terms can arise from anomaly terms with some non-Abelian gauge groups among which the best-known example is the potential of the QCD axion. Even if there is no breaking terms from gauge anomalies, there can be explicit breaking terms in the potential in which case the leading term suppressed by f determines the pseudoscalar mass scale. If the breaking term is extremely small and the decay constant is trans-Planckian, the corresponding pseudoscalar can be a candidate for a quintessential axion. In the other extreme that the breaking scales are large, still the pseudo-Goldstone boson mass scales are in general smaller than the decay constants. In such a case, still the potential of the pseudo-Goldstone boson at the grand unification scale is sufficiently flat near the top of the potential that it can be a good candidate for an inflationary model. We review these ideas in the bosonic collective motion framework.

Keywords: bosonic collective motion; "invisible" axion; string axion; quintessential axion; natural inflation; Trans-Planckian decay constant; discrete symmetries

1. Introduction

Energy of the Universe is dominated by invisibles: dark energy (DE) composing 68% and dark matter (DM) composing 27% [1]. Atoms constitute mere 5%. It has been known that bosonic collective motions (BCMs) can describe both DE and DM [2], which is the main topic in this article.

The astrophysical evidence in favor of the existence of DM has grown over the years [3]. If cold dark matter (CDM) has provided the dominant mass at the time when CDM fluctuation enters into the horizon after inflation, the fluctuation scale given by that horizon scale is typically the scale of galaxies. The time scale corresponding to this is about 3×10^{-5} the time $z \simeq 10$, i.e., at $z \simeq 3000$ which is close to the time when "matter = radiation". Before the time of "matter = radiation" the density perturbation has grown only logarithmically. After the time of "matter = radiation" the density perturbation has grown linearly, which became nonlinear around $z = 10$. Since the time of $z = 10$ on, 'galaxy formation' has started. Thus, CDM attracted a great deal of attention since 1984 [4].

The first hint for DM inferred by Zwicky in 1933 was derived from the observation of velocity dispersion of the galaxies in the Coma Cluster [5]. Still, the so-called the rotation curve in the halo forms the most convincing argument for the existence of DM. If all the galaxy mass is inside the galactic bulge, which seems to be a good ansatz for shining stars, the rotation velocity of a star at r far from the bulge goes like

$$v(r) \propto \frac{1}{\sqrt{r}}. \tag{1}$$

The feature of Equation (1) is also confirmed by the revolution speeds of gas clouds of spiral galaxies [6].

The first example for CDM was introduced in an effort to constrain the mass scale of another (heavy) lepton doublet which interacts weakly [7,8]. This showed that the multi-GeV weakly interacting particles could have closed the Universe, and opened the idea of weakly interacting massive particles (WIMPs). The symmetry for most WIMPs is "parity" or $\mathbf{Z}_2$ symmetry in which the SM particles carry parity or $\mathbf{Z}_2$ even particles and the CDM candidate particle is the lightest one among the parity or $\mathbf{Z}_2$ odd particles. This got tremendous interest in supersymmetric (SUSY) models where the R parity works for the needed symmetry [9]. The second example for CDM was found [10–12] for the "invisible" axion [13–16], which belongs to the BCM scenario.

For the observed dark energy (DE) [17,18], theoretical interpretation of can be related to the cosmological constant, originally introduced by Einstein [19]. However, there has been a theoretical prejudice that the cosmological constant (CC) must be zero, which is considered to be one of the most important problems in theoretical physics [20]. After the introduction of fundamental scalars in particle physics, the vacuum energy of scalar fields has been appreciated to contribute to the cosmological constant [21]. Therefore, the problem related to the cosmological constant must be considered together with the vacuum energy of scalar fields.

A CC dominated universe is called the LeMaître universe [22], where the scale factor expands exponentially. This idea of exponential expansion was used in early 1980s to solve the horizon and flatness problems and to dilute monopoles, which is now called "inflation" [23]. Inflation predicts that the total mass-energy density of the Universe is very close to the critical closure density. The Planck data [1] confirm that the energy density of the Universe is nearly ρ_c (spatially flat) and that the present DE is about 68% of the critical energy density of the Universe.

Under this circumstance, I present the BCM idea which can be applicable to all the problems mentioned above, CDM by the "invisible" QCD axion [13–16], DE by a quintessential axion [24,25], and inflation by "natural" inflation [26,27].

BCM is described by a scalar field. The Brout–Englert–Higgs–Guralnik–Hagen–Kibble boson, simply called the Higgs boson here, seems to be a fundamental scalar field. So, we can imagine that the QCD axion and an inflation may be fundamental fields also. Compared to spin-$\frac{1}{2}$ fermions of the canonical dimension $\frac{3}{2}$, these bosons with the canonical dimension 1 can affect more importantly to low energy physics. This has led to the Higgs boson acting as a portal to the high energy scale [28–30], to the axion scale or even to some standard model (SM) singlets in the grand unification (GUT) scale. Can these singlets explain both DE, CDM, and even inflation in the evolving Universe? In this short article, we attempt to answer to this question in the affirmative direction.

2. On Global Symmetries

For a BCM, we introduce a corresponding global symmetry. The global symmetry is that in the Lagrangian. However, global symmetries are known to be broken in general by the quantum gravity effects, especially via the Planck scale wormholes [31,32]. To resolve this dilemma, we can think of two possibilities of discrete symmetries below $M_{\rm P}$ in the top-down approach: (i) The discrete symmetry arises as a part of a gauge symmetry [33–36], and (ii) The string selection rules directly give the discrete symmetry [37]. As far as discrete symmetries are concerned, a bottom-up approach can be useful also [38]. The interesting cases are the discrete gauge symmetries allowed in string compactification. Even though the Goldstone boson directions, i.e., the longitudinal directions, of spontaneously broken *GAUGE* symmetries are flat, the Goldstone boson directions of spontaneously broken *GLOBAL* symmetries are not flat. Namely, global symmetries are always approximate. The question is how approximate it is.

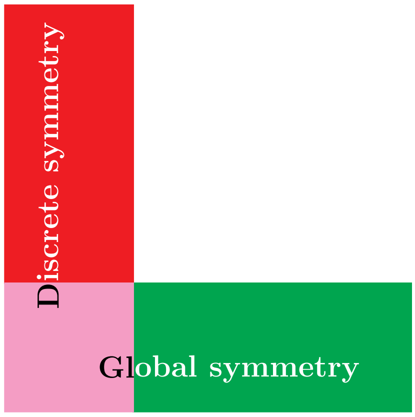

Figure 1. Terms respecting discrete and global symmetries.

In Figure 1, we present a cartoon separating effective terms according to string-allowed discrete symmetries. The terms in the vertical column represent exact symmetries such as the string allowed discrete symmetries. If we consider a few terms in the lavender part, we can consider a *global symmetry*. With the global symmetry, we can consider the global symmetric terms which are in the lavender and green parts of Figure 1. However, the global symmetry is broken by the terms in the red part in Figure 1.

2.1. The 't Hooft Mechanism

The 't Hooft mechanism is a very simple and elementary concept, but it seems that it is not known widely. In the original paper of 't Hooft [39], it was commented on breaking two continuous symmetries by one Higgs vacuum expectation value (VEV). Two continuous directional parameters correspond to one gauge transformation parameter and the other global transformation parameter. If the Higgs VEV breaks the continuous symmetries, it is obvious that the gauge symmetry is broken because the corresponding gauge boson obtains mass. Namely, only one phase (or pseudoscalar) direction is absorbed to the gauge boson, and there remains one continuous direction.

For example, let us introduce a field ϕ to study this phenomenon. Charges $Q_{\rm gauge}$ and $Q_{\rm global}$ acting on ϕ with the gauge transformation parameter $\alpha(x)$ and the global transformation parameter β, give the following transformation

$$\phi \to e^{i\alpha(x)Q_{\rm gauge}}e^{i\beta Q_{\rm global}}\phi, \tag{2}$$

which can be rewritten as

$$\phi \to e^{i(\alpha(x)+\beta)Q_{\rm gauge}}e^{i\beta(Q_{\rm global}-Q_{\rm gauge})}\phi. \tag{3}$$

Redefining the local direction as $\alpha'(x) = \alpha(x) + \beta$, we obtain the transformation

$$\phi \to e^{i\alpha'(x)Q_{\rm gauge}}e^{i\beta(Q_{\rm global}-Q_{\rm gauge})}\phi. \tag{4}$$

So, the $\alpha'(x)$ direction becomes the longitudinal mode of heavy gauge boson. Now, the charge $Q_{\rm global} - Q_{\rm gauge}$ is reinterpreted as the new global charge and is not broken by the VEV, $\langle\phi\rangle$, because out of two continuous directions one should remain unbroken. Basically, the direction β remains as the unbroken continuous direction. This is the essence of the 't Hooft mechanism: "If both a gauge symmetry and a global symmetry are broken by one scalar VEV, the gauge symmetry is broken and a global symmetry survives". The resulting global charge is a linear combination of the original gauge and global charges as shown above. This theorem has a profound effect in obtaining the intermediate scale of the "invisible" axion from string compactification [40].

2.2. Breaking Scales

The red part potential in Figure 1 breaks the global symmetry represented by the green part. In some cases, the global symmetry is broken by the anomalies of non-Abelian gauge groups. The well-known example is the Peccei-Quinn (PQ) global symmetry, broken by the quantum chromodynamics (QCD) anomaly, the $U(1)_{PQ}$–$SU(3)_c$–$SU(3)_c$ anomaly [41]. The PQ proposal assumes more in that any term is not present in the red part of Figure 1. Suppose a global symmetry $U(1)_\Gamma$. If $U(1)_\Gamma$ is spontaneously broken by a VEV, i.e., by a SM singlet VEV $\langle\sigma\rangle \equiv f/\sqrt{2}$, the pseudo-Goldstone boson mass corresponding to the spontaneously broken $U(1)_\Gamma$ is

$$m^2_{\rm pseudo} = \frac{(\text{symmetry breaking scale})^4}{f^2}. \tag{5}$$

If the non-Abelian anomalies are the sole contribution in breaking the global symmetry, the $m_{\rm pseudo} \simeq \Lambda^2/f$ where Λ is the scale of the non-Abelian gauge group. Depending on the scale of Λ, the pseudo-Goldstone is called

Name	Λ	$m_{\rm pseudo}$
QCD axion	~ 332 MeV	10^{-4} eV
KNP inflaton	10^{16}–10^{17} GeV	10^{14-15} GeV
ULA	3×10^{-7} GeV	10^{-22} eV

(6)

For the QCD axion, the strong coupling parameter $\Lambda^{(3)}_{\overline{\rm MS}} = (332 \pm 17)$ MeV for three light quarks[1] is shown above [42], which is the recent world average used in high energy scattering with the leading log expansion in powers of $\Lambda^{(3)\,2}_{\overline{\rm MS}}/Q^2$. It is basically the same as the one given for the susceptibility $\chi^{1/4} \equiv \sqrt{m_a f_a} = 75.5$ MeV [43,44], which takes into account the strong interaction coupling constant α_3 in addition to $\Lambda^{(3)}_{\overline{\rm MS}}$. For the case of ULA in Equation (6), the $SU(2)_W$ can work for the non-Abelian gauge group with a few orders of magnitude discrepancy allowed.

Even if there is no non-Abelian anomalies, still the global symmetry $U(1)_\Gamma$ is broken by the terms in the red part of Figure 1. Then, the $m_{\rm pseudo} \simeq \varepsilon^2/f$ where ε^4 in V is the leading term in the red part. Assuming that there is no non-Abelian anomalies, the pseudo-Goldstone boson mass is estimated as

Name	ε	$m_{\rm pseudo}$
ALPs	Any value	any value
N-flaton	10^{16} GeV	10^{13} GeV
ULA	3×10^{-7} GeV	10^{-22} eV
Quintessential axion	1.7×10^{-12} GeV	10^{-33} eV
Familons, etc.	unknown	unknown

(7)

The most interesting BCM is the QCD axion of (6) which will be discussed in detail in the following section as a solution of the strong CP problem and as a candidate for CDM in the Universe. A BCM for inflation can arise with non-Abelian anomalies of (6) [27] or without non-Abelian anomalies

1 With two light quarks, $\Lambda^{(2)}_{\overline{\rm MS}}$ is larger than $\Lambda^{(3)}_{\overline{\rm MS}}$.

of (7) [45]. For the ALPs, there have been numerous efforts to constrain the masses and the decay costants, as summarized in Figure 1 of Ref. [46]. A quintessential axion for the DE scale should not have the QCD anomaly and the height of the potential must be very small, i.e., with the mass of order 10^{-33} eV and a trans-Planckian decay constant [47]. Also, the ultra-light axion (ULA) should have a very small height of the potential and most probably it is in the form of (7), with its bounds given in [48–50]. However, the ULA is also included in (6) because $SU(2)_W$ anomaly can provide such a small height within a few orders. Axion-like particles (ALPs) behave like axions in the detection experiments, i.e., having the pseudoscalar–photon–photon couplings but their masses and decay constants are not related as in the QCD axion. Most probably, they arise in the form of (7). In the last row, other possibilities such as familons, archions, etc., are shown, which arise from breaking some global symmetries [51–54].

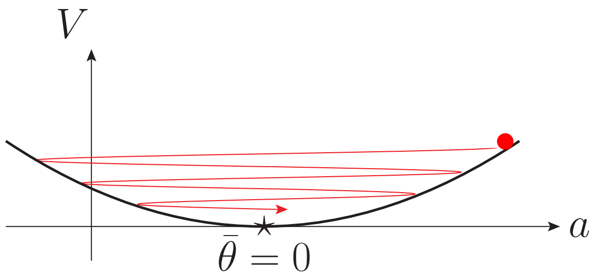

Figure 2. The almost flat axion potential.

All pseudoscalar bosons listed in (6) and (7) use the idea of collective/coherent motion of the classical part of the pseudoscalar field. The classical potential can be visualized as a very shallow potential V as shown in Figure 2. The classical vacuum shown as the red bullet starts to roll down the hill "late" in the history of the Universe. The amplitude decreases (as illustrated with the red curve) due to the Hubble expansion. At present, the amplitude remains with a non-zero value, which is the basis for attempting to detect QCD axions [2]. The names listed in (6) and (7) are made to represent the origin or the usefulness of the corresponding pseudoscalar boson.

3. The "Invisible" Axion

The strong CP problem is necessarily intertwined with the observed weak CP violation as noted in the $\overline{\theta} \equiv \overline{\theta}_{\rm QCD}$ parameter,

$$\overline{\theta} = \theta_0 + \theta_{\rm weak} \tag{8}$$

where θ_0 is the coefficient of the QCD anomaly term given above the electroweak scale and $\theta_{\rm weak}$ is the contribution generated when one introduces the weak CP violation at the electroweak scale. Since this $\overline{\theta}$ gives a neutron EDM (nEDM) of order $10^{-16}\overline{\theta}\, e$ cm, the experimental upper bound on nEDM [55] restricts $|\overline{\theta}| < 10^{-10}$. "Why is $\overline{\theta}$ so small?" is the strong CP problem.

Therefore, let us start by considering the weak CP violation by the Kobayashi–Maskawa model [56] and by the Weinberg model [57]. One may introduce the following possibilities for the weak CP violation,

(1) by light colored scalar,
(2) by right-handed current(s),
(3) by three left-handed families,
(4) by propagators of light color-singlet scalars, and
(5) by an extra U(1) gauge interaction.

The first three are those of Ref. [56], among which the second was presented in Ref. [58] also.

In 1976 the third quark family was not known in which case the weak CP violation was introduced in the Higgs potential with multi Higgs doublets [57],

$$V_{\rm W} = -\tfrac{1}{2}\sum_I m_I^2\phi_I^\dagger\phi_I + \tfrac{1}{4}\sum_{IJ}\left[a_{IJ}\phi_I^\dagger\phi_I\phi_J^\dagger\phi_J + b_{IJ}\phi_I^\dagger\phi_J\phi_J^\dagger\phi_I \right.$$
$$\left. + c_{IJ}\phi_I^\dagger\phi_J\phi_I^\dagger\phi_J\right] + {\rm H.c.}, \tag{9}$$

where the reflection symmetry $\phi_I \to -\phi_I$ is imposed. This is the discrete symmetry we mentioned by the vertical column in Figure 1. The mass parameters m_I^2 are at the electroweak scale such that the electroweak symmetry is broken at the electroweak scale. With the potential (9), three Higgs doublets are needed to introduce CP violation [57]. Not to introduce flavor changing neutral currents, Ref. [57] required that only one Higgs doublet, ϕ_1, couples to $Q_{\rm em} = \frac{2}{3}$ quarks, and another Higgs doublet, ϕ_2, couples to $Q_{\rm em} = -\frac{1}{3}$ quarks [59]. The reflection symmetries, including non-trivial transformations of the quark fields, will achieve this goal. For all scalars and pseudoscalars of ϕ_I to obtain mass, all parameters in (9) are required to be nonzero.

Peccei and Quinn (PQ) observed that if all c_{IJ} in Equation (9) are zero, which is equivalent to considering only the terms in the lavender part in Figure 1, then the discrete symmetry is promoted to a global symmetry, according to our general scheme, which we now call the PQ symmetry [41],

$$q_L \to q_L, \tag{10}$$
$$u_R \to e^{-i\alpha}u_R, \tag{11}$$
$$d_R \to e^{-i\beta}d_R, \tag{12}$$
$$\phi_1 \to e^{i\alpha}\phi_1, \tag{13}$$
$$\phi_2 \to e^{i\beta}\phi_2, \tag{14}$$

where q_L's are the left-handed quark doublets, u_R's are the right-handed up-type quark singlets, and d_R's are the right-handed down-type quark singlets. Quarks obtain masses by

$$-\bar{q}_L u_R\phi_1 - \bar{q}_L d_R\phi_2 + {\rm H.c.} \tag{15}$$

which also respects the PQ symmetry. This PQ transformation is a chiral transformation, Equations (11) and (12), creating the QCD anomaly coefficient. Therefore, this simple phase transformation is thought to be equivalent to the gluon anomaly and in any physical processes, therefore, there will not appear the phase and there is no strong CP problem, which was the argument used in Ref. [41].

The above PQ symmetry (if exact) must lead to an exactly massless Goldstone boson because quarks must obtain masses by the VEVs of ϕ_1 and ϕ_2, i.e., the PQ symmetry must be spontaneously broken. Note that the term (15) is a part in defining the PQ symmetry. However, the PQ symmetry is explicitly broken at quantum level because the QCD anomaly is present. Thus, the Goldstone boson becomes a pseudo-Goldstone boson, obtaining mass of order $\Lambda_{\rm QCD}^2/v_{\rm ew}$ by our general argument of Figure 1 [60,61]. This pseudo-Goldstone boson was named *axion*. From $V_{\rm W}$ of Equation (9), we can separate out this axion direction. If c_{12} were present, which signifies the breaking of the PQ symmetry, there will appear the phase $e^{-i(a_1/v_1 - a_2/v_2)}$. Thus, the axion direction is

$$a \propto \frac{a_u}{v_u} - \frac{a_d}{v_d} \tag{16}$$

where $\langle\phi_1\rangle = (v_u + \rho_1)e^{ia_u/v_u}/\sqrt{2}$ and $\langle\phi_2\rangle = (v_d + \rho_2)e^{ia_d/v_d}/\sqrt{2}$. This electroweak scale axion is called the Peccei–Quinn–Weinberg–Wilczek (PQWW) axion, having mass $\sim$100 keV and lifetime $\sim 10^{-8}$ second order [62] and cannot be the one considered in the cavity experiments.

In 1978 this PQWW axion was known not to exist [63], which had led to ideas on models without the PQWW axion in 1978 [64–68]. Nowadays, these are used in the Nelson–Barr type models [69,70], which are called "calculable models" [62]. The calculable models start with the CP symmetry in the Lagrangian such that $\theta_0 = 0$ in Equation (8). However, one has to introduce the weak CP violation, i.e., the CP violation is through the spontaneous mechanism [71]. These calculable models seem not working in two aspects, firstly it is very difficult to build models in which the loop corrections allow $\theta_{\rm weak}$ below O(10^{-10}) and second the observed weak CP violation is of the Kobayashi–Maskawa form [72].

However, the PQ-type solution of the strong CP problem was so attractive that the "invisible" axion with lifetime greater than the age of the Universe was reinvented with an $SU(2)_W \times U(1)_Y$ singlet scalar field [13]. Currently, this is the most studied global symmetry. For this $U(1)_{\rm PQ}$, the explicit breaking term is the QCD anomaly term

$$\frac{\bar\theta}{32\pi^2} G^a_{\mu\nu}\tilde G^{a\,\mu\nu} \tag{17}$$

where $G^a_{\mu\nu}$ ($\tilde G^{a\,\mu\nu}$) is the gluon field strength (its dual), and $\bar\theta = a/f_a$. The symmetry in (11), for example, changes $\bar\theta \to \bar\theta + \alpha$ and the anomaly term has the shift symmetry $a \to a + f_a\alpha$ if the anomaly term does not contribute to the effective potential. However, the anomaly contributes to the effective potential and the question is, "What is the value $\bar\theta$ at the minimum of the potential ?" If the potential V does not break the CP symmetry, it is easy to show that the potential generated by the anomaly term chooses $\bar\theta = 0$ as the minimum of the potential [73], which is a good cosmological solution of the strong CP problem. Even if the weak CP violation is introduced, the shift of the minimum is very tiny [74], $\Delta\bar\theta \simeq 10^{-16}$, which does not spoil the nature of the strong CP solution along this line. The axion decay constant f_a is bounded from the energy density of cold axions within a narrow window [75–77],[2]

$$10^{10}\ {\rm GeV} < f_a < 10^{12}\ {\rm GeV}. \tag{18}$$

At the minimum, the axion mass is usually calculated approximately from the cosine potential. For the two quark case, however, the axion potential mixed with π^0 is given in a closed form in the chiral perturbation theory as [43]

$$V(a,\pi^0) = m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4Z}{(1+Z)^2}\sin^2\left(\frac{a}{2f_a}\right)}\ \left[1 - \cos\left(\frac{\pi^0}{f_\pi} + \phi_a\right)\right], \tag{19}$$

$$\tan\phi_a = \frac{1-Z}{1+Z}\,\tan\left(\frac{a}{2f_a}\right), \tag{20}$$

where

$$Z = \frac{m_u}{m_d}. \tag{21}$$

From Equation (19), we obtain

$$m_a \simeq 0.570 \times 10^{-4}\,{\rm eV}\left(\frac{10^{11}\,{\rm GeV}}{f_a}\right) \tag{22}$$

which is about 5% below the value obtained by using the cosine potential. So, using the suceptibility 75.5 MeV of [43] is almost the same as using $\Lambda^{(3)}_{\overline{\rm MS}} = (332 \pm 17)$ MeV of Equation (6).

[2] If f_a is much lower than the above bound at the level of 10^7 GeV, the axion condensation can convert to thermal axions [78].

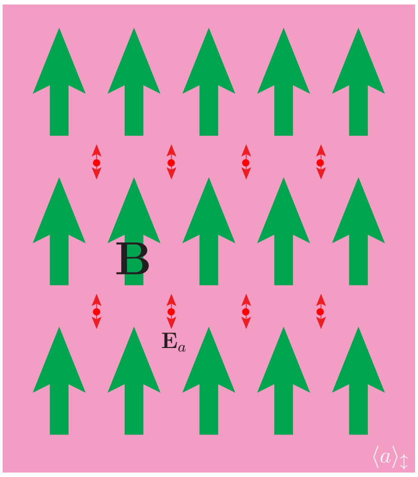

Figure 3. The resonant detection idea of the QCD axion. The **E**-field follows the axion vacuum oscillation.

In the search of the QCD "invisible" axion, the axion–photon–photon coupling $c_{a\gamma\gamma}$ is the key parameter where

$$c_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma} - c_{\text{chiral}} \tag{23}$$

where $\tilde{c}_{a\gamma\gamma}$ is given above the QCD chiral symmetry breaking scale and $-c_{\text{chiral}}$ is the contribution from the QCD phase transition, cited as -1.92 [62] and -1.96 [79]. Here, we choose it as -2 for a guidance. In a sense, the customary numbers presented in most talks are ad hoc, which can be glimpsed from those exclusion plots where the KSVZ lines are shown only for $Q_{\text{em}} = 0$ [80].

In this solution, there is only one parameter, the value of the "invisible" axion field or $\overline{\theta}$. The reason is that the form of the anomaly is completely specified at one loop and no more [81]. One parameter $\overline{\theta}$ to this interaction is one coupling G_F to the weak interaction. In both cases, details are more involved however, in the former case the detailed "invisible" axion models and in the latter case the CKM and PMNS weak gauge boson couplings to three family members. Because of this one coupling nature, one could have easily proved that $\overline{\theta} = 0$ is at the minimum of free energy [73].

The axion detection uses the following coupling [82],

$$\frac{c_{a\gamma\gamma}}{16\pi^2}\, a\, F^{\text{em}}_{\mu\nu} \tilde{F}^{\text{em}\,\mu\nu} \tag{24}$$

where $F^{\text{em}}_{\mu\nu}$ ($\tilde{F}^{\text{em}\,\mu\nu}$) is the electromagnetic field strength (its dual). Equation (24) gives the form $\mathbf{E}\cdot\mathbf{B}$ such that **E** parallel to **B** contributes to the coupling. The usual design [83] is a cavity detector immersed in a *strong constant* magnetic field. Then, **E** follows the cosmic oscillation of the classical axion field $\langle a \rangle$ generating the axions in the cavity, which is given completely in the axio-electrodynamics in [84]. [Note that similar results with the resonance condition were given earlier [85,86].] This idea of using the oscillating **E** is pictorially shown in Figure 3.

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In some GUT models, $\tilde{c}_{a\gamma\gamma}$ is related to the weak mixing angle $\sin^2\theta_{\text{W}}$. A schematic view on the gauge couplings, $\sin^2\theta_{\text{W}}$, and $\tilde{c}_{a\gamma\gamma}$ is presented as $c^0_{a\gamma\gamma}$ in Figure 5. The evolution of $\sin^2\theta_{\text{W}}$ are shown in the middle part. $\tilde{c}_{a\gamma\gamma}$ is determined by quantum numbers and does not evolve above as shown in the lower part of the figure. Axions from GUTs usually give $\tilde{c}_{a\gamma\gamma} = \frac{8}{3}$, which is not necessarily satisfied with unknown Q_{em} charges above the GUT scale, and there exist models with $\tilde{c}_{a\gamma\gamma} > \frac{8}{3}$, for example $\frac{20}{3}$ in [87].

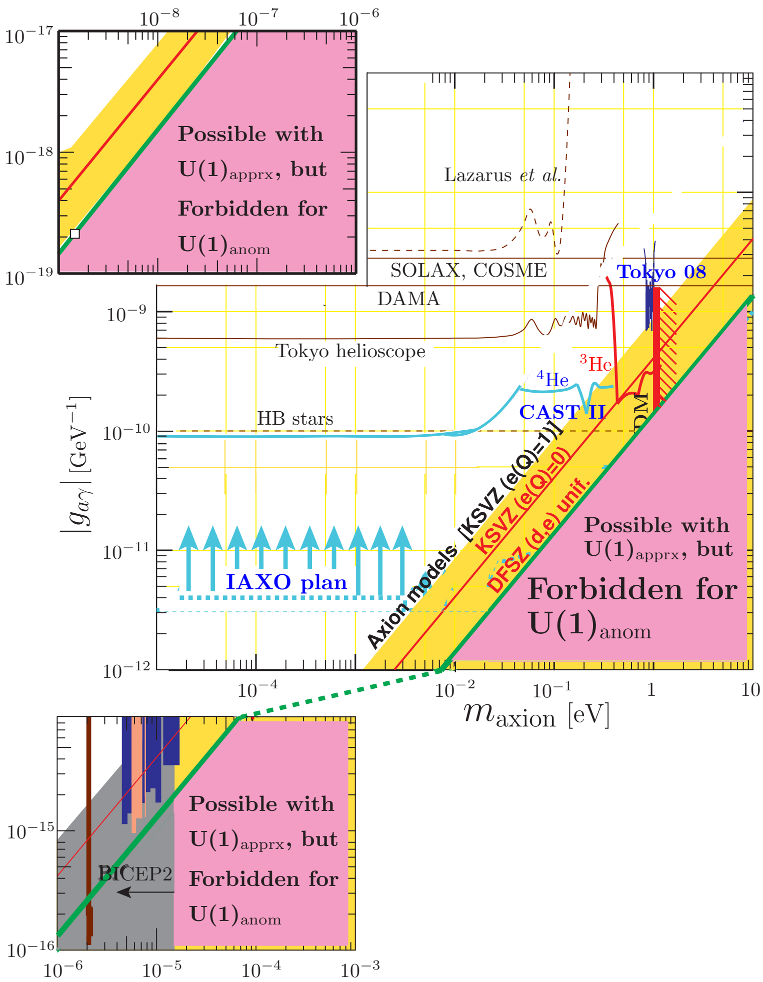

Figure 4. The $g_{a\gamma}(= 1.57 \times 10^{-10}\, c_{a\gamma\gamma})$ vs. m_a plot [88,89].

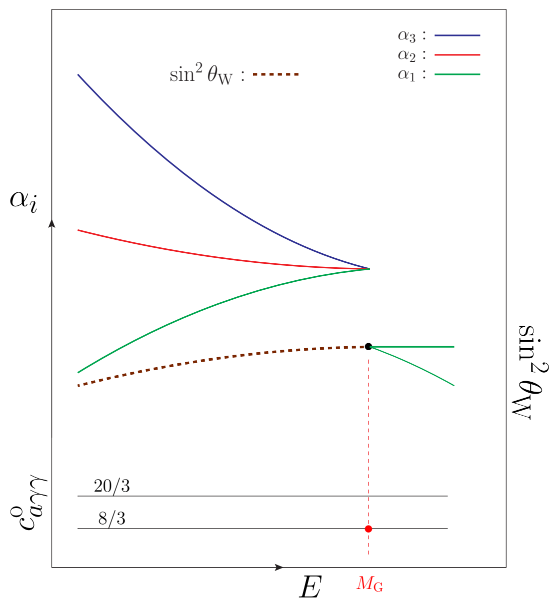

Figure 5. A schematic view on the gauge couplings, $\sin^2\theta_{\rm W}$, and $\tilde{c}_{a\gamma\gamma}$.

For a calculation of $c_{a\gamma\gamma}$, one should pay attention that the model must lead to an acceptable SM phenomenology. In Table 1, we present $c_{a\gamma\gamma}$ in several KSVZ and DFSZ models.

Table 1. The coupling $c_{a\gamma\gamma}$ for $c^0_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma}$ given in the KSVZ and DFSZ models. For the u and d quark masses, $m_u = 0.5\, m_d$ is assumed for simplicity. (m, m) in the last row of KSVZ means m quarks of $Q_{\rm em} = \frac{2}{3}\, e$ and m quarks of $Q_{\rm em} = -\frac{1}{3}\, e$. SUSY in the DFSZ models includes contributions of color partners of Higgsinos. If we do not include the color partners, i.e., in the MSSM without heavy colored particles, $c_{a\gamma\gamma} \simeq 0$.

KSVZ: $Q_{\rm em}$	$c_{a\gamma\gamma}$	DFSZ: (q^c, e_L) pair	Higgs	$c_{a\gamma\gamma}$
0	-2	non-SUSY (d^c, e_L)	H_d	$+\frac{2}{3}$
$\pm\frac{1}{3}$	$-\frac{4}{3}$	non-SUSY (u^c, e_L)	H_u^*	$-\frac{4}{3}$
$\pm\frac{2}{3}$	$+\frac{2}{3}$	GUTs		$+\frac{2}{3}$
± 1	$+4$	SUSY		$+\frac{2}{3}$
(m, m)	$-\frac{1}{3}$			

Table 2. String model prediction of $c_{a\gamma\gamma}$. In the last line, $c_{a\gamma\gamma} = (1 - 2\sin^2\theta_W)/\sin^2\theta_W$ with $m_u = 0.5\ m_d$.

String	$c_{a\gamma\gamma}$	Comments
Ref. [90]	$-\frac{1}{3}$	Approximate U(1) global symmetry
Ref. [40,89,91]	$+\frac{2}{3}$	Anomalous U(1) from string

Calculations of $c_{a\gamma\gamma}$ from ultra-violet completed models are welcome. So far there exists only one calculation on $c_{a\gamma\gamma}$ from string compactification [88,89]. In Table 2, we list two calculations of $c_{a\gamma\gamma}$. In a $\mathbf{Z}_{12-I}$ orbifold compactification present in Ref. [92], the axion-photon-photon coupling has been calculated [89],

$$c_{a\gamma\gamma} = \tilde{c}_{a\gamma\gamma} - c_{\rm chiral} \simeq \frac{-5406 - 1162 - 1960 - 784}{-3492} - 2 = +\frac{2}{3}, \tag{25}$$

which is shown as the green line in the axion coupling vs. axion mass plot of Figure 4.

3.1. Axion Inhomogeneities in Galaxies and Mini-Clusters

If there existed inhomogeneities of axion field in the galaxies, some large scales could have been formed [93]. In particular, if there existed strong primordial inhomogeneities of cold axions, their astrophysical effects can be estimated [94], such that axion perturbations on scales corresponding to causally disconnected regions at $T \sim 1\,{\rm GeV}$ can lead to very dense pseudo-soliton configurations. It is because these configurations at the $\bar{\theta}$ misalignment at $T \sim 1\,{\rm GeV}$ evolve to axion mini-clusters with present density $\rho_a > 10^{-8}\,{\rm g}\cdot{\rm cm}^{-3}$. This high density enables the process $2a \to 2a$ for the Bose–Einstein relaxation in the gravitationally bound clumps of axions, forming axion mini-clusters. During inflation, formation of primordial black holes in the mass range $(10^{-9} - 10^{-5})M_\odot$ could have been formed from the axion inhomogeneities [95].

3.2. The Domain Wall Problem in "Invisible" Axion Models

It is well-known that if a discrete symmetry is spontaneously broken then there results domain walls in the course of the Universe evolution [96]. For the "invisible" axion models, it was pointed out that the domain wall number $N_{\rm DW}$ different from 1 must have led to serious cosmological problems in the standard Big Bang cosmology [97,98]. In this regard, the standard DFSZ models with $N_{\rm DW} = 6$

should not be considered. For the "invisible" axions, we consider only $N_{DW} = 1$ models. The argument goes like this. In the evolving Universe, there always exists a (or a few) horizon scale string(s) and a giant wall attached to it [98] as shown in Figure 6a. There are a huge number of small walls bounded by an axionic string which punch holes in the giant walls as shown in Figure 6b. The punched holes expand with the light velocity and eat up the giant string-walls as shown in Figure 6c. This is the scenario in the $N_{DW} = 1$ models [99].

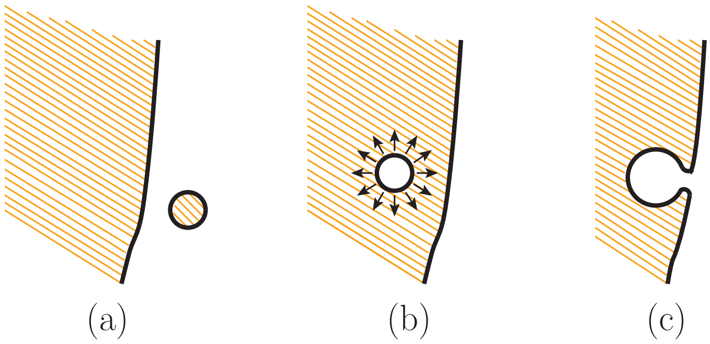

Figure 6. A horizon scale string-wall for $N_{DW} = 1$ with a small membrane bounded by string. (**a**) A (or a few) horizon scale string(s) and a giant wall attached to it [98]; (**b**) A huge number of small walls bounded by an axionic string which punch holes in the giant walls; (**c**) The punched holes expand with the light velocity and eat up the giant string-walls.

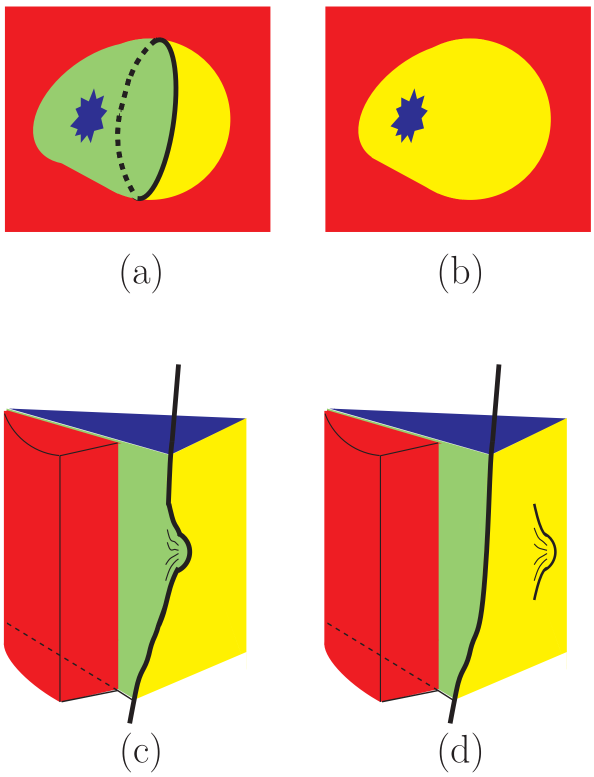

Figure 7. Small DW balls ((**a**,**b**), with punches showing the inside blue-vacuum) and the horizon scale string-wall system (**c**,**d**) for $N_{DW} = 2$: (**a**) a DW ball with a string loop, (**b**) a DW ball without a string loop, (**c**) collision with a ball of type (**a**), and (**d**) collision by a ball of type (**b**). Yellow walls are $\theta = 0$ walls, and yellow-green walls are $\theta = \pi$ walls. Yellow-green walls of type (**b**) are also present.

However, "invisible" axion models with $N_{DW} \geq 2$ have cosmological problems. For example, for $N_{DW} = 2$, a horizon scale string and wall system has the configuration shown in Figure 7.

Figure 7a,b show small balls, and Figure 7c,d show a horizon scale string-wall system after absorbing the small balls. Certainly, the horizon scale string-wall system is not erased, which is a cosmological disaster in the standard Big Bang cosmology.

With inflation, the domain wall problem has to be reconsidered. However, if the GUT scale vacuum energy were present, it is likely that the domain wall problem becomes severe. So, I want to stress that the axionic domain problem is better to be resolved without the dilution effect by inflation. One obvious model is the KSVZ axion with one heavy quark.

Another, a more sophiscated, solution is the Lazarides–Shafi mechanism in which the seemingly different vacua are identified by gauge transformation. Ref. [100] used the centers of extended-GUT groups for this purpose. Still another solution is to use Goldstone boson directions of spontaneously broken global symmetries [101,102]. There can exist a cosmological solution, using a hidden-sector confining force [103].

However, the most appealing solution is to use the model-independent (MI) axion in string compactification. It is known that the MI axion has $N_{\rm DW} = 1$ [104,105]. Also, due to the 't Hooft mechanism which we discussed in Section 2.1, the "invisible" axion scale can be lowered from the string scale down to an intermediate scale [89] and can be proved/disproved to exist in Nature.

3.3. Searches of "Invisible" Axions

The ongoing search of "invisible" QCD axions is based on the BCM in the Universe [2]. After the discovery of the Higgs boson which seems to be a fundamental elementary particle, the possibility of the QCD axion being fundamental gained some weight. The future axion search experiment can detect the CDM axion even its contribution to CDM is only 10% [106]. Because the axion decay constant f_a can be in the intermediate scale, the "invisible" axions can live up to now (for $m_a < 24$ eV) and constitute DM of the Universe. Cosmology of the "invisible" axions has started in 1982–1983 [10–12] with the micro-eV axions [13–16]. The needed intermediate axion scale, far below the GUT scale, is understood in models with the anomalous U(1) in string compactification [40].

For the detection of cosmic axions, the axion energy density ρ_a as CDM, which is assumed to be $\sim$0.3 GeV cm^{-3}, is the one to be detected. Relating ρ_a with f_a depends on the history of the evolving Universe due to the contribution to ρ_a from the annihilating string-wall system of axions. The recent numerical calculation for an $N_{\rm DW} = 1$ model shows that more than 90% of ρ_a is contributed from the annihilating string-wall system [75], which however did not include the effects shown in Figure 6.

The 2014 BICEP2 report of "high scale inflation at the GUT scale" around $>(10^{16}$ GeV$)^4$ implied the reheating temperature after inflation $>10^{12}$ GeV [107]. Then, studies on the isocurvature constraint with that BICEP2 data pinned down the axion mass [108]. However, more data, with dust contamination taken into account, has not lived up with this earlier report [109]. However, it is likely that the energy density during inflation might be near the GUT scale with a somewhat smaller tensor/scalar ratio than the value in the early report. In this case also, requiring $N_{\rm DW} = 1$ is a necessity.

4. A BCM as Dark Energy

As for the QCD axion case, the DE scale can arise via a BCM of Figure 1 [24]. The global symmetry violating terms belong to the red part in Figure 1. In the Higgs portal scenario, the BCM pseudoscalar for DE must couple to the color anomaly since it couples to the Higgs doublets and the Higgs doublets couple to the SM quarks. On the other hand, mass of the BCM pseudoscalar for DE is in the range $10^{-33} \sim 10^{-32}$ eV [47]. Therefore, the QCD anomaly term of the BCM must be forbidden to account for the DE scale of 10^{-46} GeV^4. There must be a global symmetry free of the QCD-anomaly. It is shown that such two global U(1) symmetries are achieved in general [25]. Out of the two U(1)'s, let us pick up the global symmetry U(1)$_{\rm de}$ for the DE BCM. The other U(1), carrying the color anomaly, is for the "invisible" axion. In contrast to the f_a bound for the QCD axion given in Equation (18), the U(1)$_{\rm de}$ breaking scale $f_{\rm DE}$ is of order of the Planck scale [47],

$$f_{\rm DE} \approx M_{\rm P}. \tag{26}$$

From string compactification, we argued that the U(1)$_{\rm anom}$ global symmetry via the 't Hooft mechanism is for the "invisible" QCD axion [40,89] whose decay constant f_a corresponds to (18). The global symmetry U(1)$_{\rm de}$ cannot have a QCD anomaly [25] and in the compactification of the heterotic string it must be anomaly free since there is only one U(1)$_{\rm anom}$ from the heterotic string. Thus, the VEVs $f_{\rm DE}$ and f_a breaking U(1)$_{\rm de}$ and U(1)$_{\rm anom}$ global symmetries, respectively, are clearly distinguished in the compactification of the heterotic string. However, their hierarchical values given in Equations (18) and (26) are obtained by a kind of tuning of parameters in the potential V. It is known that the anti-symmetric tensor field B_{MN} in string theory leads to string axions having GUT scale decay constants [110,111]. So, it was argued in the SUSY framework that the "invisible" QCD axion from string theory is better to arise from matter fields [112].

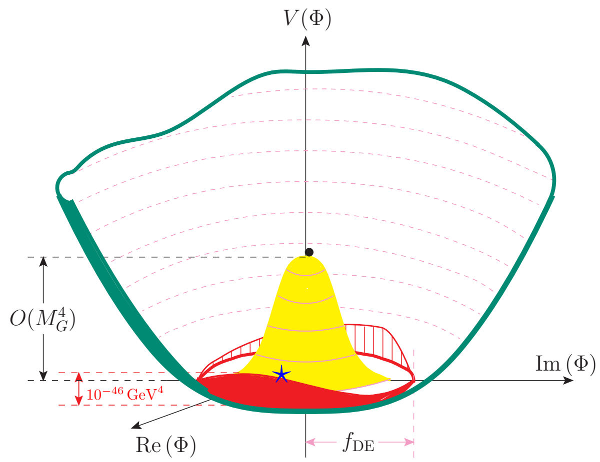

Figure 8. The DE potential in the red angle-direction in the valley of radial field of height $\approx M_{\rm GUT}^4$.

By introducing two global symmetries, we can remove the U(1)$_{\rm de}$–SU(3)$_c$–SU(3)$_c$ anomaly where SU(3)$_c$ is QCD and the U(1)$_{\rm de}$ charge is a linear combination of two global symmetry charges. The decay constant corresponding to U(1)$_{\rm de}$ is $f_{\rm DE}$. In Figure 8, we depict the scale and the breaking term $\Delta V_{\rm DE}$. Introduction of two global symmetries is inevitable to interpret the DE scale and hence in this scenario the appearance of U(1)$_{\rm PQ}$ is a natural consequence. The height of DE potential is so small, 10^{-46} GeV^4, that the needed discrete symmetry breaking term of Figure 1 must be small, implying the discrete symmetry is of high order.

For the QCD axion, the height of the potential for the "invisible" axion is $\approx\Lambda_{\rm QCD}^4$. For the DE pseudo-Goldstone boson, the height of the potential of the radial field is $\approx M_{\rm GUT}^4$, according to the Higgs portal idea, shown at the central top in Figure 8. With the exact U(1)$_{\rm PQ}$ and the approximate U(1)$_{\rm de}$ at tree level,[3] one can construct a DE model with the potential in the valley in Figure 8 from string compactification [25]. Using the SUSY language, the discrete and global symmetries below $M_{\rm P}$ are the consequence of the full superpotential W. So, the exact discrete symmetries related to string compactification are respected by the full W, i.e., the vertical column of Figure 1. The example shown in [25] has a dimension 6 superpotential for the definition of the global symmetry U(1)$_{\rm de}$ from the

[3] For the U(1)$_{\rm PQ}$, the symmetry is better to be exact at tree level or almost exact toward the solution of the strong CP problem with $|\bar\theta| < 10^{-10}$ as discussed in Section 3.

lavender part of Figure 8, and the breaking superpotential $\Delta W_{\rm DE}$ at tree level is at a dimension 10 level. For $\rm U(1)_{de}$, the global symmetry is not exact and the red part at the tree level contributes. This example shows that the definition of symmetry $\rm U(1)_{de}$ in the lavender part is at a high dimensional level and the breaking term in the red part is even more high dimensional [25]. A typical discrete symmetry $\mathbf{Z}_{10\,R}$ is considered in [25], whose level is high enough such that the defining and breaking terms appear at the high dimensional levels. The $\mathbf{Z}_{10\,R}$ charges descend from a gauge U(1) charges of the string compactification [37]. In this scheme with the Higgs portal, we introduced three scales for the VEVs, TeV scale for H_uH_d, the GUT scale $M_{\rm GUT}$ for singlet VEVs, and the intermediate scale for the "invisible" QCD axion. The other fundamental scale is $M_{\rm P}$. The trans-Planckian decay constant $f_{\rm DE}$ of (26) can be a derived scale, which can be applied also to the inflation [27].

5. Inflation and Gravity Waves in the Beginning

The inflationary idea [22] was so attractive in understanding the horizon, flatness, homegeneity and isotropy problems [23], numerous inflationary models have been considered since the early 1980s [113–115]. Even there exists a calculable framework [116,117] in inflationary models for the "quantum" density perturbation [118]. For some time, the chaotic inflation [115] attracted a great deal of attention because it can lead to a large tensor/scalar ratio, r, in particular with the earlier report of the BICEP2 collaboration [107].

The inflationary models need almost flat potentials [119]. The logarithmic form near the origin is very flat. The $\mu^2\phi_I^2$ potential with very small $\mu^2/M_{\rm P}^2$ is very flat even at some trans-Planckian values of ϕ. Sinusoidal forms are almost flat near the top of the potential. The magnitude of $\Delta T/T$ measured by COBE excluded the logarithmic form among the new inflationary scenarios [120]. It seems that the ϕ_I^2 inflation, where ϕ_I is the inflaton, is almost ruled out by the refined r measurements [109].

Thus, the sinusoidal forms are the remaining attractive possibility which belongs to the class of "natural" inflation [26]. These sinusoidal forms of a_I appear in the potential as[4]

$$V \propto 1 - \frac{1}{2}(e^{i\delta}e^{ia_I/f_I} + {\rm H.c.}) = 1 - \cos\left(\frac{a_I}{f_I} + \delta\right). \tag{27}$$

So, near $\langle a_I\rangle = f_I(\pi+\delta)$, the potential is very flat and the natural inflationary potential works for inflation. This is one of "hill-top" inflationary models that for a large r the field value $\langle\phi\rangle$ must be larger than 15 $M_{\rm P}$, which is known as the Lyth bound [121]. So, the natural inflation needs the decay constant at a trans-Planckian scale [27]. Also, a trans-Planckian scale is introduced for a quintessential DE [47]. Introducing a natural inflation with a trans-Planckian decay constant is possible if one considers two spontaneously broken global symmetries [27], which is called the Kim–Nilles–Peloso (KNP) models.

This is illustrated with an effective potential of two axions. Let us consider two non-Abelian gauge groups with scales Λ_1 and Λ_2 and two pseudoscalars a_1 and a_2 resulting from breaking two global symmetries. The effective potential of a_1 and a_2 below the confining scales is [27]

$$V \propto \Lambda_1^4\left(1 - \cos\left[p\frac{a_1}{f_1} + q\frac{a_2}{f_2}\right]\right) + \Lambda_2^4\left(1 - \cos\left[h\frac{a_1}{f_1} + k\frac{a_2}{f_2}\right]\right) \tag{28}$$

where p, q, h, and k are parameters given by the model and f_1 and f_2 are two decay constants (or VEVs of scalar fields). Diagonalization of the mass matrix of a_1 and a_2 gives the heavy and light pseudoscalar masses as [122],

$$m_H^2 = \frac{1}{2}(A+B),\ m_L^2 = \frac{1}{2}(A-B), \tag{29}$$

[4] If the symmetry breaking is soly by the non-Abelian anomaly, then we obtain $\delta = 0$.

where

$$A = \frac{p^2\Lambda_1^4 + h^2\Lambda_2^4}{f_1^2} + \frac{q^2\Lambda_1^4 + k^2\Lambda_2^4}{f_2^2}, \quad B = \sqrt{A^2 - 4(pk - qh)^2 \frac{\Lambda_1^4\Lambda_2^4}{f_1^2 f_2^2}}. \tag{30}$$

If $pk = qh$, there results a massless one with $m_L = 0$. For $pk = qh + \Delta$, there can result an effectively large decay constant f_L as shown in Ref. [27]. For simplicity, we can glimpse this phenomenon for the parameters $\Lambda_1 = \Lambda_2 = \Lambda$, $f_1 = f_2 = f$ and $p = q = h = k$ [122], corresponding to m_L,

$$f_L \simeq \frac{2|p|}{|\Delta|} f. \tag{31}$$

In this two axion case, the height of the pontential is increased roughly to $2\Lambda^4$ and the decay constant f_L is increased by the level of the quantum number discrepancy, $|p/\Delta|$.

With two confining non-Abelian gauge groups, the global charges can be assigned such that an enough trans-Planckian decay constant results. However, for inflation, not needing the vacuum angle to be zero, the explicit breaking terms of two global symmetries need not arise from gauge anomalies but can arise from ΔV in the red part of Figure 1. This possibility generalized the KNP mechanism to N spontaneously broken global symmetries under the name "N-flation" [45] which has been studied extensively in string theory [123,124]. The N-flation attempted to soften the problem $|p/\Delta| \gg 1$ of the KNP model but the height of the potential increases by a factor N.

Nevertheless, this idea of increasing N has been further pursued in obtaining a kind of cosmological generation of the weak scale from axion-type potentials, the so-called "relaxion" idea [125], which however seems not completely satisfactory [126–129].

The hilltop potential of Figure 8 is a Mexican hat potential of $U(1)_{\rm de}$, i.e., obtained from some discrete symmetry, allowed in string compactification [24]. The discrete symmetry may provide a small DE scale. The trans-Planckian decay constant, satisfying the Lyth bound, is obtained by a small quartic coupling λ in the hilltop potential V. The requirement for the vacuum energy being much smaller than $M_{\rm P}^4$ is achieved by restricting the inflaton path in the radial direction in the hilltop region, $\langle\phi\rangle < f_{\rm DE}$, to converge to an appropriate phase direction of Figure 8.

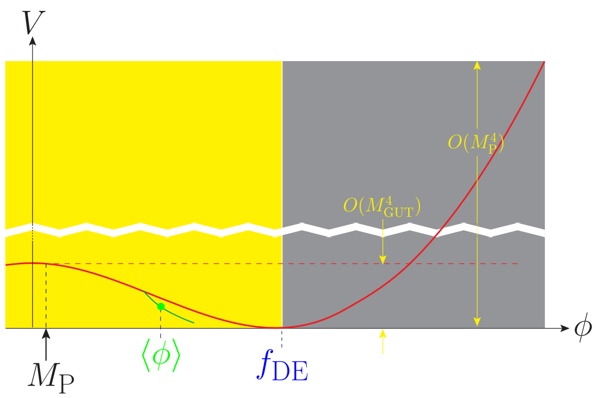

Figure 9. The trans-Planckian decay costant in the hilltop inflation.

We can compare this hilltop inflation with the $\mu^2\phi^2$ chaotic inflation. The hilltop inflation is basically a consequence of discrete symmetries [24,25,37], allowed in string compactification. If some conditions are satisfied between the discrete quantum numbers of the GUT scale fields and trans-Planckian scale fields, the hilltop potential of Figure 9 can result. On the other hand, the $\mu^2\phi^2$ chaotic inflation does not have such a symmetry argument, and lacks a rationale, forbidding higher order ϕ^n terms.

In contrast to the gravity waves generated during inflation, the gravity waves from binary black hole coalescence observed recently [130,131] occurred late in the cosmic time scale. In a recent observation of a coalescence, graviton mass is bounded as $<0.77\times10^{-22}$ eV [132]. Related to our MI "invisible" axion from superstring, for the MI axion creation to be of any observable effect along the coalescence, the black hole mass is required to be small, $\sim 2\times10^{14}$ kg [133].

6. Global Symmetries and Non-Abelian Gauge Groups

We have discussed directions in spontaneously broken global symmetries and their manifestations as pseudo-Goldstone bosons. The pseudo-Goldstone boson mass $m_{\rm pseudo}$ corresponding to a global U(1) depends on the explicit breaking scale δ^4, the red part in Figure 1, and the decay constant f: δ^2/f. The explicit symmetry breakings are broadly distinguished to two classes: (i) by small terms in the Lagrangian and (ii) by non-Abelian gauge interaction. The case (i) can be studied for the Yukawa couplings $\mathcal{L}_Y$ and small terms ΔV in V. If all terms in $\mathcal{L}_Y$ and V respect the global symmetry, then loop contributions cannot break the global symmetry. So, the breaking effect must be considered at the tree level. The case (ii) is known as the instanton effect [134] and the $\bar\theta$ vacuum discussed in Section 3 is the pseudo-Goldstone boson direction. This breaking arises at the one loop level as the (global symmetry)–(non-Abelian group)^2 anomaly.

Here, a few BCMs are commented. For the N-flation and U(1)$_{\rm de}$, the breaking terms correspond to the case (i), obtaining contribution only from V. For the "invisible" QCD axion, it belongs to the case (ii) and there should be no contribution from V. We discussed the top-down scenario for obtaining U(1)$_{\rm PQ}$ from U(1)$_{\rm anom}$ global symmetry from string compactification. For this to remain as a solution of the strong CP problem, however, QCD should be the only unbroken non-Abelian gauge group. The reason is the following.

We have already presented in (28) an effective potential of two axions a_1 and a_2 with two non-Abelian groups below their confining scales. Note that p, q, h, and k are parameters given in the model and f_1 and f_2 are two decay constants (or VEVs of scalar fields). Diagonalization of the mass matrix of a_1 and a_2 gives the heavy and light pseudoscalar masses presented in Equations (29) and (30) [122]. Here, we take different limits of parameters from the KNP inflation. Suppose that there are hierarchies $f_2 \gg f_1$ and $\frac{\Lambda_2^4}{\Lambda_1^4} \gg \frac{f_2^2}{f_1^2}$. Then, the heavy and light masses are

$$m_H^2 \simeq \Lambda_2^4\left(\frac{h^2}{f_1^2}+\frac{k^2}{f_2^2}\right),\quad m_L^2 \simeq \Lambda_1^4\left(\frac{(pk-qh)^2}{k^2f_2^2+h^2f_1^2}\right). \tag{32}$$

From Equation (32), we note that the decay constant of the heavy pseudoscalar is the smaller decay constant f_1 and the decay constant of the light pseudoscalar is the larger decay constant f_2. So, if we try to have $\Lambda_1 \to \Lambda_{\rm QCD}$, then we must choose the larger decay constant which can be a GUT or string scale. So, we fail in obtaining an intermediate scale f_a from the U(1)$_{\rm anom}$ global symmetry for the "invisible" axion. For the U(1)$_{\rm anom}$ global symmetry to lead to the "invisible" axion, we should not have any other non-Abelian gauge group above the TeV scale from string compactification. If another confining gauge group above the TeV scale is introduced, then a scenario must be introduced such that it is broken after achieving the objective.

7. Discussion and Conclusions

From theoretical and cosmological perspectives, we reviewed spontaneously broken global symmetries and the consequent pseudoscalar bosons. The PQ *global* symmetry was introduced as a prototype model, which can solve also the strong CP problem.

Theorectically, pseudoscalars can be made light if they appear as pseudo-Goldstone bosons with the decay constant f. We argued that depending on the scales of f and the explicit breaking terms, these pseudoscalars could have worked as BCMs in various stages in the evolving Universe as shown in (6) and (7). We discussed the explicit breaking terms arising from non-Abelian gauge anomalies and/or from the classical Lagrangian. These BCMs can be DE, CDM, or even an inflaton. For a possible experimental confirmation of BCM, the "invisible" QCD axion is the front runner with the decay constant f_a in the axion window (18). We also argued that the "invisible" QCD axion at the axion window is possible from string compactification.

Since the "invisible" axion determined $\overline{\theta} = 0$ as noted in Section 3, one may question a possibility of determining the weak CP violation phases, the CKM phase $\delta_{\rm CKM}$, the PMNS phase $\delta_{\rm PMNS}$, and also the CP phases in the Type-I [135] and Type-II [136] leptogenses. In principle, it can be done but these attempts cannot be present in simple forms because three quark families, for example, introduce many parameters, in contrast to the one parameter $\overline{\theta}$ in QCD [81]. Determining $\delta_{\rm CKM}$ belongs to the problem of "textures" of quark mass matrix. A hope toward this direction is to parametrize the CKM matrix in the form of Ref. [137] such that $\delta_{\rm CKM}$ is the phase in the CKM matrix itself, which may help finding out the needed texture. There exist a few attempts along this line [136,138,139].

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