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Effects of Scattering of Radiation on Wormholes

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Abstract: Significant progress in the development of observational techniques gives us the hope to directly observe cosmological wormholes. We have collected basic effects produced by the scattering of radiation on wormholes, which can be used in observations. These are the additional topological damping of cosmic rays, the generation of a diffuse background around any discrete source, the generation of an interference picture, and distortion of the cosmic microwave background (CMB) spectrum. It turns out that wormholes in the leading order mimic perfectly analogous effects of the scattering of radiation on the standard matter (dust, hot electron gas, etc.). However, in higher orders, a small difference appears, which allows for disentangling effects of wormholes and ordinary matter.

Keywords: cosmology; diffuse background radiations; microwave; cosmological wormholes

1. Introduction

The greatest development of observational techniques that we observe in modern astrophysics allows us to challenge one of the most intriguing astrophysical problem, which is the direct observation of primordial wormholes. Wormholes are objects predicted by Einstein's general relativity as early as the Schwarzschild solution itself [1,2] (see also recent review [3] and references therein).

Despite that such objects as black holes have occupied an appropriate position in astrophysics for a long time, there is still no direct observational evidence for the existence of wormholes in the universe. Such a situation looks rather strange, as there exist strong theoretical arguments for the production of some number of primordial wormholes in the very early universe.

Indeed, it has been suggested that the spacetime could have a foam-like structure at very small scales [4]. The simplest picture of the foam is described by a gas of virtual wormholes [5]. Virtual wormholes represent tunneling events, which in real space-time can be described by the creation and subsequent annihilation of real wormholes (i.e., zero-point topological fluctuations) [6]. The existence of such virtual wormholes is unavoidable, as they remove all divergencies in quantum field theory [7]. In other words, without virtual wormholes, quantum theory could not be a self-consistent theory and the vacuum state would have an infinite energy density. The stability of the vacuum (at least in causally connected regions of spacetime) guarantees that real wormholes cannot spontaneously appear in space. To create a wormhole, one has to impose a rather strong external field with a specific configuration.

From the other side, it is supposed that the very early stage of the development of the universe is described by the inflationary type of evolution, for which the scale factor increases with the exponential law. During this stage, the rate of the expansion overruns the increase in the horizon size, and, therefore, regions that were causally connected lose such a connection very rapidly. Such a rapid increase in the scale factor plays the role of the external field, which is capable of creating particles from zero-point fluctuations and real wormholes from virtual wormholes. Such a process (the creation of wormholes during of the inflationary stage) is not described rigorously yet, but there are no doubts that some

number of primordial wormholes should be created. The subsequent evolution of wormholes depends essentially on their exact structure. Spherically symmetric wormholes are highly unstable and collapse very rapidly. The presence of some amount of exotic matter at throats (the matter that violates the averaged null energy condition (ANEC)) is capable of somewhat stabilizing spherical wormholes [8]. However, for the present day, such a kind of matter remains unknown (as far as laboratory scales and astrophysical compact objects are concerned), and we still have no reasonable mechanism that may produce it. Here we ignore all pure speculative theories that have dealt with exotic forms of matter or with numerous modifications of general relativity. However, less-symmetric wormholes may exist without exotic matter (e.g., see [9]); in this case, they have throat sections in the form of tori or even more complicated surfaces. In studying possible observational effects of such wormholes, we should take into account one important fact that essentially simplifies our considerations. This is that wormholes have random orientations in space. Upon averaging over all orientations, wormholes acquire features of a spherically symmetric configuration, which means that we may consider spherical wormholes in the leading order, while their true structure will be encoded in higher orders.

Already, the simplest spherical wormholes exhibit the presence of a number of important phenomena. In the first place, they predict the essential deviation of gravity from the standard Newtonian (or Einsteinian) behavior [10,11]. In particular, wormholes produce a strong modification in the behavior of density perturbations without any modification of the basic theory itself [12]. This allows us to use wormholes in explaining the dark matter (DM) phenomenon. Indeed, in galaxies and at subgalactic scales, wormholes strongly interact with baryons, which solves the problem of cusps [13] in centers of galaxies (which form the core-type distribution of dark matter), while on larger scales, wormhole behave as very heavy dark matter particles and reproduce predictions of the Standard Model (ΛCDM) [12]. Dark matter effects and all other deviations from the standard Newtonian behavior may give only indirect evidence for the presence of primordial wormholes in the universe. In the present paper, we collect possible direct observational effects that appear as a result of the scattering of particles on wormholes [14–18]. It turns out that the scattering on wormholes mimics perfectly the scattering on the standard matter (on the dust, gas, compact objects, etc.). However some peculiarities appear, which give us hope of, in the near future, disentangling such effects from the effects of scattering on ordinary matter.

We consider some basic as well as the most important astrophysics features that can be observed due to the scattering of cosmic rays on wormholes.

First, and the simplest feature, is an additional specific damping of cosmic rays, which can be observed for any discrete source (at least, if it can be used as the standard candle). The damping is determined by the traversed path and the distribution of wormholes in space. It is determined by the optical depth related to the distribution of wormholes in space. This effect is additive to that which is caused by ordinary matter, and because wormholes contribute to dark matter, it should correlate with dark matter distribution.

The second feature is that wormholes not only capture particles by one of the ends of the throat, but they also re-emit them from the other end. Re-emitted particles possess a wider spectrum, and they form a diffuse background of a very low intensity around any discrete source (a diffuse halo). The widening of the background spectrum depends in a specific way on peculiar motions of wormholes. This leads to a scale-dependent renormalization of all discrete sources of radiation, while the diffuse halo should correlate with the dark matter distribution around the source.

The next important feature is that the scattering of radiation on a single wormhole can be observed by means of a specific interference picture. The last effect we discuss is that wormholes contribute to the distortion of the cosmic microwave background (CMB) spectrum by means of the kinematic Sunyaev–Zel'dovich effect (KSZ).

2. The Simplest Model of a Wormhole

The spherical static wormhole represents two different asymptotically flat spaces $E_\pm$ connected by a throat. This can be easily constructed by the cut-and-paste technique. In the simplest case, a static wormhole can be described by the metric

$$ds^2 = c^2 dt^2 - h^2(r)\,\delta_{\alpha\beta}dx^\alpha dx^\beta \tag{1}$$

where the scale factor is

$$h(r) = 1 + \theta(b - r)\left(\frac{b^2}{r^2} - 1\right) \tag{2}$$

and $\theta(x)$ is the step function. Here r denotes the standard radial space-like coordinate as in the ordinary Euclidean space ($r^2 = x^2 + y^2 + z^2$). Such a wormhole has a vanishing throat length. Indeed, in the region $r > b$, $h = 1$ and the metric is flat, while in the region $r < b$, the metric upon the transformation to new coordinates $x'^\alpha = \frac{b^2}{r^2}x^\alpha$ takes the same flat form as Equation (1) with $h = 1$. Therefore, the regions $r > b$ and $r < b$ represent two Minkowski spaces glued at the surface of a cylinder $S^2 \times R^1$ (which is the direct product of the time axis and a spatial sphere with the center at the origin $r = 0$ and radius $r = b$). Such a space can be described with the ordinary double-valued flat metric:

$$ds^2 = c^2 dt_\pm^2 - \delta_{\alpha\beta}dx_\pm^\alpha dx_\pm^\beta, \tag{3}$$

where the coordinates $x_\pm^\alpha$ describe two different sheets of space and cover the regions $r_\pm > b$. Here the coordinate regions $r_\pm < b$ do not correspond to any point of space, as these regions are removed.

A generalization to a more general case appears when we change the scale function $h(x)$ with any smooth function that has the properties $h(r) \to 1$ as $r \gg b$ and $h(r) \to \frac{b^2}{r^2}$ as $r \ll b$. The simplest choice $h = 1 + \frac{b^2}{r^2}$ corresponds to the Bronikov–Ellis metric. Contrary to the previous case, such a wormhole has a non-vanishing throat length. We also point out that in the most general case, one may set a non-vanishing mass on the throat. In this case, the metric sufficiently far from the throat entrances ($r \gg b$) transforms to the standard Schwarzschild metric (e.g., see [19]). If the wormhole has a non-vanishing mass, we assume that the throat radius is much larger than the respective gravitational radius ($b \gg M$); otherwise effects of such a wormhole are undistinguishable from those of black holes.

Identifying the inner and outer regions of the sphere S^2 in the space with the metric given by Equation (1) allows the construction of a wormhole that connects regions in the same space (instead of two independent spaces; see Equation (3)). This is achieved by gluing the two spaces in Equation (3) by a motion of the Minkowski space. If $R_+^\mu = (t_+, \mathbf{R}_+)$, where $\mathbf{R}_+$ is the position of the sphere in coordinates x_+^μ, then the gluing is the rule

$$x_+^\mu = R_+^\mu + \Lambda_\nu^\mu\left(x_-^\nu - R_-^\nu\right) \tag{4}$$

where $\Lambda_\nu^\mu \in SO(3,1)$, which represents the composition of a translation and a Lorentz transformation of the Minkowski space. In terms of common coordinates, such a wormhole represents the standard flat space in which the two cylinders $S_\pm^2 \times R^1$ (with centers at positions $\mathbf{R}_\pm$) are glued by the rule given by Equation (4). Thus, in general, the wormhole can be described by a set of parameters: the throat radius b, positions and velocities of throats $\mathbf{R}_\pm$, $\mathbf{V}_\pm$, the spatial rotation matrix $U_\beta^\alpha \in O(3)$, and in general some additional time shift $\Delta t = t_+ - t_-$. In the present review, we do not take into account the fact that, in general, wormholes possess non-vanishing masses M. In the problem of scattering of radiation, the masses of wormholes produce a number of additive effects, which can be considered separately. In particular, they determine light deflection and are also important when considering the back reaction (e.g., in removing cusps in galaxies [12]). However such additive effects are identical to those produced by standard forms of matter, and, therefore, they do not allow us to distinguish wormholes.

If we assume that cosmological wormholes were created during the inflationary stage of the development of the universe, the time shift should be extremely small $\Delta t \approx \ell_{Pl}$ (it does not exceed

the Planckian value) and can be neglected . Velocities of wormhole throats are non-vanishing, as they participate in the common Friedmann expansion and peculiar motions of the regions in which they are placed. The motion of one entrance (e.g., S_+^2) in space can be always excluded by the choice of the reference system, while the possible respective motion of the second throat mouth is supposed to have a small non-relativistic velocity $|\mathbf{V}_-| \ll c$.

3. Topological Damping of Cosmic Rays

In the present section, we consider the propagation of cosmic rays through the foam-like topological structure of the universe [14]. The foam-like structure is described by a static gas of wormholes embedded in the Minkowski space, while for wormholes, we use the simplest model described in the previous section.

The gas of wormholes can be characterized by at least three scales. These are an average wormhole radius, an average distance between entrances into the same wormhole (from the standpoint of the remote observer; such a scale should not be confused with the throat length) and an average distance between different wormholes (or entrances). The latter scale relates to the mean density of the gas. If a wormhole radius is very large, such a wormhole could be directly seen in the sky. We expect that the wormhole radius has the order of a solar radius. On the other hand, if we believe that wormholes may produce dark matter effects in galaxies, we should accept that the two other scales have the order of the size of a galaxy.

We show that the scattering on wormholes can be described by an additional specific linear term in the Boltzmann equation. It turns out that a single thin ray of particles emitted by a cosmic source undergoes a damping in the density of particles, which depends on the distribution of wormholes and the traversed path. The particles absorbed by wormholes are redistributed in the form of a diffuse halo around the ray. Such a halo has very low density and is difficult to observe. Because the damping (the optical depth) is determined by the distribution of wormholes, while wormholes also presumably form dark matter halos, we may state that there is a strong correlation between the dark matter distribution and the optical depth.

3.1. Boltzmann Equation

In the present section, for the sake of simplicity we consider the flat Minkowski space, while the generalization to the case of Friedman models is straightforward. Basic elements of relativistic kinetic theory can be found in standard textbooks, for example, [20]; see also [12]. We let $f(r, p, t)$ be the number of particles in the interval of the phase space $d\Gamma = d^3 r d^3 p$. This function obeys the equation

$$\frac{\partial f}{\partial t} + \dot{\mathbf{r}}\frac{\partial f}{\partial \mathbf{r}} + \dot{\mathbf{p}}\frac{\partial f}{\partial \mathbf{p}} = C[f] + \alpha(r, p, t) - |\mathbf{v}| \int \beta(\Gamma, \Gamma') f(\Gamma') d\Gamma' \tag{5}$$

where $C[f]$ stands for collisions between particles, $\alpha(r, p, t)$ stands for the rate of emission of particles in the phase volume $d\Gamma$, and $\beta(\Gamma, \Gamma')$ describes the scattering on wormholes. For the sake of convenience, we also distinguish the multiplier $|v| = p/m$. Our aim is to find an explicit expression for $\beta(\Gamma, \Gamma')$.

We consider first a single wormhole, which represents a couple of conjugated spheres $S_\pm^2$ of the radius b and with a distance $d = |\mathbf{R}_+ - \mathbf{R}_-|$ between centers of spheres. The interior of the spheres is removed, and the surfaces are glued together. The gluing procedure defines the two types of wormholes as passable (traversable) and impassable. The impassable wormhole appears when the matrix U_β^α includes the spatial reflection. The impassable wormhole works merely as a couple of independent spherical mirrors (absolute mirrors, as they reflect gravitons as well). The passable wormhole works as a couple of conjugated mirrors, so that while an incident particle falls on one mirror, the reflected particle comes from the conjugated mirror.

We consider an arbitrary point $\mathbf{r}$ on the sphere S_-^2; that is, $\mathbf{r} \in S_-^2$, and therefore $\zeta_-^2 = (\mathbf{r} - \mathbf{R}_-)^2 = b^2$. The gluing procedure transforms this point into a conjugated point $\mathbf{r}' \in S_+^2$,

which has the form $\mathbf{r}' = \mathbf{R}_+ + \boldsymbol{\zeta}_+$, where $\boldsymbol{\zeta}_+$ relates to $\boldsymbol{\zeta}_-$ by some rotation $\zeta_+^\alpha = U_\beta^\alpha \zeta_-^\beta$. Then for the wormhole we find

$$\int \beta(\Gamma, \Gamma') f(\Gamma') d\Gamma' = \left(f - f'_+\right) \delta\left(|\boldsymbol{\zeta}_+| - b\right) + \left(f - f'_-\right) \delta\left(|\boldsymbol{\zeta}_-| - b\right)$$

where we use the notations $\boldsymbol{\zeta}_\pm = \mathbf{r} - \mathbf{R}_\pm$ and $f'_\pm = f\left(r_\pm, p_\pm, t\right)$:

$$\mathbf{r}_\pm = \mathbf{R}_\mp + U^{\mp 1} \boldsymbol{\zeta}_\pm$$

$$\mathbf{p}_\pm = U^{\mp 1}\left(\mathbf{p} - 2\left(\mathbf{p}\mathbf{n}_\pm\right)\mathbf{n}_\pm\right) \tag{6}$$

and $\mathbf{n}_\pm = \boldsymbol{\zeta}_\pm / b$. This determines the scattering matrix $\beta(\Gamma, \Gamma')$ for a single wormhole in the form $\beta(\Gamma, \Gamma') = \beta_+(\Gamma, \Gamma') + \beta_-(\Gamma, \Gamma')$, where

$$\beta_\pm(\Gamma, \Gamma') = \delta\left(\boldsymbol{\zeta}_\pm - b\right)\left[\delta\left(\mathbf{r} - \mathbf{r}'\right)\delta\left(\mathbf{p} - \mathbf{p}'\right) - \delta\left(\mathbf{r}_\pm - \mathbf{r}'\right)\delta\left(\mathbf{p}_\pm - \mathbf{p}'\right)\right] \tag{7}$$

We let $F\left(R_\pm, b, U\right)$ be the density of wormholes with parameters R_-, R_+, U and b; that is,

$$F\left(R_\pm, b, U\right) = \sum_n \delta\left(\mathbf{R}_- - \mathbf{R}_-^n\right)\delta\left(\mathbf{R}_+ - \mathbf{R}_+^n\right)\delta\left(b - b_n\right)\delta\left(U - U_n\right) \tag{8}$$

Then the total scattering matrix is described by

$$\beta_\pm^{tot}(\Gamma, \Gamma') = \int \beta_\pm(\Gamma, \Gamma') F\left(R_\pm, b, U\right) d^3 R_+ d^3 R_- dU db \tag{9}$$

We note that the distribution of wormholes (Equation (8)) has in general relatively irregular and random behavior, and in practical problems, it requires some averaging out, $\bar{F}\left(R_\pm, b, U\right)$, while for a specific astrophysical object (e.g., for a galaxy), it may possess sufficiently strong fluctuations $\delta F \sim \bar{F}$.

3.2. Topological Damping of Cosmic Rays

In the present section, we consider the first terms in the topological scattering matrix (Equations (7) and (9)). These terms determine the capture of particles by wormholes, which leads to a specific damping of cosmic rays. Indeed, we neglect collisions[1] and the topological scattering in Equation (5) and consider trajectories of particles $x(t) = x(x_0, p_0, t)$, and $p(t) = p(x_0, p_0, t)$. Then we can take variables (x_0, p_0, t) as new coordinates (instead of (x, p, t)), and Equation (5) transforms to

$$\frac{df}{dt} = \alpha\left(r(t), p(t), t\right) - |v(t)| \beta_1\left(r(t)\right) f + |v(t)| \int \beta_2(\Gamma, \Gamma') f(\Gamma') d\Gamma' \tag{10}$$

where β_1 describes the capture of particles, while β_2 describes the re-emission of the same particles by wormholes. Now if we consider the case for which the source $\alpha(t)$ produces a single thin ray and assume that wormholes have an isotropic distribution around the source, then almost all particles captured by wormholes leave the ray and will radiate from another regions of space and have different directions (from the ray). Then in the first order, we can neglect the last term on the right-hand side (r.h.s.) of Equation (10) and find the solution in the form

$$f = e^{-\tau}\tilde{f} \tag{11}$$

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[1] For the topological damping, the absence of collisions is not essential however, as they merely modify the function $\tilde{f}$ in Equation (11).

where $\widetilde{f}$ obeys the standard kinetic equation with topological terms omitted (i.e., $d\widetilde{f}/dt = \partial\widetilde{f}/\partial t + \dot{r}\partial\widetilde{f}/\partial r + \dot{p}\partial\widetilde{f}/\partial p = \alpha\,(t)$), while the optical depth $\tau\,(t)$ describes the damping along the ray:

$$\tau\,(t) = \int_{t_0}^{t} \beta_1\,(r\,(t'))\,|v\,(t')|\,dt' = \int_{0}^{\ell} \beta_1\,(r\,(s))\,ds \tag{12}$$

where ℓ is the coordinate along the ray.

For astrophysical implications (when the characteristic width of rays $L \gg b$), we can replace $\delta\,(\zeta_\pm - b)$ in Equation (7) with $\pi b^2 \delta\,(\mathbf{R}_\pm - \mathbf{r})$ (which means that the absorption of particles occurs at the positions $\mathbf{R}_\pm$). Then, from Equation (9) we find

$$\beta_1\,(r) = \pi \sum_{n,s=\pm} b_n^2 \delta\left(\vec{R}_s^n - \vec{r}\right) = \pi \int b^2 n(r,b)db \tag{13}$$

where $n = n_+ + n_-$ and $n_s(r,b) = \int \delta\left(\vec{R}_s - \vec{r}\right) F\,(R_\pm, b, U)\,d^3R_+ d^3R_- dU$. The value $\beta_1\,(r)$ can be expressed via the density of wormholes as

$$\beta_1\,(r) = \pi\overline{b^2}n\,(r) \tag{14}$$

where the mean value $\overline{b^2}$ is determined according to $\overline{b^2} = \frac{1}{n(r)} \int b^2 n(r,b)db$, and $n\,(r) = \int n(r,b)db$ is the total density of the wormholes.

3.3. Topological Bias of a Point Source

We consider now the case of a stationary point-like source that radiates particles in an isotropic way; that is, $\alpha\,(r, p, t) = \lambda\,(\varepsilon)\,\delta\,(\vec{r} - \vec{r}_0)$, where $\varepsilon = \sqrt{p^2 + m^2}$ and $\lambda\,(\varepsilon)$ is the distribution of the rate of emission of particles over the momenta. Then if we neglect the external force ($\dot{p} = 0$), collisions, and the scattering on the wormholes, the stationary solution to Equation (5) is

$$f_0\,(r, p) = \frac{m\lambda\,(\varepsilon)}{p\,|r - r_0|^2}\delta\,(\cos\theta - \cos\theta')\,\delta\,(\varphi - \varphi') \tag{15}$$

where θ and φ determine the direction of the vector $(\vec{r} - \vec{r}_0)$ and θ' and φ' determine that of $\vec{p}$.

When the density of the wormholes is low enough, the topological term can be accounted for in the next order, which determines the topological bias of the source $\alpha \to \alpha + \delta\alpha_{halo}$, where the halo density is given by

$$\delta\alpha_{halo}\,(\Gamma) = |v| \int \beta^{tot}(\Gamma, \Gamma')f_0\,(\Gamma')\,d\Gamma'$$

Such a halo has the two terms $\delta\alpha_{halo}\,(\Gamma) = \delta\alpha_{1,halo} + \delta\alpha_{2,halo}$, where the first term describes the damping (Equation (14)), and the second term determines the re-emission of particles. The exact form of the halo can be found by the image method as is described in [10]. Indeed, if we continue the solution to the whole space (we recall that coordinates that cover the inner region of wormholes $\left|\vec{r} - \vec{R}_\pm^n\right| < b_n$ do not correspond to any points of space), the wormholes will produce secondary sources of particles. Thus, when we neglect the throat size ($b \ll R_\pm$) and assume an isotropic distribution over the matrix U, then upon averaging over U, every wormhole will radiate in an isotropic way, which determines the halo as

$$\delta\bar{\alpha}_{2,halo}\,(\vec{r}, \vec{p}) = \lambda\,(\varepsilon)\,B_2\,(\vec{r})$$

where

$$B_2\,(\vec{r}) = \sum_{n,s=\pm} \frac{\pi b_n^2}{\left|\vec{R}_s^n - \vec{r}_0\right|^2}\delta\left(\vec{r} - \vec{R}_{-s}^n\right) \tag{16}$$

which determines an additional distribution of particles in the form $f(r,p) = f_0(r,p) + \delta\bar{f}(r,p)$:

$$\delta\bar{f}(r,p) = \frac{m\lambda(\varepsilon)}{p} \sum_{n,s=\pm} \frac{\pi b_n^2}{\left|\vec{R}_s^n - \vec{r}_0\right|^2 \left|\vec{r} - \vec{R}_{-s}^n\right|^2} \delta\left(\cos\theta_{-s}^n - \cos\theta'\right) \delta\left(\varphi_{-s}^n - \varphi'\right)$$

The above expressions can be re-written via the distribution given by Equation (8), for example,

$$B_2(\vec{r}) = \int \frac{\pi b^2}{\left|\vec{R} - \vec{r}_0\right|^2} N(r,R,b)\, d^3R\, db \tag{17}$$

where $N(r,R,b) = N_+ + N_-$ and $N_s = \int \delta\left(\vec{R} - \vec{R}_{-s}\right)\delta\left(\vec{r} - \vec{R}_s\right) F(R_\pm, b, U)\, d^3R_+\, d^3R_-\, dU$ (we point out to the clear relation $n(r,b) = \int N(r,R,b)\, d^3R$ with the distribution $n(r,b)$ in Equation (13)).

In this manner, we see that both functions—the damping of cosmic rays, Equation (13), and the distribution of secondary sources (the halo density), Equation (17)—are determined via the same function $N(r,R,b)$, that is, the distribution of wormholes that have an irregular (random) behavior. Together with $N(r,R,b)$, the functions $\beta_1(r)$ and $B_2(r)$ acquire a random character. However, as result of the functional dependence on the only random function $N(r,R,b)$, such quantities should exhibit a rather strong correlation.

We point out that the interpretation of the cosmic ray damping possesses an ambiguity. For instance, a suppression of the cosmic ray flux could be also due to other effects, such as multiple scattering in the source itself (e.g., see [21] and references therein). Such effects produce an analogous correlation between the damping and the halo of the secondary sources. Moreover, the halo of the secondary sources (Equation (17)) is rather difficult to observe; the brightness of such a halo is very low (the intensities of the secondary sources are strongly suppressed by the factor b^2/R^2, where b is the effective section of the scatterer and R is the distance to the scatterer). However, the key point that allows us to disentangle this specific topological damping from other effects is that the same distribution of wormholes determines the distribution of dark matter, which we discuss in the next section.

3.4. Dark Matter Halos

As was demonstrated in [10] (see also discussions in [22–24]), the distribution of wormholes (Equation (8)) also determines the density of dark matter halos in galaxies, which is much easier to observe. Indeed, in the presence of the gas of wormholes, the modification of Newton's potential was shown to be accounted for by the topological bias of sources; that is, $\delta(r - r_0) \to \delta(r - r_0) + \widehat{B}(r,r_0)$, where the halo density $\widehat{B}(r,r_0)$ is determined via the same distribution of wormholes (Equation (8)) by expressions analogous to Equation (16) (e.g., see [10] for details). The form of the bias function $\widehat{B}(r,r_0)$ however admits the direct measurement by observing rotation curves of galaxies (e.g., see [25–27], and for the exact form of the bias see also [22,23]). Indeed, in galaxies, the topological bias relates the densities of dark and luminous matter by

$$\rho_{DM}(r) = \int \widehat{B}(r - r')\rho_{LM}(r')d^3r' \tag{18}$$

which for the Fourier transforms takes the form $\rho_{DM}(k) = \overline{B}(k)\rho_{LM}(k)$. And for a point mass it determines the scale-dependent renormalization of the dynamic (or the total) mass within the radius R as

$$M_{tot}(R)/M = 1 + 4\pi \int_0^R \widehat{B}(r)\, r^2 dr. \tag{19}$$

In observations the amount of DM is determined by the mass-to-luminosity ratio M/L. It is assumed that the luminosity traces the distribution of baryons ρ_{LM}, which is measured by observing the surface brightness. For example, spirals can be modeled by an infinitely thin disk with surface mass

density distribution (surface brightness) $\rho_{LM} = \sigma e^{-r/R_D} \delta(z)$, where R_D is the disc radius (the optical radius is $R_{opt} = 3.2 R_D$). The total dynamic mass is then determined by the rotation curve analysis (or by the dispersion of velocities in ellipticals) [26,27].

Observations show that the mass-to-luminosity ratio $M_{tot}(r)/L(r)$ for the sphere of the radius r increases with the distance r from the center of the galaxy in all galaxies. However if in high-surface-brightness (HSB) galaxies this ratio exceeds slightly the unity within the optical disk $M(R_{opt})/L \gtrsim 1$, which means that there is a small amount of DM, in low-surface-brightness (LSB) galaxies, such a ratio may reach $M(R_{opt})/L \sim 10^3$. Such a correlation between the surface brightness and the amount of DM in galaxies could give indirect evidence for the topological nature of DM; in accordance with Equation (14), the number of wormholes determines the damping of cosmic rays, and analogously the number of wormholes determines the amount of dark matter in galaxies [10]. However, the basic mechanism that forms such a feature is different (e.g., see [28] and references therein). Indeed in smaller galaxies, supernovae are more efficient in removing the gas from the central (stellar forming) region of a galaxy than in larger galaxies, and this gives rise to the fact that in smaller objects, the disc has a lower baryonic density (a lower surface brightness).

In the general case, the relationship between the distribution of dark matter and that of wormholes is rather complicated. Nevertheless, the renormalization of the intensity of a point-like source (Equation (19)) allows us to find a rather simple relation between the bias and the density of wormholes on scales $R \gg \bar{d}$ (where $d = \left| \vec{R}_+ - \vec{R}_- \right|$). We stress that the consideration below has a rather illustrative (or qualitative) character, while for true measurements, one has to use the exact relations in [10].

Indeed, the basic effect of a non-trivial topology is that it cuts some portion of the volume of the coordinate space. Therefore, the volume of the physically admissible region becomes smaller, while the density of particles emitted becomes higher. From the standard flat space standpoint, this effectively looks as if the amplitude of a source renormalizes Equation (19). We consider a ball of the radius r around a point-like source. For example, for an isotropic source, the number of particles emitted in the unit time in the solid angle $d\Omega = r^2 d\phi d\cos\theta$ remains constant, $dN \sim f_0 d\Omega = const$, which gives the standard distribution of Equation (15), that is, $f_0 \sim I/4\pi r^2$.

We assume that wormholes have an isotropic distribution around the source, and for the sake of illustration, we assume that the distribution also has the structure $\overline{F}(R_\pm, b, U) = g(b) F(R_\pm, U)$. Then in the presence of wormholes, the physical volume is

$$V_{ph}(r) = \frac{4}{3}\pi \left(r^3 - \Omega(r) \right)$$

where $\Omega(r) = 4\pi \int b^3 \int_0^r n(\tilde{r}, b) \tilde{r}^2 d\tilde{r} db$ determines the portion of the coordinate volume occupied by wormholes within the radius r, and the density of wormholes $n(r, b)$ is determined by Equation (13). Therefore, the true value of the surface that restricts the ball is $S_{ph}(r) = \frac{d}{dr} V_{ph}(r)$ and we find, for the density of particles, $f \sim I/S_{ph}(r)$, which determines the renormalization of the source (Equation (16)): $I(r)/I = 4\pi r^2/S_{ph}(r)$. Completely analogously we can use the Gauss divergency theorem to estimate the renormalization of the gravity source. Indeed, the Gauss theorem states that

$$\int_{S(R)} n \nabla G \, dS = 4\pi \int_{r<R} M\delta(r) \, dV = 4\pi M$$

where G is the true Green function (or the true Newton's potential). Then for the isotropic distribution of wormholes, it determines the normal projection of the force as $F_n(R) = n\nabla G = 4\pi M/S_{ph}(R)$. This can be rewritten as in the ordinary flat space (in terms of the standard Green function $G_0 = -1/r$,

i.e., the standard Newtonian law), and the coordinate surface is $S_{coor} = 4\pi R^2$) $F_n(R) = M'(R)/R^2$, where $M'(R)/M = 4\pi R^2/S_{ph}(R)$, which determines the bias function in the form of Equation (19) or

$$\widehat{B}(r) = \frac{1}{r^2} \frac{d}{dr} \frac{r^2}{\frac{d}{dr} V_{ph}(r)} \tag{20}$$

We stress again that this function admits the direct measurement in galaxies [23,25,26]. Now by making use of the above expression for $V_{ph}(r)$, we find the behavior of the dynamic mass for a point source as

$$\frac{M_{tot}(r)}{M} = 1 + \frac{\gamma(r)}{(1 - \gamma(r))} \tag{21}$$

where $\gamma(r) = \frac{4}{3}\pi \int b^3 n(r,b)\, db$, which can be estimated as $\gamma(r) \sim \frac{4}{3}\frac{\overline{b^3}}{\overline{b^2}}\beta_1(r)$. Thus, we see that both quantities, the damping (i.e., the optical depth τ) and the amount of DM (the bias $\widehat{B}$), are expressed via the same function $n(r,b)$. This means there is the presence of a tight relation between the distribution of dark matter and damping.

Astrophysical objects that may also be used to test the topological nature of DM are large-scale extragalactic relativistic jets in quasars (e.g., see [29] and references therein). The smaller jets that are widely observed in active galactic nuclei can also be used in LSB galaxies, for which the amount of DM is considerable. However the crucial step here is the exact knowledge of the launching mechanism, which may allow us to find the discrepancy between the predicted profile of a jet and the true observed profile.

4. Topological Bias of Discrete Sources

In the present section, we consider the propagation of radiation in a gas of wormholes. We show that the scattering on wormholes gives rise to a scale-dependent renormalization of all cosmic discrete sources of radiation and forms a diffuse halo around any discrete source. By virtue of their common origin, such a halo correlates strongly with the dark matter halo. The basic results of the present paper are Equations (30) and (38) for the halo and numerical estimates for the astrophysical parameters of the gas of wormholes (mean density, the characteristic scale of the throat, and relative brightness of the halo). When observing galaxies, such a diffuse halo has a low surface brightness and is usually considered as a cosmic background that is different from radiation of the galaxy's origin. We recall that generally, the observed diffuse halos in galaxies are attributed to reflection from dust, and the general diffuse component is assumed to originate from very faint and remote galaxies. This leads to an essential overestimation of the mass-to-luminosity ratio in galaxies M/ℓ, and, therefore, this is generally interpreted as the presence of dark matter. As we show in the present section, such a ratio should be larger—$M/\ell \gg 1$—in smaller objects (e.g., in dwarfs galaxies), while in larger objects (e.g., in cluster-size plasma clouds), as a result of their huge size (more than or of the order of the characteristic scale of the halo), radiation from the halo always sums up with radiation from the hot cloud itself and the parameter $M/\ell \sim 1$ (e.g., dark matter is always absent [30]).

4.1. Scattering of Waves in the Geometrical Optics Approximation

We consider a unit discrete source of radiation and the problem of scattering on the gas of wormholes. For the sake of simplicity, we consider the case of a static gas; that is, we assume that wormholes do not move in space. In this case, the scattering is not accompanied with the frequency shift. We also point out that our results can easily be generalized to the case of the expanding universe.

The problem of the modification of Newton's law in the presence of the gas of wormholes (i.e., origin of the topological bias) was first solved in [10]. It turns out that in the case of radiation, we can also speak of the topological bias of sources. We are interested in the behavior of the bias on scales

$L \gg b$, where b is the characteristic size of a wormhole throat, and, therefore, in such an approximation, the throat resembles a point object. Our aim is to find the Green function to the wave equation:

$$\left(k^2 + \nabla^2\right) G(r, r_0) = 4\pi \delta(r - r_0) \tag{22}$$

in the presence of the wormhole. We recall that that equation describes the distribution of radiation at the frequency $\omega = kc$ (i.e., any component E, H or the vector potential A) produced by a unit stationary source $j \sim e^{-i\omega} \delta(r - r_0)$. Indeed, in the Lorentz gauge $\partial_i A^i = 0$, the Maxwell equations take the form

$$\frac{\partial^2}{\partial x^k x_k} A^i = \frac{4\pi}{c} j^i \tag{23}$$

where j^i is a 4-current. Using now the linear relation between the strength tensor and the vector potential $F_{ik} = \partial_i A_k - \partial_k A_i$, we see that both $E_i = F_{0i}$ and $H_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$ obey the above equation with the clear replacement $j^i \to J_{ik} = \partial_i j_k - \partial_k j_i$. We note that in the case of an arbitrary source, the vector potential (and respectively field strengths) can be expressed via the retarded Green function as follows:

$$A^k = \frac{1}{c} \int G(r, r_0) j^k(r_0) d^3 r_0 \tag{24}$$

In the case of the flat space, the (retarded) Green function is known to have the standard form $G_0(R) = e^{ikR}/R$ (where $R = |r - r_0|$). As a result of the conformal invariance of the Maxwell equations, the same function can be used for the class of conformally flat metrics.

    The simplest wormhole can be constructed as follows. Consider two spheres $S_\pm$ of radius b and with distance $d = |R_+ - R_-|$ between their centers. The interior of the spheres is removed, and the surfaces of the spheres are glued together. Such spheres $S_\pm$ can be considered as conjugated mirrors, so that while the incident signal falls on one mirror, the reflected signal outgoes from the conjugated mirror. Thus, every wormhole is determined with a set of parameters b, $\mathbf{R}_\pm$, and U, where b is the radius of the throat, $\mathbf{R}_\pm$ stands for positions of centers of spheres (i.e., of throats), and U stands for the rotation matrix, which determines the gluing procedure for the surfaces of the spheres. In the approximation used below, the dependence on U disappears and is not accounted for.

    The exact solution of the scattering problem in the case of a unique wormhole is rather tedious and will be presented elsewhere. For astrophysical needs, it is sufficient to consider diffraction effects in the geometrical optics limit $kb \gg 1$. According to the Huygens principle, the scattering on a wormhole can be prescribed to the presence of secondary sources on throats, which can be accounted for by additional terms:

$$G(R) = G_0(R) + u_R^+ - u_A^+ + u_R^- - u_A^- \tag{25}$$

where the terms $u_{A,R}^\pm$ describe absorption and reflection by throats $S_\pm$, respectively. Every such term can be described by the surface integral (e.g., see the standard book [31]):

$$u_\alpha^\pm(r, r_0) = \frac{k}{2\pi i} \int\limits_{S_\pm^\alpha} G_0(r', r_0) \frac{e^{ikR'}}{R'} df_n \tag{26}$$

where $R' = |r' - r|$, and $S_\pm^{R,A}$ denotes the light and dark sides of throats respectively[2]. If we neglect the throat size, then the integration gives the square πb^2. Then we find terms that describe the reflection and absorption of the signal and correspond to secondary sources placed on the throats:

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[2]   We note that the light side of a throat $S_\pm$ is turned by the matrix $U^{\pm 1}$ with respect to the dark side, so that in general, we have the union $S^R \cup S^A \neq S$.

$$u_A^\pm = \frac{k}{2\pi i} \pi b^2 G_0(R_\pm - r_0) G_0(r - R_\pm) \tag{27}$$

$$u_R^\pm = \frac{k}{2\pi i} \pi b^2 G_0(R_\mp - r_0) G_0(r - R_\pm) \tag{28}$$

which are spherical waves from the two additional sources at the positions $\vec{R}_\pm$. In this manner, we see that the scattering on wormholes can be prescribed to additional sources, that is, to the bias of the point source in Equation (22) of the form

$$\delta(r - r_0) \to \delta(r - r_0) + \widehat{B}(r, r_0) \tag{29}$$

where $\widehat{B}(r, r_0)$ is the bias function. Indeed, in this case, the Green function remains formally the same as in the flat space $G_0(R)$, while the scattering on the topology is described by the bias of sources $J(r) \to J(r) + \int \widehat{B}(r, r') J(r') d^3 r'$. Thus, in the case of the static gas of wormholes, the bias function takes the form

$$\widehat{B}(r, \omega) = \frac{\omega}{2\pi i c} \sum_m \pi b_m^2 \left(\frac{e^{ikR_-^m}}{R_-^m} - \frac{e^{ikR_+^m}}{R_+^m} \right) \left[\delta(\vec{r} - \vec{R}_+^m) - \delta(\vec{r} - \vec{R}_-^m) \right] \tag{30}$$

where for the sake of simplicity, we set $r_0 = 0$, and the index m enumerates different wormholes.

Thus the presence of the gas of wormholes leads to the origin of a specific radiating halo (Equation (30)) around every point source. As a result of the randomness of phases in multipliers in Equation (30), such a halo has an incoherent (or diffuse) nature.

Using the density of the distribution of wormholes $F(R_-, R_+, b)$ and transforming sums in integrals, the above equation can be cast to the form

$$\widehat{B}(r, \omega) = \frac{\omega}{2ic} n \int \left(\frac{e^{ikR}}{R} - \frac{e^{ikr}}{r} \right) \left[g(R, r) + g(r, R) \right] d^3 R \tag{31}$$

where $g = \frac{1}{n} \int b^2 F(R_-, R_+, b) \, db$ and n denotes the mean density of wormholes in space. In the case of a homogeneous distribution of wormholes, the function g depends only on $d = |R_+ - R_-|$. Then the bias function takes the simplest form for the Fourier transforms:

$$\widehat{B}(k, \omega) = \frac{\omega}{ic} n \frac{4\pi(g(k) - g(0))}{k^2 - \frac{1}{c^2}(\omega + io)^2} \tag{32}$$

where $g(k) = (2\pi)^{-3/2} \int g(r) e^{-ikr} d^3 r$ is the Fourier transform for the function $g(\vec{d})$.

4.2. Lorentz Invariance and the Dispersion Relations

We note that because we are working in the geometric optics limit, the Lorentz invariance and the standard dispersion relations for photons are not violated. Indeed, in the optical range of wavelengths, the bias $\widehat{B}(k, \omega)$ tends to zero, as the characteristic scale of the distribution of wormholes $L = 2\pi/k$ has the order of a few Kpc. In other words, it is in the optical range $g(k) \to 0$. Nevertheless, the violations surely take place at the scales for which the geometric optics does not work, namely, at sufficiently large distances at which topological defects start to show up (e.g., dark matter starts to show up at $\lambda = 2\pi/k \gtrsim 5Kpc$ [22], which is much greater than any wavelength $\lambda \sim c/\omega$ of a photon).

The violation of the Lorentz invariance has been intensively discussed in the literature [32]. All the existing estimates coincide in the order of magnitude and concern only extremely small scales [33–35]. In particular, in a recent paper [35], the strongest limit for the "first" correction to the standard dispersion relation was established (e.g., see also [34,36,37]). Namely, if $\omega^2 = k^2(1 + kl_1 + k^2 l_2^2 + ...)$, then $l_1 < L_{Pl}$, where L_{Pl} is the Planckian length. We point out that such

a correction corresponds to the decomposition over the small parameter $kl \ll 1$, where l has the sense of a characteristic scale connected to wormholes[3], which leads to a very strong limit on the existence of microscopic wormholes (i.e., we may admit only the existence of wormholes at scales $l_1 < L_{Pl}$; see also analogous estimates in [36,37]).

However (we hope that this should not be a surprise for readers), the same restriction can be used to estimate parameters of wormholes having astrophysical meaning (scales). Indeed, in the presence of wormholes of astronomical scales ($L \sim$ a few Kpc), the small parameter is already the ratio of the photon wavelength to the characteristic scale of a wormhole, which is the inverse parameter $1/(kL)$. Therefore, in the dispersion relation, the first non-trivial corrections should have the form $\omega^2 = k^2 \left(1 + \frac{1}{kL_1} + \frac{1}{k^2L_2^2} + ...\right)$. Then, if we use the above restrictions on the absolute value of the existing correction to the dispersion relation (e.g., $k \sim 10^6 cm^{-1}$ and $kl_1 < 10^{-27}$), then we find that the characteristic scales of astrophysical wormholes should obey the "restrictions" $L_1 > (1 \div 5)Kpc$. These are just the scales at which dark matter effects start to display themselves, and according to our interpretation of dark matter effects [22], the density of wormholes should achieve the value $nL_1^3 \sim 1$. We also point out the fact that at galaxy scales, the Lorentz invariance is also violated as a result of relativistic gravitational effects (i.e., due to general relativity). In other words, to essentially improve the above restrictions on the dispersion relations' violation is hardly possible.

As was pointed out above, the reason for the absence of the Lorentz invariance violation, for example, in the optic range, is very simple. For rather short wavelengths, the forming halo secondary sources (cf. Equation (30)) have random phases and, therefore, do not contribute to the amplitude of a signal. In other words, the halo carries a diffuse character. In such a case, it is more correct to consider the expression for the energy flux, which comes to the point r and which for the diffuse field becomes an additive quantity.

4.3. The Diffuse Halo

We consider now the renormalization of the intensity of radiation. By virtue of the diffuse character of the current, the intensity of radiation is determined by the square of the current, for example,

$$\overline{I(R)I^*(R')} = |I(R)|^2 \delta(R - R') \tag{33}$$

where the averaging out should be thought of as either over the period of the field $T = 2\pi/\omega$ or over the random phases. When we do not account for the scattering on topology, that is, in the ordinary flat space, the intensity of the radiation $W = \frac{E^2+H^2}{8\pi}$ is determined by the intensity of the current $|I|^2$ as $W(r) \sim \int \frac{|I(r)|^2}{|r-r'|^2}d^3r'$, and, in particular, for a point source $|I(r)|^2 = |I_0|^2\delta(r - r_0)$, the intensity of the energy flow is $W(r) = |G_0(r - r_0)|^2|I_0|^2$. The scattering on wormholes leads to the replacement $G_0 \to G$, which effectively can be described as the origin of the bias (Equations (29) and (30)) or as the origin of an additional halo that has the property pointed out above to be δ-correlated and which leads to a renormalization of the current intensity $|I_0|^2 \to |\tilde{I}(r)|^2$. We use the Green function; that is, we consider the sum

$$G(r)G^*(r) = |G(r)|^2 = |G_0(r)|^2 + \sum_{s=A,R;\ p=\pm} \left|u_s^p\right|^2 \tag{34}$$

which gives

$$|G(r)|^2 = \frac{1}{r^2} + \frac{\omega^2}{4\pi^2c^2} \sum_m \pi^2 b_m^4 \left(\frac{1}{(R_-^m)^2} + \frac{1}{(R_+^m)^2}\right) \left(\frac{1}{|R_-^m - r|^2} + \frac{1}{|R_+^m - r|^2}\right) \tag{35}$$

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[3]　We recall that wormholes relate to three basic parameters. These are the density of wormholes $n^{-1/3}$, the mean throat size b, and the mean distance between throats $d = |R_+ - R_-|$.

In the above expression, as a result of the randomness of phases, the intersection terms are omitted. Then the above expression determines the bias for the intensity of a unit source in the form

$$|G(r)|^2 = \frac{1}{r^2} + \int \frac{\widetilde{B}^2(R)}{|R-r|^2} d^3R \tag{36}$$

where

$$\widetilde{B}^2(R) = \frac{\omega^2}{c^2} \frac{n}{4} \int \left(\frac{1}{R^2} + \frac{1}{X^2} \right) (\tilde{g}(R,X) + \tilde{g}(X,R)) \, d^3X \tag{37}$$

and $\tilde{g}(R_+, R_-) = \frac{1}{n} \int b^4 F(R_-, R_+, b) \, db$. For an isotropic distribution of wormholes, $\tilde{g} = \tilde{g}(|R_+ - R_-|)$, and therefore we find the bias function in the form

$$\widetilde{B}^2(R) = \frac{k^2}{2} n \int \left(\frac{1}{R^2} + \frac{1}{|X+R|^2} \right) \tilde{g}(X) \, d^3X \tag{38}$$

In this manner, the relation between the intensity of the true $|I_0(r)|^2$ and the apparent $|\tilde{I}(r)|^2$ currents (or, equivalently, between the true ℓ_0 and observed ℓ luminosity) is determined by the distribution of wormholes in space and is given by the expression

$$|\tilde{I}|^2(r) = |I_0|^2(r) + \int \widetilde{B}^2(r-r')|I_0|^2(r') d^3r \tag{39}$$

4.4. Estimates

We consider now some simple estimates. In order to find estimates for the renormalization of the surface brightness of a source, we consider the case in which all wormholes have the same value for $d = \left| \vec{R}_- - \vec{R}_+ \right| = r_0$. In this case, we can take $\tilde{g}(X) = \frac{b^4}{4\pi r_0^2} \delta(X - r_0)$, and for a point source, we obtain the bias in the form

$$\widetilde{B}^2(R) = \frac{k^2}{2} n \overline{a^4} \frac{1}{R^2} \left(1 + \frac{R}{2r_0} \ln \left| \frac{R+r_0}{R-r_0} \right| \right) \tag{40}$$

We note that the characteristic behavior $\tilde{b}^2 \sim 1/R^2$ of the halo density is the specific attribute of the point-like structure of a source. In the case of real sources, such a halo acquires the cored character $\widetilde{B}^2 \sim \widetilde{B}^2(l) \sim const$, where l corresponds to the linear size of the source.

To obtain the estimate for the number density of wormholes is rather straightforward. First, wormholes appear at scales for which dark matter effects start to display themselves, that is, at scales of the order $L_1 \sim (1 \div 5)Kpc$, which gives, in that range, the number density

$$n \sim (3 \div 0.024) \times 10^{-66} cm^{-3} \tag{41}$$

The characteristic size of throats can be estimated as follows [10,38]. As was pointed out above in the case of a homogeneous distribution of wormholes, the value of $\bar{b}$ determines the amount of dark energy in the universe. Indeed, we consider a single wormhole in the flat (Minkowski) space. Then the metric can be taken in the form of Equation (1):

$$ds^2 = dt^2 - h^2(r)(dr^2 + r^2\sin^2\vartheta d\phi^2 + r^2 d\vartheta^2)$$

with $h(r) = 1 + \theta(b-r)(\frac{b^2}{r^2} - 1)$. Both regions $r > b$ and $r < b$ represent portions of the ordinary flat Minkowski space, and, therefore, the curvature is $R_i^k \equiv 0$. However on the boundary $r = b$, it has the singularity that determines the scalar curvature as $R = -8\pi GT = \frac{2}{b}\delta(r-b)$, where T stands for the trace of the stress energy tensor, which one has to add to the Einstein equations to support such a wormhole. It is clear that such a source violates the weak energy condition and, therefore, it reproduces the form of dark energy (i.e., $T = \varepsilon + 3p < 0$). If the density of such sources (and respectively the

density of wormholes) is sufficiently high, then this results in an acceleration of the scale factor for the Friedmann space as $\sim t^{\alpha}$ with $\alpha = \frac{2\varepsilon}{2\varepsilon + T} > 1$.

Every wormhole gives a contribution $\int Tr^2 dr \sim \bar{b}$ to the dark energy, while the dark energy density is $\varepsilon_{DE} \sim (8\pi G)^{-1} n\bar{b}$. Because the density of dark energy has the order $\varepsilon_{DE} \sim 0.75\varepsilon_0$, where ε_0 is the critical density, then we immediately find the estimate $\bar{b} \sim (1 \div 125) \times 10^{-3} R_{\odot}$, where $R_{\odot}$ is the solar radius. Now by means of use of Equation (40), we find the estimate for the relative brightness of the halo:

$$\frac{\ell}{\ell_0} \sim \frac{k^2}{2} n\bar{b}^4 l \sim 4\frac{l}{R_{\odot}} \left(\frac{k}{k_0}\right)^2 \left[1 \div 2 \times 10^6\right] \times 10^{-14} \tag{42}$$

Here l is the linear size of the source around which the diffuse halo forms, and k_0 determines the wavelength λ_{max} that corresponds to the temperature $T_{\odot} = 6 \times 10^3 K$. It is clear that the relative brightness of the halo is small: $\ell/\ell_0 \ll 1$; it reaches the order of unity only for sufficiently extended objects of the characteristic size $\left[0.5 \times 10^{-6} \div 1\right] \times 10^{14} R_{\odot}$. We also point out that outside the radiating region, the halo brightness decays according to Equation (40) as $\sim 1/R^2$.

5. Generation of an Interference Picture

The problem of the scattering of radiation on a static gas of wormholes has been considered in the previous section. It was demonstrated that any discrete source turns out to be surrounded with a diffuse halo, which should be correlated with an analogous halo of dark matter. However, we used there an approximation for which the size of wormholes was negligible. Here we consider some general features of the scattering by a single wormhole of a finite size and account for the vector character of radiation. We demonstrate that the scattering of a plane electromagnetic wave by a wormhole leads to a partial depolarization and a specific interference picture in the scattered signal. The interference picture depends on the distance to the observer and specific parameters of a wormhole, giving thus an instrument for the possible search of wormholes. We also point to [39], where some aspects of the scattering by an Ellis geometry are also considered.

Scattering of electromagnetic waves in the short-wavelength limit by a conducting sphere is the classical problem of electrodynamics [40]. We do not repeat all derivations here referring to the above textbook but explain additional features that come from a wormhole.

We let $\mathbf{E} = \mathbf{E}_0 e^{-i\omega t + i\mathbf{k}_0 \mathbf{r}}$ be the incident field; then the surface of a sphere can be divided into illuminated and shadow regions, which produce contributions to the scattered field as

$$\mathbf{E}_{sh} \approx ikb^2 \frac{J_1(kb\sin\theta)}{kb\sin\theta} \frac{e^{-i\omega t + ikr}}{r} \frac{[[\mathbf{k}\mathbf{E}_0]\,\mathbf{k}]}{k^2}$$

from the shadow region and

$$\mathbf{E}_{ill} \approx -\frac{b}{2} E_0 \frac{e^{-i\omega t + ikr}}{r} e^{-2ikb\sin\frac{\theta}{2}} \mathbf{e}_0$$

from the illuminated region. Here b is the radius of the sphere, $\mathbf{e}_0 = (2(\mathbf{n}_0\mathbf{E}_0)\mathbf{n}_0 - \mathbf{E}_0)/E_0$, $\mathbf{n}_0$ is the unit vector along $\mathbf{k} - \mathbf{k}_0$, $\mathbf{k} = k\mathbf{r}/r$, $\cos\theta = (\mathbf{k}\mathbf{k}_0)/k^2$, and J_1 is the Bessel function. It is also supposed that the sphere is at the origin. The contribution from the shadow region represents the standard diffraction, which does not depend on the nature of the obstacle (it depends on the projected area only) and gives a very narrow beam along the incident signal $\theta \lesssim 10/kb$, while the illuminated region gives an isotropic intensity of radiation. The cross-section of such scattering is given by two terms:

$$\frac{d\sigma_{sh}}{d\Omega} = \sigma_0 \frac{(kb)^2}{4\pi} \left|\frac{2J_1(kb\sin\theta)}{kb\sin\theta}\right|^2 \tag{43}$$

where $\sigma_0 = \pi b^2$, and the isotropic term from the illuminated region:

$$\frac{d\sigma_{ill}}{d\Omega} = \sigma_0 \frac{1}{4\pi} \tag{44}$$

Now we add features from the structure of a wormhole. It turns out that a general wormhole can be considered as a couple of dielectric spheres S_+ and S_- glued along the surface. Thus the incident signal generates illuminated and shadow regions on the outer side of the surface and analogous illuminated and shadow regions on the inner side of the surface. As a result of gluing, the inner side of S_+ corresponds to the outer side of S_-, and vise versa. Because points on the spheres are glued, both throats radiate the scattered signal in the same manner, and we may consider radiation from one throat. The incident field is partially reflected by the throat (in the illuminated region) and partially goes through the throat. The reflection and transmittance coefficients $|r|^2 + |t|^2 = 1$ depend on the specific structure of the wormhole and, in particular, on the matter content filling the throat and surrounding the wormhole, (in the vacuum case, $r(b,k) \to 0$ as $bk \gg 1$; e.g., see [39]). Thus the total scattered signal comprises an additional term as follows:

$$\mathbf{E} = \mathbf{E}_{sh} + r\mathbf{E}_{ill} + t'\mathbf{E}'_{ill} \tag{45}$$

We let S_+ be the throat at the origin. The additional term describes the wave transmitted through the throat (i.e., radiation of the wave adsorbed on the conjugated throat S_-). This term is equivalent to the classical reflection by the sphere S_+ of an additional wave $\mathbf{E}' = \mathbf{E}'_0 e^{-i\omega' t + i\mathbf{k}'_0 \mathbf{r} + i\psi}$, where $\psi = -\omega\Delta t + \mathbf{k}_0\Delta\mathbf{R}$ $(\Delta\mathbf{R} = \mathbf{R}_- - \mathbf{R}_+)$ is the phase difference that the incident field has in the center of the throat S_-, and $\mathbf{E}'_0$, ω', and $\mathbf{k}'_0$ are related to $\mathbf{E}_0$, ω and $\mathbf{k}_0$ by a Lorentz transformation Λ^μ_ν that determines the gluing (Equation (4)).

The shadow contribution $\mathbf{E}_{sh}$ gives a very narrow beam along $\mathbf{k}_0$, while the illuminated part determines an omnidirectional flux $\mathbf{E}^{tot}_{ill} = r\mathbf{E}_{ill} + t'\mathbf{E}'_{ill}$, which is depolarized (as in general, $\mathbf{e}_0 \neq \mathbf{e}'_0$). The intensity of the flux $I = (\mathbf{E}\mathbf{E}^*)$ is given by

$$I = \frac{b^2}{4}\frac{E_0^2}{r^2}\left(1 + A\cos\left\{\psi - \Delta k\,(ct - r) - \left(2kb\sin\frac{\theta}{2} - 2k'b\sin\frac{\theta'}{2}\right)\right\}\right) \tag{46}$$

Here $\cos\theta = (\mathbf{r}\mathbf{k}_0)/rk_0$, $\cos\theta' = (\mathbf{r}\mathbf{k}'_0)/rk_0$, $\Delta k = c\,(\omega - \omega')$, and $A = (rt'^* + r^*t')\,(\mathbf{e}_0\mathbf{e}'_0)$.

We see that the strongest interference picture appears when $A \simeq 1$ (recalling that A depends on the specific structure and the matter composition of the wormhole) and is given by the phase $\Delta k\,(ct - r)$, which comes from the possible respective motions of throats $\Delta\omega \simeq k\Delta V$ (here $\Delta\mathbf{V} = \mathbf{V}_+ - \mathbf{V}_-$ and c is the speed of light); $\Delta\omega$ is the standard Doppler shift. In this case, the phase ψ also has a dependence on time (via $\mathbf{R}_\pm(t)$). However the additional phase in Equation (46) forms a more peculiar and complex interference picture with a much longer wavelength $\delta k \ll \Delta k$, which in principle can be measured while the Earth is orbiting the Sun. We point out that these oscillations are not pure harmonic oscillations.

We consider for simplicity the case in which the Lorentz transformation reduces to a pure spatial rotation U^α_β and the frequency remains the same: $\omega' = \omega$. Then expanding Equation (46) by $\delta\mathbf{r}$ near the observer at the position $\mathbf{r}$, we find the expression

$$I = \frac{b^2}{4}\frac{E_0^2}{r^2}\left(1 + A\cos\left\{\varphi_0 + (\delta\mathbf{k}\delta\mathbf{r})\right\}\right)$$

where the constant phase $\varphi_0 = \psi - 2kb\left(\sin\frac{\theta}{2} - \sin\frac{\theta'}{2}\right)$, and

$$\delta\mathbf{k} = \left[\frac{1}{\sin\frac{\theta}{2}} - \frac{1}{\sin\frac{\theta'}{2}}\right]\frac{2b}{r}\mathbf{k}_0 \tag{47}$$

Thus the intensity has specific additional oscillations with a very long wavelength proportional to the ratio $\lambda_0 r/b \gg \lambda_0$, where λ_0 is the wavelength of the incident signal. We suppose that the interference picture described above should be taken into account in analyzing observations and may be useful in the direct search for wormholes.

6. Distortion of CMB Spectrum by Wormholes

In the present section, we consider the scattering of CMB radiation on wormholes and show that this can be observed by means of the effect analogous to the KSZ [41,42]. The KSZ signal is based on the inverse Compton scattering of relic photons on a moving electron gas. It represents one of the main tools in studying the peculiar motions of clusters and groups of galaxies (e.g., see [43–45], and see also more applications in a recent review [18]). It is produced by any kind of matter that scatters CMB (not only by a hot electron gas). As was shown in [18], in the first order in v/c, the contribution of wormholes in the KSZ cannot be separated from that of the electron gas in clusters and groups. Therefore, there are two possibilities. The first is to look for such an effect in those spots in the sky in which the baryonic matter is absent, for example, in voids where the number density of wormholes should have the largest value and the leading contribution will come from wormholes alone. The second possibility is to study next-order corrections and peculiar features of the scattering of CMB on wormholes. It turns out that already in the second order, the KSZ on wormholes differs from that on other sorts of matter.

We recall that spherical wormholes require the presence of exotic matter [8]. While the natural sources of the exotic matter are not found, we should state that the relic spherical wormholes collapse very rapidly and have hardly survived until the present days. Remnants of such wormholes cannot be distinguished from primordial black holes. Stable cosmological wormholes may exist without exotic matter, if their throat sections have the form of tori [9]. In the sky, such a torus-like throat will be seen as a geodesic rectangle. In studying the KSZ on wormholes, the exact form of the throat is not important.

The simplest model of a spherical wormhole is given by a couple of conjugated spherical mirrors; when a relict photon falls on one mirror, it is emitted, upon the scattering, from the second (conjugated) mirror. Such mirrors represent two different entrances into the wormhole throat, and they can be separated by an arbitrary large distance in outer space. The cross-section of such a process has been described in the previous section and can be summarized as follows. Let an incident plane wave (a set of photons) fall on one throat. Then the scattered signal has two components. The first component represents the standard diffraction (which corresponds to the absorption of CMB photons on the throat) and forms a very narrow beam along the direction of the propagation. This is described by the cross-section

$$\frac{d\sigma_{absor}}{d\Omega} = \sigma_0 \frac{(kb)^2}{4\pi} \left| \frac{2J_1(kb\sin\theta)}{kb\sin\theta} \right|^2 \tag{48}$$

where $\sigma_0 = \pi b^2$, b is the radius of the throat, k is the wave vector, θ is the angle from the direction of propagation of the incident photons, and J_1 is the Bessel function. Together with this part, the second throat emits an omnidirectional isotropic flux with the cross-section

$$\frac{d\sigma_{emit}}{d\Omega} = \sigma_0 \frac{1}{4\pi} \tag{49}$$

Both total cross-sections coincide:

$$\int \frac{d\sigma_{absor}}{d\Omega} d\Omega = \int \frac{d\sigma_{emit}}{d\Omega} d\Omega = \sigma_0$$

which expresses the conservation law for the number of absorbed and emitted photons. In the absence of peculiar motions (a static gas of wormholes), every wormhole throat-end absorbs photons as the absolutely black body, while the second end re-radiates them in an isotropic manner (Equation (49)) with the same black body spectrum. It is clear that no distortion of the CMB spectrum will appear at all. In the presence of peculiar motions, the motion of one end of the wormhole throat with respect to CMB causes the angle dependence of the incident radiation with the temperature

$$T = \frac{T_{CMB}}{\sqrt{1-\beta^2}\,(1+\beta\cos\theta_+)} \simeq T_{CMB}\left(1 - \beta\cos\theta + \frac{1}{2}\left(1 + 2\cos^2\theta\right)\beta^2 + ...\right)$$

where $\beta = V/c$ is the velocity of the throat-end, $\beta \cos \theta = \left(\vec{\beta} \vec{n} \right)$, and $\vec{n}$ is the direction for incident photons. Therefore, the absorbed radiation has the spectrum

$$\rho(T) = \rho(T_{CMB}) + \frac{d\rho(T_{CMB})}{dT} \Delta T + \frac{1}{2} \frac{d^2 \rho(T_{CMB})}{dT^2} \Delta T^2 + \dots$$

where $\rho(T)$ is the Planckian spectrum and $\Delta T = T - T_{CMB}$. As was discussed previously by [18], in the first order in β, the above anisotropy does not contribute to the re-radiation of relic photons from the second end according to Equation (49). Indeed, integration over the incident angle θ gives $\langle \Delta T \rangle = -\frac{T_{CMB}}{4\pi} \int \beta \cos \theta d\Omega = 0$. In other words, in the first order in β, the peculiar motions of the absorbing ends of wormholes can be ignored. In this case, the KSZs is caused by wormholes and by the standard baryonic matter mix and cannot be disentangled. The difference however appears in the second order in β. Indeed, considering the second order, we find

$$(\Delta \rho)_2 = \frac{\beta^2}{6 T_{CMB}^3} \frac{d}{dT} \left(T_{CMB}^5 \frac{d\rho(T_{CMB})}{dT} \right)$$

where $(\Delta \rho)_2 = \langle \rho(T) - \rho(T_{CMB}) \rangle_2$ and we have used the property $\langle \cos^2 \theta \rangle = \frac{1}{3}$.

This means that together with the standard Planckian spectrum $I(T_{CMB}) = c\rho(T_{CMB}) = I_0 \frac{x^3}{e^x - 1}$, where $I_0 = \frac{2h}{c^2} \left(\frac{k_B T_{CMB}}{h} \right)^3$ and $x = h\nu / k_B T_{CMB}$, every wormhole emits the additional isotropic flux of photons with the spectrum

$$\left(\frac{\Delta I}{I_0} \right)_2 = \frac{1}{6} \frac{x^4 e^x \left(3e^x - 3 + x \left(e^x + 1 \right) \right)}{\left(e^x - 1 \right)^3} \beta^2 = f(x) \beta^2 \tag{50}$$

We point out that in this case, the distortion of the spectrum does not reduce to a frequency-invariant shift of the temperature. The estimate of the relative integrated amplitudes of the radiation emitted by a single wormhole is given by

$$\frac{(\Delta I)_2}{I_{CMB}} = 5.33 \times \beta^2 \tag{51}$$

When considering a cloud of wormhole throats, in addition to the standard CMB, every throat radiates photons with the flux $(\Delta I)_2$. In the presence of peculiar velocities, the CMB part undergoes the Doppler shift (which is the complete analog of the KSZ):

$$\frac{\Delta T_{KSZ}}{T_{CMB}} = \beta_p \tau_w$$

Here β_p is the projection of the peculiar velocity of the cloud along the line of sight and the optical depth τ_w determined as

$$\tau_w = \int \pi b^2 n(r, b) db d\ell \tag{52}$$

where the integration is taken along the line of sight and $n(r, b)$ is the number density of wormholes measured from the center of the cloud and depending on the throat radius b. The optical depth τ_w is interpreted as follows. Let L be the characteristic size of the cloud of wormholes. Then in the sky, it will cover the surface $S \sim L^2$, while the portion of this surface covered by wormhole throats is given by

$$\tau_w = \frac{N \pi \overline{b^2}}{L^2} = \pi \overline{b^2} \overline{n} L$$

where N is the number of wormhole throats in the cloud and $\overline{n}$ is the mean density. In a sufficiently dense cloud $\tau_w \sim 1$, this effect produces simply a hot or a cold (depending on the sign of β_p) spot on

the CMB maps. It is important that the KSZ corresponds to a frequency-invariant temperature shift, which leaves the primary CMB spectrum unchanged.

The second-order effect discussed earlier does not depend on velocities of throats in the cloud. It depends however on velocities of conjugated entrances into throats and is given by

$$\frac{(\Delta I)_{2KSZ}}{I_{CMB}} = 5.33 \times \int \beta^2 \pi b^2 n(r, b, \beta^2) db d\ell d\beta \sim 5 \left\langle \beta^2 \right\rangle \tau_w \tag{53}$$

In general, such an effect is very small, as the typical values does not exceed the value $\left\langle \beta^2 \right\rangle \sim 10^{-4}$. It is however measurable for sufficiently dense clouds $\tau_w \sim 1$; it is important that it cannot be reduced to a shift of the CMB temperature, and, therefore, it does slightly change the primary CMB spectrum according to Equation (50). This gives a new tool that allows us to distinguish the contribution of wormholes to the KSZ from that of the rest of matter.

7. Conclusions

In this manner, we have shown that wormholes produce a number of effects that can be disentangled from effects produced by ordinary matter. The first important effect is the absorbtion of particles, which causes an additional damping of cosmic rays. In observing damping, the optical depth always contains two terms $\tau = \tau_b + \tau_w$, where the term τ_b appears as a result of scattering on the ordinary matter (dust, electron gas, etc.) and the contribution τ_w, which comes from wormholes. The key difference between wormholes and standard matter is that in the second case, the absorbtion causes the heating of the matter, which can be directly observed. To extract the contribution of wormholes τ_w requires hard work. Indeed, one has to take a discrete source that may serve as a standard candle, evaluate all standard contributions τ_b on the line of sight and measure the difference. The value τ_w should correlate with the amount of dark matter on the line of sight. We also point out that the damping can be observed in large-scale extragalactic relativistic jets in quasars [29].

The second important effect is that wormholes re-emit, in an isotropic way, particles captured. This forms a diffuse halo around any discrete source. The key moment here is that in the absence of peculiar motions of wormholes, they do not change the initial spectrum of particles. Even if sources of radiation have an inhomogeneous distribution in space, their diffuse halos merge and form a low-intensity isotropic and homogeneous background of radiation. This shows that the presence of the isotropic flux in any energy range does not guarantee the homogeneity in the distribution of sources. At first glance, it is impossible to use such an effect and extract some information about the presence of cosmological wormholes. However, wormholes, as well as ordinary matter, are involved in peculiar motions. It turns out that already in higher orders, in v/c, peculiar motions do change the spectrum of emitted particles. The largest value has the first order in the v/c effect, which causes the Doppler shift of the spectrum. This should contribute to the high-energy cosmic-ray spectrum [46], while in the case of CMB, it corresponds to the KSZ. It is important that the KSZ from wormholes always mixes with the scattering on electron gas. Nevertheless there is the possibility to disentangle the contribution of wormholes and baryonic matter. Indeed, the KSZ from electrons is always accompanied with the thermal Sunaev-Zel'dovich effect, while the KSZ from wormholes is not. Moreover, if we observe the KSZ signal from voids in which baryonic matter is practically absent, this should indicate the presence of wormholes.

Another possibility is to investigate such an effect in the second order in v/c. It gives a more subtle contribution but it differs essentially from the analogous effect of ordinary matter. Roughly in the second order, peculiar motions produce a widening of the spectrum (e.g., see Equation (50)).

The last important effect that we may look for in observational data is the generation of an interference picture. Here we have considered the interference on a single wormhole. However in general, we have a distribution of wormholes, and, therefore, we have to study the multibeam interference picture. In particular, as a result, supernovae explosions should always be accompanied with the multibeam or stochastic interference that can be observed as a sudden increase of the energy

flux in some (random) regions of space. Moreover, such a stochastic signal may overrun the basic wave front.

In conclusion, we consider the simplest estimates for the parameters of cosmological wormholes [47]. Real wormholes seem to be responsible for dark matter. Therefore, obtaining an estimate for the number density of wormholes is rather straightforward. First, wormholes appear at scales on which dark matter effects start to display themselves, that is, at scales of the order $L \sim (1 \div 5)Kpc$, which gives the number density of wormholes (Equation (41)):

$$n \sim (3 \div 0.0245) \times 10^{-65} cm^{-3}$$

The characteristic size of throats can be estimated from ε_{DE}. Every wormhole gives a contribution $\int Tr^2 dr \sim \bar{b}$ to the dark energy, while the dark energy density is $\varepsilon_{DE} \sim (8\pi G)^{-1} n\bar{b}$. Because the density of dark energy has the order $\varepsilon_{DE}/\varepsilon_0 = \Omega_{DE} \sim 0.75$, where ε_0 is the critical density, then we immediately find the estimate

$$\bar{b} \sim \frac{2}{3}(1 \div 125) \times 10^{-3} R_{\odot} \Omega_{DE} h_{75}^2$$

where $R_{\odot}$ is the solar radius, $h_{75} = H/(75km/(sMpc))$, and H is the Hubble constant. We also recall that the background density of baryons ε_b generates a non-vanishing wormhole rest mass [12] $M_w = \frac{4}{3}\pi\bar{b}^3 a^3 \varepsilon_b$, where $a(t)$ is the scale factor of the universe and therefore M_w remains constant. In the Friedman model, if we use the cut-and-paste technique, such a mass should be added to every wormhole in order not to destroy the homogeneity of space. This produces the dark matter density related to wormholes, $\varepsilon_{DM} \simeq M_w n$. Thus the typical mass of a wormhole M_w is estimated as

$$M_w \sim 1,7 \times (1 \div 125) \times 10^2 M_{\odot} \Omega_{DE} h_{75}^2$$

where $M_{\odot}$ is the solar mass. We point out that this mass is not directly related to the parameters of the wormhole gas. However, it determines the time at which wormhole throats separated from the cosmological expansion.

The last parameter is the averaged distance between entrances into the same wormhole (between two ends) $d = |R_+ - R_-|$. This parameter can be retrieved from the distribution of dark matter in galaxies. Indeed, in galaxies, the distributions of dark and luminous matter are strongly correlated [48]. This indicates the existence of a rigid relation $\rho_{DM}(k) = b(k)\rho_{vis}(k)$, where $\rho(k)$ are Fourier transforms for dark and visible matter densities. Then from the observed distribution of dark matter in galaxies (e.g., see [25,26]), we may retrieve Newton's potential as [22,23]:

$$\phi_{emp} = -\frac{4\pi G M_{gal}}{k^2}(1 + \delta(k))$$

where M_{gal} is a galaxy mass, and the term $\delta(k)$ describes the deviation from Newton's law. At small scales $Lk \gg 1$ (where $L \sim 5Kpc$), $\delta(k) \to 0$ gives the standard Newton's law, while at large scales $Lk \ll 1$, it modifies it. In the case of a homogeneous distribution of wormholes, the correction can be evaluated as follows [10]:

$$\delta(k) = \frac{4\pi}{k^2} n\bar{b}(\nu(k) - 1)$$

where $\nu(k)$ is the characteristic function for the distribution over the distances d between throat entrances. This allows us to relate the correction $\delta(k)$ observed in galaxies to the distribution over the distances d. This problem however requires the further study.

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